FINAL REVIEW CHECKLIST

1. Format of the final exam

The final exam will take three hours. It will consist of two parts: one part consists of 5 short questions, and the other part consists of 3 long questions. In each of the long questions, you will be given an ODE and most¹ parts of the question will be related to some of the themes discussed in Section 3 below.

2. Topics covered in Weeks 8-9

For the topics that are covered in the two midterms, you are referred to the respective midterm review sheets for discussions. We will restrict our discussion in the section on topics covered during Weeks 8 and 9. By far the most important results that we have discussed in these two weeks are

- The stable manifold theorem.
- Poincaré–Bendixson theorem.

On the way, we have developed some results, which are also important in their own right:

- Theorem on stability in first approximation.
- Properties of an \( \omega \)-limit set \( \Omega \): \( x \in \Omega \implies \varphi_t(x) \in \Omega \), \( \Omega \) bounded \( \implies \Omega \) connected.
- An \( \omega \)-limit set intersects with a transversal line segment at no more than one point.

Here are some things that we should be able to do using the techniques and methods that we have discussed:

- Determine whether a hyperbolic equilibrium point is stable (and/or asymptotically stable).
- Find the stable manifold near a hyperbolic equilibrium point (when applicable).
- Determine the \( \omega \)-limit set of a given solution to a given ODE.
- Make arguments about planar ODEs using transversal lines.
- Use the Poincaré–Bendixson theorem to prove the existence of equilibrium points and/or periodic solutions for planar ODEs.

Finally, you are encouraged to review all the homework problems, problems on the midterm, as well as problems that are marked as Exercise in the notes.

3. Summary of the themes of the course

The following are some of techniques we have discussed for studying a given ODE:

- Determine whether a unique local solution exists (for instance by checking if the Picard–Lindelöf theorem applies).
- (If a unique local solution exists,) determine the maximal interval of existence (for instance by using the extension theorem).
- For a linear ODE, solve the ODE explicitly.
- When separation of variables applies, solve the ODE explicitly.
- Determine whether the ODE is a Hamiltonian system, and if so, find a Hamiltonian.
- Find first integral of motions (or at least check that a given function is a first integral of motion).
- Find all equilibrium points.
- For each equilibrium point, determine the corresponding linearized system.
- Determine the stability/instability of all hyperbolic equilibrium points.
- Find the stable manifold corresponding to a hyperbolic equilibrium point (when applicable).
- For equilibrium points which are not hyperbolic, prove that they are stable using the Lyapunov theorem (when applicable).
- Sketch a phase portrait.
- Given a solution, find its \( \omega \)-limit set.

¹but not necessarily all!
• Determine whether the ODE admits periodic solutions.