FINAL REVIEW CHECKLIST

1. Format of the final exam

The final exam will take two hours. It will consist of two parts: one part consists of 3 shorter questions, and the other part consists of 2 longer questions. In each of the longer questions, you will be given an ODE and most parts of the question will be related to some of the themes discussed in Section 3 below.

The final exam will mostly (but not exclusively!) concern topics not covered by the midterms.

2. Topics covered in the last part of the term

For the topics that are covered in the two midterms, you are referred to the respective midterm review sheets for discussions. We will restrict our discussion in the section on topics that are not covered in the two midterms.

The most important results that we have discussed are

- Stability theorem when all eigenvalues of the linear part have negative real parts (Theorem 4.2).
- Lyapunov theorems (Theorems 4.11 and 4.12).
- The stable manifold theorem (Theorem 4.10, parts (ii) and (iii)).
- Poincaré–Bendixson theorem.

On the way, we have discussed the following, which are also important in their own right:

- Positively invariant sets (Theorem 5.2, Corollary 5.3).
- Properties of an $\omega$-limit set $\Omega$ (Propositions 5.6, 5.7, 5.8).
- Global Lyapunov theorem (Proposition 5.10).
- An $\omega$-limit set intersects with a transversal line segment at no more than one point.

Here are some things that we should be able to do using the techniques and methods that we have discussed:

- Determine whether an equilibrium point is hyperbolic.
- Determine whether a hyperbolic equilibrium point is stable (and/or asymptotically stable).
- Find the stable manifold near a hyperbolic equilibrium point (when applicable).
- Use the Lyapunov theorems to prove stability of equilibrium points.
- Apply the Lyapunov theorems in the case of the Newton’s equation, or more generally Hamiltonian systems.
- Determine whether a given set is positively (or negatively) invariant.
- Determine the $\omega$-limit set of a given solution to a given ODE.
- Make arguments about planar ODEs using transversal lines.
- Use the Poincaré–Bendixson theorem to prove the existence of equilibrium points and/or periodic solutions for planar ODEs.

Finally, you are encouraged to review all the homework problems and problems on the midterm.

3. Summary of the themes of the course

The following are some of techniques we have discussed for studying a given ODE:

- Determine whether a unique local solution exists (for instance by checking if the Picard–Lindelöf theorem applies).
- (If a unique local solution exists,) determine the maximal interval of existence (for instance by using the extension theorem).
- For a linear ODE, solve the ODE explicitly.
- When separation of variables applies, solve the ODE explicitly.
- Determine whether the ODE is a Newton's equation, and if so, find the potential. (Also, more generally, do this for a Hamiltonian system.)
• Find all equilibrium points.
• For each equilibrium point, determine the corresponding linearized system.
• Determine the stability/instability of all hyperbolic equilibrium points.
• Find the stable manifold corresponding to a hyperbolic equilibrium point (when applicable).
• For equilibrium points which are not hyperbolic, prove that they are stable (or asymptotically stable) using the Lyapunov theorem (when applicable).
• Determine whether a set is positively invariant.
• Sketch a phase portrait.
• Given a solution, find its $\omega$-limit set.
• Determine whether the ODE admits periodic solutions.