Spring 2020 Math 256A: Homework Assignment 1 (Due 4/21 during lecture)

In the following, Simon 1.4, Simon 2.2 etc. refers to the problems in the notes posted on canvas.

1. Simon 1.4
2. Simon 2.2
3. Simon 2.5
4. Simon 2.7

5. Simon 3.3 [There is a typo in the book. The solution should instead be \( \varphi = d(c(d - a))^{-1}(1 - bx - \sqrt{(1 - bx)^2 + 2(c - a)y}) \).]

6. Consider initial value problem for the heat equation in one dimension.

\[
\begin{align*}
\partial_t u &= \partial_{xx}^2 u, \\
u(0, x) &= \frac{1}{1 + x^2}.
\end{align*}
\]

Prove that this is no analytic solution in any neighborhood of \((t, x) = (0, 0)\). [Hint: if it has an analytic solution, what would the power series look like? Does it converge?]

This example was given already by Kolvalevskya.

7. Simon 4.4