Spring 2017 Math 63CM: Remarks on Homework Assignment 6

• On problem 4, the result of course follows directly from what is proven in class. You are not supposed to use that for this problem. Recall that the proof in class relies on the fact that $\mathbb{R}^n$ and $\mathbb{R}^m$ are homeomorphic if and only if $m = n$, i.e., there exists a continuous map $f : \mathbb{R}^n \to \mathbb{R}^m$ with continuous inverse if and only if $m = n$. We did not prove this fact in class. The point of this problem is that in the particular case in question, we do not need to use this unproven fact.

• Problem 5 may seem a little bit unmotivated. The point is that the following theorem holds (and will be proven next week):

**Theorem 1.** Consider the ODE

$$u'(t) = Au(t) + G(u(t)),$$

**where**

- $A$ is an $(n \times n)$ real matrix such that all eigenvalues satisfy $\text{Re}(\lambda_i) < 0$, and
- $G : \mathbb{R}^n \to \mathbb{R}^n$ is a $C^1$ function satisfying $\lim_{\|x\| \to 0} \frac{\|G(x)\|}{\|x\|} = 0$.

Then 0 is an asymptotically stable equilibrium solution.

The main point of this problem is to show that if we replace $\text{Re}(\lambda_i) < 0$ by $\text{Re}(\lambda_i) \leq 0$, then the conclusion of the theorem no longer holds.