1. Two problems:

(a) (Darboux’s theorem.) Let \( f : \mathbb{R} \to \mathbb{R} \) be differentiable. Suppose that \( x < y \) and \( f'(x) < \lambda < f'(y) \). Show that there exists a \( z \in (x, y) \) so that \( f'(z) = \lambda \).

(b) Let

\[
f(x) = \begin{cases} 
-1 & \text{if } x < 0 \\
0 & \text{if } x = 0 \\
1 & \text{if } x > 0 
\end{cases}
\]

and let \( F(x) = \int_{-1}^{x} f(t) \, dt \). Compute \( F(x) \). Is \( F \) differentiable? Does this contradict the fundamental theorem of calculus?

2. Two problems:

(a) Prove the formula for integration by parts: if \( F \) and \( G \) are \( C^1 \) functions on \([a, b]\) and \( f = F' \), \( g = G' \), then

\[
\int_{a}^{b} F(x)g(x) \, dx = F(b)G(b) - F(a)G(a) - \int_{a}^{b} f(x)G(x) \, dx.
\]

(b) Compute \( a_n = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x^n e^{-x^2/2} \, dx = \lim_{L \to \infty} \left( \frac{1}{\sqrt{2\pi}} \int_{0}^{L} x^n e^{-x^2/2} \, dx \right) + \lim_{L \to \infty} \left( \frac{1}{\sqrt{2\pi}} \int_{-L}^{0} x^n e^{-x^2/2} \, dx \right) \). You may take for granted that \( a_0 = 1 \); we’ll prove this in a future section.

3. Suppose that \( \gamma : [a, b] \to \mathbb{R}^n \) is a curve of finite length. Prove that if \( c \in (a, b) \) then \( \ell(\gamma|[a,c]) + \ell(\gamma|[c,b]) = \ell(\gamma) \).

4. Let

\[
g(x) = \begin{cases} 
0 & \text{if } x \text{ is irrational;} \\
1 & \text{if } x \text{ is rational.} 
\end{cases}
\]

Show that \( f \) is not Riemann integrable.

5. Let

\[
h(x) = \begin{cases} 
0 & \text{if } x \text{ is irrational;} \\
1/q & \text{if } x = p/q \text{ in lowest terms.} 
\end{cases}
\]

Show that \( f \) is Riemann-integrable on \([0, 1]\) and compute \( \int_{0}^{1} f \).

6. Find two functions \( f \) and \( g \) so that \( f \) and \( g \) are Riemann integrable but their composition \( f \circ g \) is not.