MIDTERM 2 REVIEW CHECKLIST

The midterm will only concern material regarding linear algebra and linear ODEs that we discussed during weeks 4-7. I have heard that rumors have been spread that equivalence of linear ODEs will not be covered - this is not the case. Here is a list of things that we have discussed:

Here is a list of main results:

• On linear algebra, we proved
  – exp is well-defined; \( e^{A+B} = e^A e^B \) if \( AB = BA \); \( e^A = \lim_{n \to \infty} (1 + \frac{A}{n})^n \).
  – Cayley–Hamilton theorem.
  – Decomposition of \( \mathbb{C}^n \) into generalized eigenspaces for any given complex matrix.
  – Any matrix can be decomposed uniquely into its diagonalizable and nilpotent part, and the two parts commute.
  – Every nilpotent matrix is equivalent to one in a block diagonal form where each block is one of the following:

\[
J_m := \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 & 1 \\
0 & \cdots & \cdots & 0 & 0
\end{bmatrix}
\]

  – Every complex matrix is equivalent to a Jordan canonical form.

• On linear ODEs, we proved
  – \( u'(t) = Au(t), \ u(0) = u_0 \) can be solved by \( u(t) = e^{At}u_0 \).
  – General form of solutions to linear ODEs.
  – For linear real ODEs \( u'(t) = Au(t) \) such that \( A \) has no eigenvalues on the imaginary axis, \( \mathbb{R}^n \) decomposed into stable and unstable subspaces.
  – Necessary and sufficient conditions for linear, topological and smooth equivalence of linear real ODEs.

Here are some things that we should be able to do using the techniques and methods that we have discussed:

• Compute operator norms of matrices and/or proving properties of operator norms.
• Given a (complex) matrix \( A \),
  – Determine whether it is diagonalizable.
  – Find its eigenvalues, eigenvectors and generalized eigenvectors.
  – Find its Jordan canonical form (and find a basis such that it is in Jordan canonical form after a change of basis).
  – Find \( B \) and \( N \) such that \( A = B + N \), \( B \) diagonalizable, \( N^n = 0 \) and \( BN = NB \).
  – Compute \( e^{At} \).
• Solving linear ODEs using exponentials of matrices.
• Determine whether 0 is a stable (and/or asymptotically stable) equilibrium to a linear ODE.
• Determine whether two given real linear ODES are lineary/topologically/smoothly equivalent.

Finally, you are encouraged to review all the homework problems, as well as problems that are marked as Exercise in the notes.