The pressure explicit expression for the fugacity in the mixture is

$$\ln\phi_i = \frac{g_i^{\mathrm{R}}}{RT} = \int_0^P \left[\left(\frac{\partial Nz}{\partial N_i}\right)_{T,P',\{N_{j\neq i}\}} - 1\right]\frac{\mathrm{d}P'}{P'}.$$

To convert to volume-explicit expression, we substitute the definition $Nz \equiv PV/(RT)$ to the above, and get

$$\ln\phi_i = \int_0^P \left[\frac{P'}{RT}\left(\frac{\partial V}{\partial N_i}\right)_{T,P',\{N_{j\neq i}\}} - 1\right]\frac{\mathrm{d}P'}{P'}.$$

Noticing that the volume V depends on both P' and N_i, we convert the first term in the bracket to its volume-explicit expression using the triple product rule,

$$\left(\frac{\partial V}{\partial N_i}\right)_{T,P',\{N_{j\neq i}\}} = -\left(\frac{\partial P'}{\partial N_i}\right)_{T,V,\{N_{j\neq i}\}}\left(\frac{\partial V}{\partial P'}\right)_{T,P',\{N_j\}}.$$

This leads to

$$\begin{aligned}\ln\phi_i &= -\frac{1}{RT}\int_0^P \left(\frac{\partial P'}{\partial N_i}\right)_{T,V,\{N_{j\neq i}\}}\left(\frac{\partial V}{\partial P'}\right)_{T,P',\{N_j\}}\mathrm{d}P' - \int_0^P \frac{\mathrm{d}P'}{P'}\\ &= -\frac{1}{RT}\int_\infty^V \left(\frac{\partial P'}{\partial N_i}\right)_{T,V',\{N_{j\neq i}\}}\mathrm{d}V' - \int_0^P \frac{\mathrm{d}P'}{P'}\\ &= \int_V^\infty \left(\frac{\partial Nz}{\partial N_i}\right)_{T,V',\{N_{j\neq i}\}}\frac{\mathrm{d}V'}{V'} - \int_0^P \frac{\mathrm{d}P'}{P'}\end{aligned}$$

Now the final term can be simplifed using $V\mathrm{d}P = \mathrm{d}(PV) - P\mathrm{d}V$, which leads to

$$\int_0^P \frac{\mathrm{d}P'}{P'} = \int_0^P \left(\frac{\mathrm{d}(P'V)}{P'V} - \frac{P'}{V}\mathrm{d}V\right) = \int_0^P \left(\frac{\mathrm{d}(zN)}{zN} - \frac{1}{V}\mathrm{d}V\right) = \ln(zN/N) - \int_\infty^V \frac{\mathrm{d}V'}{V'}$$

The term $\ln(zN/N)$ is derived from the integration of zN, and the fact $zN \to 1$ as $P' \to 0$. Combining all the pieces, we finally get

$$\ln\phi_i = \int_V^\infty \left[\left(\frac{\partial Nz}{\partial N_i}\right)_{T,V',\{N_{j\neq i}\}} - 1\right]\frac{\mathrm{d}V'}{V'} - \ln(z).$$