Autumn 2019 Math 61CM: Homework Assignment 7 (Due 11/11 during lecture)

You are strongly encouraged to work with fellow students on the problems but (1) please write down the names of students whom you discussed with and (2) you must write up your own solutions. Please do not search for solutions on the internet or use online forums.

1. Compute the inverse of

\[ A = \begin{pmatrix}
1 & -1 & -1 & -1 & -1 \\
0 & 1 & -1 & -1 & -1 \\
0 & 0 & 1 & -1 & -1 \\
0 & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}. \]

2. Simon 3.5.1
3. Simon 3.5.2
4. Simon 2.6.1
5. Simon 2.6.2
6. Simon 2.7.1
7. For \( f : U \to \mathbb{R}^m \) (with \( U \) open in \( \mathbb{R}^n \)) differentiable at a point \( a \in U \), consider the affine approximation \( \mathcal{A}(h) \equiv f(a) + Df(a)h \), and the graphs \( G(f), G(A) \subset \mathbb{R}^{n+m} \) defined by

\[ G(f) = \left\{ \left( \begin{array}{c} a + h \\ f(a + h) \end{array} \right) : a + h \in U \right\} \quad \text{and} \quad G(A) = \left\{ \left( \begin{array}{c} a + h \\ A(h) \end{array} \right) : h \in \mathbb{R}^n \right\}. \]

(a) Show that \( G(A) \) is exactly the affine space \( \left( \begin{array}{c} a \\ f(a) \end{array} \right) + C(J) \), where \( C(J) \) is the column space of the \((n + m) \times n\) matrix \( J \) given by \( Jh = \begin{pmatrix} h \\ Df(a)h \end{pmatrix} \) (i.e. the first \( n \) rows of \( J \) are the \( n \times n \) identity matrix, and the last \( m \) rows = \( Df(a) \), the \( m \times n \) derivative matrix \((D_1f(a), \ldots, D_nf(a))\) of \( f \) at \( a \)).

The translate to the origin, i.e. the subspace \( C(J) \), is called the tangent space of \( G(f) \) at \( \left( \begin{array}{c} a \\ f(a) \end{array} \right) \).

(b) Find the tangent space to the graph of \( f \) at the point \( \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \) in case

\[ f(x) = \sin x_1 \cos x_2 \text{ for } x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2. \]