1. Reed–Simon II.4. (Hint for part (b): define an “inner product” by the polarization identity and check that it has the desired properties. Additivity, i.e. \( \langle x, y + z \rangle = \langle x, y \rangle + \langle x, z \rangle \), can be proven using the parallelogram law. For \( \langle x, \lambda y \rangle = \lambda \langle x, y \rangle \), first show it for \( \lambda \) rational.)

2. Reed–Simon II.8.


4. Reed–Simon II.15.

5. Construct an example of an inner product space \((V, \langle \cdot, \cdot \rangle)\) and a closed subspace \(M \subset V\) such that \(V \neq M \oplus M^\perp\). (Hint: One construction is as follows. Let \(V \subset \ell^2\) be 
\[
V = \{(a_1, a_2, \ldots) : \exists N \text{ such that } a_n = 0, \forall n \geq N\}.
\]
Choose \(x \in \ell^2\) appropriately and let \(M = \{v \in V : \langle v, x \rangle = 0\}\). Show that \(V\) and \(M\) have the desired properties.)