Are Close Elections Random?*

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January 28, 2011

Abstract

Elections with small margins of victory represent an important form of electoral competition and, increasingly, an opportunity for causal inference. When scholars use close elections for examining democratic competition or for causal inference, they impose assumptions about the politics of close contests: campaigns are unable to systematically alter the vote total. This paper calls into question this model and introduces a new model that accounts for strategic campaign behavior. We draw upon the intuition that elections that are expected to be close attract greater campaign expenditures before the election and invite legal challenges and fraud after the election. Our theoretical models predict systematic differences between winners and losers in extremely close elections. We test our predictions using all House elections from 1880-2008, finding that structurally advantaged candidates are more likely to win close elections. Our findings suggest a new research agenda and may diminish the normative appeal of marginal elections.

*We thank Dan Lee for helpful discussant comments and participants at the Midwest Political Science Association Annual Conference and seminar participants at Stanford and Harvard University. For helpful discussions and data we thank Lisa Blaydes, Daniel Butler, Gary Cox, Andy Eggers, James Fearon, Jens Hainmueller, Daniel Hopkins, Guido Imbens, David Laitin, David Lee, Simon Jackman, Holger Kern, Gary King, Clayton Nall, Jonathan Rodden, Erik Snowberg, Jim Snyder, Jonathan Wand and Arjun Wilkins. All remaining errors, omissions, and interpretations remain ours.

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Competitive majoritarian elections comprise perhaps the defining feature of democratic republics. The question of whether these elections are truly competitive has become a central criterion in the assessment of democracy. Robert Dahl described a fundamental of democracy as free, fair and competitive elections on a regular schedule (Dahl, 1970). Analysts both qualitative (Bensel, 2004) and quantitative (Gasiorowski, 1996; Vanhanen, 2000; Przeworski et al., 2000) have expanded upon this insight. The idea is rather simple and compelling; if those who hold power have little chance of becoming unseated, whether through elections or other means, then the political system tends toward autocracy in fact, whatever its formal institutions may suggest.

Not even the world's mature democracies can take for granted the prevalence of electoral competition. The existence of competitive elections depends not merely upon institutions such as universal adult suffrage, open candidate qualification, reduced barriers to entry, and free press and speech protections, but also on how elections unfold behaviorally. A powerful idea entertained by political scientists for decades is that close or "marginal" contests are competitive and supply proper electoral incentives. Some scholars render this point more continually, as they argue that the closer the margin of election (the lower the votes or percentage separating winner from loser or second-place contestant), the greater the incentive of the elected representative to pay attention to constituent preferences and demands (Levitt 1996, Stokes 1999, 125). Dahl went so far as to claim that in the presence of strong electoral competition, "it may not matter" if parties themselves are democratic or oligarchic or authoritarian (Dahl, 1970, 5). Dahl's reasoning invoked a fundamental mapping from electoral competition to the probability of losing power: "If parties are actively competing for votes, then a party that fails to respond to majority concerns will probably lose elections."

In many cases, however, formally democratic systems fail to exhibit a marked degree of genuine competition. In the United States, scholars have puzzled over the disappearance of "marginal elections" (Fiorina, 1977), or close contests in which each candidate or party would have plausible incentives to show responsiveness to voter preferences and concerns. The vast literature on the "incumbency advantage" in American congressional elections is, in part, a reflection on this reduced electoral competition (Ansolabehere, Snyder and Stewart, 2000). Some critics have gone so far

as to suggest that the lack of electoral competition makes the concept of democracy problematic itself. Elections for political office may not, in and of themselves, suffice for representative government; indeed, elections without genuine competition may create fictions of popular sovereignty (McCormick, 2001).

Despite their historically increasing scarcity, marginal elections have become important in another way. In recent years economists, political scientists, statisticians and other scholars have begun to exploit the properties of marginal elections for purposes of causal inference (Thistlethwaite and Campbell, 1960; Lee, 2008). Using a sophisticated technology of statistical inference and the intuition that close elections are near-randomly determined, these scholars have essentially treated the winners and losers of marginal elections as randomly assigned to "election winner" (treatment) and "election loser" (control) groups. As the margin gets close, in other words, the winner of the election is determined as if it were the result of a fair coin toss. In quite powerful analyses, these scholars have shown theoretically that only very simple and easy-to-satisfy assumptions are needed to identify causal effects of interest (Hahn, Todd and van der Klaauw, 2001; Lee, 2008). Drawing upon these methods, causal inference designs from marginal elections have been skillfully used to demonstrate incumbency advantage (Lee, 2008), policy responsiveness (Lee, Moretti and Butler, 2004), rents from office holding (Eggers and Hainmueller, 2009), spillover effects in elections (Hainmueller and Kern, 2008), and the effect of mayors on budgetary decisions (Gerber and Hopkins, 2011).

When scholars point to marginal elections – for purposes of normative justification of elected representatives, or for causal inference – they implicitly or explicitly adopt a model of the politics of close contests: the closest elections are assumed free of systematic sorting or manipulation. In this paper we consider properties of marginal elections that cast some doubt on this portrait and suggest a different model of how the closest elections are decided. We draw upon a basic intuition of strategic electoral politics: in single non-transferable vote systems where the "winner takes all" – where the value from votes garnered in a close but losing effort is zero – the effort and advantages to be deployed by a candidate or party will be much more effective in a close election than in a rout. In other words, close elections are those where differences of campaign resources, structural

advantages, and even fraud should most show themselves. As a result, marginal elections are the ones that will attract the greatest campaign effort and resources, and close contests will also attract the deployment of structural advantages.

If our hypotheses are correct about the effects of this resource flood, then close elections may fall disproportionately to the candidate with certain structural advantages. This result carries substantive importance, theoretical relevance and methodological implications. If close elections are systematically determined at the margin, then mere attention to the margin of victory in an election will constitute radically insufficient information for whether the election was in fact a competitive contest. And while analyses of the "declining marginals" (Mayhew 1974, Fiorina 1977) may be informative in and of themselves, they may obscure a set of richer dynamics that make apparent close contests rather uncompetitive. And if certain candidates have powerful structural advantages in close elections, then the near-randomness of these contests – and their utility for causal inference – must be called into question. So too might the conclusions of regression discontinuity designs be revisited. If, for instance, it is shown that the winners of close elections are more likely than the losers to go onto richer earnings (Eggers and Hainmueller, 2009; Snyder and Querubin, 2008), one might ask whether the effect is due to winning office, or whether some property of the candidate that correlates with winning elections is the same property that leads to higher post-career earnings. For example, winning candidates may have better class-position, higher skill levels, or better access to the party elite. The idea that winning marginal elections reflects resource and structural advantages may also help explain why these individuals are reelected at higher rates in subsequent contests (Lee, 2008). Candidates better able to exploit their party's structural advantages may also be better able to exploit the tools of incumbency once they arrive in Washington or have increased access to fundraising opportunities before the next election.

We also believe there is a genuine puzzle here, and a research agenda across various domains of political science. At one level, our findings constitute a negative result for the use of close elections as a source of natural experiments in US Congressional elections. Yet our theoretical expectations and empirical results also open a new line of inquiry into the determinants of close elections in different contexts. Our theoretical intuition is built upon the American case, where partisan control

over election administration and partisan strength in a district exercise influence over results in the closest elections. But the conditions that determine this influence vary across states and countries: different institutions imply a differential ability to manipulate to determine who wins the closest elections. We view a productive new line of inquiry that examines the determinants of the closest elections. This can take the form of a comparison within the United States, analyzing how different institutional features predict imbalances in close elections within a state, or changes in structural advantages over time. Or, these studies could take a cross-national form, analyzing how electoral institutions contribute to the determination of the closest elections.

To formalize our hypotheses about close elections, we begin with two types of models of electoral "manipulation", one model of campaigning before Election Day, one model of legal challenges and fraud after. Our first model makes the intuitive prediction that campaign expenditure will depend upon the predicted margin of the race. The model formalizes the intuition that equilibrium campaigning decreases as the expected margin of a race increases. For marginal elections, then, any asymmetries in campaign resources, skills, structural advantages and other candidate properties will become *magnified*. This implies that there will be systematic differences within narrow bandwidths of the break-even point (or, for statistical analysts, the supposed "discontinuity" provided by close elections). Our second model examines manipulation of electoral results after an election, making the prediction that systematically manipulated elections will give the appearance of the razor-thin differences necessary for valid RDDs. Our models predict that candidates with structural advantages are better able to manipulate votes after the election, leading to the prediction that the winners of close elections differ systematically from the losers. In either case – the case of imbalances between winners and losers within the bandwidth of a close margin (model one), or the case of elections stolen after the votes have been cast (model two) – the dynamics we describe will likely confound the estimates from RDDs. We aim for the simplest possible formal models to yield our predictions, suggesting richer models of dynamic electoral competition as an important agenda for further research.

We test the predictions of our theoretical models using a data set of U.S. House elections from 1880-2008. We aggregate data that are indicative of structural advantages in a district. Specifically,

we employ data on the party controlling the Governor's office at the time of the election, the party controlling the election administration–such as the Secretary of State's office, and partisan control of the state house and state senate. Our analyses indicate that candidates with structural advantages in a district hold a systematic advantage in extremely close elections. In some instances, these candidates are over *ten percentage points* more likely to win the election. This is indicative of the systematic determination of extremely close elections. This builds upon observations about who wins close elections first made in Snyder (2005), while also offering a theoretical logic for the systematic determination of close elections.

Before proceeding, we offer two qualifications. First, our analyses do not by themselves form the basis for any sort of general critique of elections and competitive democracy. More research would be needed to follow upon the inquiries here, yet the idea that close elections may be less stochastic than commonly presumed opens both normative and positive questions, to which we return in our conclusions. Second, our analyses do not suggest that regression discontinuity designs are necessarily invalid. In cases where the distribution of election outcomes does not satisfy the properties we attribute theoretically and empirically to marginal elections, RDD designs may stand as robust designs for causal inference. So too, one interpretation of our findings is that analysts simply need to take into account these structural advantages in a matching design where scholars match on partisan advantages. Still, the theoretical basis of our paper suggests that there may be unobservable differences in candidates in close elections, differential advantages for which statistical analysts cannot fully measure or account.

1 Marginal Elections and Their Properties

Normative analysts of elections, quantitative scholars examining election margins and the disappearance of "marginal seats" and scholars of causal inference who examine close elections all rely upon a basic intuition – as the margin separating winner from loser in a two-candidate race gets smaller, the election becomes more "competitive" and its outcome more probabilistic.[1] Analysts

[1] Throughout our discussion, we are imagining a setting of single non-transferable votes, in which the well-known result of Duverger's Law applies. Hence our two-candidate assumption –

may be invoking a normative claim about a pattern of elections that are getting more or less competitive. Or scholars may be estimating the partial association of legislator behavior (voting, campaigning, other features) and their margin of victory in the last election. Or scholars may rely explicitly on an assumption that when the margin if victory (the "bandwidth") is small, elections are near-randomly determined. In all cases, the smaller margin denotes greater electoral competition and often embeds notions of "fairness" and "fair chances." As the election margin gets closer, the incentives induced by competition get larger. And at the limit, it is claimed, observers will witness near-randomness of the eventual outcome as the margin approaches zero.

We begin with the "randomness" description of close elections used in regression discontinuity designs (RDD), because it is the most extreme – and currently quite popular – description of close elections. While some of the following discussion is therefore focused upon the explicit model of close elections in RDD analyses, much of the discussion and its implications applies as well, and we return in the middle of the essay and in our conclusion to implications of non-random close elections for normative and quantitative analyses of elections.

1.1 Regression Discontinuity Designs

The idea that close elections embed a random component that pushes a winner "over the top" is made as a useful statistical assumption. But underlying this statistical assumption are several assumptions about the *politics* of close elections. We begin our analysis of close elections by recounting the model of close elections used explicitly (and implicitly) in regression discontinuity designs (RDD), for two reasons. First, the RDD assumptions now compromise the dominant model used when exploiting close elections. Second, the statistical assumptions in the RDD model have clear empirical implications, which will provide useful insights into our alternative model of how competition occurs in close elections.

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which structures both of our formal models – is reasonable and indeed commonly used in political science and political economy. Our questions and methods are, however, extensible to elections with three or more candidates.

The use of regression discontinuity for causal inference requires assumptions about how competition occurs in elections. In a world of two candidates and one office, a really competitive race is one that both candidates have a shot at winning. Taken to the extreme, this assumption about competition presumes that as the race gets close to equal vote shares, the outcome is determined as if a fair coin were tossed. This randomness creates opportunities for what is commonly called a "natural experiment." If winning a marginal election is determined by the flip of a coin, then the background characteristics of candidates, parties, and districts that normally confound analyses are rendered orthogonal. This enables a study of a wide-range of consequences from winning office–rents, subsequent election advantages, a portfolio of policy choices, and policy outcomes–that are otherwise deeply confounded.

When employing RDD for causal inference, we are primarily interested comparing two counterfactual states of the world (Hahn, Todd and van der Klaauw, 2001). For a running example in this section, we are interested in measuring the *incumbency advantage* or the effect of incumbency status on electoral support (for example, Erikson 1971 ; Gelman and King 1990). We follow Lee's (2008) example and consider the effect of incumbency on support for Democrats in Congressional districts. To measure the incumbency advantage, we need to compare the percent of the vote for Democrats in district i under "treatment" $Z_i(1)$, with a Democrat incumbent in district i, and the percent of the vote for Democrats in district i under control $Z_i(0)$, or without a Democrat incumbent in the district. The fundamental problem of causal inference ensures that for each district i we observe only response under treatment or response under control (Holland, 1986), $Z_i = D_i Z_i(1) - (1 - D_i) Z_i(0)$, where D_i is equal to 1 if the Democrat candidate wins the election and 0 otherwise. Given the impossibility of identifying individual level treatment effects, the goal of many causal studies is to identify the *Average Treatment Effect* (ATE), or the average response to treatment for a population of Congressional districts, $\text{ATE} = \text{E}[Z(1) - Z(0)]$.[2]

In general, the systematic selection that plagues observational data will make identifying the ATE difficult, if not impossible. Recognizing this, political scientists regularly employ regression

[2] Throughout this section, we will suppose that the expectation is over the relevant districts.

models or use matching procedures in an attempt to remove confounding. But both methods rely upon *selection on observables*: the assumption that we have the exact set of covariates that remove all systematic differences between incumbents and challengers (Morgan and Winship, 2007). Further, unless exact stratification on the covariates is possible, we also must assume that we have identified the proper functional form for a regression, the correct specification of a propensity score (Rosenbaum and Rubin, 1983), or a combination of other matching algorithms that lead to comparable treatment and control groups (Sekhon, 2010; Hainmueller, 2010). Certainly the careful application of regression, matching, and their combination can reduce the confounding, but exact identification of any causal effect remains unlikely (Ho et al., 2007).

The insight of the regression discontinuity design is that identification of a *local* average treatment effect is possible, even from observational data that are otherwise deeply confounded. RDDs focus on identification of a treatment effect at a covariate level that constitutes a threshold for treatment assignment: below the threshold level of the covariate the subjects are assigned to control, above the threshold they are assigned to treatment. In electoral studies that employ RDDs, it is common to focus on vote share in the previous election, x, with studies attempting to identify the causal effect of incumbency *at the discontinuity*, or at the level of voter support that determines the election winner, $x = \frac{1}{2}$. We will denote the causal effect at the threshold of $\frac{1}{2}$ of vote share by,

$$\text{ATE}_{1/2} \quad = \quad \text{E}[Z(1) - Z(0)|x = 1/2],$$

or the average difference between electoral support for Democrats in districts with a Democrat incumbent, less the electoral support for Democrats in districts without a Democrat incumbent, given that the vote share in the previous election was $x = 1/2$.

Identification of $\text{ATE}_{1/2}$ from observational data requires two continuity assumptions. Specifically, RDD assumes that $\text{E}[Z(0)|x]$, expected support Democrats in districts without an incumbent, given previous vote share x; and $\text{E}[Z(1)|x]$, expected support for Democrats in districts with an incumbent, given previous vote share x, are continuous in x (Hahn, Todd and van der Klaauw, 2001; Lee, 2008; Imbens and Lemieux, 2008).[3] The continuity assumptions identify the

[3] This is stronger than actually needed to identify the causal effect of interest, as both Imbens

causal effect of interest by overcoming of the fundamental problem of causal inference, but only at the threshold. As we approach 0.5 from either side, the continuity of the functions ensures that $\mathrm{E}[Z(0)|X = 0.5] = \lim_{x\uparrow 0.5} \mathrm{E}[Z(0)|X = x]$ and that $\mathrm{E}[Z(1)|X = 0.5] = \lim_{x\downarrow 0.5} \mathrm{E}[Z(1)|X = x]$. And therefore,

$$\begin{aligned} \mathrm{E}[Z(1) - Z(0)|X = 0.5] &= \lim_{x\downarrow 0.5} \mathrm{E}[Z(1)|X = x] - \lim_{x\uparrow 0.5} \mathrm{E}[Z(0)|X = x] \\ &= \mathrm{ATE}_{1/2} \end{aligned}$$

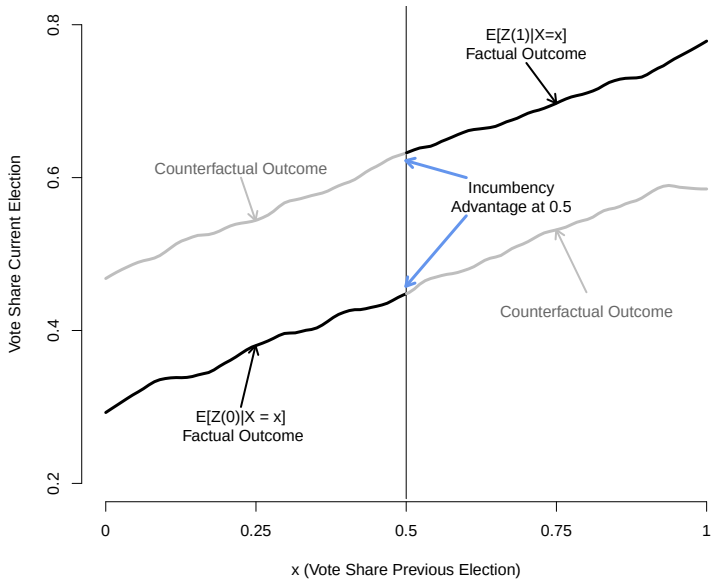
In other words, the continuity assumptions allow us to simultaneously observe $\mathrm{E}[Z(1)|X = 0.5]$ and $\mathrm{E}[Z(0)|X = 0.5]$.

To better understand this assumption, Figure 1 provides a graphical depiction. In Figure 1 the black lines represent the observed conditional expectations and the gray lines are the counterfactual conditional expectations, those that are not observed. Notice that the black and gray lines are connected continuously at 0.5. This continuity implies that there are no systematic differences between the treatment and control groups, immediately around the discontinuity. This then implies that, as we approach 0.5 from below in the limit, the expected value of the control observations provide the correct counterfactual value for the treated observations. Likewise, in the limit as we approach the discontinuity from above, the treated observations provide the correct counterfactual responses for the control units. The result is that the difference, $E[Z(1)|X = 0.5] - E[Z(0)|X = 0.5]$ identifies $\mathrm{ATE}_{1/2}$.

The continuity assumptions at the marginal elections is the key to RDDs identifying $\mathrm{ATE}_{1/2}$.

and Lemieux (2008) and Lee (2008) observe. However, the more general assumptions preserve the basic intuition that we motivate here and suffer from similar vulnerabilities. In general, we can restrict the continuity assumption to the discontinuity (Imbens and Lemieux, 2008). Even more generally, we might suppose that we observe vote share x, but fail to observe some effort level W. Then, it need only be the case that the cdf of x conditional on w, $F(x|W)$ is continuously differentiable in x at $x = 1/2$. As we will see all the assumptions rely on the critical assumption that, at the discontinuity, observations are just as likely to be above the threshold as they are to be below the threshold (which is why the continuity assumptions are so critical).

Figure 1: Graphical Presentation of Assumptions to Identify $\text{ATE}_{1/2}$



This figure provides a graphical demonstration of the assumptions used to identify $\text{ATE}_{1/2}$ in regression discontinuity designs. The black lines represent the observed relationship between electoral support as a non-incumbent ($\text{E}[Z(0)|X = x]$) and electoral support as an incumbent ($\text{E}(Z(1)|X = x)$). The gray lines are the counterfactual, or unobserved functions. The critical assumption is that both conditional-expectation functions are continuous. In the limit as we approach the discontinuity, there are no systematic differences the incumbent party and challenger party, otherwise, there would be a discontinuity in the conditional-regression functions. The absence of these discontinuities implies the identification of $\text{ATE}_{1/2}$.

These assumptions, and their more general variants, are regularly trumpeted as weak assumptions that provide robust identification in many different contexts. In political terms, these assumptions require that political resources are unable to systematically determine who wins extremely close elections. In the next section we argue that this critical assumption is unlikely to be satisfied for US House elections.

1.2 Why Close Elections Are Unlikely to Be Randomly Determined

Recent applications of RDDs draw heavily upon the continuity logic developed in Hahn, Todd and van der Klaauw (2001) and Lee (2008), and therefore impose the same basic assumptions about how close elections are determined. We highlight two potential problems with the continuity logic: one practical, one theoretical. In practice, the key problem in application of RDD designs to election is that data constraints and statistical power requirements means that too few elections with razor-thin margins are available for most analyses. Hence the analyst must choose a bandwidth – for purposes of election analyses, a margin of victory into which the sample cases fall, thus specifying a sample from which races with margins larger than the bandwidth are excluded (Green et al., 2009).

The selection of bandwidths represents a disconnect between the theoretical results that justify the use of regression discontinuity designs and their actual application and a problem for the assertion that marginal elections are the elections that are actually competitive. Regression discontinuity proofs are based on an assumption of an infinite (or extremely large) sample that allows for no extrapolation at the discontinuity (for example, Lee 2008). In any application, however, there will be insufficient data at the margin to perform the described limit and still retain enough statistical power to reject any null hypotheses. This forces the selection of a bandwidth and the borrowing of information across the bandwidth to extrapolate to the discontinuity. If factors are balanced at the discontinuity, but imbalanced in areas very close to the discontinuity and within the bandwidth, then the result could be a badly biased estimate of the $\text{ATE}_{1/2}$. Our theoretical model below predicts that this imbalance around the discontinuity should occur. As the campaigns allocate more resources to districts expected to be competitive, the structural advantage of one candidate is amplified. The result, are systematic differences in partisan strength between winners and losers of close elections.

A second problem is the possibility of sorting around a discontinuity. Once an initial ballot count is announced in a close race all sides know, with certainty, how many votes they will need to legally challenge or how many ballots they will need to stuff in order to win the election. This enables stealing of elections with extremely small margins. Building on this intuition, below, we present a game of post-election manipulation that predicts candidates will use their resources

Table 1: Summary of Assumptions and Potential Issues with RDD Models of Marginal Elections

1) Treatment is essentially randomized to winners and losers only in the limit, yet researchers must choose a bandwidth. In this bandwidth, there should be differences in party strength.
2) There are no post-assignment (post-voting) discontinuities such as legal challenges or fraud that may affect assignment to winners and losers.

to systematically secure office. The manipulation will result in candidates doing just enough to "steal" an election from their opponent–creating the impression of marginal elections that are actually systematically determined. If candidates can deterministically sort around the boarder, RDDs no longer provide valid estimates of $\text{ATE}_{1/2}$ or another causal effect of interest. Intuitively, sorting represents a type of selection, breaking the protocol of an experiment. More technically, sorting creates a discontinuity in $\text{E}[Z(1)|X = x]$ and $E[Z(0)|X = x]$ functions.[4] The result is that $\text{E}[Z(0)|X = 1/2]$ no longer provides a valid estimate of the counterfactual losing response for candidates that just happen to win. The result is bias in an unknown direction and of unknown size.

In the following sections we provide a theoretical logic why both problems discussed here are likely to manifest in Congressional election data and empirical evidence that they do. Narrowing bandwidths around the discontinuity focuses on elections that are, by definition, marginal. These marginal elections will attract greater campaign investments, such as advertising, deployment of structural advantages, and mobilization efforts. Indeed, as the margins get smaller, our models suggest that candidates will invest more of these resources in the race. Any systematic differences in candidate resources, quality, advantages and other variables will then be magnified. The result is a systematic advantage for one candidate in close elections, which manifests in systematic differences

[4] In the more general proof in Lee (2008) we can think of the discontinuity occurring in the measure on the unobserved (effort) variable W. If $g(w)$ is continuous, then each observation is just as likely to be in the treated arm or the control arm at the discontinuity. If there is a discontinuity, however, some observations are systematically more likely to be in treatment than control. This breaks the weighted average conditions in Lee's (2008) Proposition 2b and 3b.

between candidates in the closest elections. And even if the conditions are met for randomization at the discontinuity, close elections are the most likely to be subjected to legal challenges and most at risk for electoral fraud. Post-election manipulations of vote results are deterministic, resulting in sorting around the discontinuity. And as a result, winners of close Congressional elections are systematically different than losers.

2 How Do Campaigns Purposefully Sort Around the Discontinuity?

Politicians do not participate in elections only as candidates; they also have a hand in managing nearly every decision of the electoral process, from deciding the boundaries of electoral jurisdictions to the system of voter registration, from the format of the ballot to the mobilization of supporters. Moreover, some politicians, namely those associated with the dominant political party in their respective states and districts, play a far greater role in the process than their competitors. Consequently, we consider the potential for the origin of structural advantages in districts and the potential for purposeful sorting around the discontinuity.

Dominant parties may have a very good sense of how close a given election is going to be ahead of time. These parties may understand the pulse of the voters and the landscape of the district. If the election does not look close, they need not waste their resources. If it looks very close, they may employ massive resources to put themselves over the 50% mark. And immediately after Election Day, but before the results are certified, parties know with certainty the number of votes necessary to win an election. Dominant parties are able to use their influence on legal proceedings, the ability to certify electoral results, or even their opportunity to commit fraud to tip electoral results.

We consider two possible pathways for manipulation by dominant parties in close elections, one before the election, one after.[5] Long before Election Day, dominant parties are able to craft Congressional districts to accomplish their electoral goals. If one political party dominates a state's

[5] There are, of course, many more pathways for manipulation possible.

political offices, it can reap significant advantages by creating favorable legislative districts. Strategic redistricting was one of the first causes hypothesized for the decades-long trend of fewer and fewer close elections in the U.S. Congress (Tufte, 1973). But a growing consensus has emerged that redistricting is not the cause of the vanishing marginals (e.g., Ferejohn 1977; Abramowitz, Alexander and Gunning 2006, Ferejohn 1977), because political parties rarely construct safe districts. Rather, the optimal strategy for a dominant party is to create districts in which its candidates can all win by slight margins, allowing the party to gain more seats overall (Gopoian and West, 1984; Campagna and Grofman, 1990; Desposato and Petrocik, 2003). The result are systematic differences in narrow bands around a discontinuity, although there will still be balance at the discontinuity.

After Election Day, but before the certification of electoral results there is the opportunity for electoral fraud and legal challenges. The dominant party or candidate, likely in control of key functions of election administration, clearly has more opportunities to perpetuate fraud than out-partisans. Caro (1990) recounts how Lyndon Johnson exploited his connections in Texas to steal a Senate primary election from Coke Stevenson, producing just enough fraudulent ballots to defeat his opponent (this is also recounted in Snyder (2005)). Similarly, structural partisan advantages shaped the outcome of the Florida recount during the 2000 presidential election. The Republican Secretary of State, Katherine Harris, certified candidate George W. Bush as the winner under a cloud of partisan favoring. The Florida Supreme Court, filled with Democrat appointees, extended recounts, raising the suspicion that the Court was aiding Gore's effort. And, of course, the United States Supreme Court's 5-4 decision that ended the post-election dispute was vilified as partisan.

2.1 A Case Study: 2008 Minnesota Senate Race

As a more detailed example of systematic manipulation, consider the recent post-election dispute between 2008 Minnesota U.S. Senate candidates Norm Coleman and Al Franken. The first stage of the post-election dispute involved a recount of contested paper ballots that were submitted at the polls on Election Day. Questionable ballots were reviewed and a non-partisan committee determined whether each vote was properly cast. These were basically the equivalent of "hanging-

chad" issues: ballots that were marked, but not marked exactly right. The result of the ballot review was a success for Franken. The Election Day count was Coleman up by 215; after these ballots were sorted through, Franken took a lead by 49 votes.[6]

But the second stage of the recount reveals how structural advantages can determine the outcomes of very close elections. A number of absentee ballots were not initially counted because local election offices determined they were not properly submitted. The Coleman and Franken campaigns agreed to open up the envelopes of 953 of these contested absentee ballots and count the votes inside. And this agreement had the appearance of unbiasedness: each campaign had the power to veto absentee ballots that they thought were invalid, but had to raise the objection *before* the envelopes were opened. The recount went exceedingly well for the Franken campaign, whose lead jumped to 225 votes after this stage of the recount, essentially ensuring that Franken would win the election.

How did the Franken campaign gain such a huge lead from a set of votes that both parties could have rejected? The key is that the absentee ballots came in the mail, revealing the names of the voters and their address information on the envelopes. This enabled the campaigns to perpetrate two forms of cherry-picking. First, the campaigns could selectively contact voters who had submitted absentee ballots but whose votes were not counted and encourage them to complain and/or provide them with legal aid. The two campaigns demanded lists from the election office of people who requested absentee ballots but who were not marked as having voted. They then could merge these records with their statistical model predicting each person's level of support in the Senate race (presumably based on voter demographics, campaign contacts, and other micro-targeted information) and selectively call citizens favoring their respective candidates and encourage them to complain. If the Democrats had access to a superior voter file than the Republicans, this could have helped them gain votes.

The second form of cherry-picking is that the competing campaigns sorted through the absentee ballots together, and each campaign could veto the inclusion of disputed ballots they thought should

[6] Rachel E. Stassen-Berger, "Franken Leads by 50," *St. Paul Pioneer Press,* December 29, 2008.

not be counted. Nate Silver, then of the website *fivethirtyeight.com*, suggested that the Franken campaign may have been seriously advantaged in this veto process. The Coleman campaign vetoed ballots based on the partisan composition of the precinct or county where the ballots were cast. The Franken campaign vetoed ballots based on the individual characteristics of the actual voter whose ballot was in dispute. Based on the counties that the 953 absentee ballots came from, observers predicted that Franken would receive 52% of the recounted absentee ballots. In fact, he received 61% of them.[7]

The Minnesota recount demonstrates how structural advantages determine close elections. Franken likely won the recount because Democrats had a better voter list, better access to the list, or a better model to identify likely supporters than Republicans. The recount also demonstrates the possibility for post-election manipulation, even in elections with national implications, and even in very recent contests. This recount was extremely high profile, receiving attention from both the liberal and conservative leaning media; yet the Franken campaign was able to deploy its advantages to win the election.

3 Theoretical Model

We now formalize this intuition about campaigning and post-election manipulation and juxtapose these predictions to those from RDD models. The idea that the expected margin of an election can draw greater effort from its contestants and their allies can be usefully formalized; the formalization not only ratifies the intuition but also draws attention and lends clarity to the underlying variables that matter most in examining these elections. There are, of course, many models of elections –

[7] Details about the Minnesota senate recount, cherry-picking, and selective vetoing of absentee ballots are described in news articles and web blogs, such as: Nate Silver, "Franken Jumps Out to 225-Vote Lead on Strength of Absentee Ballots," *fivethirtyeight.com*, January 3, 2009; Nate Silver, "In Minnesota, End of Beginning Starts Today, *fivethirtyeight.com*, January 3, 2009; Bob Collins, "Recount Q & A," *Minnesota Public Radio News*, January 3, 2009; Eric Kleefeld, "Friendly Coleman Witness: They Cherry-Picked Me," *Talkingpointsmemo.com*, January 29, 2009; "Senate Contest Day 4: 'Cherrypicking,"' *The Uptake*, January 30, 2009.

such as spatial models of vote choice – but the essential properties of the models we seek are not those that examine voter choice or aggregation, nor the production of information (as in models of negative advertising). Instead, we seek simple but generalizable models that describe campaign dynamics, both before and after an election.

To that end, we build upon Erikson and Palfrey (2000) and consider a model of two candidates who observe a pre-election poll. In response to this information, the candidate (and/or the parties) spend costly resources in an attempt to increase their vote shares. These attempts meet with stochastic success, a random component still partially determines the outcome of the election. Under equilibrium campaigning in this model, resources are directed into districts that pre-election polls reveal to be competitive. This magnifies structural advantages and subsequently causes systematic differences between winners and losers within narrow bandwidths around the discontinuity. In our supplemental appendix we generalize this model using a differential game and demonstrate that our same predictions hold in this much more general model.

Our second model formalizes post-election challenges that are an important element of marginal elections. In this model, candidates observe the post-election, but pre-certification, vote totals. Then both candidates employ a set of tools to modify the final electoral total, similar to the strategies used in vote-buying games (Groseclose and Snyder, 1996). Under equilibrium in this model, we show that resource advantaged candidates are able to "steal" elections from their disadvantaged opponents. This causes systematic sorting around the discontinuity and therefore systematic determination of close elections.

Both models sacrifice a focus upon information production (the equilibria and dynamics are not Bayesian), but they are useful for describing the dynamics of campaigns and the behavior of contestants as margins get smaller or larger both before and after the election. Both models preserve the rational choice properties of campaigns while permitting fully dynamic modeling that embeds candidates' valuations of the future.[8]

[8] Models with greater behavioral realism are possible and desirable, but are beyond the scope of analysis here.

3.1 A Simple Model of Campaigning

We begin our analysis with a simple model of resource investment during campaigns (Erikson and Palfrey, 2000). Our model demonstrates that resources from both parties will converge upon close elections and that institutional advantages for one party will make them systematically more likely to win close elections. The result is that the parties that hold an institutional advantage in a state will be systematically more likely to win close elections, in contrast to the expectations from RDD.

We suppose that there are two candidates, 1 and 2, who are competing in an election. Our game proceeds in two stages. First, a poll that reveals to the candidates the current vote share in the election x_0. After observing this poll the candidates make a decision about how much to invest in the campaign. Let c_1 denote the resources for candidate 1 and c_2 denote the resources for candidate 2. After the candidates make their investment decision, the final vote share is revealed, with the vote share for candidate 1 given by

$$x_1 = \gamma_1 c_1 - \gamma_2 c_2 + w \tag{3.1}$$

where γ_1 and γ_2 represent a multiplier on the campaign's investments and w is a draw from a Normal(x_0, σ_0^2). The vote share for candidate 2 is given by x_2 = 1- x_1. γ_1 and γ_2 capture one manifestation of candidates' institutional capacity during an election. Candidates with stronger party backing may be able to receive more return for their investments than their opponent.

Candidates' utilities are a combination of the cost of the campaign and their probability of obtaining the returns from office. Let k_1 and k_2 be multipliers that capture how efficiently candidates are able to invest their money during an election. Then, the candidates' utility functions are given by,

$$\begin{aligned} U_{\text{cand1}}(c_1, c_2) &= \text{Prob}(x_1 \geq 0.5) - k_1 \exp(c_1) \\ U_{\text{cand2}}(c_1, c_2) &= \text{Prob}(x_2 \geq 0.5) - k_2 \exp(c_2) \end{aligned}$$

To summarize, our game proceeds in three stages:

1) A poll result x_0 is revealed to the candidates

2) Candidates make their campaign investments c_1 and c_2

3) Vote share is revealed and payoffs are realized

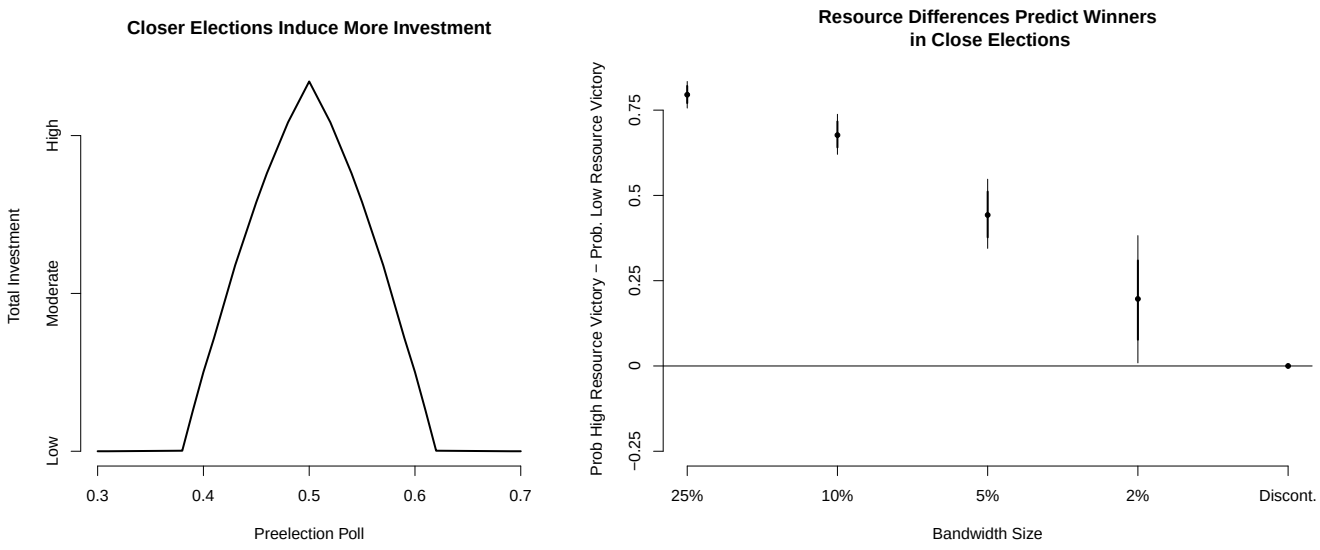
Proposition 1 in the appendix proves that there is a pure strategy symmetric Nash equilibrium. To provide comparative statistics on this equilibrium we employ two simulations to demonstrate two primary points of our analysis. First, we show that an equilibrium response from both candidates is to invest more in closer elections.[9] For both simulations, we will analyze an election where Candidate 1 has a resource advantage over Candidate 2, $\gamma_1 > \gamma_2$. Our first simulation demonstrates that, in the equilibrium, candidates invest more in close elections. The left-hand plot in Figure 2 shows that closer preelection polls induce more investment from candidates. To demonstrate this, we varied the preelection poll from 0.5–indicative of a very close election–to 0.7 and 0.3–indicative of an uncompetitive election.

As Figure 2 illustrates, the closer election induces more investment from both candidates. The result of this increased investment is systematic differences in who wins elections. In the right-hand plot in Figure 2 shows that equilibrium strategies predict that candidates with resource advantages will be systematically more likely to win close elections, even within very small bandwidths. This figure varies the size of the bandwidth along the horizontal axis, from wider (a 25% bandwidth) to more narrow (using the predictions from a polynomial regression model at the discontinuity). The vertical axis presents the average difference in resources between candidates who win and those that lose.

The right-hand plot in Figure 2 shows that our model predicts systematic differences exist between winners and losers, even in very close elections. Even elections decided by less than 2 percentage points, we expect that those with greater resources will be systematically more likely to win. This has two important implications. First, this implies that marginal elections may mask

[9] A formal comparative static will likely reveal that the amount invested in any one election is non-decreasing, because some elections an equilibrium response is to not campaign.

Figure 2: Close Elections Induce Greater Campaigning



This figure demonstrates two predictions from the simple campaigning model. The left-hand plot shows that the game predicts more resources invested in close elections. The right-hand plot presents the prediction of systematic differences in winners and losers in even close elections.

candidates' structural advantages, rendering these elections less competitive than they appear. Second, RDD estimates that rely upon wide bandwidths will provide poor estimates of $\text{ATE}_{1/2}$. But, because of the randomization after the candidates invest their resources, the model predicts that the resources will be balanced at 0.5, which is demonstrated with the zero estimate at the far right.

This model predicts, therefore, that systematic differences will exist between winners and losers even within narrow regions around a discontinuity, even though there is no difference (on average) at 0.5. This model predicts the emergence of an imbalance as a direct result of differential partisan strength in a district.

3.2 Systematic Differences at the Discontinuity

Our model of campaigning predicts that candidates with an institutional advantage in a district are systematically more likely to win close elections, even within very narrow bandwidths. But the model does predict that at 0.5 partisan advantages should not determine who wins the closest elections. The randomness inherent in each model predicts that the estimate at the discontinuity will be an unbiased estimate of the treatment effect at the discontinuity, so long as there are sufficient observations to estimate the effect exactly at the threshold for winning the elections. The important substantive implication is that partisan differences may swing narrow elections, but the closest elections are determined without systematic manipulation. The key statistical implication is that commonly used bandwidths are unable to identify the desired treatment effect. In principle, however, enough data could be collected to identify the desired causal effect if sufficiently narrow bandwidths are employed.

Campaigns represent only one method candidates and parties can employ to affect vote totals. After an election, they are able to employ legal and illegal means to alter the official tally. This manipulation represents a type of sorting, a violation of the assumptions necessary for RDD to identify valid causal effects. In extremely close elections, both parties will file legal complaints, demand recounts, challenge ballots and use their resources to obtain a desired certified vote total. Parties and candidates are able to use more nefarious methods to obtain their desired results. Candidates can stuff ballot boxes, use the votes of citizens long deceased, or commit a variety of other components of fraud that will systematically alter the outcome of the close election. For example, Caro (1990) details how the leading candidate in Texas elections would "hold out" their fraudulent ballots to ensure that they remain ahead of their opponent (Caro, 1990)[310].

In this section we discuss a simple game that captures this post-election manipulation. We model a sequence of "legal" challenges and show that candidates with a resource advantage are able to systematically claim elections using legal challenges that their opponent would have won in the absence of such challenges.[10]

[10] We use legal challenges to avoid appropriating fraudulent motivations or deeds to party officials. But certainly, our model is intended to include both legal and illegal methods of post-election vote

To generate theoretical expectations about post-election manipulation, we analyze a modified version of the game employed in Section 3.1. We modify the game to include two new features. First, we remove the random component from the previous game. After an election, both parties know with certainty the number of votes they will need to tilt the election in their favor. Second, we introduce a sequential structure to this game, similar to the sequential structure employed in similar vote buying models in legislatures for analytic tractability (for example, Groseclose and Snyder (1996)).[11]

To state the model, suppose that a campaign has occurred and both candidates have observed the vote share x_c. After observing this electoral result, the game proceeds in three stages. In the first stage of the game, the candidate ahead after the campaign (if $x_c > 0.5$, Candidate 1, if $x_c < 0.5$ Candidate 2) makes a decision about how much to invest in post-election manipulation. In the second stage of the game the other campaign decides on how much to invest in their legal challenges. We will denote both campaigns investment by l_1 and l_2. The final stage of the game is the realization of election results, which we assume are a consequence of the following process,

$$x_1 = \eta_1 l_1 - \eta_2 l_2 + x_c$$

where η_1 and η_2 represent Candidate 1 and 2's institutional capacity to manipulate post-election results, respectively. If $\eta_1 > \eta_2$, a candidate is more effectively able to manipulate election results. After deciding on the amount to invest, payoffs are realized.

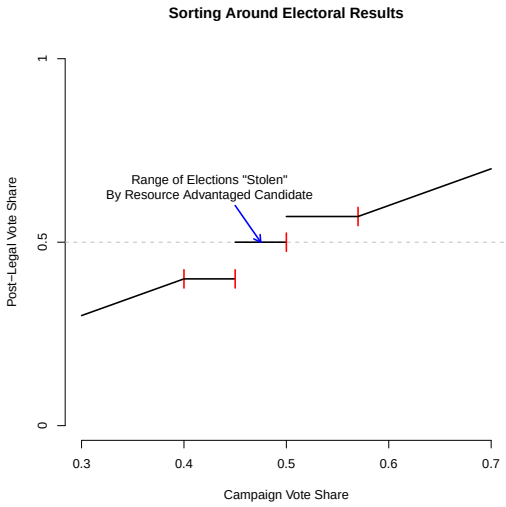
To finish specifying the game, the utility function for the candidates are,

$$U_1(l_1, l_2) = \begin{cases} -k_1 \exp(l_1) & \text{if } x_1 \leq 0.5 \\ 1 - k_1 \exp(l_1) & \text{if } x_1 > 0.5, \end{cases} \quad U_2(l_1, l_2) = \begin{cases} 1 - k_2 \exp(l_1) & \text{if } x_1 \leq 0.5 \\ -k_2 \exp(l_1) & \text{if } x_1 > 0.5, \end{cases}$$

manipulation.

[11] As with vote buying in legislatures, we introduce the sequential structure to avoid the use of mixed strategies in an equilibrium.

Figure 3: Resource Advantages Allow Candidates to Steal Election



This figure presents the equilibrium predictions from the simple post-election manipulation game, predicting that candidates can employ their resource advantages to systematically win extremely close elections.

where k_1 and k_2 encode the cost multiplier to both candidates.

Proposition 2 in the Appendix describes a pure-strategy sub-game perfect Nash Equilibrium to this game. It predicts that a candidate with a resource advantage will be able to manipulate election results after the fact, ensuring her final victory even though she was behind on election day. In this way the candidate is able to "steal" the election: even the public voted for Candidate 2 in the campaign, Candidate 1 emerges victorious through post-electoral manipulation. Figure 3 displays this dynamic demonstrating the area of vote stealing. The horizontal axis presents the pre-election vote share, the vertical axis is the vote share after the legal manipulation. The thick line through the plot presents the equilibrium election results, with the vertical red-lines denoting changes in the equilibrium strategy.

Figure 3 shows clearly that the resource advantaged candidate is able to use legal challenges to secure victory in marginal election that originally favored their opponent. This represents *sorting* around the discontinuity. Substantively, this suggests that there will be systematic characteristics

that predict the winners of even the closest elections. Statistically, this equilibrium result violates the assumptions necessary for RDD to identify valid causal effects. If candidate's resource advantages help to determine whether they are able to steal marginal elections and subsequently affects their behavior in office, then the continuity assumption is violated. Specifically, candidates who just happen to win extremely close election will, on average, hold a resource advantage over the candidates that happen to just lose an election. This systematic difference then implies that $\lim_{x\uparrow 0.5} \mathrm{E}[Z(1)|X = x] \neq \lim_{x\downarrow 0.5} \mathrm{E}[Z(1)|X = x]$ and that $\lim_{x\uparrow 0.5} \mathrm{E}[Z(0)|X = x] \neq \lim_{x\downarrow 0.5} \mathrm{E}[Z(0)|X = x]$.

4 Empirical Analysis of Close Elections

Our theoretical models predict that there will be systematic differences in resources in very close elections and differences at the discontinuity in close elections if sorting occurs. If the differences in resources are correlated with the dependent variable, this will result in RDD failing to identify $\mathrm{ATE}_{1/2}$. Substantively, this implies that indicators of partisan or candidate strength in an error should systematically predict who wins the narrowest elections. In this section we show that there are systematic differences in who wins very close U.S. House elections and these differences are indicative of the importance of structural advantages.

Our empirical analysis requires data on election returns and measures of party control that serve as indicators of a party's structural advantages in a state. We employ a wide ranging data set of House elections from 1880-2008, first introduced in Ansolabehere and Snyder (2002). For these races, we ask if measures of partisan strength in a state predicts the winner of the closest elections. To measure party strength in a state we employ party control of four key institutions of state government at the time each election was held: the governorship, the election administration, the state lower legislative chamber, and the state upper legislative chamber. An implication of our theoretical model of close elections is that there should be systematic differences in the rate of party agreement between winners and losers, even in the closest elections.

Using these data, the left-hand plot in Figure 4 demonstrates that winners of extremely close U.S. House elections also tend to hold structural political advantages. Along the horizontal axis

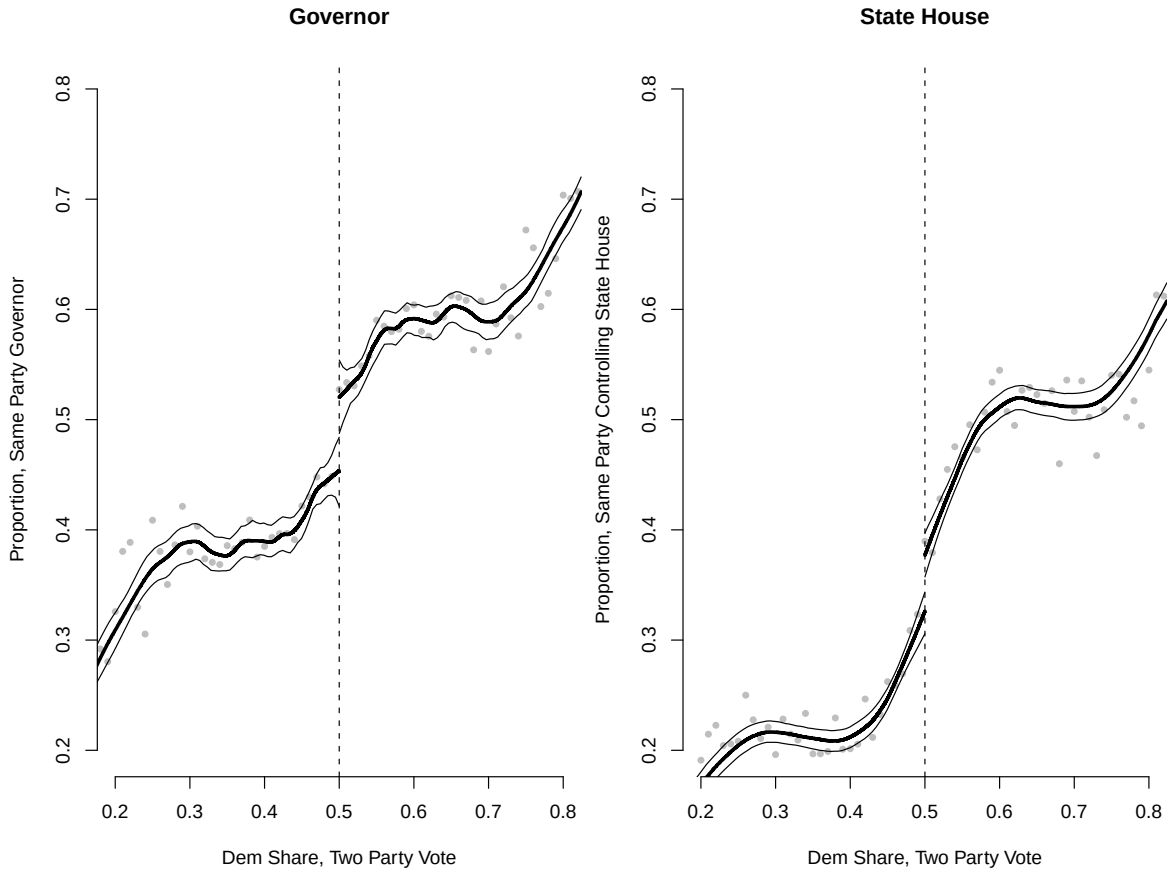
is the share of the two-party vote. As we move towards the center, we observe increasingly close elections, until we reach the dashed line which represents the discontinuity, or the 50% of votes necessary to win the election. The vertical axis measures the proportion of candidates from the same party as the Governor and the gray-dots create bins of legislators based on their vote share and measure the proportion of candidates with the same party as the Governor within each bin. The black-lines are nonparametric regressions of the proportion of candidates from the same party as their Governor against the two-party vote share.[12] If marginal elections are essentially decided by a coin flip, we would expect the line to the left of 50% and the line to the right to meet exactly at the discontinuity.

But the large gap between the regression lines demonstrates that structural advantages are correlated with who wins extremely close elections. Candidates who barely won the election were almost 7 percentage points more likely to belong to the same party as the Governor than candidates who barely lost. And the significant gap in the binned estimates of agreement between candidates and governors suggests that this finding will be robust to a wide variety of modeling choices (we demonstrate this below). The right-hand plot in Figure 4 shows that winners were also systematically more likely to belong to the same party as the party controlling the state House. Winners of the closest elections were 5 percentage points more likely to belong to the party controlling the state House then candidates who lost the closets elections.

The differences we observe in Figure 4 are replicated across all four offices representing partisan advantages, across a wide range of different model specifications (Green et al., 2009). Figure 5 summarizes the systematic differences between winners and losers in very close elections. To do this, each plot compares the average party agreement between winners and losers in the U.S. House contests across offices representing party control (the different plots) and different bandwidths (the lines in each plot). In each plot, moving left to right we move from a wide bandwidth (25%

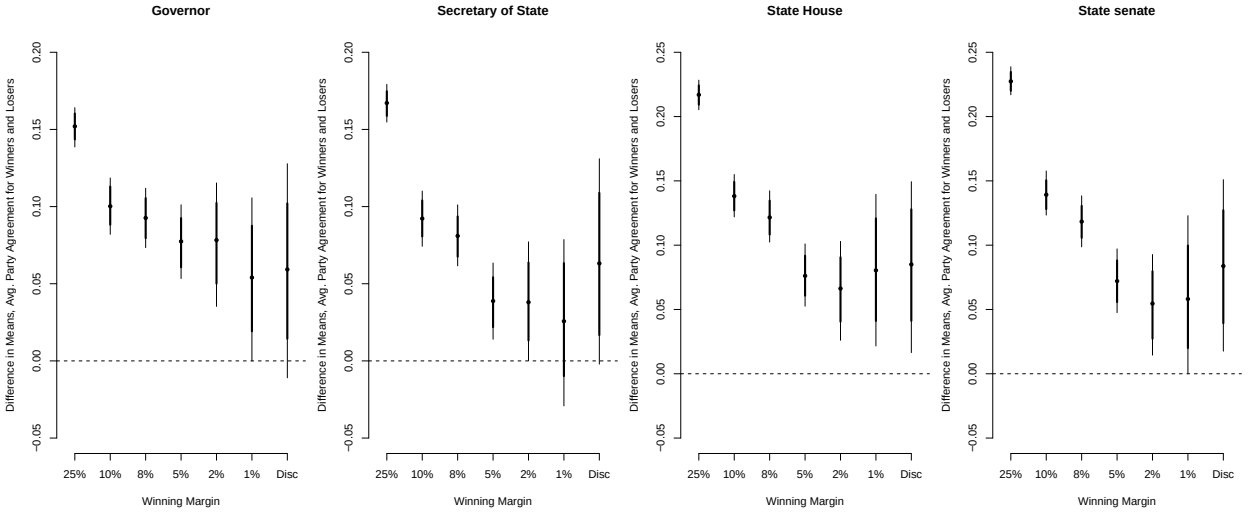
[12] The bandwidths in this plot are fairly narrow (approximately 6.02% of observations at discontinuity) and were chosen to be illustrative, below we select bandwidths using well established selection criteria that validates the point here.

Figure 4: Gubernatorial and State House Control is Correlated with Winning Close Elections for the U.S. House



This figure demonstrates that U.S. House candidates who win very close elections are systematically more likely to belong to the same party as the Governor and as the majority party in the state legislature's lower chamber. The large gaps at the discontinuities show that structural political advantages predict who win close elections. This is evidence that pre-election campaigning and post-election legal challenges are inducing differences in marginal elections.

Figure 5: Winners of Close U.S. House Elections Hold Systematic Structural Advantages



Each plot represents the proportion of U.S. House winners who are of the same party as the state office (e.g. governor) minus the proportion of U.S. House losers who are of same party as the state office. Statistically significant positive values suggest systematic differences between winners and losers in very close U.S. House elections.

(62.5-37.5) or closer) to a narrow bandwidth (a 1% (50.5-49.5 or closer)) bandwidth, and finally an estimate at the discontinuity (50%) using a third-order polynomial, fit within a 10% (55-45 or closer) bandwidth (Lee, 2008; Green et al., 2009). The dots are the point estimates and the thick and thin lines are 80 and 95 percent confidence intervals. (Table 4 provides numerical values for these figures, along with the number of observations used to compute each difference).

The left-hand plot in Figure 5 shows that winners of elections are systematically more likely to belong to the same party as the governor than the losers of close elections. The winners of extremely close House elections (within a 1% band around 50%) are 5.6 percentage points more likely to share the same party as the governor ($p<0.05$). There is also a systematic difference at the discontinuity: according to the model, winners at the discontinuity are 5.8 percentage points more likely to belong to the same party as the governor ($p<0.07$). The other plots reveal similar differences at the discontinuity: winners are systematically more likely to belong to the same party

as the secretary of state (6.1 percentage points more likely than losers, second plot from left), share the same party label as the party controlling the state House (8.6 percentage points, second plot from right), and belong to the same party as the party controlling the state Senate (8.3 percentage points, far right plot). And the figures show that all these differences are statistically significant at standard levels.

4.1 Sensitivity

Figure 5 provides strong evidence that winners of close elections are from strong parties. But a concern when presenting these results, as with estimates from an RDD, is that they might depend on the modeling assumptions employed. To explore the sensitivity of our results to models we use to obtain them, we follow the advice in Green et al. (2009) and compute our estimates of differences between winners and losers at a wide range of bandwidths. We present the results of this sensitivity analysis are contained in Table 2. For all four examples of structural advantage, we fit a third order polynomial on different subsets of the data, ranging from all observations to a narrow, 10% bandwidth (this estimate is placed in Figure 5). We also use a local linear regression (Loader, 1999) with bandwidths selected using leave one out cross-validation (Ludwig and Miller, 2007) and a formula derived asymptotic theory (Imbens and Kalyanaramang, 2009), with bootstrapped confidence intervals.

This sensitivity analysis demonstrates the robustness of our findings to a variety of modeling specifications and bandwidth choices: winners are systematically more likely to hold structural advantages over losers. The stability of our estimates is exemplified in the findings for governors. All models estimate that winners are 6-7 percentage points more likely to share the same party label as governors than losers, with all differences significant. And the same stable and large estimates are observed across models and bandwidths for party agreement with the state House and state Senate. The sensitivity analyses do reveal that the correlation between winners of close elections and party control of secretaries of state is more fragile and model dependent. But overall, these analyses reveals that winners are different than losers in politically important ways.

Table 2: Sensitivity Across Offices

Method	Band Width	Point Estimate	SE
	Governors		
Polynomial	Global	0.07	0.01
Polynomial	25%	0.06	0.02
Polynomial	10%	0.06	0.04
Local Linear	7.74% (IK Method)	0.06	0.02
Local Linear	16% (LM Cross-Valid)	0.07	0.01
	Secretaries of State		
Polynomial	Global	0.01	0.01
Polynomial	25%	0.01	0.02
Polynomial	10%	0.06	0.03
Local Linear	6.68%(IK Method)	0.01	0.02
Local Linear	23% (LM Cross-Valid)	0.07	0.01
	State House		
Polynomial	Global	0.06	0.01
Polynomial	25%	0.03	0.02
Polynomial	10%	0.09	0.03
Local Regression	6.67% (IK Method)	0.04	0.02
Local Regression	24% (LM Cross-Valid)	0.12	0.01
	State Senate		
Polynomial	Global	0.05	0.01
Polynomial	25%	0.01	0.02
Polynomial	10%	0.08	0.03
Local Linear	6.04% (IK Method)	0.02	0.02
Local Linear	24% (LM Cross-Valid)	0.12	0.01

This table follows the advice of Green et al. (2009) and fits a variety of models to assess systematic advantages at the discontinuity. We fit a third order polynomial to varying bandwidths (from all observations to a 10% band around 50%) and a local linear regression with bandwidth selected using cross-validation (Ludwig and Miller, 2007) (LM in the table) and a formula derived asymptotically (Imbens and Kalyanaramang, 2009) (IK in the table). This demonstrates that our findings are robust across modeling specifications.

4.2 The Power of State Office in Close Elections

Readers might fairly ask why the party of the Governor or election administrator or the party that dominates the state legislature might reflect advantages for a candidate for federal office in that

state. We argue that the advantages are manifold, and that it is difficult if not impossible to break out different paths of causation (although in our conclusion we discuss empirical implications that are testable). Some of the effect might have less to do with the officials themselves and more to do with what control of the offices represents: an indicator of underlying latent strength. The fact that a party controls the Governorship is surely a correlate of the strength of its state party organization. In close races, candidates and their party allies will seek to capitalize on superior (lower cost, higher leverage) mobilization, publicity and turnout operations that are available. So too, a Governor may be able to use her or his personal appeal and name recognition to campaign for candidates in close races. As an incumbent with superior access to fundraising (contributors often rationally invest in incumbents), the Governor may be able to redirect funds to a same-party candidate in a close election.

Like the holders of statewide offices, state legislators each have small teams of volunteers and committed enthusiasts whom they can mobilize for electioneering purposes. There are, after all, countless opportunities for local activists to participate in the political process in such a highly decentralized election system as is found in the United States. The thousands of citizens who populate the election boards and party ward committees, the precinct caucuses and town councils and polling locations are not a random subset of individuals solely committed to administrative functions. They are often the friends, family, and supporters of local politicians like state legislators. These legislators and local party chieftains can instruct their loyalists in a variety of ways to influence the electoral process.

If partisan control of state offices is an indicator of latent party strength, then each office provides an error filled measurement of latent party strength. If we treat partisan control of state offices as indicators of an underlying latent index of partisan strength, then we can reduce our error in measuring this index through the creation of a partisan strength index. Two methods have become popular for retrieving these latent indices: either summing across offices (Ansolabehere, Rodden and Snyder, 2008) or through the use of statistical dimension reduction techniques, such as factor analysis models (Clinton, Jackman and Rivers, 2004) or principal component analysis (Tipping and Bishop, 1999). Using these latent measures of partisan strength, Table 3 compares

the average partisan strength for the parties of winners and losers in extremely close House elections (50.5-49.5, or closer) using three different indices, with each index created using different statistical assumptions. The first index is created by summing across offices considered (Ansolabehere, Rodden and Snyder, 2008). A candidate will score a zero on this index if their party does not control the Governor's mansion, the secretary of state office, or either house in the legislature. And a candidate's party will score a four if their party holds all four offices.

Table 3: Winners of Close House Elections Are From Stronger Parties

Ind. Method	Point Estimate	Standard Error
Sum	0.23	0.09
Prin. Comp.	0.12	0.04
IRT	0.13	0.05

This table uses latent indices of partisan strength to show that winners of close elections (50.5-49.5 or closer) are from much stronger parties than the losers of close elections. Three different methods are used to measure latent partisan strength: summing across party agreement of the four offices considered here, principal components applied to the matrix of office agreement, and an item response theory model. For all three indices, we find that winners are, on average, from much stronger parties than losers.

The point estimates and the accompanying small standard errors reveals that winners of the closest elections are from substantially stronger parties. Winners of these very closest elections score an average of 0.22 points higher on this index. The same imbalance is used if we employ more sophisticated methods to construct a latent indicator of partisan strength.[13] And using these other indices, we reach the same basic conclusions: the winners of close elections belong to parties that are much stronger than the losers of close elections. The second-row of the table shows that the

[13] The limited number of offices considered here insures that each index cannot deviate far from the summation index. The principal component and the IRT scores were rotated so that higher scores imply greater party strength.

substantial difference in partisan strength between winners and losers is replicated using indices generated using principal components (Tipping and Bishop, 1999). And the third row shows that we reach the same conclusion with indices of partisan strength created using a one-dimensional item response theory (IRT) model (Clinton, Jackman and Rivers, 2004).[14]

There are also more insidious possibilities that are not based on the offices as latent indicators of partisan strength. Governors and Secretaries of State may be able to influence election administration, may be able to deploy state officials and even government workers to dampen turnout in opposing constituencies while enhancing turnout in friendly constituencies. Or there may be partisan advantages in the counting and certification of ballots. It was a common joke among Democrats after the Bush-Gore election in 2000 that "it ain't over until your brother counts the votes." The presidential election that year was decided by a razor-thin margin in the state of Florida, where the state's Governor, Jeb Bush, was the brother of Presidential candidate George W. Bush. The advantages of holding the Secretary of State's office are more straightforward. Secretary's of State design election rules that might dampen turnout for opponents. After the election, secretaries of state often conduct recounts and decide when to certify elections. Partisan control over these pivotal steps in the election process invites fraud and manipulation of post-election results.

The 2000 Florida dispute highlights most dramatically how these offices are used to advantage candidates. Through each step of the post-election dispute, key movers, like the Governor and Secretary of State, but also the state legislature, state Supreme Court, and U.S. Supreme Court, were all accused of using their authority to advance the chances of their preferred candidate's success. The very fact that in the United States the election authorities are themselves elected as partisan actors or are appointed by partisan actors means that the closest elections may be anything but randomly decided. The differences we present in this section suggest that there are systematic components to elections that are assumed to be random. Even in elections decided by razor-thin margins, we observe important differences between winners and losers. And this result

[14] In results not presented here, we show that the differences in partisan strength across winners and losers, is replicated across a variety of modeling assumptions and statistical methods.

is extremely robust. Because these differences are likely to be correlated with many outcomes of interest, this represents an important objection to the use of RDDs to estimate causal effects.

5 Discussion

Close elections may not reflect randomness and are the precise venue where candidate and partisan advantages in organization most show themselves. In the single non-transferable vote systems employed in the United States, where the plurality winner takes the full value of the seat, it is the marginal elections that will consume the focus of candidate and party resources during the campaign and even after the votes have been cast. Our analysis suggests that further research is in order before analysts confidently use close elections as quasi-experiments or as benchmarks for necessary democratic competition.

We have introduced a new theoretical model to explain how campaigns behave both before and after extremely close elections. Our theoretical results point to an expectation that these elections will be systematically determined. Our empirical results uncover one form of this systematic determination: the winners of close elections belong to parties that are substantially stronger. This finding is significantly robust, across models and different measures of partisan strength.

Our results have substantive importance, methodological relevance, and normative implications. Substantively, this paper suggests the need for a new literature examining who wins close elections. There are both immediate extensions of our work and more general implications that point to a new and important puzzle in the study of elections. Our theoretical model posits two sources for manipulation. Before elections, the increased attention to marginal districts amplifies differences in partisan strength. And after an election, candidates engage in systematic manipulation that privileges candidates from stronger parties. An important line of inquiry involves adjudicating between these two potential sources of influence. One possible research design is an analysis of vote total changes from election day to the certified total. Other designs could be used to determine the extent to which preelection differences in partisan strength swing elections towards a systematically advantaged candidate.

Beyond this immediate extension, we view our results as opening an important line of inquiry

into the relationship between electoral institutions, party influence and the systematic determination of close elections. Our theoretical models are based on intuition from the American case, with electoral institutions that are subject to partisan manipulation. Of course, the extent to which parties hold control over electoral institutions varies across American states and countries. A host of questions emerge naturally from the recognition of variation in electoral institutions and party politics. In the American context, does partisan control of gerrymandering influence how well parties can determine close elections? Are incumbents better equipped to win razor-thin elections than non-incumbents? Voting technology, campaign style, and party influence have changed over the course of American history; when have close contests appeared to be least randomly determined? From a comparativist perspective, are similar differences found in proportional representation systems as in the American system? Does bureaucratic administration of elections (such as Germany's electoral system) dampen a party's ability to swing an election? With these (and many other) questions in mind, a comparative study that demonstrates how incidence of different electoral institutions correlates with systematic differences in close elections will provide insights into how candidates determine the closest elections.

In addition to the substantive implications, our work has wide-ranging implications for the use of close elections in causal inference. Our empirical analyses demonstrate that close elections are in fact systematically determined, in large measure because these contests attract disproportionate investment by the candidates. Our theoretical models provide two explanations for this result. First, RDD analysts rely upon a theoretical result about the causal assignment mechanism at the discontinuity point (one-half of the votes cast), but in practice they must choose a bandwidth which includes races with larger margins. These margins will include races where one candidate has systematic advantages over another candidate, and there are many reasons to believe that these advantages are correlated with downstream variables like later earnings, later voting patterns and policy outcomes. Second, elections do not end when people are finished casting their ballots. Ballots have to be counted and certified, and election results must be declared legal and legitimate. Candidates can also deploy advantages at this post-voting stage, breaking the continuity of the regression function at the 50 percent threshold.

Our theoretical arguments and empirical evidence imply that more care is needed before natural experiments are used to identify causal effects. Across applications, we believe that assumptions made to identify causal effects using a natural experiment carry with them important political assumptions. Therefore, to identify and exploit natural experiments, it is incumbent upon the researcher to understand and examine the underlying political process that determines the assignment mechanism. This includes more than quantitative demonstrations of balance in covariates across treatment and control groups. It may also require qualitative, ethnographic or historical study to show that how observations are assigned to treatment and control are not systematically related to the outcome of interest. To that end, we believe that future work should examine not only the practice of RDDs, but also the equally vital question of who wins close elections. Depending on the margin, what are predictors of winning these elections? How often do legal challenges occur and with what conditional probability of success? How can the various causal pathways of candidate advantage be disentangled from one another?

Finally, our theoretical results and evidence we present also calls into question whether elections decided by razor-thin margins are truly marginal. Marginal elections, where either party has a chance to win, represent an important source of voter influence on national government (Mayhew, 1974). Certainly the marginals are vanishing, a large literature argues, but at least some "toss-up" seats remain. But our analyses present a bleaker portrait of party competition for seats. Even the closest elections are determined, at least in part, by systematic structural advantages of one party. This blunts the effectiveness of close elections as a tool to translate voter preferences into national government.

A Proofs Appendix

Proposition 1. *A pure strategy symmetric Nash-Equilibrium exists to the this game, with Candidate 1's equilibrium investment strategy given by*

$$\begin{aligned} c_1 &= \frac{1}{2(\gamma_1-\gamma_2)^2}(\gamma_1-\gamma_2-2\sigma_0^2-2\gamma_1x_0+2\gamma_2x_0+2(\gamma_1-\gamma_2)\gamma_2\log[\frac{k_1}{\gamma_1 t}] \\ &\quad +2\gamma_1\gamma_2\log[\frac{k_1}{\gamma_1 t}]+2\gamma_1\gamma_2\log[\frac{\gamma_2 t}{k_2}]-2\gamma_2^2\log[\frac{\gamma_2 t}{k_2}] \\ &\quad +2(\sigma_2(-\gamma_1+\gamma_2+\sigma_0^2+2\gamma_1x_0-2\gamma_2x_0-2\gamma_1(\gamma_1-\gamma_2)\log[\frac{k_1}{\gamma_1 t}]+2\gamma_2(\gamma_2-\gamma_1)\log[\frac{\gamma_2 t}{k_2}])^{1/2} \end{aligned} \tag{A.1}$$

where $t=\frac{1}{2\pi\sigma_0^2}$.

Proof. We begin by calculating Candidate 1's probability of Winning.

$$\begin{aligned} \text{Prob}(x_1\geq 0.5) &= \text{Prob}(\gamma_1c_1-\gamma_2c_2+w\geq 0.5) \\ &= \text{Prob}(-\gamma_1c_1+\gamma_2c_2+0.5\leq w) \\ &= \int_{-\gamma_1c_1+\gamma_2c_2+0.5}^{\infty} f(w|x_0,\sigma_0^2)dw \end{aligned}$$

where $f(\cdot|\cdot)$ represents the normal density function. Of course, $\text{Prob}(x_2\geq 0.5)=1-\text{Prob}(x_1\geq 0.5)$. To find the symmetric pure strategy Nash, we'll solve the first order conditions for both candidates, which sets up the following equations

$$\begin{aligned} \gamma_1 t\exp\left[-\left(\frac{(-\gamma_1c_1+\gamma_2c_2+0.5-x_0)^2}{2\sigma_0^2}\right)\right] &= k_1\exp[c_1] \\ \gamma_2 t\exp\left[-\left(\frac{(-\gamma_1c_1+\gamma_2c_2+0.5-x_0)^2}{2\sigma_0^2}\right)\right] &= k_2\exp[c_2] \end{aligned}$$

And all that remains is to solve the simultaneous equations. □

Proposition 2. *Without loss of generality assume that* $\eta_1\log\left(\frac{1+k_1}{k_1}\right)>\eta_2\left(\frac{1+k_2}{k_2}\right)$. *A pure strategy*

sub-game perfect Nash-Equilibrium to the game is characterized by the following investments,

$$
\begin{aligned}
l_1 = 0, l_2 = 0 \quad & \textit{if} \quad x_c \leq 0.5 - \eta_1 \log\left(\frac{1+k_1}{k_1}\right) \\
l_1 = 0, l_2 = x_c - \left(0.5 - \eta_1 \log\left(\frac{1+k_1}{k_1}\right)\right) \quad & \textit{if} \quad 0.5 - \eta_1 \log\left(\frac{1+k_1}{k_1}\right) < x_c \leq 0.5 + \eta_2 \log\left(\frac{1+k_2}{k_2}\right) - \eta_1 \log\left(\frac{1+k_1}{k_1}\right) \\
l_1 = 0.5 - x_c, l_2 = 0 \quad & \textit{if} \quad 0.5 + \eta_2 \log\left(\frac{1+k_2}{k_2}\right) - \eta_1 \log\left(\frac{1+k_1}{k_1}\right) < x_c \leq 0.5 \\
l_1 = 0.5 + \eta_2 \log\left(\frac{1+k_2}{k_2}\right) - x_c, l_2 = 0 \quad & \textit{if} \quad 0.5 < x_c \leq 0.5 + \eta_2 \log\left(\frac{1+k_2}{k_2}\right) \\
l_1 = 0, l_2 = 0 \quad & \textit{if} \quad 0.5 + \eta_2 \log\left(\frac{1+k_2}{k_2}\right) < x_c
\end{aligned}
$$

Proof. Candidates can always guarantee $-k_1$ and $-k_2$, respectively, without investing. Therefore, the maximum possible equilibrium investment for candidate 1 is $l_1 = \log\left(\frac{1+k_1}{k_1}\right)$, with a total possible effect of $\eta_1 \log\left(\frac{1+k_1}{k_1}\right)$. For candidate 2, the maximum equilibrium investment is $l_2 = \log\left(\frac{1+k_2}{k_2}\right)$, with a total possible effect of $\eta_2 \log\left(\frac{1+k_2}{k_2}\right)$

For the candidate who is behind after election day, the subgame perfect strategy is to invest if a win is possible and to not invest if the legal challenges are too expensive. Therefore, in elections where the election is close, but x_c favors the low resource candidate, $0.5 - \eta_1 \log\left(\frac{1+k_1}{k_1}\right) < x_c \leq 0.5 + \eta_2 \log\left(\frac{1+k_2}{k_2}\right) - \eta_1 \log\left(\frac{1+k_1}{k_1}\right)$ the winning candidate must invest to prevent the higher resource candidate from stealing the election (which is a credible threat).

Consider x_c such that $0.5 + \eta_2 \log\left(\frac{1+k_2}{k_2}\right) - \eta_1 \log\left(\frac{1+k_1}{k_1}\right) < x_c \leq 0.5 + \eta_2 \log\left(\frac{1+k_2}{k_2}\right)$ and first suppose that the resource advantaged candidate is behind on the election day total, $x_c < 0.5$. In this case, the sub-game perfect response for the advantaged candidate is to invest enough to steal the election, $0.5 - x_c + l_2$. Since the resource advantaged candidate is able to outspend the first moving candidate, her equilibrium response is to not invest. If $x_c > 0.5$ then the resource advantaged candidate just needs to invest enough to deter the credible threat from her opponent.

This describes the complete sub-game perfect Nash equilibrium.

□

B Numerical Estimates of Results

Table 4: Table Summary of Figure 5

Method/Bandwidth	Point	SE	N
Governor			
25%	0.15	0.01	24972
10%	0.1	0.01	11664
8%	0.09	0.01	9506
5%	0.08	0.01	6074
2%	0.08	0.02	2412
1%	0.05	0.03	1276
Polynomial	0.06	0.03	11664
Secretary of State			
25%	0.17	0.01	24972
10%	0.09	0.01	11664
8%	0.08	0.01	9506
5%	0.04	0.01	6074
2%	0.04	0.02	2412
1%	0.03	0.03	1276
Polynomial	0.06	0.04	11664
State House			
25%	0.24	0.01	24972
10%	0.15	0.01	11664
8%	0.13	0.01	9506
5%	0.08	0.01	6074
2%	0.07	0.02	2412
1%	0.08	0.03	1276
Polynomial	0.08	0.04	11664
State Senate			
25%	0.23	0.01	24972
10%	0.14	0.01	11664
8%	0.12	0.01	9506
5%	0.07	0.01	6074
2%	0.06	0.02	2412
1%	0.07	0.03	1276
Polynomial	0.08	0.04	11664

This table provides the numerical estimates that comprise the plots in Figure 5.

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