MIDTERM 2 REVIEW CHECKLIST

The midterm will only concern material regarding linear algebra and linear ODEs that we discussed. Here is a list of things that we have discussed:

Here is a list of main results:

• On linear algebra, we proved
  – exp is well-defined: $e^{A+B} = e^A e^B$ if $AB = BA$.
  – Any matrix is similar to an upper triangular matrix.
  – Diagonalizable matrices are dense.
  – Cayley–Hamilton theorem.
  – Decomposition of $C^n$ into generalized eigenspaces for any given complex matrix.
  – Any matrix can be decomposed uniquely into its diagonalizable and nilpotent part, and the two parts commute.
  – Every nilpotent matrix is equivalent to one in a block diagonal form where each block is one of the following:

$$J_m := \begin{bmatrix}
0 & 1 & 0 & \ldots & 0 \\
0 & 0 & 1 & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & 0 \\
\vdots & \ddots & \ddots & 0 & 1 \\
0 & \ldots & \ldots & 0 & 0
\end{bmatrix}$$

  – Every complex matrix is equivalent to a Jordan canonical form.

• On linear ODEs, we proved
  – $x'(t) = Ax(t)$, $x(0) = x_0$ can be solved by $x(t) = e^{At}x_0$.
  – General form of solutions to linear ODEs.
  – For linear real ODEs $u'(t) = Au(t)$ such that $A$ has no eigenvalues on the imaginary axis, $\mathbb{R}^n$ decomposed into stable and unstable subspaces.

Here are some things that we should be able to do using the techniques and methods that we have discussed:

• Compute operator norms of matrices and/or proving properties of operator norms.

• Given a (complex) matrix $A$,
  – Determine whether it is diagonalizable.
  – Find its eigenvalues, eigenvectors and generalized eigenvectors.
  – Find its Jordan canonical form (and find a basis such that it is in Jordan canonical form after a change of basis).
  – Find $L$ and $N$ such that $A = L + N$, $L$ diagonalizable, $N'' = 0$ and $LN = NL$.
  – Compute $e^{At}$.

• Given a matrix $A$, or properties about its eigenvalues/eigenfunctions, sketch phase portraits for the corresponding linear ODE $x'(t) = Ax(t)$.

• Solving linear ODEs using exponentials of matrices.

• Determine whether 0 is a stable (and/or asymptotically stable) equilibrium to a linear ODE.

Finally, you are encouraged to review all the homework problems, as well as problems that are marked as Exercise in the notes.