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FISCAL AND MONETARY STABILIZATION POLICIES IN A MODEL OF CYCLICAL GROWTH

By

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PREFACE

The purpose of this paper is to investigate the impacts of fiscal and monetary stabilization policies in a model of endogenous cyclical growth. The first three chapters are devoted to introducing the problem of cyclical growth and setting up an analytical framework. If the reader is already familiar with these problems and has a knowledge of the Phillips' model of cyclical growth, he might skim these chapters and proceed on to the rest of the paper. Chapters four and five contain the derivation of the various policies to be used, the application of these policies to the model, and finally the investigation of the impacts on growth and stability.

I am particularly grateful to Mr. Richard Cornwall for helpful reading of first drafts and for suggestions as to what should be included in the final draft, and to Mr. E. Philip Howrey for suggesting the topic and indicating how I might proceed in the analysis. I am also grateful to Mr. I. Thomas Cundiff of the Princeton University Computer Center for valuable assistance in using his programming package for simulation.

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CHAPTER 1

INTRODUCTION

In recent years, various articles and papers have been written on the effects of government fiscal and monetary stabilization policies. Both theoretical and empirical, these studies have been mainly concerned with the effects of such policies on short run fluctuation in national income. For this reason, they have been limited to economic models which are only valid in a short period analysis. That is, they are concerned only with oscillations about a stationary level of income. These studies have demonstrated the various effects of contracyclical policies on the short run oscillations of the model.

Because these analyses have been limited to short run models, they have not rigorously investigated the possible effects of government regulation on economic growth and fluctuations in the longer run. The models assume fluctuations about a constant level of income, and cannot be extended over a longer period where growth in the economy must be considered. Thus there has been a gap left in the theoretical examination of government regulation. This paper attempts to fill that gap by applying fiscal and monetary policies to a model which endogenously produces both oscillations and growth. In this way, it will examine the influences of government regulation on economic growth as well as any additional influences on fluctuations as related to economic growth.

Before proceeding, it will be helpful to briefly consider the plan of study. Chapter 2 will examine the problem of growth and fluctuation. We will see why most conventional models will produce either oscillations or growth but not both together. Some of the attempts which have been made to unite both factors in a single model will then be mentioned. Starting with some earlier attempts, such as superimposing a growth trend exogenously into a short term analysis; we will proceed to some models which include both growth and fluctuations endogenously.

Chapter 3 will develop at length one of these models of cyclical growth. The model to be used in this study was developed by A. W. Phillips.[1] We will set up the model and demonstrate how it operates in an unregulated economy.

Chapter 4 will examine possible mathematical representation of fiscal and monetary policy. Here again we will lean heavily on an earlier work by Phillips[2] where such representations of fiscal policy were first applied to economic models. These policies will need some modification for application to the model of cyclical growth. In addition a similar type of monetary regulation will be developed.

In Chapter 5, we will then apply these policies to the model of cyclical growth and examine the effects on fluctuations in the levels of income and growth. Some of this analysis can be handled analytically with mathematical techniques for investigating linear second order

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[1] A. W. Phillips, "A Simple Model of Employment, Money, and Prices in a Growing Economy," Economica (November, 1961).

[2] A. W. Phillips, "Stabilisation Policy in a Closed Economy," Economic Journal (June, 1954) and "Stabilisation Policy and the Time Form of Lagged Responses," Economic Journal (June, 1957).

differential equations. However, as time lags and more complex policies are considered, the equations of income are of third order and with growth fifth order. In such cases the techniques of mathematical analysis become too cumbersome. Therefore, two simulation techniques are used. In the first, the equations are simulated on a Model TR-20 electro-analog computer where electric circuits are set up to represent the various integrations. The second method of simulation is performed on the IBM Model 7094 digital computer where the paths of the variables are projected in time with various initial conditions. Numerical values are assigned to the parameters of the model for this simulation.

Because operation of a small electro-analog computer is much less expensive than a digital computer, the former was used mainly as a preliminary investigating tool. After general behavior patterns were observed, the digital simulation was then used for a more precise analysis. Since the techniques are of some interest in themselves they are outlined in the appendices.

Because of the large number of variables and parameters used in this paper, uniform notation is difficult. Where possible the notation is like that of Phillips, but there are some necessary differences. These will have to be remembered when referring back to the original work.

CHAPTER 2

THE PROBLEM OF GROWTH AND FLUCTUATIONS

It is a well known fact that long term growth and business cycles exist simultaneously in a capitalist economy. The problem of cyclical growth is concerned with explaining in dynamic economic models how both growth and fluctuations exist together. There have been a number of theoretical and empirical studies which attempt to explain each of these behaviors separately. Business cycle theories of the "multiplier-accelerator" type have shown how the level of income can fluctuate about a stationary level. The Samuelson[1] model is representative of this type of theory. On the other hand growth theories have mainly been concerned with steady growth and have said little about how the level of income and the growth rate can fluctuate. The Harrod-Domar[2] models are representative of this type. Theories which explain how growth and cycles can occur together have been infrequent and until recently not satisfactory because they have resorted to exogenous trends.

This chapter will deal with some theories used to explain this dynamic problem. Section A begins by demonstrating how a simple model will give either growth or fluctuation but not both. Section B will then summarize some of the various attempts at bridging the two.

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[1] P. A. Samuelson, "Interactions Between the Multiplier Analysis and the Principle of Acceleration," Review of Economics and Statistics (May, 1939).

[2] R. F. Harrod, Towards a Dynamic Economics (London: Macmillan and Co.,1948) and Evsey D. Domar, "Expansion and Unemployment," American Economic Review (March, 1947).

A. The Multiplier-Accelerator Model: Growth or Fluctuations But Not Both

In order to demonstrate the inability of a simple model to explain both problems, consider the following multiplier-accelerator model with three equations and three variables.[3] We will assume that there are two lags:[4] an investment lag with time constant of unity, and an output lag with time constant T.

The demand equation can be written as

$$Z = (1-s)Y + I \tag{2.1}$$

where Z is total demand, Y is output or income, I is net investment, and s is the constant savings ratio. Consumption is equal to (1-s)Y.

The investment function is in the form of an accelerator with an exponential lag:

$$I = \frac{1}{D+1}\, aDY \quad , \tag{2.2}$$

where a is a constant accelerator coefficient, and D = d/dt is the first derivative with respect to time. Thus net investment is lagged on the rate of change in output. Finally, we assume that output is lagged on demand:

$$Y = \frac{1}{TD+1}\, Z \; , \tag{2.3}$$

where T is the time constant of the lag. By substituting equations (2.1) and (2.2) into (2.3) the model can be reduced to a second order linear differential equation:

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[3] R. G. D. Allen, Macroeconomic Theory (London: Macmillan and Co., 1967), p.328.

[4] For an explanation of these lags see Chapter 3 of this work.

$$Y = \frac{1}{TD+1} [(1-s)Y + \frac{1}{D+1} aDY] \quad . \tag{2.4}$$

Multiplying through by (D+1)(TD+1) equation (2-4) reduces to recognizable differential form:

$$TD^2Y + (T+s-a)DY + sY = 0 \quad . \tag{2.5}$$

Depending on the values of the parameters this equation will generate either a steady path of Y, or oscillation of Y about Y = 0. To see this we solve the equation for Y. The solution will be of the form

$$Y = A_1e^{q_1t} + A_2e^{q_2t}$$

where q_1 and q_2 are the roots of the characteristic equation:

$$q^2 + [(T+s-a)/T]q + s/T = 0 \quad .$$

When q_1 and q_2 are real, then it can be shown that they are either both positive or both negative. If they are both positive then Y increases steadily over time; if they are both negative then Y decreases steadily over time. Thus Y moves in a steady path.

When q_1 and q_2 are complex, then the solution will be of the form

$$Y = A_3e^{\alpha t} \cos \omega t$$

where α and ω are the real and imaginary parts respectively of the complex roots. Thus Y will oscillate over time with frequency ω and damping factor α . This is the case of fluctuation about the stationary level Y = 0.

Because complex roots always come in pairs it is impossible to have a situation where Y oscillates around an increasing level. Therefore, the model can produce either a steady path (growth) or oscillations, but not both at the same time.

This example reduces to the Harrod-Domar type model when T = 0. In that case the solution is of the form $Y = Y_0 e^{s/(a-s)t}$, where s/(a-s) is the warranted rate of growth. Such a solution cannot produce oscillations in Y. It provides no explanation of how Y returns to this path if it is displaced. In fact with the Harrod-Domar model any deviation from the warranted rate of growth will result in a larger increasing deviation.

When $T \neq 0$ and the parameters are such that the roots of the characteristic equation are complex, the model oscillates around a stationary level.[5]

We will now examine some models which attempt to explain the simultaneous occurence of fluctuations and growth.

B. Models Displaying Both Growth and Fluctuations

Economists have been concerned with this problem for a long time and many have proposed models which produce both growth and oscillation.

> Since the very beginnings of speculation on the problem of the Trade Cycle, the cyclical swings of the economic system have been regarded as being inherently connected with the essentially "dynamic" process of economic growth.[6]

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[5] Samuelson, op. cit. arrives at this type of conclusion using discrete variables and difference equations.

[6] Nicholas Kaldor, "The Relation of Economic Growth and Cyclical Fluctuations," Economic Journal (March, 1954), p. 53.

Joseph Schumpeter[7] was one of the first economists to produce a theory which would explain the connection. In his model the fluctuations in economic activity are an integral part of the process of economic growth in a capitalist economy. The theory involves the nature of technical innovation. In Schumpeter's model innovations come in spurts. When one entrepreneur adopts a major innovation he is quickly immitated by a great number of other entrepreneurs. This results in an investment boom and a surge in economic growth. Once the innovation is completely utilized, the investment boom will stop and the economy begins a depressed period. The depression is ended when another innovation is discovered and adopted. The concept of technical progress is central to economic cycles, and for this reason fluctuations in economic activity are a necessary by-product of growth.

However Schumpeter's theory is difficult to represent in a mathematically simple economic model.[8] Therefore several economists have attempted to represent both cycles and growth by starting with a model of oscillations (multiplier-accelerator) and superimposing an exogenous growth trend. Finally in recent years attempts have been made to explain both facts in a linear model by including both product and money markets. We will briefly describe seven models which are representative of the different techniques of generating growth and oscillations in a single model. It is impossible to do justice to any one of the following models in this short chapter, but it is hoped that a brief description will serve as an introduction to the problems of developing a model of cyclical growth such as the one used in this paper.

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[7] J. A. Shumpeter, Business Cycles (New York: McGraw Hill, 1939).

[8] R. M. Goodwin, "A Model of Cyclical Growth," in The Business Cycles in the Postwar World, proceedings of International Economic Association Conference, edited by E. Lundberg (London: Macmillan and Co., 1955), p. 37.

1. Superimposition of exogenous trends

In order to generate cyclical growth J. R. Hicks[9] employs a simple multiplier-accelerator model similar to that mentioned above and adds an exogenous trend to the investment accelerator. Thus equation (2.2) becomes:

$$I = \frac{1}{D+1} [aDY + Ne^{\lambda t}] \tag{2.2.1}$$

where N is a constant and λ is the growth rate of the exogenous investment.[10] Thus the differential equation of the model reduces to:

$$TD^2Y + (T + s - a)DY + sY = Ne^{\lambda t} \tag{2.5.1}$$

The homogeneous solution to this equation is just that of equation (2-5). However, now we will also have a particular solution for Y which is given by

$$Y = He^{\lambda t} \quad \text{where} \quad H = N/[T^2\lambda + (T+s-a)\lambda + s]$$

For the general solution of equation (2.5.1) this particular solution is added to the homogeneous solution. Suppose that the homogeneous solution has complex roots so that Y oscillates; then in the general solution Y will oscillate around a steadily increasing growth rate $He^{\lambda t}$. Hicks calls the steady path the equilibrium rate of growth around which the

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[9] J. R. Hicks, A Contribution to the Theory of the Trade Cycle (Oxford: The Oxford University Press, 1950) and "Mr. Harrod's Dynamic Theory," Economica (May, 1949).

[10] Hicks uses a discrete analysis so that his trend term is of the form $N(1+g)^t$.

actual rate of growth oscillates. Thus the model has generated both growth and oscillations.

Hicks then goes on to say that the increasing exogenous investment $Ne^{\lambda t}$ also determines a full employment ceiling and a lower equilibrium floor both of which are also steadily increasing. In his complete model, the floor and ceiling constrain the fluctuation of Y about the equilibrium path. This can be seen in figure (2.1) where the scale is logarithmic on the vertical axis Y.

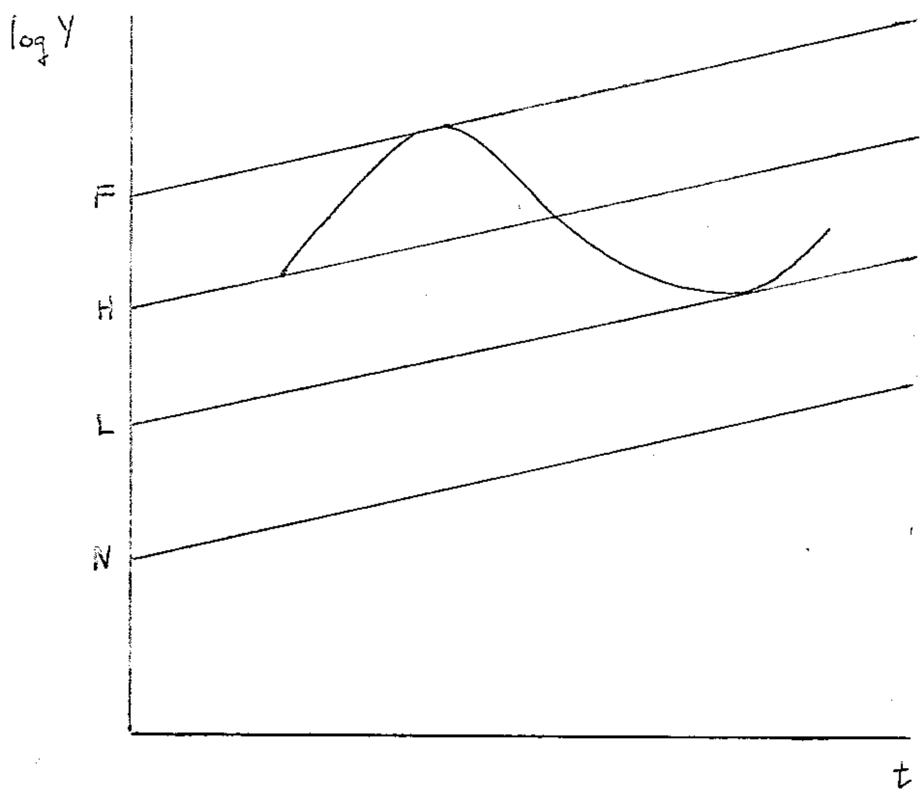

Figure 2.1. Hicks' Growth Trend with Floor and Ceiling.

F is the full employment ceiling and L is the lower equilibrium. The path of Y "must turn down when it gets to the top, and when it approaches the bottom it must turn up."[11]

The model is successful in producing cyclical growth. However, it leaves the reason for the growth trend unexplained in terms of the model. The constantly increasing term added to the investment function is exogenous. The model does not explain why this term is steadily increasing. For a more satisfactory model the growth trend should be explained endogenously.

Goodwin[12] has developed a model similar to that of Hicks. By introducing nonlinear elements he demonstrates that cycles cannot exist without economic growth. In the Hicks' model the existence of fluctuations is independent of growth, although there is an influence on amplitude and frequency. The Goodwin model is therefore closer to the Shumpeterian hypothesis that cycles are a by-product of economic growth. However, Goodwin's theory also leaves the source of economic growth unexplained in terms of his model. The trend in growth is explained exogenously by population growth and technical progress.

A third model which uses a rather different technique of introducing trend is that of Minsky.[13] Using the multiplier-accelerator model Minsky introduces a growth trend by intermittently imposing new initial conditions.

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[11]Hicks, op. cit. (1949), p. 331.

[12]Goodwin, op. cit. and also "The Problem of Trend and Cycle," Yorkshire Bulletin of Economic and Social Research (August, 1953).

[13]H. P. Minsky, "A Linear Model of Cyclical Growth," Review of Economics and Statistics (May, 1959).

The initial conditions are introduced in such a way that the supply of income increases at a given rate. This provides a ceiling. The floor is then determined by the rate of capital consumption.

Minsky uses the multiplier-acceleration model in finite terms, so that the equation of the model is a difference equation rather than a differential equation. The solution of the difference equation (analogous to equation (2.5) for the continuous case) is:

$$Y = A_1 q_1^t + A_2 q_2^t + K_0 \qquad (2.6)$$

where q_1 and q_2 are the roots of the characteristic equation. The coefficients A_1 and A_2 are determined by the initial conditions and K_0 is interpreted as the income Y at which consumption equals income. A_1 and A_2 can be altered by changing the initial conditions, so that the model can display cyclical growth. The values of A_1 and A_2 are changed when income deviates too far from equilibrium; that is, when Y hits a floor or a ceiling. The model is successful in generating a cyclical growth which is similar to certain occurrences in U.S. economic history; in particular the great postwar boom, where wartime shortages made ceiling income much greater than actual income. In the Minsky model this allows actual income to rise for a considerable period before turning down again.

However, there is still the same disadvantage mentioned in the Hicks and Goodwin models: the growth trend is supplied exogenously and is not explained by features of the model.

2. Endogenous models of cyclical growth

A few economic models have been developed which can generate both fluctuation and growth without the superimposition of exogenous trends.

We will briefly mention four models which demonstrate some of the methods and complexities which must be employed to produce endogenous cyclical growth.

Smithies[14] has developed a model of economic fluctuations and growth which differentiates between actual income and a full capacity income. Both of these can fluctuate and grow at different rates. The essential feature of the Smithies model is a ratchet effect which means that the highest level of income yet achieved influences the level of income at the present time. This ratchet effect keeps the economy from falling greatly below past peak levels of income and permits growth and fluctuation to occur at the same time, without the addition of outside trends. (Smithies does include trend in part of his analysis but this is to include changes that occur independently of the economy, i.e. population growth. The model will produce cyclical growth without these trends.)

The ratchet effect enters the model in the consumption function and consequently in the savings function. Because the equality of savings and investment is assumed without lag, the level of investment is also determined by the ratchet effect. The consumption function is defined in finite term as:

$$C = (1-a_1)Y_A + a_2\bar{Y}_A$$

where Y_A is actual income and $\bar{Y}_A$ is the highest level of income yet attained. When Y_A is less than $\bar{Y}_A$ the ratchet keeps consumption from falling as far

[14] Arthur Smithies, "Economic Fluctuations and Growth," Econometrica (January 1957).

as it otherwise would. The rationale behind the ratchet is that once consumers have attained a certain level of income it is difficult for them to readjust to a lower one. Therefore, their level of consumption does not decrease in proportion to Y_A.

The level of full capacity income Y_F is proportional to net investment and to the difference between the level of full capacity income and the level of actual income in the last period. If actual income was greater than full capacity income in the previous period then there will be more investment in the present period.

Because of the nonlinearity of the ratchet assumption, the model requires complicated solutions. One set of solutions apply when the ratchet is not operating ($Y_A > \bar{Y}_A$) and the other set when the ratchet is operating ($Y_A < \bar{Y}_A$). The complexity of the solutions makes the model difficult to use for more involved analysis such as government stabilization policies. Smithies mentions stabilization, but the investigation is not rigorous.

Another model which attempts to explain endogenous cyclical growth is that of Kurehara.[15] The distinguishing feature of this model is a nonlinear investment function. This investment function is explained endogenously and is "the common maker of cycles and growth."[16] This investment function can be written as:

$$I_t = f(Y_{t-1}) - bK_{t-1} \qquad (2.7)$$

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[15] K. K. Kurihara, "An Endogenous Model of Cyclical Growth," Oxford Economic Papers (October, 1960).

[16] Ibid., p. 243.

where Y is income and K is the capital stock. Using the multiplier analysis we can write:

$$Y = \frac{1}{s} I \quad \text{or} \quad \Delta Y = \frac{1}{s} \Delta I \ .$$

We assume a constant output-capital ratio so that a full capacity level of income could be written as:

$$Y_F = vK \quad \text{or} \quad \Delta Y_F = v\Delta K$$

where v is the output-capital ratio. To see how this model generates cyclical growth first assume that the economy is in the state of "balanced" growth where $\Delta Y = \Delta Y_F$. Then:

$$\Delta I = sv\Delta K$$

where sv can be interpreted as the Harrod-Domar warranted rate of growth. But this situation is unlikely. We must examine what happens when equilibrium does not hold. If $\Delta I > sv\Delta K$ then ΔI will decrease because the impact of capital accumulation is greater than the impact of income expansion in equation (2.7). The cycle turns down. If $\Delta I < sv\Delta K$ then ΔI will increase because the impact of capital accumulation is less than the impact of income expansion. The cycle turns up. Thus, this model can display oscillation about a balanced growth path.

One disadvantage of this model is that the exact nature of the nonlinear investment function is not stated. For extended analysis an explicit function must be chosen and this requires a justification for this particular type of investment relation. Despite this disadvantage the model does illustrate one way to endogenously generate cyclical growth.

Another way to produce both cycles and growth is to include price levels and a money market in a conventional analysis of product markets. This was done by Phillips[17] and with further additions by Bergstrom.[18] Since the Phillips model is explained in detail in Chapter 3 we need only state that it has the advantage of using linear relations which do not require complicated mathematical treatment. By working in terms of the ratios of the variables, the model reduces to linear differential equations which display cyclical growth.

The Bergstrom model is an extension of the Phillips model. The significant difference is that the Bergstrom model determines separate paths for full employment and full capacity output. The full employment level is restrained by the labor force growth. However, this extension greatly increases the complexity of the model and the differential equations are nonlinear.

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[17] A. W. Phillips, op. cit. (1961).

[18] A. R. Bergstrom, "A Model of Technical Progress, the Production Function, and Cyclical Growth," Economica (November, 1962).

CHAPTER 3

A MODEL OF CYCLICAL GROWTH

As mentioned in Chapter 2 the model of cyclical growth which we will use in this analysis closely resembles that of Phillips. By including both product and money markets in the system we are able to generate cyclical growth. The model is demand oriented and continuous. It reduces to a system of linear differential equations whose variables will be the ratios of the real variables of the model. If we consider the case of no government regulation the differential equations are of second order.

An important feature of the model is the concept of full-capacity output (Phillips calls this "normal capacity output"). Full capacity output is to be distinguished from actual output. Actual output can easily be defined as the national product or income of the economy. (There is no lag assumed between output and income.)

Full capacity output is defined as "the output that would be obtained if firms were operating with that percentage utilisation of available physical resources which they would consider to be the most satisfactory average percentage utilisation over a period of years."[1] This definition is similar to that of Smithies "the output that the existing stock of equipment is intended to produce under normal working

[1] Phillips, op. cit. (1961), p. 360.

conditions."[2] Notice that with these definitions actual output can be greater or less than full capacity output. In fact we would expect the actual output to fluctuate about the full capacity level.

We will find it useful to consider the ratio of actual output to full capacity output in this model, that is $Y_A/Y_F = X$. The relationship between actual and full capacity output is the value of the variable X. A value of X near unity would indicate full capacity production. We would expect values less than unity to represent depressed levels of output, and values greater than unity tending toward over-utilization or inflation. The value X-1 could indicate a proportional "gap".

The model does not explicitly distinguish between full capacity output and full employment output. However, it is likely that the rate of employment will fluctuate in much the same manner as the full capacity ratio. We would expect a low level of employment when actual output is well below full capacity output, and this level should increase as the economy approaches full capacity. As the economy pushes past full capacity the level of employment should increase still farther until bounded by structural restraints and resulting inflation. Studies have shown that the fluctuations between the two are highly correlated, with the fluctuation in the ratio of actual to full capacity about five times greater than fluctuation in employment rate.[3]

In a recent paper, Tinsley[4] mentions some of the possible empirical measures of full capacity and full employment outputs. The concepts can

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[2] Smithies, op. cit., p. 8.

[3] Phillips, op. cit., p. 361.

[4] Peter A. Tinsley, Potential GNP and Discretionary Fiscal Policy, Ph.D. Dissertation: Princeton University (1965).

both be combined in the frequently-used term "Potential GNP". There are various definitions and measures of this term, ranging from Okun's law to engineers' surveys. The concept is not exact but is necessary to any thorough examination of growth.

Now let us go on to model. There are six basic equations. The first is a simple production function which defines the full capacity output as a constant ratio to the capital stock:

$$Y_F = vK \tag{3.1}$$

where K is the capital stock and v is the constant output-capital ratio which will not change in the model.

Secondly, there is a simple consumption function where C is assumed to be a constant proportion (1-s) of Y_A

$$C = (1-s)Y_A$$

s is of course the familiar savings ratio. Substituting this consumption function into an actual income equation (remember we now assume no government) we get the following relation:

$$Y_A = (1-s)Y_A + I \tag{3.2}$$

Thirdly, the capital stock is defined as accumulated net investments over time:

$$K = \int_0^t I dt \qquad \text{or equivalently} \quad I = DK \tag{3.3}$$

where $D = d/dt$ is the first derivative with respect to time.

In order to complete the definition of the product market (as distinguished from the money market) we need an investment function. This investment function will define the disequilibrium condition of the model. The investment function can be written:

$$I = \frac{1}{T_I D+1} K \left[E + \gamma(X-1) + \rho(c-r) \right] \quad (3.4)$$

Thus, net investment is dependent on the capital stock and three other relationships. Let us examine each in turn. E represents business expectation, i.e. the expected rate of future output. That is, entrepreneurs will invest according to their expectation of future business conditions. Phillips suggests that the expected rate of future output depends mainly on past rates of change, and goes on to define such a relationship.[5] However, throughout the bulk of his analysis he assumes that E is a constant. In fact, the possibility of his suggested relation is only mentioned briefly toward the end. Since the assumption that E is constant greatly facilitates the analysis of the model (which is almost mandatory when government expenditure is considered) we will make that assumption.

The second term in the brackets represents the reactions of businessmen to greater than or less than full capacity operation. For example, if entrepreneurs observe that they have been operating at less than full capacity, they will adjust their investment plans so as to reduce this unused capacity. Conversely, if they observe that they have been

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[5] Phillips, op. cit., p. 361.

producing at over capacity they will adjust their investment plans in order to create greater capacity. This adjustment in investment plans does not have its full effect immediately. Instead the entrepreneurs plan to increase or decrease investment by a certain proportion γ each year. At the end of $1/\gamma$ years they will have completely made the adjustment. Thus we can call γ the <u>speed</u> of the adjustment process and $1/\gamma$ the <u>fixed time interval</u> over which capacity is planned to be increased or reduced to the desired level.

The third term represents the effect of interest rates on investment. The parameter c is defined as the marginal efficiency of capital; that is, the rate of return, or yield, on capital investment. c is a positive constant of the model. The market rate of interest r is a variable to be defined below. When the rate of interest is greater than the marginal efficiency of capital, entrepreneurs will invest less according to the proportion ρ of the difference. When the marginal efficiency is higher entrepreneurs will find it profitable to increase their capital investments.

Finally, actual investment is lagged on the amount indicated by the three terms of the investment decision. The assumed lag is of exponential form with a time constant T_I and speed of response $1/T_I$.

To understand the mechanism of this lag it will be helpful to examine its derivation. It is a special form of the distributed lag:

$$I(t) = \int_0^\infty w(\tau)\bar{I}(t-\tau)d\tau \qquad \text{where} \quad \int_0^\infty w(\tau)d\tau = 1$$

That is I depends on the weighted sum of the past values $\bar{I}$. $w(\tau)$ is a weighting function. In the case of exponential lag this weighting function is an exponential: $w(\tau) = \frac{1}{T_I} e^{-\tau/T_I}$ where $T_I \int_0^\infty e^{-\tau/T_I} d\tau = 1$

so that we can write:

$$I(t) = \frac{1}{T_I} \int_0^\infty e^{-\tau/T_I} \bar{I}(t-\tau)d\tau$$

which can be reduced to a simple differential equation:

$$I(t) = \frac{1}{T_I} e^{-\tau/T_I} \int_{-\infty}^{t} e^{1/T_I(t-\tau)} \bar{I}(t-\tau)d\tau$$

$$T_I e^{\tau/T_I} I(t) = \int_{-\infty}^{t} e^{1/T_I(t-\tau)} \bar{I}(t-\tau)d\tau$$

differentiating and dividing by $T_I e^{\tau/T_I}$ we arrive at

$$DI = \frac{1}{T_I} (\bar{I}-I)$$

or $$I = \frac{1}{T_I D+1} \bar{I}$$

which is the form found in (3.4). We now can see that the lag has the effect of adjusting the difference between $\bar{I}$ and I over a time period T_I.

These first four equations (3.1), (3.2), (3.3), and (3.4) represent the product market. Now we will examine the money market and the two equations which determine interest rates and price changes.

In developing a suitable equation for the rate of interest, we will consider the demand for cash balances to be a function of money income (as opposed to real income) and the interest rate.[6] The demand for money can then be written as $L(PY_A, r)$ where P is the price level and r the interest rate. A function L which satisfies these requirements and which is simple enough to use in the model, can be written as:

$$L(PY_A, r) = kPY_A e^{-r/\mu}$$

where k and μ are positive constants. The demand for money is therefore an increasing function of money income and an exponentially decreasing function of the interest rate. We can satisfy the supply and demand conditions in the money market by equating this expression for demand with the supply of money M.

$$kPY_A e^{-r/\mu} = M$$

By taking the natural logarithm of both sides, we obtain an expression for the interest rate

$$\log k + \log PY_A - r/\mu = \log M \qquad \text{or}$$

$$r = k' + \mu(\log P + \log Y_A - \log M) \tag{3.5}$$

where $k' = \mu \log k$.

With no contracyclical monetary policy, M is supplied exogenously at a constant proportional rate $m = \frac{DM}{M} = D \log M$. We now can complete the model with an expression for the rate of change of prices.

[6] A good explanation of this interest rate equation is found in Allen, op. cit. (1967), p. 391.

The rate of change in the price level is assumed to take the following form:

$$DP/P = \beta(x-1) - DY_F/Y_F + \delta \qquad (3.6)$$

The rationale behind this expression is that the proportional rate of change of prices should be related to the proportional full capacity "gap" minus the change in productivity. Consider the gap first. As mentioned above this can be written as $x-1 = (Y_A - Y_F)/Y_F$. We would expect that prices would tend to rise faster when $Y_A > Y_F$ or $x > 1$ (periods of inflation) and rise slower or fall when $Y_A < Y_F$ or $x < 1$ (depressed periods). Since there is an upward trend in prices over time, we include the constant term δ to account for this. δ indicates the tendency of prices to rise when $x = 1$.

We assume that changes in productivity can be directly measured by the proportional rate of change in full capacity output DY_F/Y_F. This will be true under the assumptions that the labor force and the number of hours worked per week is constant. If we want to include a constantly changing labor force and work week we can simply adjust the proportional rate DY_F/Y_F by a constant. For example, if the labor force is constantly increasing we would subtract a positive constant equal to the proportional rate of growth. We will assume that the constant term δ includes this adjustment.

The model (with no government sector) is now complete. There are six equations and six variables. Y_F, Y_A, K, I, P, and r.

$$Y_F = vK \tag{3.1}$$

$$Y_A = (1-s)Y + I \tag{3.2}$$

$$K = \int_0^t I dt \quad \text{or} \quad DK = I \tag{3.3}$$

$$I = \frac{1}{T_I D+1} K \left[E + \gamma(x-1) + \rho(c-r) \right] \tag{3.4}$$

$$r = k' + \mu(\log P + \log Y_A - \log M) \tag{3.5}$$

$$DP/P = \beta(x-1) - DY_F/Y_F + \delta \tag{3.6}$$

where $x = Y_A/Y_F$. In order to visualize these relationships a block diagram has been provided; the boxes indicate arithmetic operations, the lines are either flow conditions or definitional relations between the variables.[7] (Figure 3-1)

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[7] A description of such block diagrams is found in Chapter 9 of R. G. D. Allen, Mathematical Economics (New York: Macmillan Co., 1960).

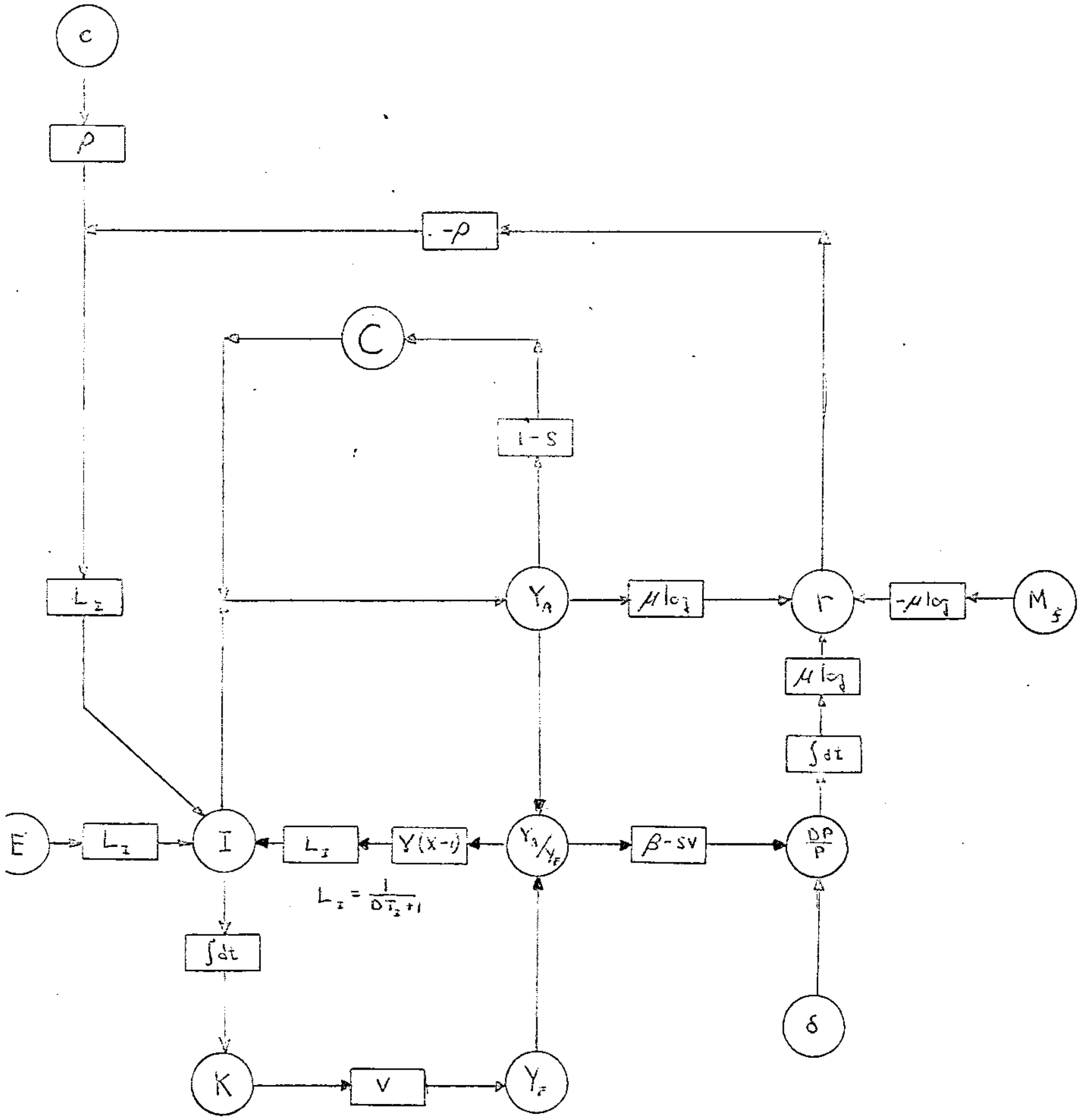

Figure 3.1 Block Diagram of the Unregulated Cyclical Growth Model.

We now proceed to show how these equations can be developed into a linear second order differential equation. There are two important variables to remember. One has already been mentioned. It is the ratio $Y_A/Y_F = x$. The other is the proportional rate of change of full capacity output $DY_F/Y_F = D \log Y_F$. Let $DY_F/Y_F = y_F$. These two variables are related by equations (3.1), (3.2) and (3.3). From (3.1) we know that:

$$DY_F = vDK$$

and from (3.2) and (3.3)

$$DK = I = sY_A$$

so that:

$$DY_F = vsY_A$$

dividing by Y_F we get an equation for the proportional growth rate

$$y_F = svx$$

Thus by obtaining an expression for x we can obtain an expression for the proportional rate of growth y_F. We will now reduce the model to a differential equation in terms of x.

From (3.1) and (3.2) we know that $I = svKx$. This can then be substituted into (3.4) to obtain (cancelling K)

$$svx = \frac{1}{T_I D+1} [\Xi + \gamma(x-1) + \rho(c-r)]$$

or

$$svT_I DX + (sv-\gamma)x = E - \gamma + \rho(c-r) \quad . \tag{3.7}$$

Notice that with a constant rate of interest this will be a simple first order equation which can display either steady explosion or steady decline in x. In order to include oscillations and cyclical growth we must include the money market and a variable rate of interest.

If we differentiate equation (3.5) we obtain

$$\frac{1}{\mu} Dr = D \log P + D \log Y_A - D \log M \, . \tag{3.5.1}$$

Since D log P = DP/P we can obtain from equation (3.6) an expression for the first term on the right hand side of (3.5.1). D log M has been assumed constant (D log M = m) so that we need only find an expression for the second term. To do this we need to make an approximation.

By definition:

$$\log Y_A - \log Y_F = \log [\, 1+(x-1)] = (x-1) + \frac{1}{2}(x-1)^2 + \ldots$$

The expansion is approximately equal (x-1) if x is near 1, which we assume to be true. Thus:

$$\log Y_A \approx \log Y_F + (x-1)$$

and differentiating:

$$D \log Y_A = D \log Y_F + Dx \qquad \text{or}$$

$$D \log Y_A = y_F + Dx \tag{3.8}$$

Substituting this approximation and (3.6) into (3.6.1) we obtain:

$$Dr = \mu(Dx + \beta(x-1) + \delta - m) \qquad (3.9)$$

Now by differentiating (3.7) and substituting for Dr, from equation (3.9) we get:

$$svT_I D^2 x + (sv-\gamma)Dx + \rho Dr = 0$$

$$svT_I D^2 x + (sv-\gamma+\rho\mu)Dx + \beta\rho\mu x = \rho\mu(\beta-\delta+m) \qquad (3.10)$$

which is a linear second order differential equation in terms of x. An equation for y_F is found by the simple substitution $y_F = svx$.

Since the equation is of second order it is not difficult to examine analytically. The solution can be found by solving the quadratic characteristic equation. Depending on the values of the roots we can determine how the model will behave over time. Let us first examine the steady state solutions; that is the values of x and y_F which the model will eventually approach if the system is stable. These can be found by setting all the time derivatives in (3.10) to zero.

We find that the steady state solution of x is $x_s = 1-(m-\delta)/\beta$. Thus, when the rate of change in the money supply is equal to the rate of change in prices at full capacity operation ($m = \delta$), we will have actual output equal to full capacity output. The proportional steady state growth rate is $y_{Fs} = svx_s = sv[1-(m-\delta)/\beta\ ]$. This rate will be a constant which is dependent on the proportion of used capacity. We can also find a steady state solution for the price level from equation (3.6)

$$(DP/P)_s = \beta(x_s-1)y_{Fs} + \delta = m-sv$$

From this relation we can see that prices will be constant if $m = sv$.

The value sv is recognized as the Harrod-Domar "warranted" rate of growth. If x_s is unity then the economy will approach the warranted rate of growth $Y_F = Y_{FO}e^{svt}$. However, this will be true if and only if $m = \delta$. In order to have constant prices with the warranted rate of growth we need to satisfy the additional condition that $m = sv$; that is the rate of increase in the money supply must equal the warranted rate. For warranted growth with constant prices both conditions must hold $\delta = m = sv$. The rate of money supply could conceivably be set equal to either δ or sv, but there is nothing about the model which guarantees that $\delta = sv$. Allen points out that the equality of all three would be a "kind of 'accidental' agreement among the parameters."[8]

Now let us go on to see how the model behaves before it reaches the steady state values. This will be of most interest in this study since we want the model to display varying or cyclical growth over time. We can see how the model generates cyclical growth by looking at the homogeneous solution of (3.10)

$$D^2x + \frac{1}{svT_I}(sv - \gamma + \rho\mu)Dx + \frac{\beta\rho\mu}{svT_I}x = 0 \tag{3.10.1}$$

The characteristic equation is:

$$q^2 + \frac{1}{svT_I}(sv - \gamma + \rho\mu)q + \frac{\beta\rho\mu}{svT_I} = 0 \tag{3.10.2}$$

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[8] Allen, op. cit (1967), p. 396.

the roots of the differential equation are therefore:

$$q_1, q_2 = - \frac{1}{2svT_I} (sv-\gamma + \rho\mu) \pm \frac{1}{2} \sqrt{\left(\frac{1}{svT_I}\right)^2 (sv-\gamma + \rho\mu)^2 - 4\frac{\rho\mu\beta}{svT_I}} .$$

The values of these two roots will determine how the system will behave over time. We are most interested in the case of complex roots because these will give the desired cyclical growth. But first consider the case of real roots.

The roots will be real if $\beta < \frac{1}{4}\frac{(sv-\gamma+\rho\mu)^2}{sv\rho\mu T_I}$.

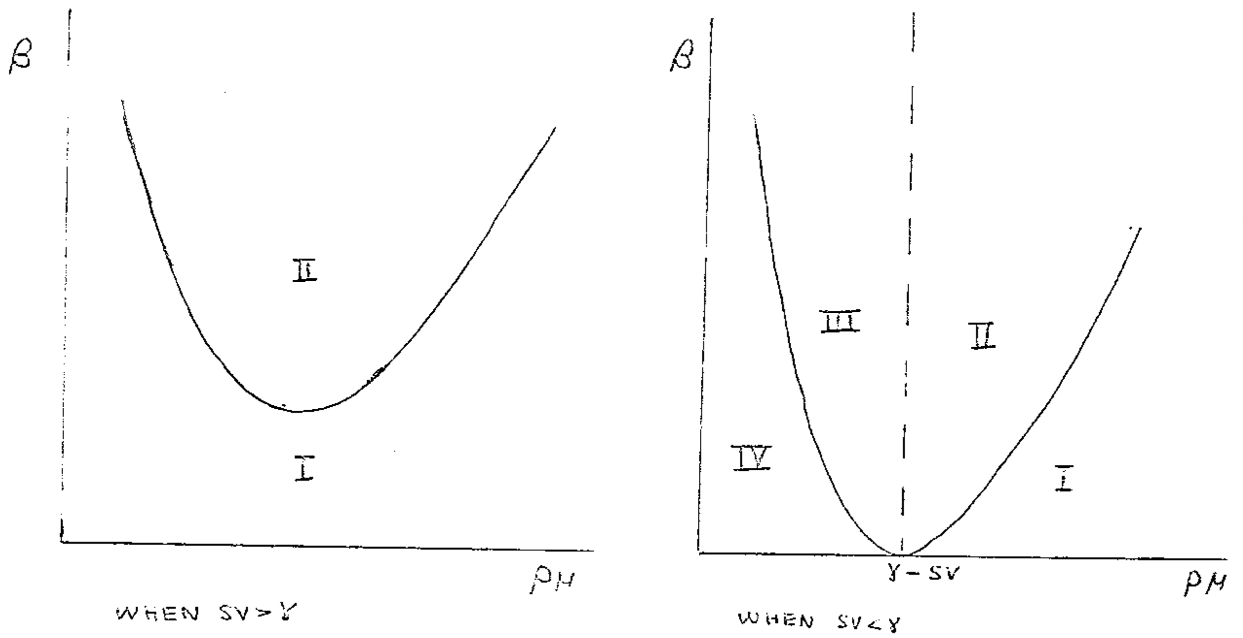
Since $q_1 \cdot q_2 = \frac{\beta\rho\mu}{svT_I} > 0$ the roots will either be both positive or both negative. Thus there will either be steady explosion or steady decline to the steady state solution. Explosion will occur if $sv-\gamma+\rho\mu < 0$ (roots positive) and will be damped if $sv-\gamma+\rho\mu > 0$ (roots negative)

The more interesting (and more realistic case) is when the roots are complex. In this case we will have oscillations in x and therefore in y_F giving cyclical growth. The roots are complex if $\beta > \frac{1}{4}\frac{(sv-\gamma+\rho\mu)^2}{sv\rho\mu T_I}$.

The oscillations will be demped if $(sv-\gamma+\rho\mu) > 0$ and will explode when the opposite is true.

Notice that if $sv > \gamma$, (where γ is the speed of entrepreneurs adjustment to under or over capacity operation) the model will always be damped (either oscillating or steady). On the other hand if $sv < \gamma$, then the model will be demped or explosive depending on value of $\rho\mu$.

The various possibilities in either case can be seen in Figure (3.2)[9] which plots β on the vertical axis and $\rho\mu$ on the horizontal axis.



I: Steady Approach
III: Explosive Oscillation
II: Damped Oscillation
IV: Steady Explosion

Figure 3.2. Range of Values of β and $\rho\mu$ Which Generate Each Type of Behavior.

$\rho\mu$ can be interpreted as a combined monetary influence on net investment: μ operating through the interest rate by way of the quantity of money and ρ determining the magnitude of the influence of the interest rate. A high $\rho\mu$ would indicate strong monetary influences. We will recall that β indicates the influence of under or over capacity operation on prices. An example behavior of the model for each possibility is then plotted in Figure (3.3) where we assume that $x_0 < x_s$; the path is exactly reversed when $x_0 > x_s$.

[9] Allen, op. cit. (1967), p. 400.

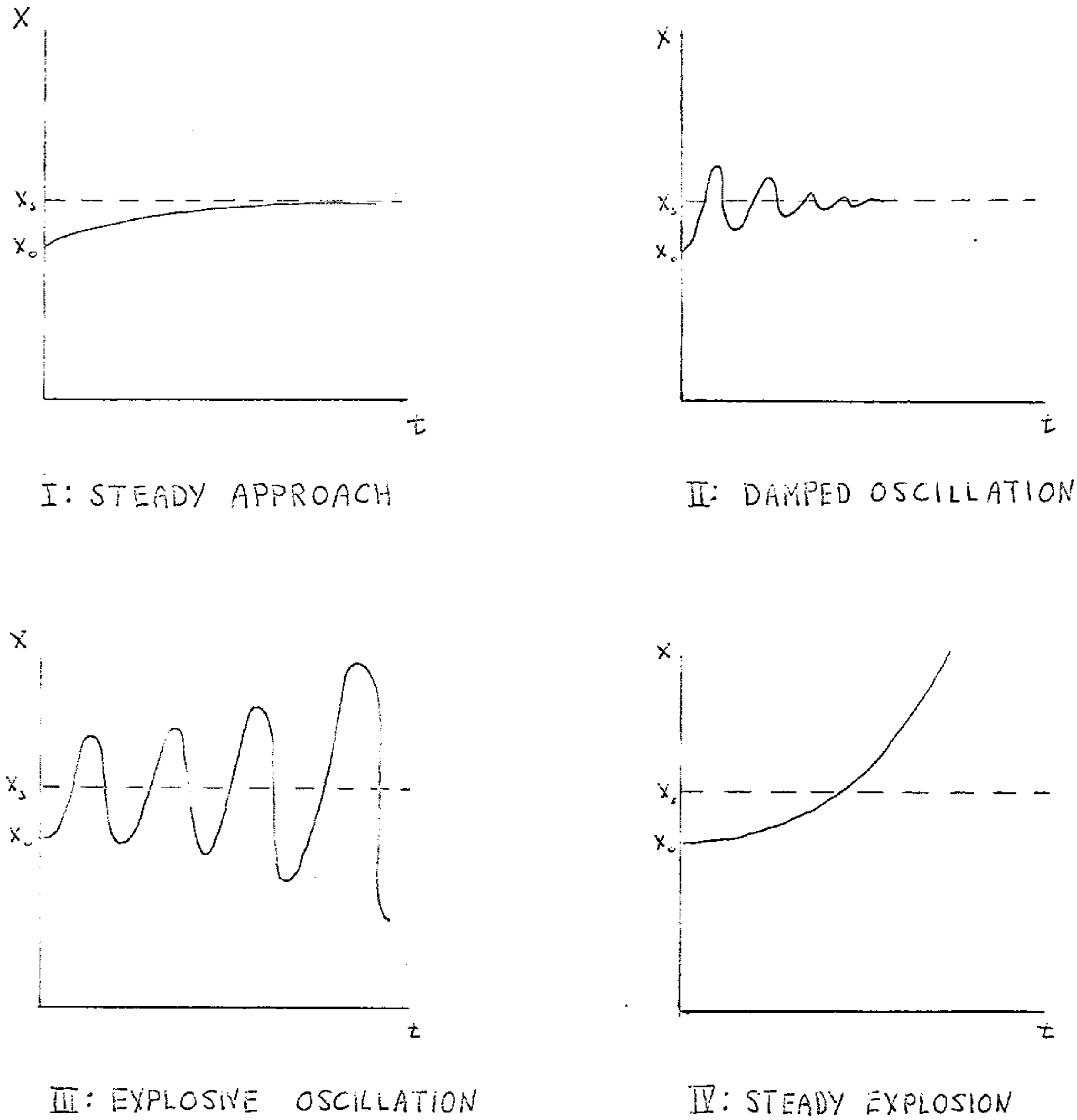


Figure 3.3. Time Paths of Actual to Full Capacity Ratio (x) for Each Type of Behavior.

Let us examine the oscillations case more closely to see how the model is actually displaying cyclical growth. We know that x oscillates and that therefore $y_F = svx$ oscillates with the same frequency. Both will oscillate about their respective steady state values mentioned above. In terms of the actual variables of the model, Y_F fluctuates about the steady state growth path $Y_{Fs} = Y_0 e^{y_{Fs} t}$. These fluctuations will either be damped or explosive depending on the values of the parameters mentioned previously. Actual output Y_A will have a double fluctuation, since $Y_A = xY_F$ and both x and Y_F are oscillating. Possible paths for the damped case are plotted in Figure (3.4).

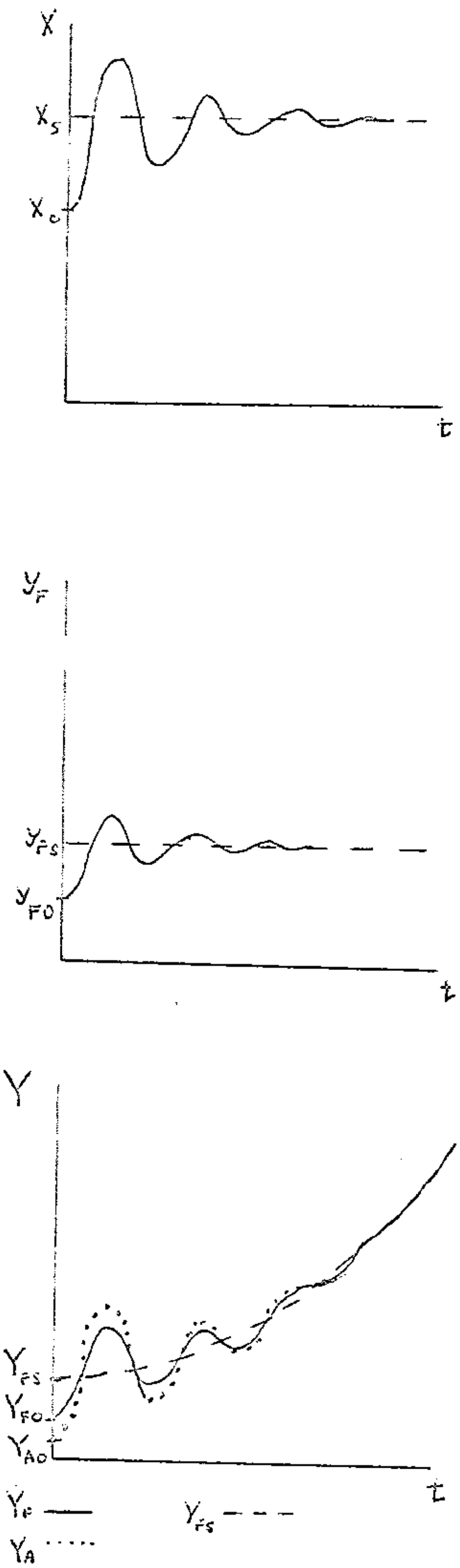


Figure 3.4. Time Paths of Y_F and Y_A Compared with y_F and x for the Case of Damped Oscillations.

Having examined the nature of this model of cyclical growth we now proceed to an analysis of possible government fiscal and monetary policies. We will then determine the effect of these policies on fluctuations and the growth rate. Using the model we can investigate such questions as whether there is a trade off decision between fluctuations and desired growth because of the negative effect of stabilization policies on the growth rate.

CHAPTER 4

GOVERNMENT STABILIZATION POLICIES

The contracyclical policies which are used in this model of cyclical growth are modified versions of policies mentioned by Phillips in his article on economic stabilization in a closed economy.[1] These policies are analogous to feedback methods of control which engineers have used in stabilizing electrical systems. Phillips applied these methods to a disequilibrium multiplier-accelerator model similar to that discussed in the first section of Chapter 2. The aim of this chapter is to illustrate the methods of fiscal stabilization used by Phillips, show how they should be modified to fit in the model of cyclical growth, and finally propose some related methods of monetary stabilization which will also be applied to the model.

A. Fiscal Policies

By fiscal stabilization policies we shall mean policies through which the government creates an addition (or subtraction) to the total demand of the economy by adjusting expenditures, tax rates, or transfer payments. Such policies have the aim of moving the output of the economy closer to a full-capacity (or full employment) level of income and to offset undesirable income trends so as to keep the economy stabilized

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[1] A. W. Phillips, *op. cit.* (1954).

about this desirable full capacity level. It is claimed that such stabilization policies will reduce the size and frequency of fluctuations about full capacity and as a consequence increase the rate of growth of the economy. This last conclusion rests on the assumption that economic fluctuations are not necessary for a high rate of economic growth, and in fact that they tend to lower the rate. This analysis will help examine that last assumption.

In the following representations of fiscal policy there will be no distinction among the actual methods used by the government to create demand. Instead expenditures, taxes, and transfer payments will be lumped under one concept: budget surplus (+) or deficit (-). An increase in the budget surplus or a decrease in the deficit can occur through a decrease in expenditures and transfer payments or through an increase in taxes. Conversely, a decrease in budget surplus or an increase in the deficit can occur through an increase in expenditures and transfer payments or a decrease in taxes. Therefore when the government is adding to official demand we can say it is increasing the deficit; or when subtracting from official demand, it is decreasing the deficit. For simplicity we will speak of increases in the deficit as government spending and remember that this can mean increased transfer payments or lower taxes.

The policies discussed are of a continuous type; that is we assume that the government can continuously adjust the budget according to present levels and trends in income. Continuous policy can be distinguished from "one shot policy."[2] With a one shot policy the government will adjust

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[2] Howard Pack, Formula Flexibility: A Quantitative Analysis, Ph.D. Dissertation: Massachusetts Institute of Technology (1964), p. 19.

the budget deficit by a certain amount determined by economic indicators, and then wait until the indicator returns to its normal level before cutting spending. It has been suggested that one shot policy will aggravate fluctuations because it is initialized after the trend has passed and is cut off after a new trend is getting started.[3] This is partly due to the large effective time lag and the inaccuracy of indicators which are used only when they reach extreme points. This disadvantage is partly avoided by continuous policy because the indicators are constantly being observed and adjustments are made in the deficit at each observed change rather than only at extreme points. The time lag between the actual behavior of the economy and the initiation of corrective policy is thus shorter.

We will consider three forms of continuous government fiscal policy: proportional, integral, and derivative. The effect of each of these on actual demand will be lagged.

1. Proportional fiscal policy

Using this type of policy the government will spend in proportion to the difference between a desired level of income (e.g. full capacity) and the actual level. If we let the desired level of income be full capacity then we can write this policy as $-g_p(Y_A-Y_F)$, where g_p is fiscal derivative coefficient. For example, if g_p is 0.5, when there is a 2% gap between actual and full capacity income, the government spends 1% of national income. The policy has the aim of bringing the economy closer to the desired level of income.

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[3] Milton Friedman, "The Effects of Full Employment Policy on Economic Stability: A Formal Analysis," in Essays in Positive Economics (Chicago: University of Chicago Press, 1953).

This is what Phillips has called the potential policy demand using a proportional policy. If there is no time lag then the potential policy demand will equal the actual policy demand. However, it is realistic to incorporate a distributed exponential lag between potential and actual policy. The lag is the same form used as the investment lag in equation (3.4). The time constant T_G will indicate the average period between the actual change in the economy and the initiation of the corrective policy. Thus we can write

$$G_p = \frac{1}{T_G D+1} [-g_p(Y_A - Y_F)] \tag{4.1}$$

This lag is the time required for observation of the indicators, for adjusting the corrective action accordingly, and for the corrective action to produce its full effect on demand.[4]

When we apply a policy like this to our cyclical growth model the most important change is that Y_F is now a variable. Thus, government spending may change when the growth of actual income remains steady. An example of such a situation could be a time saving technological innovation suddenly introduced throughout the economy. This would cause Y_F to increase more rapidly relative to Y_A and the proportional policy would indicate an increase in the government deficit in order to increase Y_A towards the relatively larger Y_F. In the proportional policy of Phillips, such a development is not possible because Y_F is assumed to be a constant. We will find that more realistic assumption that Y_F is a variable, has important consequences for stabilization policy.

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[4] Phillips, op. cit. (1954) assumes that the desired level of income is zero. This is possible by simply placing Y_F at the origin, since Y_F is a constant. Such a simplification is not possible when Y_F is a variable.

The notation of this policy will be slightly different when we apply it to the cyclical growth model. The equivalent expression can be found by factoring out Y_F from equation (4.1)

The result is:

$$G_p = \frac{1}{T_G D+1} [-g_p Y_F (x-1)] . \qquad (4.2)$$

This will be the proportional policy which we shall use. As we shall see in Chapter 5, this expression in terms of Y_F and x can be incorporated directly into the cyclical growth model.

2. Fiscal integral policy

A proportional policy alone cannot bring the economy completely to the desired level of output in the multiplier-accelerator model. This is due to the fact that proportional spending gets smaller as we approach the desired level of income. The level will be approached but never reached. Therefore a second type of policy is introduced which is designed to eliminate the gap completely.

This policy is called integral stabilization policy. It is applied to the cumulative gaps in income over time. We will assume the same exponential lag in this case. The lagged policy can be written as follows:

$$G_i = \frac{1}{T_G D+1} [-g_i \int_0^t (Y_A - Y_F) dt] \qquad (4.3)$$

The government demand is proportional to an integrated sum of differences between Y_A and Y_F starting at the time the system is initiated (t=0). This will completely eliminate the gap because no matter how small a gap persists, the time integral of that gap in the corrective factor will be increasing. However, this policy has a disadvantage of making the multiplier-accelerator model unstable. This can be seen by observing that the corrective action will continue even after the gap has been eliminated. The combination of the proportional and integral policies will eliminate some of these fluctuations.

This policy will have to be modified slightly for application to the model of cyclical growth. As in the proportional case, we first factor Y_F from the integrand of equation (2.3). This results in:

$$G_i = \frac{1}{T_G D+1} \left[-g_i \int_0^t Y_F(x-1)dt \right] \qquad (4.4)$$

This expression will need further modification before it can be mathematically inserted into the model. It will be desirable to have the variable Y_F on the left-hand side of the integral. Doing this equation (4.4) becomes:

$$G_i = \frac{1}{T_G D+1} \left[-g_i Y_F \int_0^t (x-1)dt \right] \qquad (4.5)$$

Equation (4.5) is obviously different mathematically from equation (4.4), and when the definite integrals are evaluated their magnitudes will differ. However, the relationship between the gap and the aim of the policy will remain roughly the same. The impact of the policy on demand will still be the cummulative sum of the gaps between actual and full capacity output. Furthermore, the difference in the magnitudes of the two representations will be small unless Y_F increases at an unusually high rate. Therefore, we will assume that the effect of equation (4.5) on the model would be similar to that of (4.4). The necessity for this modification in the integral policy arises because of the more realistic feature that Y_F is a variable rather than a constant target. The advantage of being able to treat Y_F as a variable has already been mentioned, and still exists with the necessary modification of the integral policy.

3. Fiscal derivative policy

The integral policy is necessary for a complete elimination of the gap in the multiplier-accelerator model, but it has the disadvantage of decreasing stability. To offset this tendency, a third policy has been proposed called a derivative stabilization policy. In the multiplier accelerator model, using the same fiscal lag as earlier, this takes the form

$$G_D = \frac{1}{T_G D+1} [-g_D D(Y_A - Y_F)] \qquad (4.6)$$

With the assumption of constant Y_F this reduces to

$$G_D = \frac{1}{T_G D+1} [-g_D D Y_A] \qquad (4.6.1)$$

The policy demand is opposite in sign and proportional to the rate of change of actual income. The effect is to offset income trends by deficit spending when Y_A is falling and increasing the budget surplus when Y_A is rising. The effect of this policy is stabilizing in the sense that the damping of the oscillations is increased. However, as Baumol has pointed out such policy is destabilizing in another sense: it increases the frequency of the oscillation.[5] Thus, the net effect of the policy may be mixed.

When we consider this type of policy in the model of cyclical growth, the assumption that Y_F is a constant must be dropped and we again have equation (4.6). Now the policy demand is opposite in sign and proportional to the rate of change in the gap between actual and full capacity income. If the gap is growing larger (either through a relative increase in the growth rate of Y_F or a relative decrease in the growth rate of Y_A) then the government will offset the movement by deficit spending.

The derivative policy will also be modified so that it may fit mathematically into the cyclical growth model. As before we factor out Y_F to get an expression in terms of x and Y_F. Thus equation (4.6) becomes:

$$G_D = \frac{1}{T_G D+1} \left[-g_D D Y_F (x-1) \right] \tag{4.7}$$

Again, we would like to have the factor Y_F "outside" of the time derivative. Doing this we obtain the following

$$G_D = \frac{1}{T_G D+1} \left[-g_D Y_F D x \right] \tag{4.8}$$

The difference between (4.7) and (4.8) is the difference between $Y_F Dx$ and $DY_F(x-1)$. Completing the second differentiation we obtain

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[5] William J. Baumol, "Pitfalls in Contracyclical Policies: Some Tools and Results," *Review of Economics and Statistics* (February 1961). A similar effect is found for proportional policy.

$$DY_F(x-1) = Y_F Dx + (x-1)DY_F$$

Thus the difference between the two equations for derivative policy is the term $(x-1)DY_F$. Several things can be said about this term. First, it will always be small relative to $Y_F Dx$. For example, in a discrete interval of one year, typical values and changes might be $x = 1.04$, $Y_F = 500$, $\Delta x = 0.01$, $\Delta Y_F = 20$. Then $Y_F \Delta x = 5.0$ while $(x-1)\Delta Y_F = 0.8$. Secondly even though the term is small and might be ignored, it is not clear whether we want it anyway. For example if $x > 1$ and Y_F is increasing then the effect will be that the government will spend less as a result of this term; this might decrease Y_A and increase the gap even further. A more realistic policy with the term $(x-1)$ is found in the proportional policy mentioned above. There we have $(x-1)Y_F$ rather than $(x-1)\Delta Y_F$. For these reasons we will assume that the term is not present in the derivative policy, and shall use equation (4.8).

These are the three fiscal policies which we shall consider. They can all be used at the same time and with various values for the fiscal coefficient g. When they are all used together we can write:

$$G = \frac{1}{T_G D+1} \left[-g_F Y_F(x-1) - g_i Y_F \int_0^t (x-1)dt - g_D Y_F Dx \right] \qquad (4.9)$$

Several points should be mentioned in reference to the modifications made in the fiscal policy as applied to the model of cyclical growth. The elimination of the assumption that Y_F is a constant (as Phillips assumed), has obvious advantages in the examination of policy effects on growth. However, the inclusion of Y_F as a variable makes the expressions more difficult to handle mathematically and apply to the cyclical growth model. The resulting necessary modifications are therefore made in the integral and derivative policies. In both cases the modifications are small and do not change the purpose of the policies. That is, the modified stabilization policies still work to offset the size of the gap (integral) and trends in the gap (derivative). The policies are still intuitively realistic. It now remains to apply these policies to the model of cyclical growth. But first let us consider some possible monetary policies that the government might undertake to stabilize fluctuations.

B. Monetary Policies

We will assume that the central bank of the economy can increase or decrease the supply of money through actions on the open market. By selling securities the monetary authority can decrease the money supply (which will raise interest rates in the model) and conversely by buying securities the authority can increase the supply of money (decrease in interest rates). We will assume that these operations on the open market can be carried on continuously, and that the correct action is calculated by observation of economic indicators (e.g. GNP and potential GNP).

The structure of the monetary policies is similar to that of the fiscal policies already mentioned. The monetary authority will adjust the money supply depending on three variable factors: proportional, derivative, and integral policies.[6] The authority will be adjusting

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[6] Phillips, op. cit. (1961), p. 367, mentions the possibility of a derivative monetary policy. We shall consider all three possibilities.

the rate of growth of the money supply around some constant equilibrium rate. Using logarithmic notation so that the expression is in proportional terms, we can write an equation for the money supply which is dependent on this government regulation:

$$\log M = \log M_f - g_{mp}(x-1) - g_{mD}Dx - g_{mi} \int_0^t (x-1)dt \qquad (4.10)$$

Thus the monetary authority proportionally adjusts the monetary supply about a level M_f (which increase steadily over time) according to the proportional size of the gap, the rate of change of the gap, and the cumulative sum of the proportional gap over time. The parameters g_{mP}, g_{mD} , g_{mi} indicate the amount by which each term affects the money supply. For example, government monetary proportion policy will alter the money supply by g_{mp} times the proportional gap (x-1).

The variable M_f can be interpreted as the money supply when there is no monetary stabilization policy ($g_{mp} = g_{mD} = g_{mi} = 0$) or when the net effect of the monetary policy is zero. We will assume that the central bank increases M_f by a constant rate so that $D \log M_f = DM_f/M_f = m_f$, which is a constant. A constant m_f when there is no monetary contracyclical policy, is the kind of monetary action that Friedman has suggested.[7] That is, the money supply is increased at a constant rate regardless of whether there is a deflationary or inflationary gap. Thus we are able to separate his proposal from the contracyclical policies.

[7] Milton Friedman, A Program for Monetary Stability (New York: Fordam University Press, 1960).

An appropriate lag could be incorporated in this contracyclical monetary policy similar to that in the fiscal policy. However, such a lag greatly complicates the analysis and necessitates further assumptions for proper mathematical investigation. Therefore, we will assume that there is no lag in the monetary policy. This can be partially justified by assuming the monetary authority is less subject to political restraints and delays than a fiscal authority; and also that the full effect of monetary action on the money supply will occur more quickly than the full effect of fiscal action on final demand.

Equations (4.9) and (4.10) are the fiscal and monetary policies which shall be applied to the model. We can investigate different mixes of each type of policy by varying the values of the parameters g_p, g_i, g_D (fiscal policy) and g_{mp}, g_{mi}, g_{mD} (monetary policy). In addition the lag of the fiscal policy can be varied by altering the value of T_G, the time constant of the policy lag.

CHAPTER 5

THE GOVERNMENT REGULATED MODEL OF CYCLICAL GROWTH

A. The Differential Equation of the Regulated Model

To regulate the model of cyclical growth explained in Chapter 3, we must add two additional variables defined by two equations. These variables (G = government deficit spending, or surplus collections, and M = a variable money supply) have been defined in the previous chapter by equations (4.9) and (4.10). When added to the model, it becomes a system of eight equations and eight variables. In addition, the income equation (3.2) must now include the variable G. Equation (4.10) for the money supply can be considered as an extension of the assumption that M is increasing at a constant rate. Now it will be increasing at a variable rate. To investigate this regulated system we will reduce it to a set of differential equations in terms of the variables x and y_F. These will be interpreted exactly as in the regulated case. When all policies are considered, the system including expressions for y_F, will be of fifth order.

Let us begin by summarizing the model:

$$Y_F = vK \tag{3.1}$$

$$Y_A = (1-s)Y_A + I + G \tag{3.2}$$

$$K = \int_0^t I dt \quad \text{or} \quad DK = I \tag{3.3}$$

$$I = \frac{1}{T_I D+1} K \; [\; E + \gamma(x-1) + \rho(c-r) \;] \tag{3.4}$$

$$r = k' + \mu(\log P + \log Y_A - \log M) \tag{3.5}$$

$$DP/P = \beta(x-1) - DY_F/Y_F + \delta \tag{3.6}$$

$$G = \frac{1}{T_G D+1} \; [\; -g_\rho Y_F(x-1) - g_i Y_F \int_0^t (x-1)dt - g_D Y_F Dx \;] \tag{4.9}$$

$$\log M = \log M_f - g_{mp}(x-1) - g_{mi} \int_0^t (x-1)dt - g_{mD} Dx \tag{4.10}$$

The parameters and variables of this system are listed below:

Variables:

Y_A = actual level of income

Y_F = full capacity level of income

K = stock of capital

I = net investment

r = rate of interest

P = price level

G = government demand

M = variable money supply

M_f = that part of the money supply which increases at a fixed rate

$x = Y_A/Y_F$

$y_F = DY_F/Y_F$

Parameters:

v = output-capital ratio

s = marginal propensity to save

γ = speed of entrepreneurs investment adjustment to reduce excess capacity or increase insufficient capacity

c = marginal efficiency of capital

ρ = influence of interest rate on investment

μ = influence of price level, actual income, and money supply on interest rate

β = influence of over or under capacity output on prices

T_I = time constant of investment lag

E = business expectations (assumed constant)

δ = constant price trend

Government Stabilization Policy Coefficients:

g_p = proportional fiscal policy

g_i = integral fiscal policy

g_D = derivative fiscal policy

T_G = fiscal lag

g_{mp} = monetary proportional policy

g_{mi} = monetary integral policy

g_{mD} = monetary derivative policy

A block diagram for this government regulated model is provided in Figure 5.1.

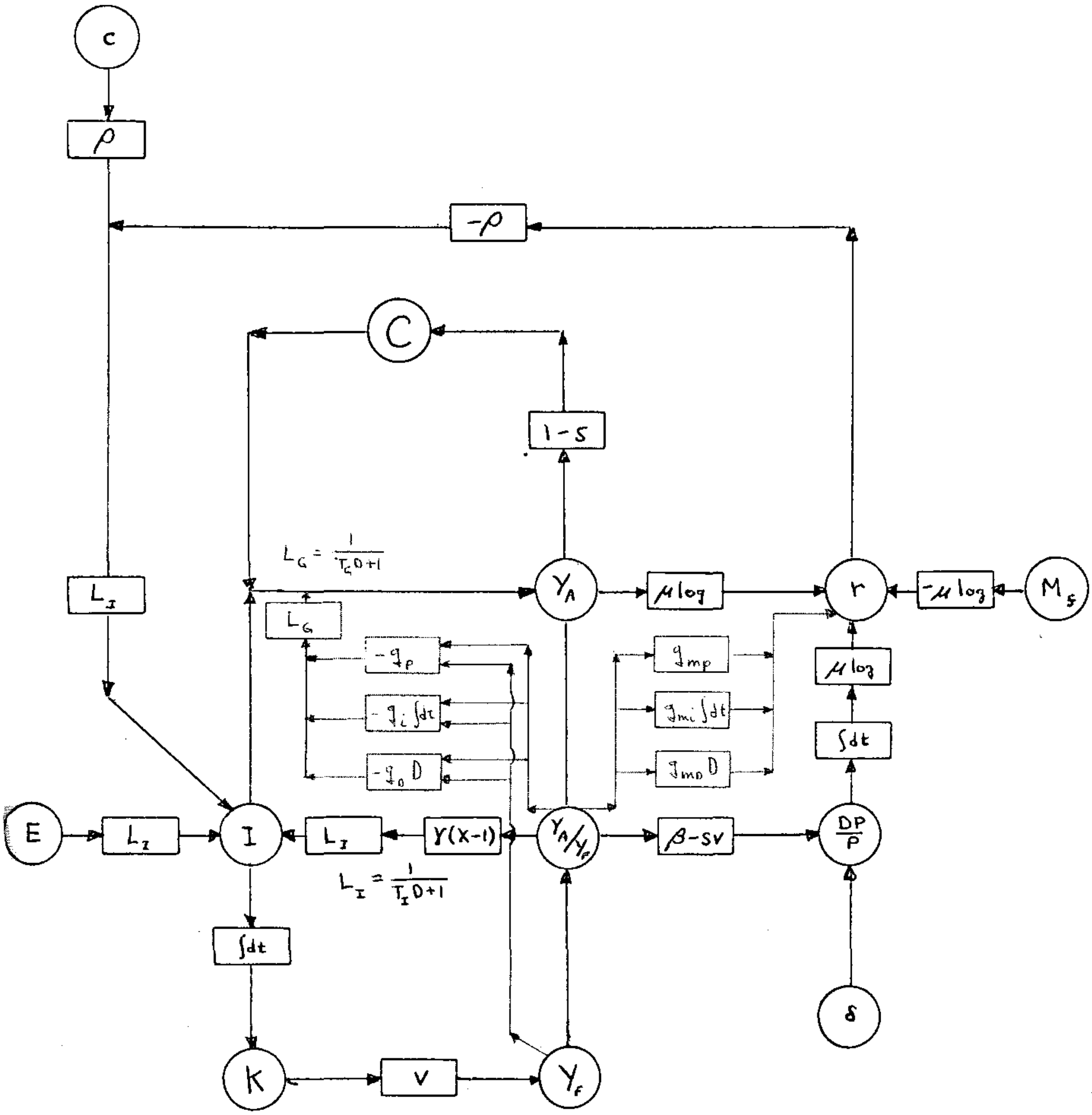

Figure 5.1. Block Diagram of the Cyclical Growth Model when Regulated by Fiscal and Monetary Policies

This is analogous to figure (3.1) with the addition of government policy. The lines of flow and the particular type of operation used in each policy are shown in red. It can be seen that the direct effect of fiscal policy is to add or subtract an additional quantity to Y_A depending on the magnitudes of $x = Y_A/Y_F$. The direct effect of monetary policy is on the interest rate r.

Now let us derive the differential equation of the model. The derivation is more complicated than in the unregulated system, because of the higher order equations. However, the modifications in fiscal policy outlined in chapter 4 help to simplify the problem and leave the equations in linear form. From equations (3.1), (3.2) and (4.9) we derive an expression for I in terms of x and K. From equation (3.2):

$$I = sY_A - G \quad .$$

Using the identity $Y_A = xY_F$ this can be written as:

$$I = sxY_F - G \tag{5.1}$$

Substitution for G in (5.1) from equation (4.9) gives:

$$I = sxY_F - \frac{1}{T_G D+1}\left[- g_p Y_F(x-1) - g_i Y_F \int_0^t (x-1)dt - g_D Y_F Dx \right]$$

$$= sxY_F + \frac{Y_F}{T_G D+1}\left[g_p(x-1) + g_i \int_0^t (x-1)dt + g_D Dx \right] \tag{5.2}$$

Now from equation (3.1) we can substitute for Y_F in equation (5.2). This gives the desired equation for I in terms of x and K:

$$I = svKx + \frac{vK}{T_G D+1} \left[g_p(x-1) + g_i \int_0^t (x-1)dt + g_D Dx \right] \qquad (5.3)$$

Equation (5.3) can now be substituted directly into the investment equation (3.4), which after cancelling K, results in a differential equation in terms of x and r:

$$svx + \frac{1}{T_G D+1} v \left[g_p(x-1) + g_i \int_0^t (x-1)dt + g_D Dx \right] =$$

$$\frac{1}{T_I D+1} \left[E + \gamma(x-1) + \rho(c-r) \right]$$

After multiplying through by $(T_G D+1)(T_I D+1)$ and collecting terms we get:

$$(svT_I T_G + T_I g_D v)D^2 x + [sv(T_I+T_G) + T_I g_p v + g_D v - T_G \gamma] Dx$$

$$+ [sv + T_I g_i v + g_p v-\gamma] x + g_i v \int_0^t (x-1)dt + T_G \rho Dr + \rho r$$

$$= T_I g_i v + g_p v + E-\gamma + \rho c \qquad (5.4)$$

In order to convert this equation to strict linear differential form we differentiate with respect to time so as to eliminate the integral expression. Doing so we arrive at the following third order equation in terms of x and r:

$$(svT_I T_G + T_I g_D v)D^3 x + [sv(T_I+T_G)+T_I g_p v + g_D v-T_G \gamma]D^2 x$$

$$+ (sv+T_I g_i v + g_p v-\gamma)Dx + g_i vx + T_G \rho D^2 r + \rho Dr-vg_i = 0 \qquad (5.5)$$

In order to obtain the final differential equation of the model we must substitute suitable expressions for D^2r and Dr in equation (5.5). The money market with the proposed monetary stabilization policy must now be considered. We begin by substituting for log M in equation (3.5) from the money equation (4.10) and differentiating with respect to time:

$$Dr = \mu(D \log P + D \log Y_A - D \log M_f + g_{mp}Dx + g_{mi}(x-1) + g_{mD}D^2x) \qquad (5.6)$$

By substituting for D log P from equation (3.6) and for D log Y_A from the approximation outlined in Chapter 3, and collecting terms, we arrive at the following equation for Dr in terms of x:

$$Dr = \mu[g_{mD}D^2x + (g_{mp}+1)Dx + (\beta+g_{mi})x-\beta+\delta-m_f-g_{mi}\ ] \qquad (5.7)$$

where m_f = D log M_f is the constant rate of increase in the money supply. Differentiating we get an expression for D^2r

$$D^2r = \mu[\ g_{mD}D^3x + (g_{mp}+1)D^2x + (\beta + g_{mi})Dx\ ] \quad . \qquad (5.8)$$

Equations (5.7) and (5.8) can now be used to substitute for D^2r and Dr in equation (5.5). The result is the linear third order differential equation of the model in terms of the variable x. This equation will give us the path of x over time when the system is regulated by monetary and fiscal policy with a lag:

$$[svT_IT_G + T_Ig_Dv + T_G\,\rho\mu g_{mD}\,]\,D^3x$$

$$+ [sv(T_I+T_G) + T_Ig_pv + g_Dv - T_G\,\gamma + T_G\,\rho\mu(g_{mp}+1)+\rho\mu\,g_{mD}\,]D^2x$$

$$+ [sv + T_Ig_iv + g_pv-\gamma + T_G\,\rho\mu(\beta + g_{mi}) + \rho\mu(g_{mp}+1)\,]\,Dx$$

$$+ [g_iv + \rho\mu(\beta + g_{mi})]\,x = vg_i + \rho\mu[\beta + m_f-\delta+g_{mi}] \qquad (5.9)$$

In order to complete the equations of the model we must show the relationship between x and y_F. In the unregulated case we found that $y_F = svx$. But this will no longer be true when the system is regulated by fiscal policy. From equation (3.1), (3.2) and (3.3) we know that:

$$DY_F = vDK = vI$$

$$= v(sY_A-G)$$

$$y_F = DY_F/Y_F = v(sY_A-G)/Y_F \qquad (5.10)$$

Substituting for G from equation (4.9) we get

$$y_F = svx + \frac{1}{T_GD+1}\;v[\,g_p(x-1) + g_i\int_0^t (x-1)dt + g_DDx]$$

or

$$T_GDy_F + y_F = T_GsvDx + svx + g_pv(x-1) + g_iv\int_0^t (x-1) + g_DvDx \qquad (5.11)$$

By differentiating equation (5.11) we can eliminate the integral and get a differential equation for y_F in terms of x:

$$T_GD^2y_F = Dy_F = (T_Gsv + g_Dv)D^2x + (sv + g_pv)Dx + g_ix-g_i \qquad (5.12)$$

This equation is of second order in y_F. When combined with the variable x and its time derivatives the entire system considering both x and y_F (with government monetary policy and fiscal policy with lags) becomes a fifth order equation. By examining the behavior of equations (5.9) and (5.12) over time we can determine the effects of fiscal and monetary policy.

In the following two sections of this chapter the impacts of such policies are examined. In section B the examination is mathematical and in section C simulation techniques are used.

B. Analytic Investigation

If we ignore fiscal lag and assume that there is no fiscal derivative policy ($T_G = g_D = 0$), then equation (5.9) is reduced to a linear second order differential equation:

$$D^2x + \frac{sv + T_I g_i v + g_p v - \gamma + \rho\mu(g_{mp}+1)}{T_I v(s+g_p) + \rho\mu\ g_{mD}} Dx$$

$$+ \frac{g_i v + \rho\mu(\beta + g_{mi})}{T_I v(s+g_p) + \rho\mu\ g_{mD}} x = \frac{vg_i + \rho\mu(\beta + m_f - \delta + g_{mi})}{g_i v + \rho\mu(\beta + g_{mi})} \qquad (5.13)$$

By examining the roots of equation (5.13) we can determine the effects of all policies without lags except fiscal derivative . The criterion for stability and the steady state values of the variables can be determined directly from (5.13). In this way, we can determine the general path of the variables. The more specific paths of the variables are then determined in section C by simulation.

We can rewrite the homogeneous part of equation (5.13) as:

$$D^2x + \alpha Dx + \sigma x = 0 .$$

The roots of the characteristic equation are then:

$$q_1, q_2 = -\frac{1}{2}\alpha \pm \frac{1}{2}\sqrt{\alpha^2 - 4\sigma}$$

where α determines stability in terms of damping and $\sqrt{4\sigma - \alpha^2}$ gives the frequency in the case of oscillations. α is the damping factor; when positive the system is stable; when negative the system is explosive. When $\alpha = 0$ the system oscillates with constant amplitude. We will now proceed to examine the impacts of each policy alone on stability and growth under the assumption of zero lag.

1. Fiscal proportional policy

When proportional policy is used alone the damping factor from equation (5.13) becomes:

$$\alpha = \frac{sv + g_p v - \gamma + \rho\mu}{svT_I + T_I g_p v}$$

If we take the partial derivative of α with respect to g_p we get:

$$\frac{\partial \alpha}{\partial g_p} = \frac{\gamma - \rho\mu}{T_I v(s+g_p)^2}$$

Thus proportional policy will increase stability by increasing the magnitude of the damping factor α if $\gamma > \rho\mu$, and conversely it will decrease damping if $\gamma < \rho\mu$. To understand the significance of this consider the unregulated model. If the unregulated system is unstable then, from Chapter 3, $\gamma > sv + \rho\mu$ which implies that $\gamma > \rho\mu$. Therefore, if the unregulated system is unstable, then the initiation of proportional fiscal policy will increase stability. If the unregulated system was already stable, then the proportional policy will either increase or decrease damping depending on whether γ is greater than or less than $\rho\mu$ If $\gamma = \rho\mu$ then the proportional policy will not change the stability of the system. In that case α will remain equal to $1/T_I$ no matter how much we increase g_p. If $\gamma > \rho\mu$ then $\alpha < 1/T_I$ and the initiation of g_p will move α toward $1/T_I$ and thus increase damping. If $\gamma < \rho\mu$ then $\alpha > 1/T_I$ and the initiation of g_p will decrease damping. We can see the various cases by looking at Figure 5-2, where α is the ordinate and g_p is the abscissa.

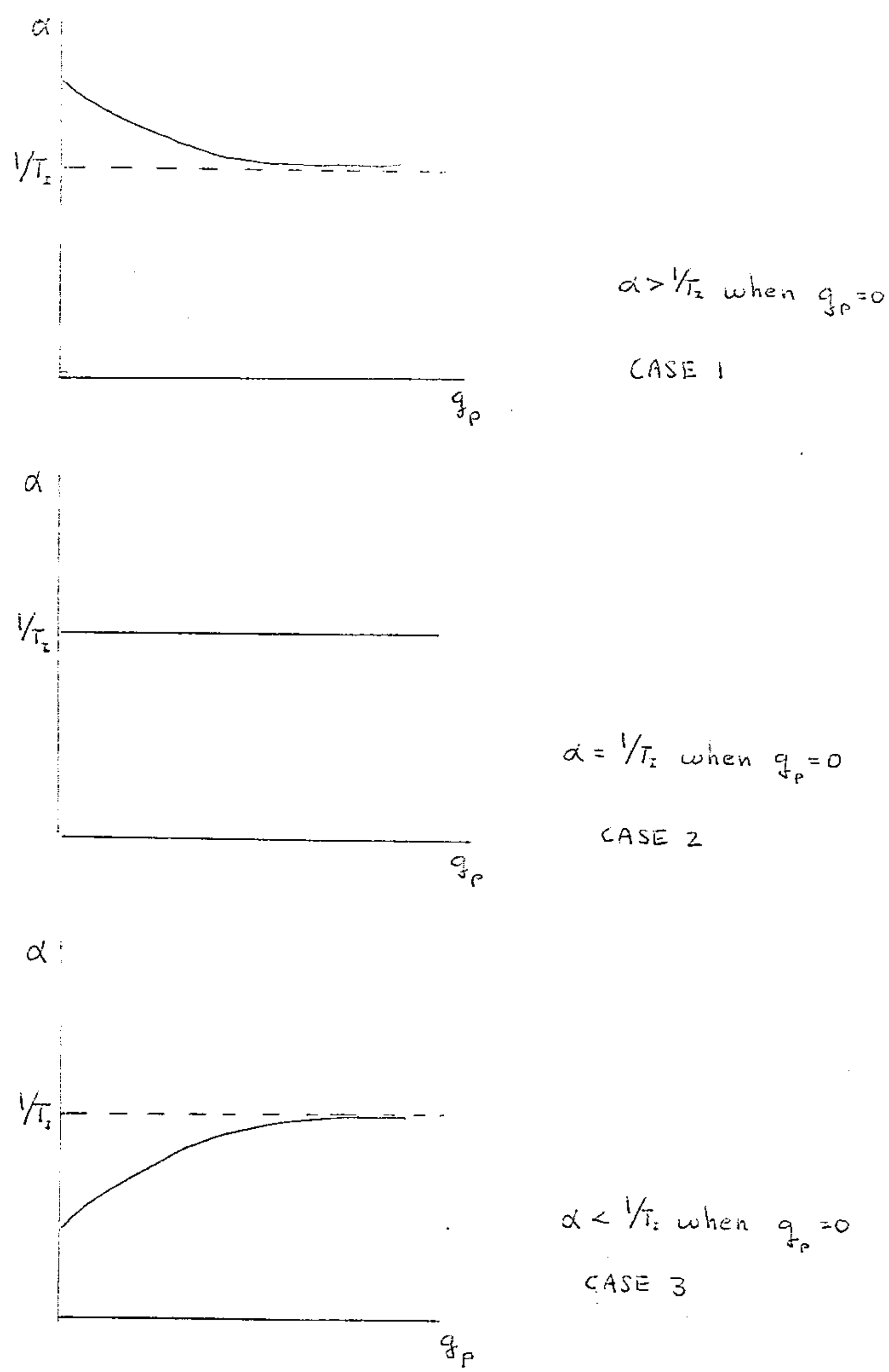

Figure 5.2. The Effect of Proportional Fiscal Policy on the Damping Factor α for Three Cases

The value of α at the vertical axis indicates the damping of the unregulated system. The three cases of $\alpha \gtreqless 0$ at $g_p = 0$ are considered and in each an increase in g_p has a different effect. The only case where government proportional policy will increase stability is case 3. Notice that this includes the situation where the unregulated system is unstable ($\alpha < 0$ when $g_p = 0$). Since $1/T_I > 0$ proportional fiscal policy alone will never make the system unstable, but it could reduce damping.

Considering the effects of proportional policy on the growth rate we see that equation (5.11) reduces to:

$$y_F = svx + vg_P(x-1).$$

In the unregulated model, $y_F = svx$ so that proportional policy has the effect of adding an additional term to the growth equation. However, the impact of the policy can not be determined simply by comparing these two expressions for growth, because the value of x will be different in each expression. To see the impact on growth, consider a situation where x is less than unity in the unregulated system; that is, $Y_A < Y_F$. In this case the proportional policy will increase demand by deficit spending since $G = -vg_p(x-1) > 0$; thus total demand Y_A will be increased. This will result in a higher value for x which will in turn induce investment causing a higher rate of growth. This effect is represented by the term svx. But we must also consider the second term. If x remains less than unity (although now greater than before) the term $g_p v(x-1)$ will have the effect of not letting the growth rate increase quite as much. This is due to the fact that government spending, as we have defined it, does not <u>directly</u> add to investment. Government spending has been entirely on consumption; the addition to investment came indirectly through the increase in the term x of the investment function. If the increase in Y_A had been accomplished in the private sector then investment would have increased by the normal amount and the growth rate would be svx. As we shall demonstrate in Appendix 2, the government deficit would increase y_F by a greater amount if G is in the form of investment, rather than consumption spending. The sum effect of this government policy has therefore been to increase the rate of growth by increasing demand and thus increasing x, but not by the same amount as an equivalent increase in demand in the private sector. The argument is similar when $x > 1$; in that case the government decreases the rate of growth by decreasing demand and thus decreasing x, but not by the same amount as an equivalent decrease in demand in the private sector.

Now let us look at the goal of full capacity output (i.e. full employment) in light of this effect of government policy on growth. The goal of full capacity is the goal of $x = Y_A/Y_F = 1$. Following the same procedure as above, if $x < 1$ then government increases Y_A. But if Y_F increases by the same amount then the gap is not decreased. However, government policy has a tendency to not let Y_F increase by the same amount as Y_A, because the term $vg_p(x-1)$ has a negative effect on y_F. If the term g_p is large enough to make Y_F increase at a slower rate than Y_A (when $x < 1$) then the economy will approach full capacity $x = 1$. Conversely when $x > 1$ we want to decrease Y_A but not by the same amount as Y_F. Therefore the positive effect of the term $vg_p(x-1)$ is added to y_F. If the term g_p is large enough to make Y_F decrease at slower rate than Y_A, then the economy will approach full capacity.

We now begin to see that effect of g_p on the growth equation is necessary if the stability of the system is to be increased by proportional policy. The effect on growth is desirable if we want x = 1. This conclusion is important because it could not be seen in the multiplier-accelerator model where Y_F is a constant.

The last point to consider about proportional fiscal policy is the effect on the steady state ratio of actual to full capacity output. By setting all derivatives in equation (5.13) to zero, the steady state ratio in the absence of other policies is

$$x_s = 1 + \frac{m_f - \delta}{\beta}$$

This is exactly what we found in the regulation case, so that the proportional policy has no effect on the steady state ratio. If we want the steady state to be consistent with the desired level x = 1, then we must adjust m_f, the constant rate of increase in the money supply. Thus we might say that the target of x = 1 is established by adjusting m_f, and proportional policy can be used to get us to that target faster.

2. Fiscal integral policy

Using the same type of analysis as used for proportional policy, we see that integral fiscal policy alone without lag has a damping factor:

$$\alpha = \frac{sv - \gamma + \rho\mu + T_I g_i v}{T_I vs}$$

Taking the partial derivative with respect to g_i we get

$$\frac{\partial \alpha}{\partial g_i} = \frac{1}{sv}$$

which is always positive. Thus the integral policy without lag will always increase the stability of the system by increasing the damping factor. If the system is already stable then the initiation of integral fiscal policy will bring the economy to the full capacity output more quickly than it otherwise would. As the value of g_i is increased the system will approach this desired level of income at faster rates. Thus, in the absence of lags integral policy is a much stronger tool than proportional policy which could only attain a maximum level of damping ($\alpha = 1/T_I$).

The effect of proportional policy on the growth equation can be written as:

$$y_F = svx + vg_i \int_0^t (x-1)dt \quad .$$

To evaluate the impact we must compute the integral. This is not possible unless we know the exact path of x, but we can consider some cases which help explain the behavior. Let us assume that $x < 1$ and that the integral policy completely stabilizes the system so that there is a steady increase in x towards 1. As in the proportional case there is an increase in the term svx. But now the negative effect of the integral policy is much larger, and in fact could be larger than the positive effect of svx. Conversely, if $x > 1$ and the system is steadily damped towards 1, then svx will have a negative influence on the growth rate and the integral expression will have a positive influence. The impact of the integral can be seen in figure (5.3) as the area between the

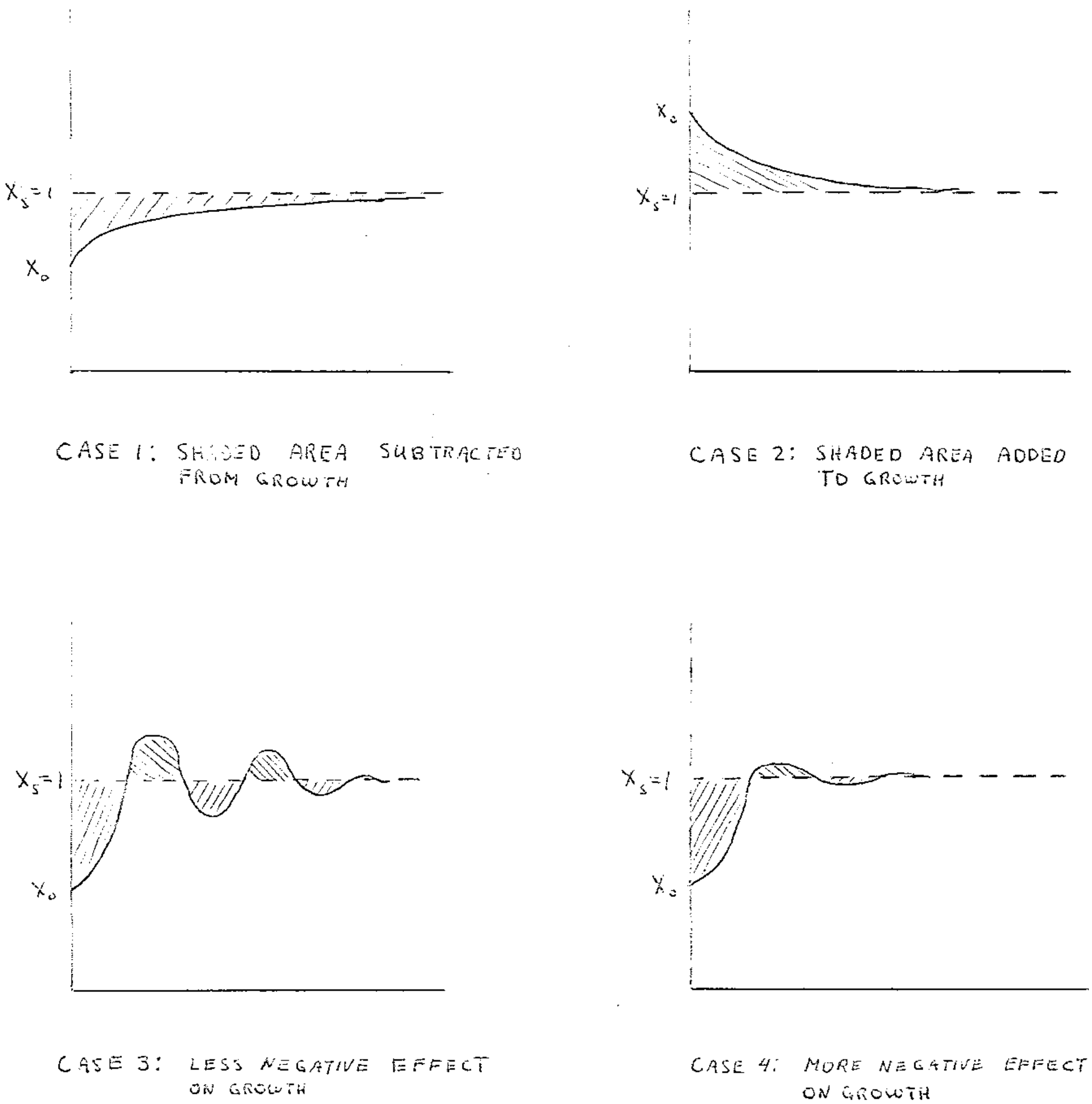

Figure 5.3. Possible Effects of Integral Fiscal Policy on the Rate of Growth.

curve x and the line x = 1. To pursue this, the bottom two graphs in figure (5.3), show that the greater the damping (i.e., the larger g_i) the greater the effect of the integral term.

It seems possible, therefore, that the integral policy as we have defined it could drastically reduce the growth rate of the model if it is initiated when the economy is in a depression ($x < 1$). (This will be shown by simulation in the next section.)

An explanation for this behavior can be found by examining the operation of the system with integral policy after the steady state has been reached. If we assume that the steady state value of x is 1, then the value of the integral will be constant (equal to the area in figure (5.3), case 1). That is, G will be a positive constant in the income equation. But I in the income equation will not be increased by G at all as can be seen in figure (5.1). The value of x is 1 so that entrepreneurs will keep investment at a constant level.[1] The result is that government policy is having no positive effect on the rate of change of Y_F: the value of DY_F is what it was before the integral policy was initiated. But Y_F is now larger because Y_A is larger and $Y_A = Y_F$. Therefore, the proportional rate of growth $DY_F/Y_F = y_F$ is smaller.

The negative effect indicates a <u>trade-off</u> consideration between stability and growth when using integral policy during a period of depression. The policy might be effective in bringing Y_A into equality

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[1] We are now assuming that the effect of interest rate on investment is negligible. If it is included then investment would be even lower because r is greater with a higher value of Y_A.

with Y_F, but the economy would be growing at an extremely slow rate afterwards. This point will be pointed out in the next section.

Integral policy also has an effect on the steady state ratio x_s. With integral policy this ratio is:

$$x_s = \frac{\rho\mu(\beta+m_f-\delta) + vg_i}{\rho\mu\beta + vg_i}$$

Taking the partial derivative of this expression with respect to g_i we get:

$$\frac{\partial x_s}{\partial g_i} = \frac{-\rho\mu(m_f-\delta)}{(\rho\mu\beta+vg_i)^2}$$

This is positive if $m_f < \delta$ (which is when $x_s < 1$ in the unregulated case) and is negative if $m_f > \delta$ (which is when $x_s > 1$ in the unregulated case). Thus, the addition of integral fiscal policy will bring the steady state ratio closer to 1. That is, the higher the value of g_i the closer the economy will be to full capacity operation in the steady state. This desirable effect must also be evaluated in light of the negative effect on the growth rate.

3. Fiscal derivative policy

When fiscal derivative policy is added to the model the differential equation becomes third order. This is true even in the case of zero lags. Therefore, an analytic investigation is not possible in simple mathematical terms. Section C.2 provides an investigation of this case by simulation.

In the limiting case fiscal derivative policy has no effect on the steady state ratio of actual to full capacity output. This can be seen by setting all the derivatives in equation (5.9) equal to zero and solving for x.

4. Monetary proportional policy

In the absence of the other policies the damping factor for proportional monetary policy is

$$\alpha = \frac{sv-\gamma + \rho\mu(g_{mp}+1)}{T_I vs}$$

Differentiating with respect to g_{mp} gives

$$\frac{\alpha}{g_{mp}} = \frac{\rho\mu}{T_I vs} > 0$$

so that the addition of proportional monetary policy with no lags will always increase stability.

As with the other <u>monetary</u> policies, the growth relationship $y_F = svx$ still holds as in the unregulated case. Thus the effect of monetary policy on growth is dependent on the effect on x. Proportional policy will stabilize the oscillation in the rate of growth, since it stabilizes x.

When $x < 1$ the monetary authority will increase the money supply which will cause interest rates to fall and increase investment. The result is an increase in the rate of growth. In our model this increase in the rate of growth is represented by an increase in x. x rises because an increase in investment causes an increase in Y_A relative to Y_F. Thus the proportional growth rate $y_F = svx$ also increases. We see then that a reduction in interest rates can increase the rate of growth in this model. Conversely if $x > 1$ the monetary authority will decrease the money supply causing interest rates to rise and investment to fall. Thus y_F decreases. This chain of events can be seen in the block diagram in Figure 5.1.

5. Monetary integral policy

Since there is no term g_{mi} in the damping coefficient, the integral monetary policy has no effect on the stability of the system. However, it will increase the frequency of the oscillations in x and thus in y_F.

As in the case of fiscal integral policy, the monetary integral policy brings the steady state ratio closer to unity:

$$x_s = 1 + \frac{m_f - \delta}{\beta + g_{mi}}$$

Therefore as g_{mi} is increased the economy will be nearer to full capacity operation in the steady state.

	NO REGULATION	FISCAL PROPORTIONAL $-g_p Y_F (X-1)$	FISCAL INTEGRAL $-g_i Y_F \int (X-1)dt$	MONETARY PROPORTIONAL $-g_{mp}(X-1)$	MONETARY INTEGRAL $-g_{mi}\int (X-1)dt$	MONETARY DERIVATIVE $-g_{mD} Dx$
DAMPING FACTOR c	$\frac{SV-\gamma+\rho\mu}{SVT_I}$	$\frac{SV-\gamma+\rho\mu+g_p V}{SVT_I+T_I g_p V}$	$\frac{SV-\gamma+\rho\mu+T_I g_i V}{T_I VS}$	$\frac{SV-\gamma+\rho\mu+\rho\mu(g_{mp}+1)}{T_I VS}$	$\frac{SV-\gamma+\rho\mu}{T_I VS}$	$\frac{SV-\gamma+\rho\mu}{T_I VS+\rho\mu g_{mD}}$
$\frac{\partial c}{\partial g}$	—	$\frac{\gamma-\rho\mu}{T_I V(S+g_p)^2}$	$\frac{1}{SV}$	$\frac{\rho\mu}{T_I VS}$	0	$\frac{T_I VS+\rho\mu g_{mD}-(SV-\gamma+\rho\mu)\rho\mu}{(T_I VS+\rho\mu g_{mD})^2}$
STABILITY INCREASED IF:	—	$\gamma>\rho\mu$	ALWAYS	ALWAYS	NO EFFECT	$\alpha<0$
GROWTH RATE y_F	SVX	$SVX+Vg_p(X-1)$	$SVX+Vg_i\int_0^t (X-1)dt$	SVX	SVX	SVX
STEADY STATE RATIO X_s	$1+\frac{m_s-\delta}{\beta}$	$1+\frac{m_s-\delta}{\beta}$	$1+\frac{\rho\mu(m_s-\delta)}{\rho\mu\beta+Vg_i}$	$1+\frac{m_s-\delta}{\beta}$	$1+\frac{m_s-\delta}{\beta+g_{mi}}$	$1+\frac{m_s-\delta}{\beta}$

Figure 5.4. The Effects of Various Policies with Zero Lag: Analytic Results.

6. Monetary derivative policy

In this case the damping factor is:

$$\alpha = \frac{sv-\gamma + \rho\mu}{T_I vs + \rho\mu g_{mD}}$$

The partial derivative of α with respect to g_{mi} is positive if $\alpha < 0$ and negative if $\alpha > 0$. Thus if the system is explosive the introduction of monetary derivative policy will make the system less explosive, but it will never stabilize the system. If the model is stable already, then the addition of derivative policy will decrease damping.

When Y_A is increasing relative to Y_F then the derivative policy will increase the money supply and thus reduce the interest rate. This will result in increased investment and a higher rate of growth y_F. The converse is true when Y_A is decreasing relative to Y_F. This will stabilize, or destabilize the system depending on the parameters.

The examination of all policies together is more difficult, because of the wide variety of combinations which might be tried. Therefore simulation is used to investigate the combined effects. This will be done in the following section. The results of section B are summarized in Figure 5.4.

C. <u>Investigation By Simulation</u>

In this section the model is simulated with numerical values given to all the parameters. The simulation techniques used are outlined in Appendix 1. There are several advantages in using such simulation techniques. First, the exact paths of the variables of the model can be determined so that we can find their values at any point in time, not just the general behavior and the steady state solution as was done in section B. This will give us more insight into the behavior of the model even when it is of second order. A second advantage is that we can determine the behavior of the model when the differential equation is higher than second order. The behavior would be almost impossible to determine analytically for the fifth order equation which arises when fiscal lags and derivative policy are considered.

The numerical values chosen for this simulation are those suggested by Phillips and Allen in their descriptions of the cyclical growth model.[2] The values are such that the time unit assumed is one year. For example, the value of the output-capital ratio assumes yearly output. Some of these parameters will remain unaltered throughout the analysis. These will be:

$$s = 0.1$$
$$v = 0.25$$
$$T_I = 1.0$$
$$\gamma = 0.1$$
$$\delta = 0.03$$

The other parameters will be adjusted to consider the various general cases of the model's behavior. By considering different values of $\rho\mu$ we can make the unregulated system stable or unstable; variations in β can determine either a steady or oscillating path.

In addition, by changing the value of m_f we can determine the effect of a change on the constant rate of money supply. The value of T_G can be varied to examine the effects of fiscal lags. Finally, the values of the stabilization policy parameters can be altered: g_p, g_i, g_D, g_{mp}, g_{mi}, g_{mD}.

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[2]Allen, op. cit. (1967), has a good description of the Phillips model in his last chapter on cyclical growth.

1. The unregulated system

In Chapter 3 we discussed the model of cyclical growth as developed by Phillips when there was no government stabilization policies. We saw how different values of the parameters would produce different time paths for the variables. The first simulation presented is of this unregulated model. Using the parameter values given above, the model should be explosive if $\rho\mu < 0.075$; and damped if $\rho\mu > 0.075$. In addition it will oscillate or move steadily depending on the value of β. Trying each of these four possibilities we see in Figure 5.5 that the simulation supports our analytic conclusions. These exact paths can be compared to the general paths of x in Figure 3.3. In addition we can also follow the paths of the proportional growth rate and the interest rate. Finally as an example of what the output actually looks like, a graph plotted by the digital computer is provided in Figure 5.6. Since the smallest increment on this graph is one print line, the curves seem discontinuous. However, the actual values of each parameter are also given by the computer and represent much smoother curves. The value of $\rho\mu$ in Figure 5.6 is 0.075. Therefore, the oscillations are neither damped nor explosive but of constant amplitude. The output ratio is plotted with the character (x), growth rate y_p = (*), and interest rate r = (R). The values of the growth rate and the interest rate are much smaller in magnitude than the ratio of actual to full capacity output. Therefore, they are plotted on a magnified scale relative to x.

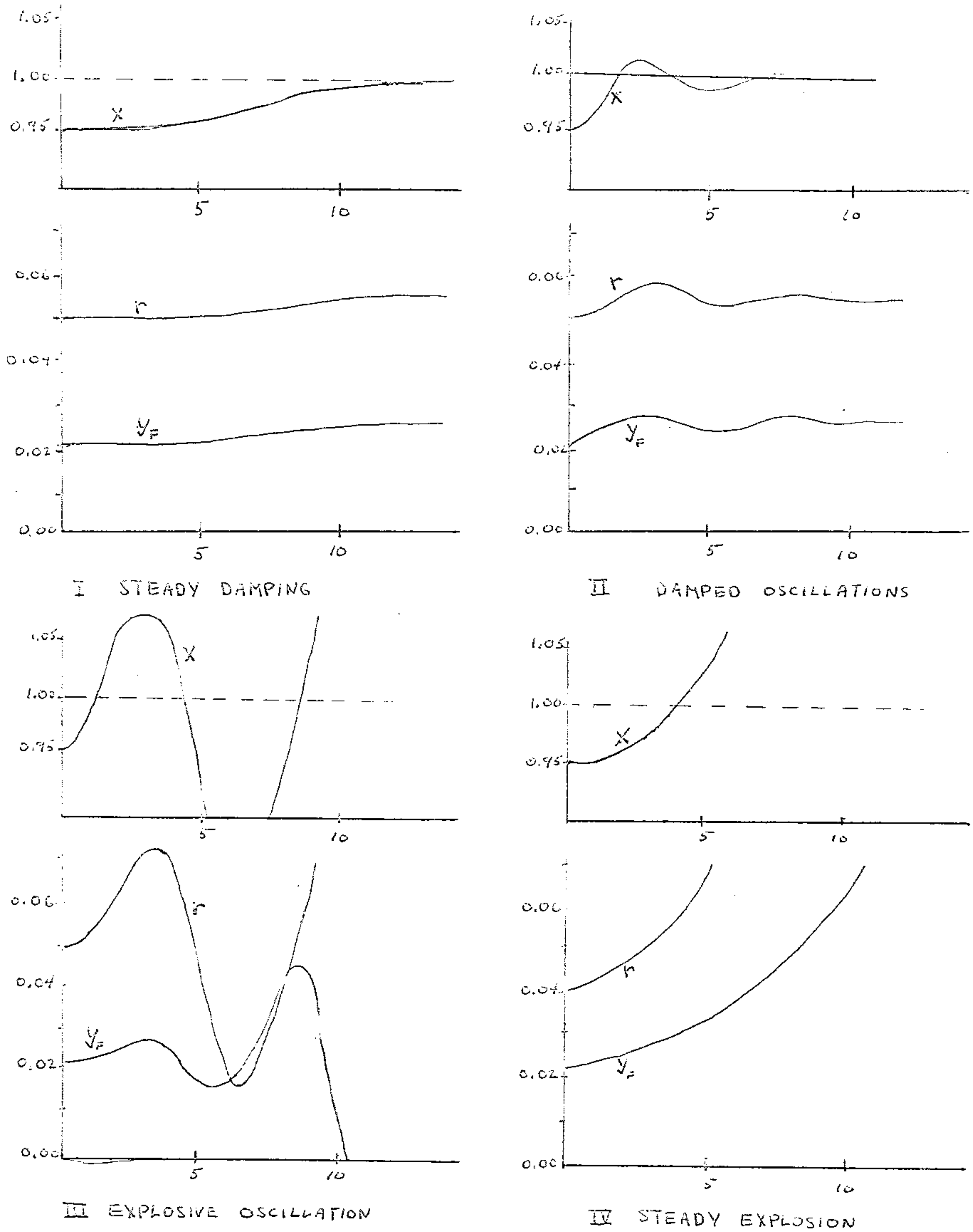

Figure 5.5. Simulation of the Four Types of Behavior of the Unregulated System.

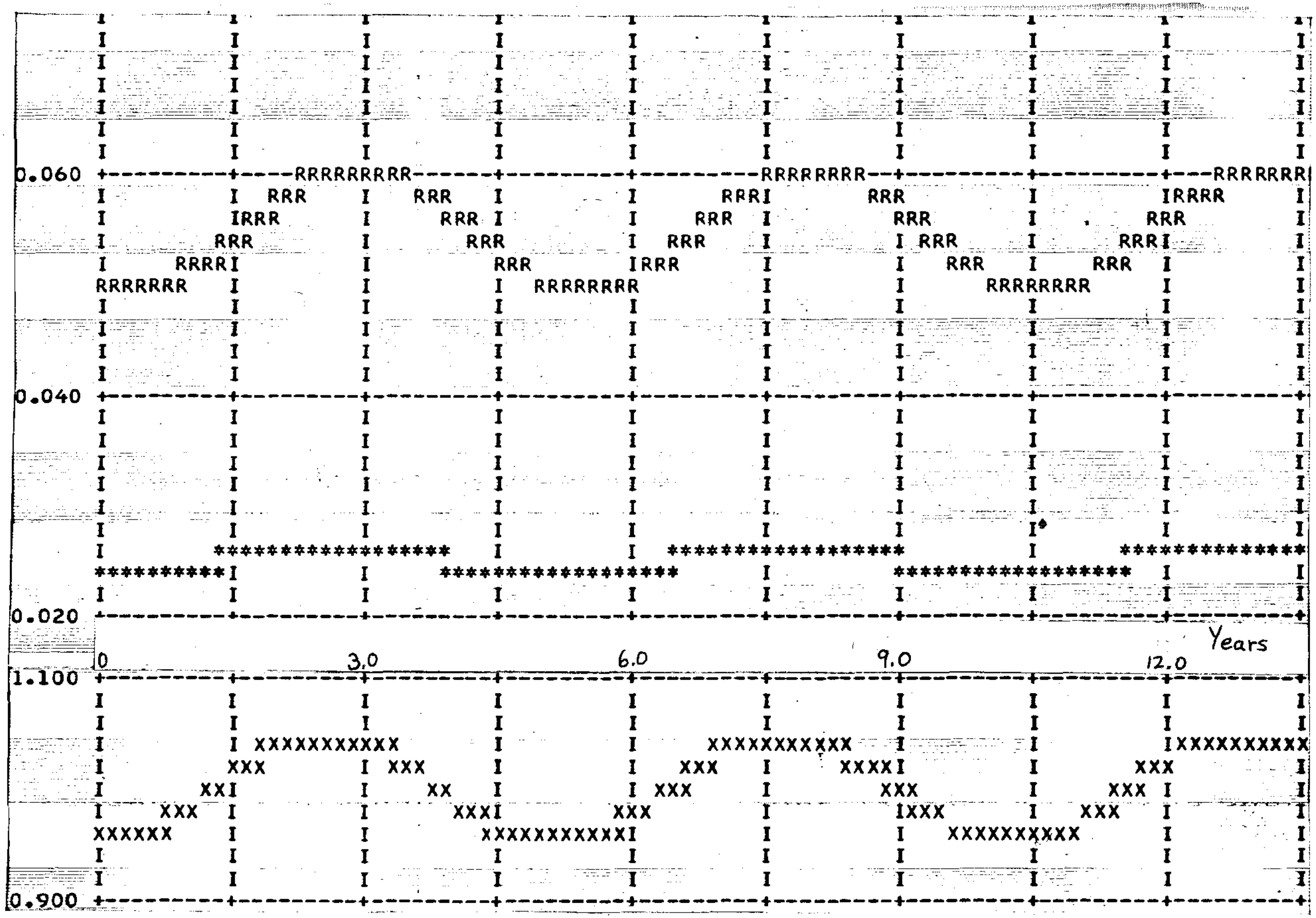

Figure 5.6. Simulation of the Unregulated Economy with a zero Damping Factor.

The initial values[3] for these simulations are

$$x(0) = 0.95$$
$$Dx(0) = 0$$
$$y_F(0) = svx(0) = 0.02375$$
$$r(0) = 0.05$$

$D^2x(0)$ is then determined from these according to the values of the parameters. Notice that the period of all these oscillations is about 5 years, except that r is slightly lagged behind x and y_F. This can be seen in equation (3.9) where the rate of change of r depends on (x-1) as well as the rate of change of x. When x starts its downturn there is still a net positive effect on Dr, because the positive effect of (x-1) is greater than the negative effect of Dx. As x and Dx continue to decrease the net effect on Dr will become negative and r will then begin its downturn. There is a similar effect when x begins its upturn at the bottom of the cycle.

The economic aspect of this point is that high interest rates persist for a while after a boom and similarly low interest rates persist for a while after a depression. This might indicate that interest rates are not good indicators of future economic behavior.

These are the possible types of behavior of the unregulated model. We will now proceed to examine the effects of stabilization policies. The policies will aim at reducing the amplitude and frequency of a system like Figure 5.6. Graphs similar to Figure 5.6 are provided as illustrations in the more interesting cases.

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[3] Different initial values would alter the amplitude and frequency of the oscillations. For example with x(0) = 0.9 the amplitude would be about twice as large.

2. The simulation of the regulated cases discussed in section B

In section B of this chapter the impact of the fiscal and monetary policies with no lags was examined analytically. Using simulation we can extend that discussion by examining the path of the variables before the steady state is reached.

First, consider the case where the unregulated economy is completely explosive. The impacts of the various policies on stability are shown in Figure 5.7. This is in agreement with the analysis of section C. Fiscal integral policy (g_i) has the most desirable effect: the unstable system is quickly brought to full capacity output as $x \rightarrow 1$. Monetary and fiscal proportional (g_{mp} and g_p) policies also stabilize the system but require a much longer time to reach full capacity. Monetary derivative policy increases damping but not enough to stabilize the economy. Finally monetary integral policy does not influence stability at all; in fact it has the undesirable effect of increasing frequency.

In the unstabilized model the effect of stabilization policies on the growth rate is difficult to evaluate because the system never reaches an unregulated steady state. Of course those policies which stabilize the system have a beneficial stabilizing effect on growth.

For a more meaningful evaluation of the impacts of fiscal and monetary policy on the proportional growth rate we will choose parameters which make the unregulated system produce cyclical growth of constant amplitude, i.e. when the damping factor is zero. This case was simulated in Figure 5.6. The model oscillates around steady state values of $x_s = 1$, $y_{Fs} = 0.025$, and $r_s = 0.055$. Stabilization policies are applied to the model and the effects on damping and the time paths of the variables are determined.

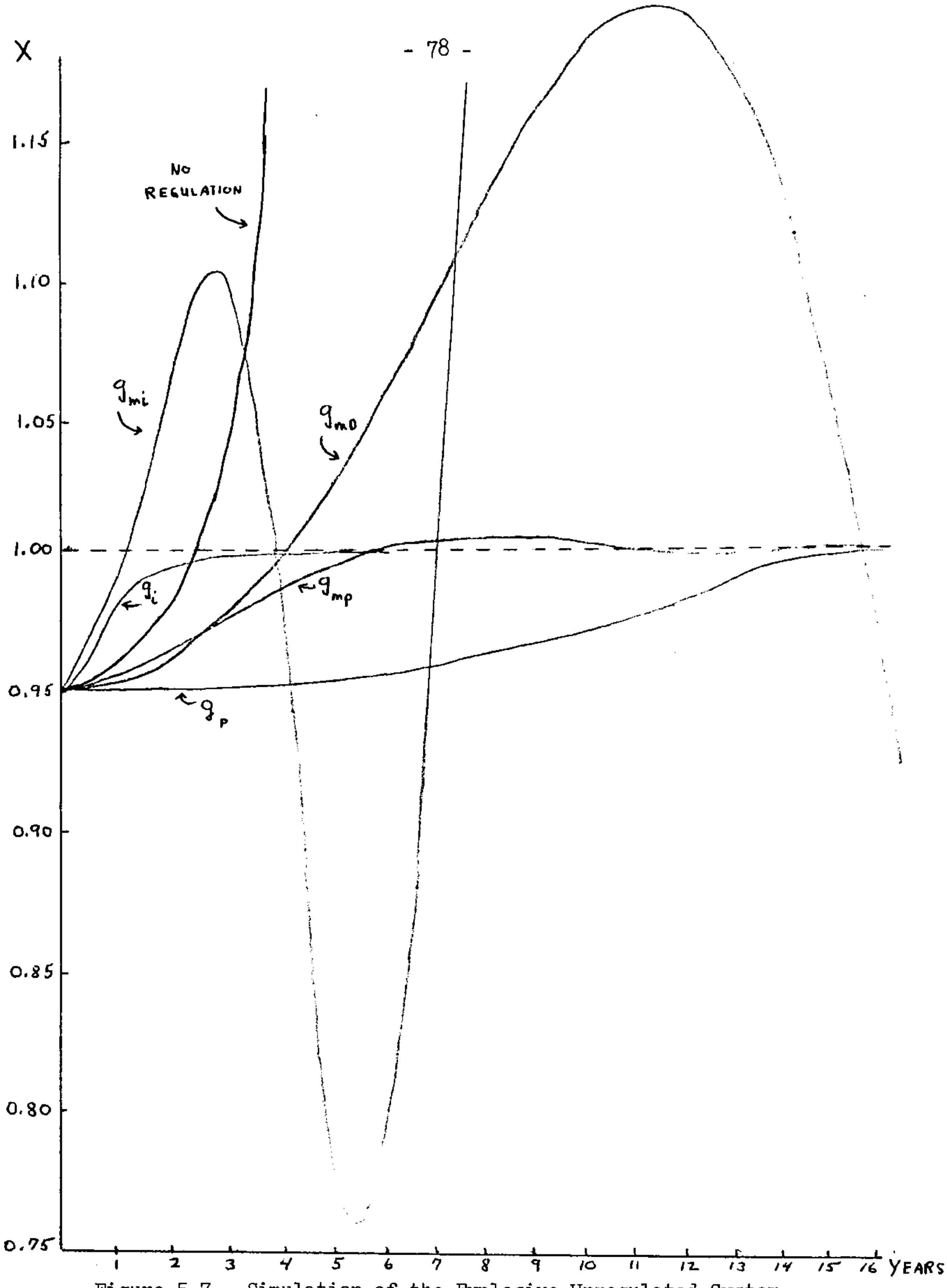

Figure 5.7. Simulation of the Explosive Unregulated System Compared with the Effect of the Various Stabilization Policies. Zero lag.

Several magnitudes of the policy coefficient were tried for each type of policy. In general it was found that the higher the value of the coefficient (i.e. more powerful government policy) the more magnified were the impacts on stability and growth; if the impacts were harmful it was found that a more powerful policy was more harmful; and conversely, if the impacts were desirable, a powerful policy was more desirable. In some cases the impacts on stability could not be increased beyond a certain limit, as was shown for fiscal proportional policy in section B.1.

Figure 5.8 shows the effect of a fiscal proportional policy with coefficient $g_p = 0.5$. This and the graphs for the other policies can be compared with Figure 5.6 of the unregulated economy. Fiscal proportional policy has very little effect on damping x, however it does decrease the frequency of the oscillations. The period here is about 24 years, as compared with 5 years in the unregulated model. (Figure 5.8 only shows a little more than half of one cycle.[4]) The most interesting effect here is the greatly increased amplitude and the higher steady state value for the proportional growth rate. The reason for the increased amplitude as discussed in section B, is that the policy results in an larger increase in the rate of growth of Y_F when $Y_A > Y_F$ or $x > 1$; and larger decrease in the rate of growth of Y_F when $Y_A < Y_F$. In other words the effective coefficient of x in the growth equation is increased, so that with similar fluctuations in x the fluctuations in y_F will be much larger.

[4]The solution was carried out over the whole cycle but only this section is plotted.

This higher value for $y_{Fs} = 0.056$ is very appealing despite the increase in the amplitude of the fluctuations. But this value is partially due to the way in which the proportional policy with no lag was initiated at $t = 0$. What we have assumed is that the unregulated economy was fluctuating as in Figure 5.6 when suddenly the government policy was added. This sudden initiation occurred at $t = 0$ in Figure 5.8, so that we can look at the economy to the left of $t = 0$ as that represented in Figure 5.6. The value of y_F to the immediate left of $t = 0$ is given by $y_F = svx = 0.2375$. Therefore we set y_F to the same value at $t = 0$. However, now the equation for growth is $y_F = svx + vg_i(x-1)$; the result is that there is a discrepancy between the value of y_F given by the new growth equation and y_F given at $t = 0$. The numerical simulator eliminated the discrepancy by adding a constant to the growth equation. Thus:

$$y_F = 0.2375 = svx + vg_i(x-1) + 0.025.$$

This constant 0.025 is then the amount by which the steady state value of y_F is raised. In economic terms the effect of this constant can be interpreted as assuming that the full effects of the fiscal policy throughout the economy occur instantaneously. In other words this large increase in the growth rate is partially a consequence of our assumption of zero lag. As we shall see in the next section when lags are considered, the effect on the growth rate is still positive but not by the same amount. We therefore interpret the increase in y_F, as shown in Figure 5.8, to be an exaggeration of the actual value.

Fiscal proportional policy increases the amplitude of the interest rate fluctuations, with no change in the steady state value.

It was mentioned earlier that fiscal integral policy can have a negative effect on the rate of growth. This is clearly demonstrated in Figure 5.9. The policy is effective in completely stabilizing the economy at full capacity output, but the consequence is a drastically decreased rate of growth. The more powerful fiscal integral policy gets, the more pronounced is this negative effect. Thus, there is a definite <u>trade off consideration</u> between fluctuations and growth when a fiscal integral policy is used when the economy is below full employment. The implications on public policy could be important in this case.

By combining fiscal proportional and integral policy we find that much of this negative effect on the growth rate is eventually eliminated. However, as mentioned above some of positive impact of fiscal proportional policy on growth is exaggerated because of the assumption of zero lag. In Figure 5.10 the paths of the variables are plotted for this combined policy.

The most promising type of monetary policy as indicated by simulation was a monetary proportional policy. As shown in Figure 5.11, the policy effectively damps the oscillations with no negative effect on the growth rate or the interest rate in the steady state. The fluctuations of the growth rate and the interest rate are also reduced..

The impacts of monetary integral and derivative[5] policies are minimal, except on frequency. The monetary derivative policy tends to lower the frequency and monetary integral tends to increase it.

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[5]Monetary derivative policy has no effect because the unregulated system has $\alpha = 0$ damping factor, and therefore g_{mD} does not affect stability as shown in section B.

As a final example of these policies with no lags, Figure 5.12 represents a combination of fiscal and monetary policies. Comparing this with Figure 5.10 which had the same fiscal policy with no monetary policy we see that the combined effect of the monetary policy is almost insignificant when compared with the fiscal policies. Both graphs are approximately the same. However, the actual numbers which represent these plots indicate that the growth rate and the interest rate have a tendency to be slightly higher with the addition of the monetary policy.

For more accurate evaluation of these policies we must consider the effect of lags. But first let us consider fiscal derivative policy which was not examined analytically.

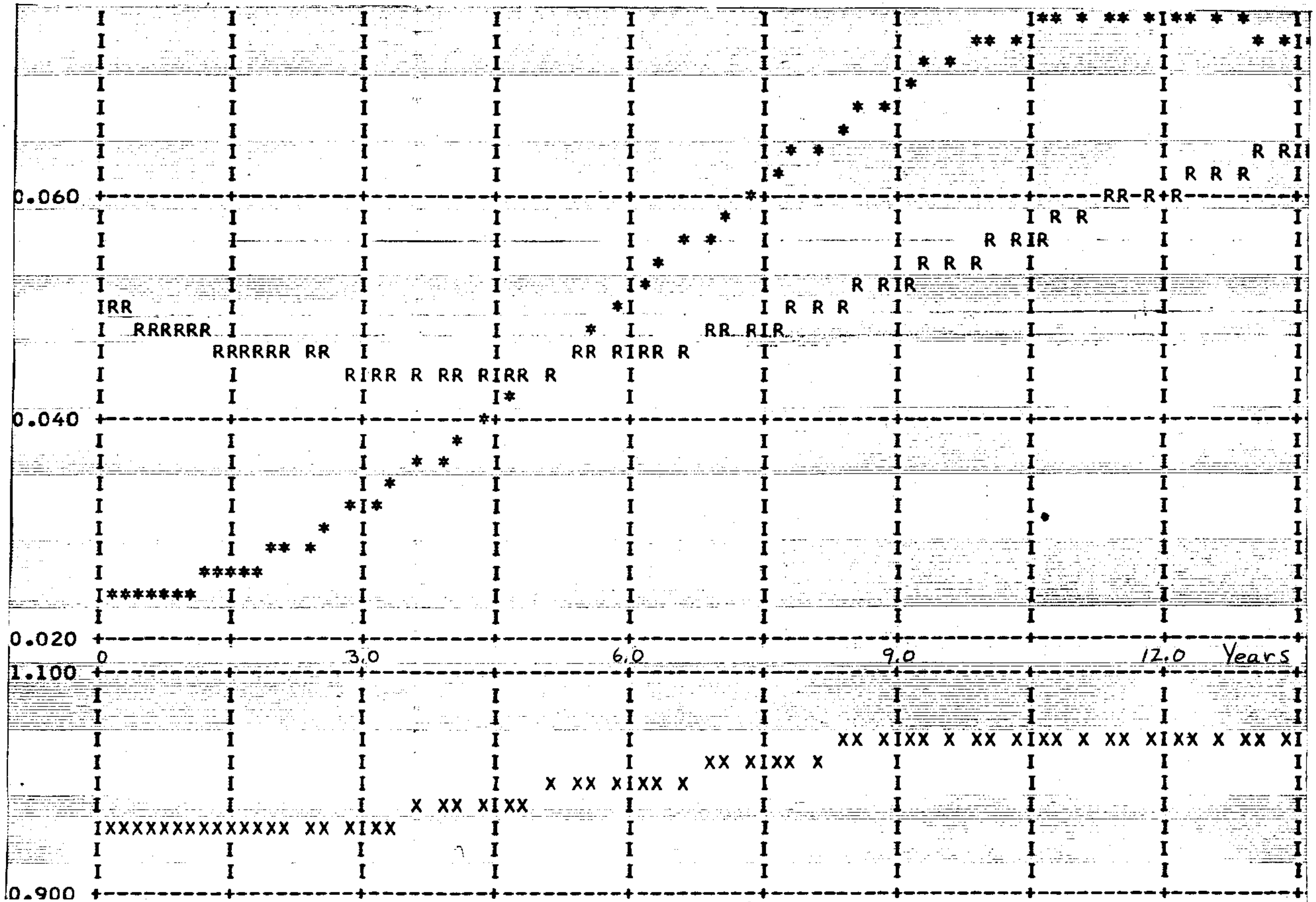

Figure 5.8. Simulation of the Model Regulated By Fiscal Proportional Policy. Zero lag. $g_p = 0.5$.

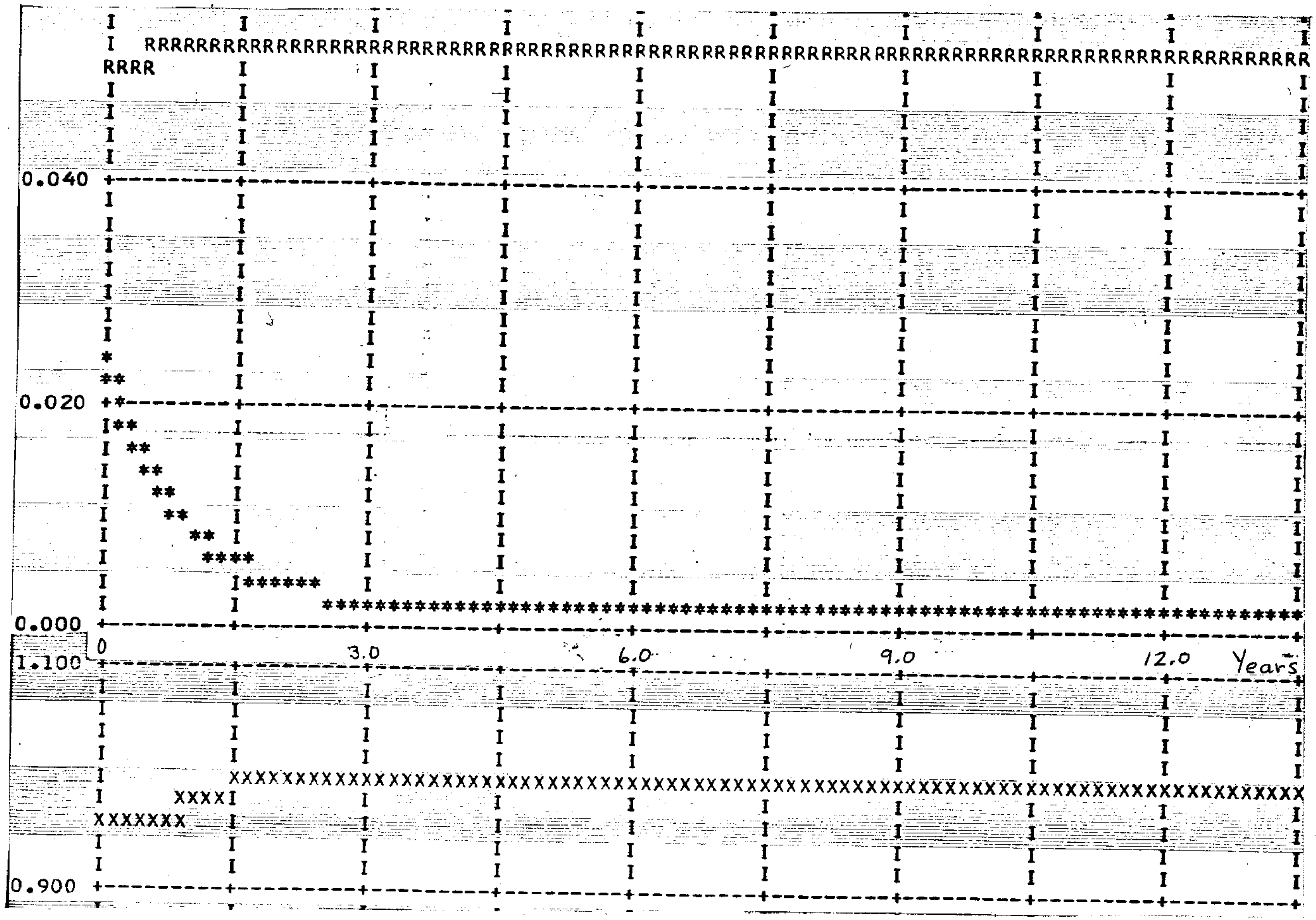

Figure 5.9. Simulation of the Model Regulated By Fiscal Integral Policy. Zero lag. $g_i = 0.5$.

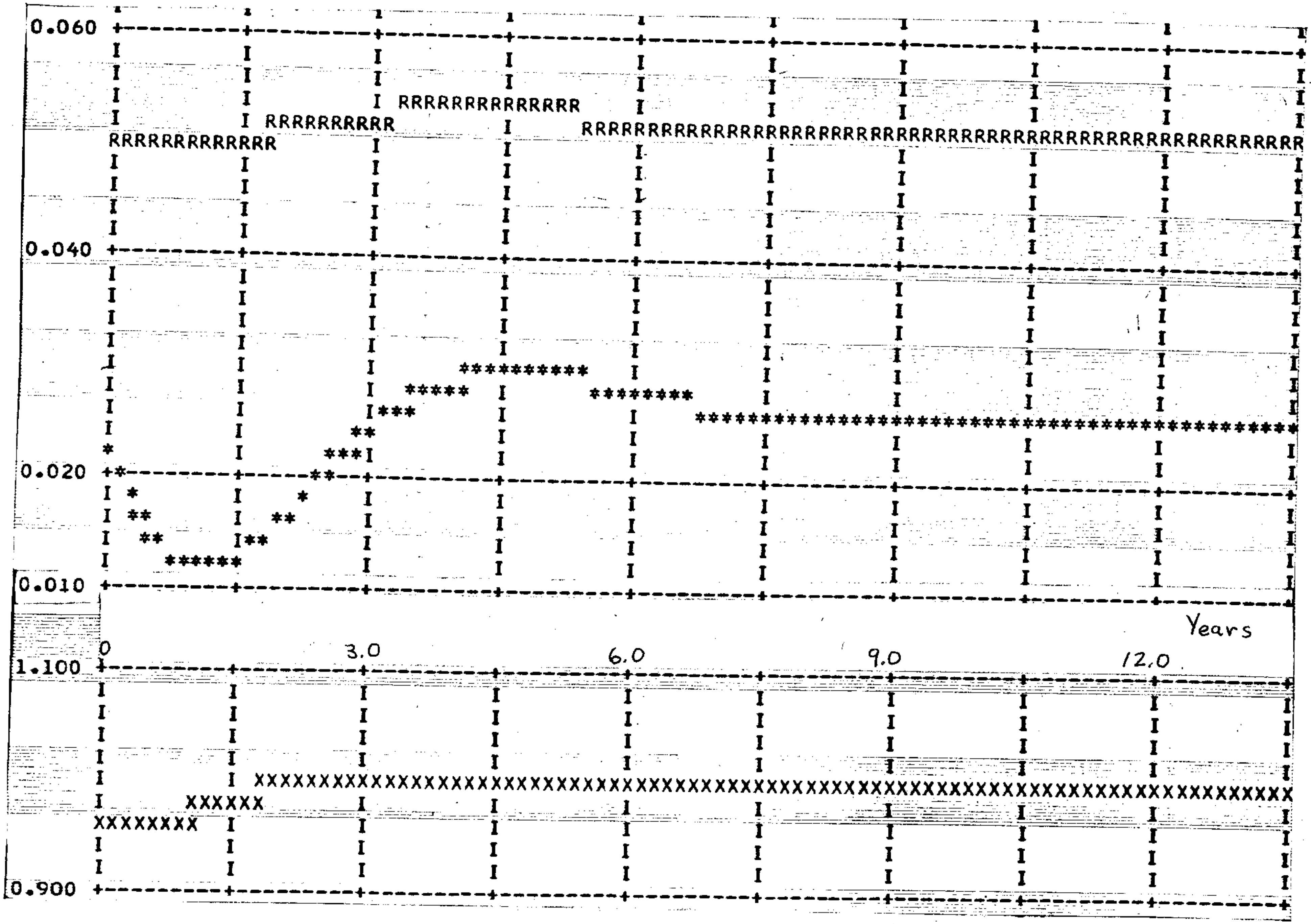

Figure 5.10. Simulation of the Model Regulated By Fiscal Proportional and Integral Policies. Zero lag. $g_p = g_i = 0.5$.

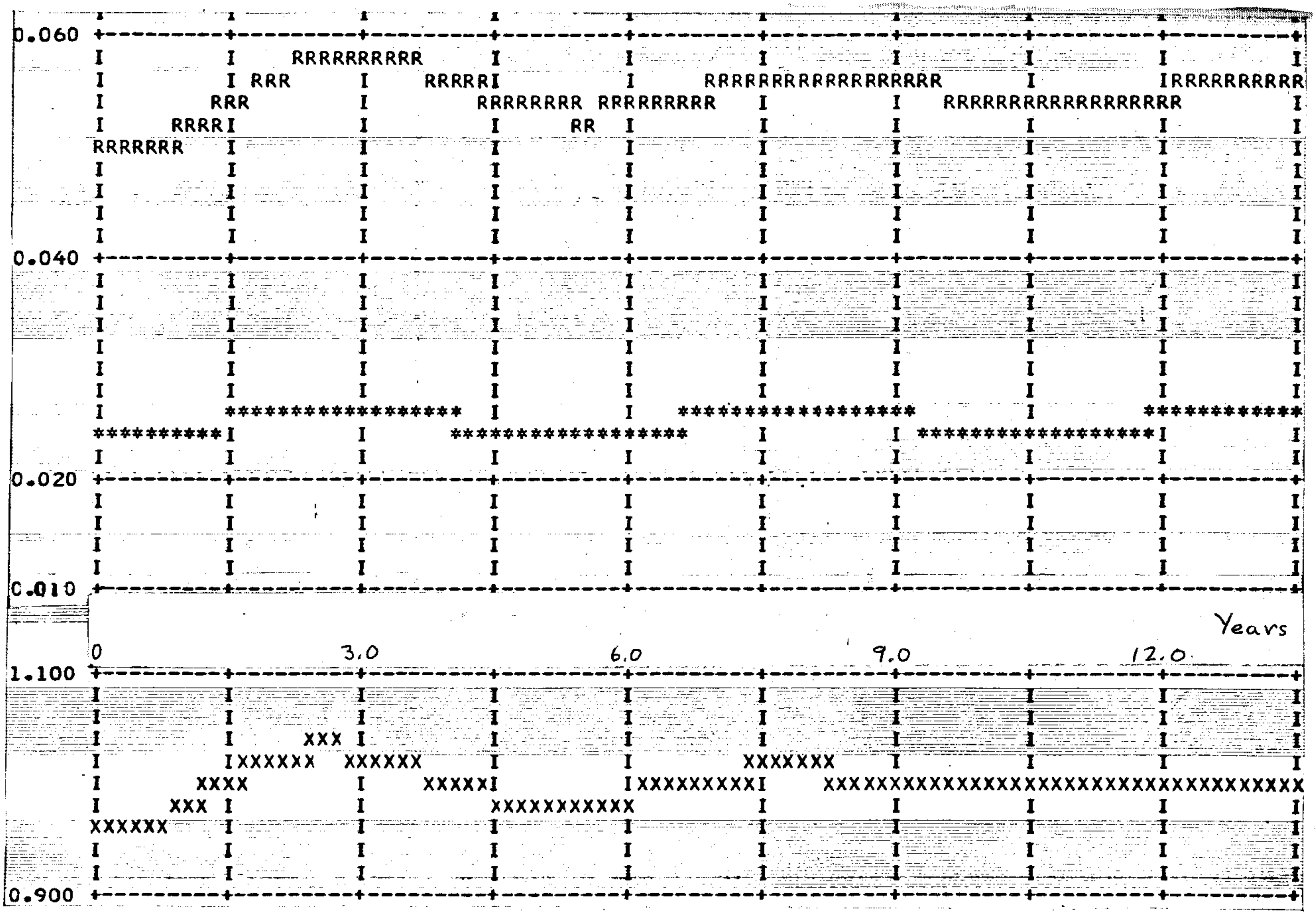

Figure 5.11. Simulation of the Model Regulated By Monetary Proportional Policy. Zero lag. $g_{mp} = 0.5$.

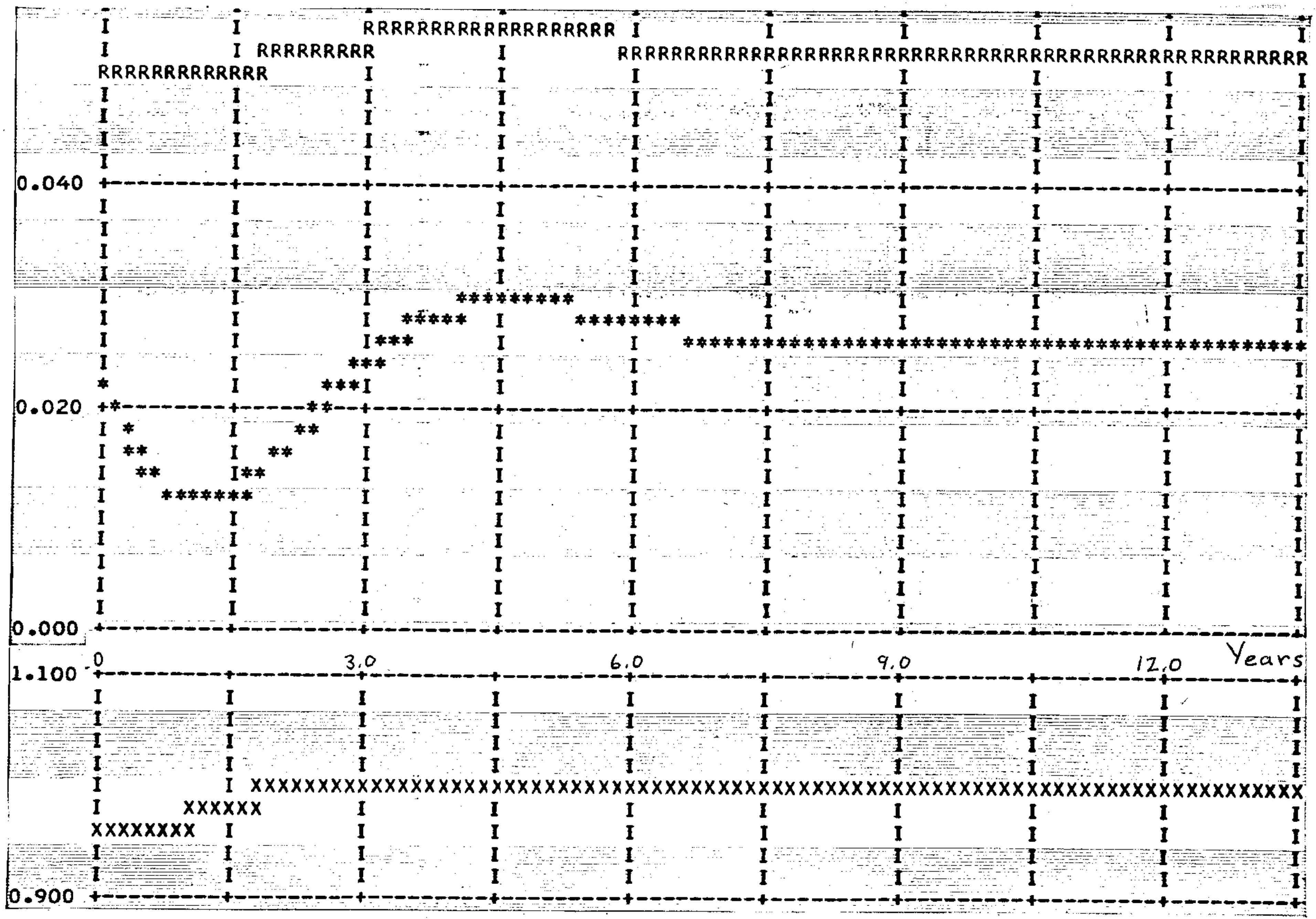

Figure 5.12. Simulation of the Model Regulated by Fiscal and Monetary Policy. Zero lag.
$g_p = g_i = g_{mp} = g_{mi} = g_{mD} = 0.5$.

3. Fiscal derivative policy with no lag

When this type of government regulation is applied to the model, the differential equation is of third order so it is desirable to use simulation to examine the impacts. Figure 5.13 shows the effect of fiscal derivative policy equal to 0.5. The policy will damp the system

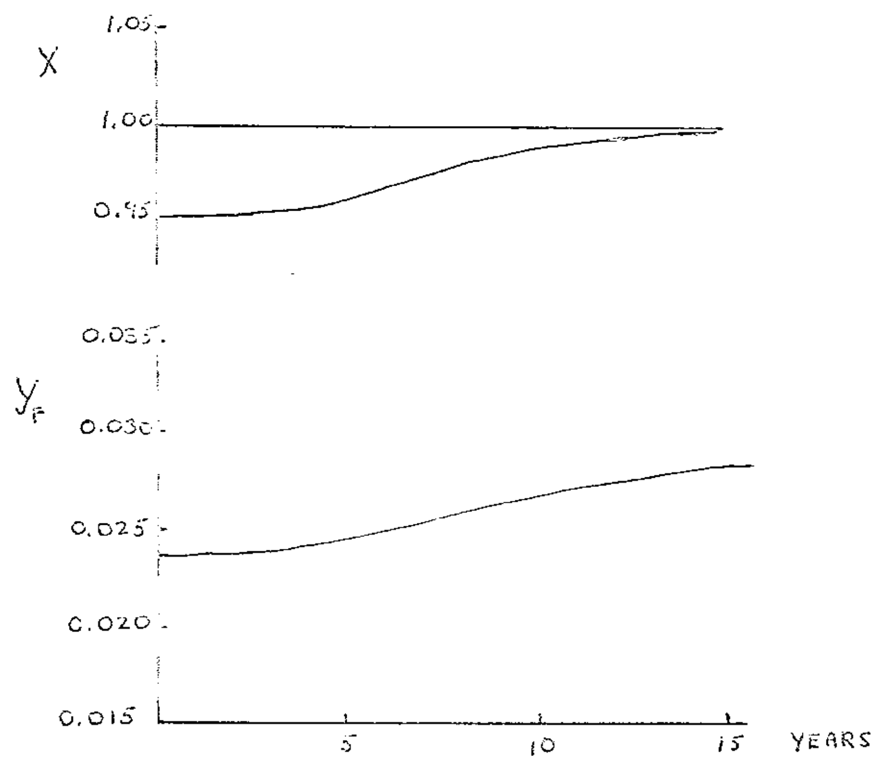

Figure 5.13. Simulation of the Model Regulated by Fiscal Derivative Policy with Zero Lag. $g_D = 0.5$.

considerably. However, there is a very slow approach to full capacity level of output. It takes almost 15 years before actual output is significantly close to full capacity output! In an equivalent time, the unregulated model has undergone three business cycles so that an average value of $x = 1$. Here the average value is less than one. The derivative policy can iron out fluctuations but it leaves the economy at a less than full capacity level for a long time. Higher values of g_D resulted in similar though slightly improved results. The derivative policy will add to the growth rate as x increases because Dx is positive. This factor will eventually die out as x approaches unity and Dx approaches zero. The derivative fiscal policy is not therefore significantly beneficial when used alone. The long time necessary to achieve full capacity operation is a disadvantage. However, when derivative policy was applied to the multiplier-accelerator models with no growth, it was done so in conjunction with the other policies. We will consider this aspect below when discussing fiscal lags.

4. The effects of lags in fiscal policy

When we drop the assumption that there are no lags in fiscal policy, the differential equation of the model becomes third order and growth equation is of fifth order. Using simulation we can examine the effect of lags of varying length, by adjusting the value of T_G, the time constant of the exponential fiscal lag.

When fiscal proportional policy with a lag is introduced to the system the behavior of the model is different than in the no lag case. The frequency of the oscillation is decreased with the addition of a lag and as the value of the lag is increased the frequency continues to decrease. The amplitude of the oscillations is not influenced by the length of the lag. Thus the path of x is much as in Figure 5.8 except that there are oscillations. For a lag of 6 months ($T_G = 0.5$), the frequency is about twice that of Figure 5.8, i.e. the period is about 15 years. For a lag of 3 years the period is about 5 years, or approximately what it was with no regulation at all.

The addition of a lag also reduces the increase in the steady state growth rate for proportional fiscal policy. With a lag of 1 year the system was initiated as in Figure 5.8 and the steady state value of y_F was 3.2% as compared with 5.3% with no lags. Nevertheless the value is still higher than without regulation.

The effect of lag in the fiscal integral policy is to greatly increase the frequency of the oscillations in the variables, and to reduce the growth rate even more drastically than without lag. As the length of the lag increases, the frequency of the oscillations is increased even further. Figure 5.14 illustrates fiscal integral policy with a lag of one year. The period of each business cycle (oscillation of x) is two and a half years. Comparing this with Figure 5.9 we see that the addition of a lag in the integral policy makes quite a difference in the behavior of the system. The growth rate in this case is reduced to 0.1% per annum!

It was found that the effect of lags on fiscal derivative policy is small. As the lag is increased the system begins to oscillate at higher frequencies, but even with a lag of three years the periods of oscillation are as large as fifteen years. The derivative fiscal policy is capable of damping the system if the system is already stable. Simulation shows that the growth rate is slightly reduced with the introduction of fiscal derivative policy.

When all three fiscal policies are used in conjunction we can arrive at some desirable stabilization effects even with fiscal lags. Figure 5.15 illustrates the behavior of the model when $g_p = g_i = g_D = 0.5$. with a fiscal lag of one year. This combination will effectively damp the system and eliminate the business cycle completely after only four years. However, there is a great negative effect on the growth rate as the simulation shows. The resulting lower growth rate would be such a severe disadvantage that such a policy as this could never be seriously considered.

If we eliminate, or reduce, the size of the coefficient of the fiscal integral policy, this negative effect on growth disappears. But the resulting combination of fiscal derivative and proportional policy does not damp the system, and we have cycles of similar amplitude but much longer periods as in the unregulated case.

The addition of monetary policy (all $g_m = 0.5$) does not improve the growth rate significantly when integral policy is used in the system. When all six policies are used together with a fiscal lag of one year, the behavior of the system is very similar to that of Firure 5.15 where we had no monetary policy, except that the damping is slightly increased.

In an effort to alleviate this situation of a lower growth rate, the constant rate of money supply (around which monetary policy fluctuates) was increased from 3% to 5%. The result of this action is favorable on the growth rate. Figure 5.16 shows that when m_r is increased to this value that the system as regulated by all policies (all coefficients 0.5) with a fiscal lag of one year will eventually produce a higher rate of growth than with the lower value of m_f. The interest rate is greatly decreased as a result of this action. Simulation of the rate of price change indicated that prices went up considerably which is a logical result of such a large increase in the money supply.

The most important conclusion from this simulation is that there is no simple method of determining the most desirable policies to use for fluctuations and growth. There are great numbers of combinations of policies which can be considered and the impacts of these policies are altered by various assumptions about lags. The simulation supported our analytic examination and gave additional insight into the operations of the model as regulated by fiscal and monetary policy.

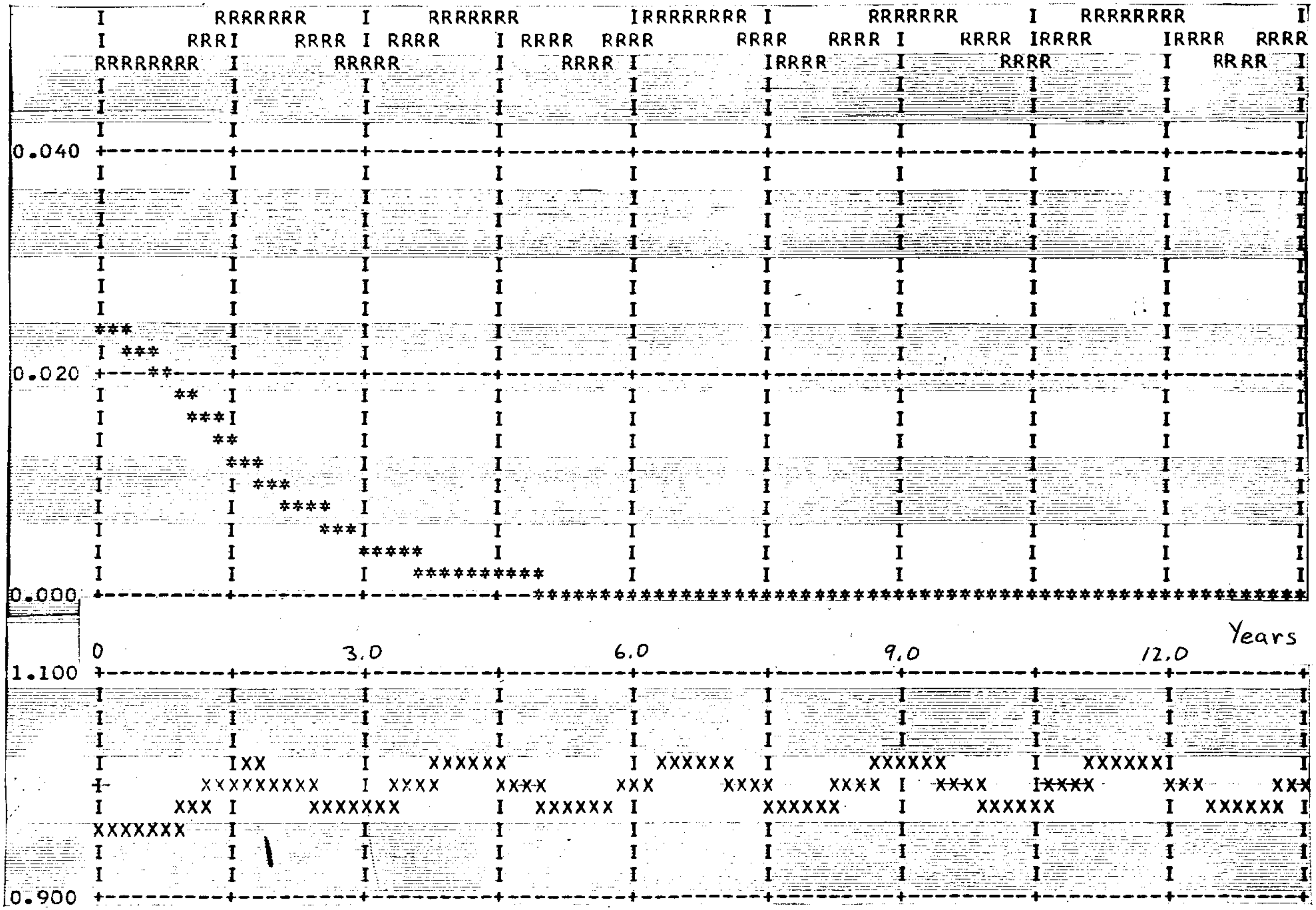

Figure 5.14. Simulation of the Model Regulated by Fiscal Integral Policy with one year lag. $g_i = 0.5$.

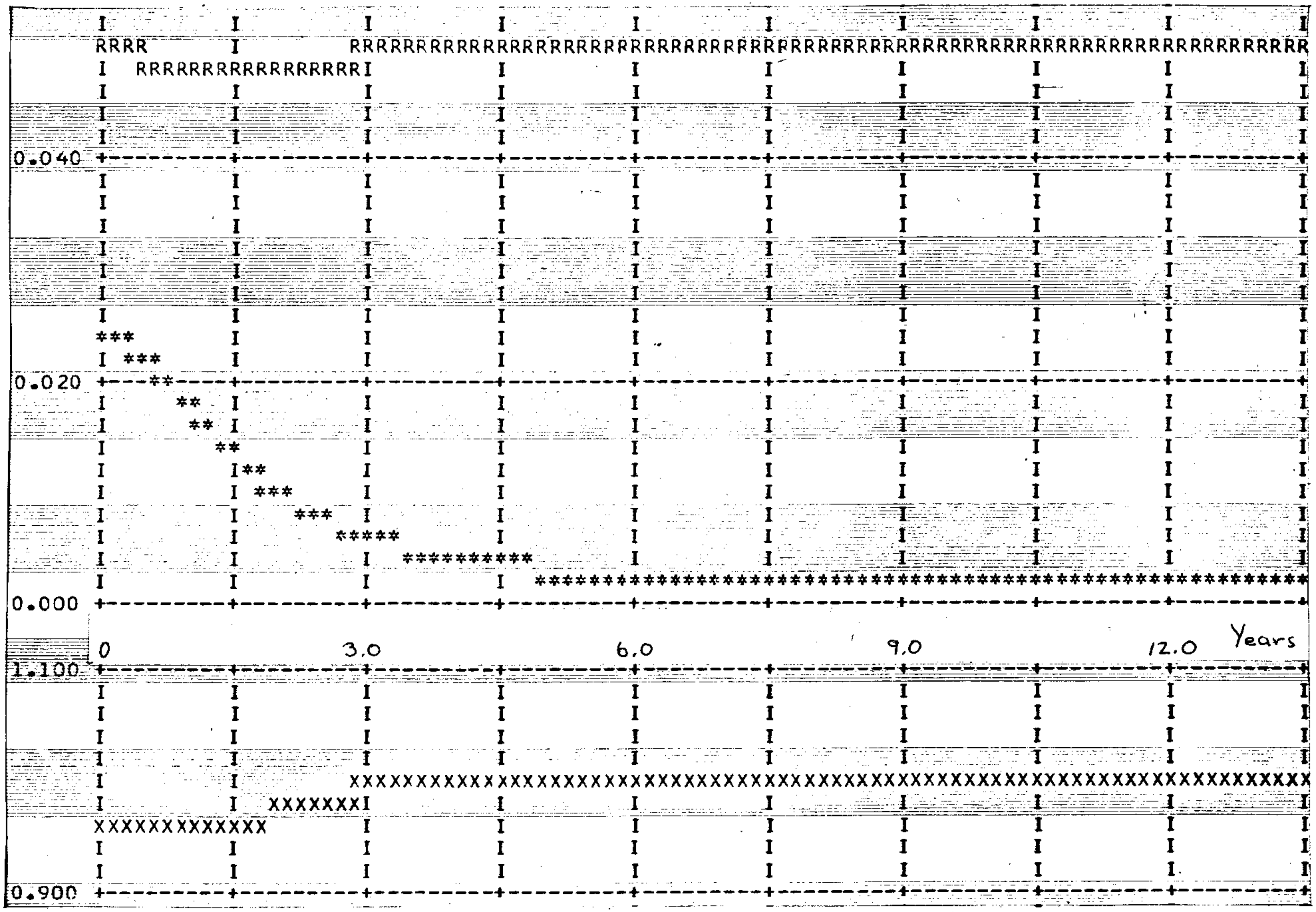

Figure 5.15. Simulation of the Model Regulated by Full Fiscal Policy with one year lag. $g_p = g_i = g_D = 0.5$.

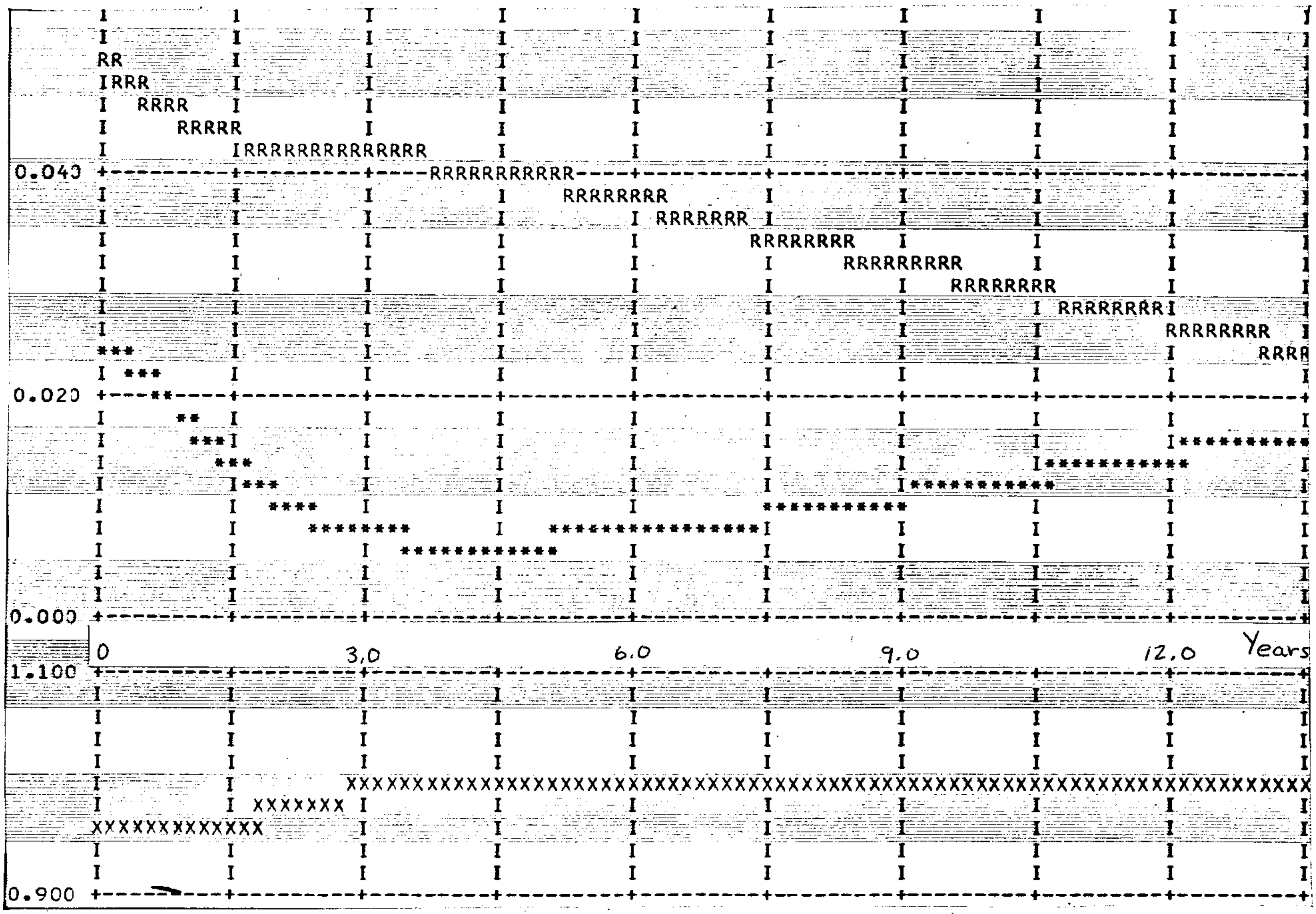

Figure 5.16. Simulation of the Model Regulated by Full Fiscal Policy and Monetary Policy with an Increased Rate of Money Supply.

CHAPTER 6

CONCLUDING REMARKS

In the introduction to this study, it was mentioned that a relatively unexplored area existed in the theoretical examination of government policies for stability and growth. This study has attempted to investigate that area by applying government policies to a model which explains both growth and fluctuations. Let us briefly review what has been done.

In Chapter 2 we examined the problem of cyclical growth and mentioned some of the various attempts to represent both growth and fluctuations together. This was done as an introduction to the Phillips model which was set up in Chapter 3.

In Chapter 4 we showed how various types of continuous stabilization policies would be represented when a variable growth rate was assumed. Because actual output is distinguished from full-capacity output, each policy is a function of two variables. In stationary models the policies are a function of only one variable. Therefore, the broader assumption necessitated more complicated representations of fiscal policies, which were modified for convenient mathematical analysis. Three similar types of monetary policy were also introduced and these were added to the money and interest rate equations.

When these stabilization policies were added to the model of cyclical growth in Chapter 5, the order of the differential equation was altered. In particular, the equation for the ratio of actual to full capacity output became third order rather than second. The equation for the proportional rate of growth was now a differential equation of second order; whereas, in the unregulated model, growth was a simple linear relationship of the actual to full capacity ratio.

To investigate the behavior of the model, when regulated by these stabilization policies, we made an analytic investigation of the general behavior of the differential equations. Finally, to examine the impacts of lags and to further support the analytic investigation, the model was simulated with various numerical values of the parameters.

Several interesting results were discovered, some of which could have policy implications. Every type of policy considered has some effect on the rate of growth. For fiscal proportional policy, the growth rate is altered depending on whether the economy is at over or under capacity operation. In depressed periods this policy raises the growth rate of full capacity output, but not by the same amount that private demand could raise it. This effect is related to stability. For example, the deflationary gap will not be eliminated if full capacity output rises as fast, or faster than, actual output. The proportional policy can act as a stabilizer by not letting full capacity increase as fast as actual capacity. The gap is reduced. The argument is similar when there is an inflationary gap. This particular effect on growth is therefore necessary if the proportional policy is to stabilize the economy at full capacity or full employment output.

Fiscal integral policy has a strong impact on the proportional rate of growth. If initialized during a depressed period, this policy will quickly stabilize the economy at full capacity output, but with a drastic reduction in the rate of growth. This suggests that there is a trade off consideration between growth and fluctuations, when integral policy is used during a period of under capacity output. Of course, the reverse is true if the policy is used when the economy is operating at over capacity. But such a situation is not usual in an economy like the United States, where recent economic history shows that under capacity is a more prevalent state of output.

The monetary policies influence the rate of growth through an influence on the interest rate. However, the influence is not as strong as that of fiscal policies. A possible reason for the lesser influences is the opposite effect which a change in actual income has on interest rates. For example, if the monetary authority decreases the money supply during inflation, then interest rates will rise. This will reduce investment and thus the growth rate will be lower. But at the same time the reduced investment makes actual income lower which in turn will tend to pull interest rates down again. Thus the eventual effect is that interest rates are at, or near, their original level, and the growth rate is not significantly changed.

Monetary policies are not as strong as fiscal policies in their effect on stability. This was found by simulating fiscal regulation of the model with and without monetary regulation. The path of the economy was almost identical in both cases.

The analytic investigation showed that some fiscal and monetary policies can be either stabilizing or destabilizing depending on the parameters of the unregulated model. Some of these stability conclusions were due to the broader assumption that full capacity output and actual output are two different variables. The multiplier-accelerator analysis would yield different stability conclusions, because it assumed a stationary level of desired or full capacity output.

Finally, simulation showed that when lags are introduced to fiscal policy, the impacts on stability and growth are changed. For some policies the frequency of the oscillations was increased as a result of lags; for others, the impacts on growth were magnified. These results were given in the last section of Chapter 5, and indicate that further studies might deal with the problem of fiscal and monetary lags in a cyclical growth model.

Endogenous cyclical growth models, like that of Phillips, are a relatively new development of economic theory. With further research such models could open a whole new area for theoretical economics. It is hoped that this study might indicate how a cyclical growth model can be a useful framework from which to extend the theories of positive economics to the problems of normative economics, including the problems of government policies for growth and stability.

APPENDIX 1

SIMULATION TECHNIQUES

Simulation was used to investigate the behavior of the regulated model of cyclical growth when the differential equations became too cumbersome or impossible to solve analytically. Two techniques were used: (1) numerical solutions of the differential equation using a IBM Model 7094 digital computer, and (2) electro-analog solutions using an Electronic Associates TR-20 electro-analog computer. The latter method was used primarily as a preliminary investigating tool and as an aid in understanding the behavior of the model. Both methods are briefly described.

A. Simulation by Numerical Solution on a Digital Computer

In order to determine the numerical solution of these equations, a package of computer programs was used. The name of this package is ODEPAC[1] and is currently available on the IBM Model 7094 at the Princeton University Computer Center. Given the set of differential equations, the values of the parameters, and the initial conditions, the program will solve the system giving values for all variables at any desired points in time. The techniques of numerical solution used in the program are

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[1]See I. Thomas Cundiff, "On the Automation of the Numerical Solution of Ordinary Differential Equations," (Washington, D. C., August, 1967) for a discussion of the programming methods involved in this package.

similar to those discussed by Henrici.[2] Basically the method involves successive integration of the equations at certain variable intervals or steps.

In order to change the differential equation of the model of cyclical growth into a desirable form for numerical solution, it is written in Fortran IV as a set of simultaneous first order differential equations. In the most involved case this involved three equations for the variables $x(= Y_A/Y_F)$, plus two for the variable growth rate, y_F, and finally one for the rate of interest r. Thus, the set consisted of six first order differential equations. The values of these variables were then given at various intervals in time. For example, if the interval is set at $\frac{1}{4}$ t, then the values of each variable are given at quarterly intervals in the economic terms of the model. If the interval is 2t, then the values are given for every two years. By changing the parameters of the equations, we can investigate the behavior of the system under different types of government regulation.

It was found that the output, consisting of a great volume of figures, was difficult to analyze, especially when many different values for the parameters were tried. Therefore, a plotting routine was installed into the program to graph the output. The plotting routine is currently stored in the computer and its use only involved some simple CALL statements. Selected results of this plotting are presented in the text. Because the printer is limited to increments of one print line the graphs seem discontinuous; the curves they represent are much

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[2] P. Henrici, Discrete Variable Methods in Ordinary Differential Equations (New York: Wiley 1962).

smoother. After examining the behavior of these graphs, the actual numbers can then be consulted for more exact analysis.

This method of simulation was a great help in investigating the model, and can be a useful tool in examining other economic models which can be represented as differential equations. The high accuracy of the numerical solutions was demonstrated when some of the more simple simulations were compared with analytic solutions (e.g. the second order case). The actual numbers can then be consulted for more exact comparison.

B. Electro-Analog Simulation

The usefulness of electro-analog methods in economic analysis has been demonstrated in a number of articles.[3] In this paper the analog computer was used as a tool in understanding the behavior of the government regulated model.

The idea behind electro-analog computing is that certain combinations of electrical circuitry can perform mathematical operations. For example, if we let voltage be a variable then we can sum several different voltages by joining them together in an electrical circuit. Similarly, integrations can be performed by a combination of a capacitor and an amplifier in parallel.

It is beyond the scope of this paper to explain the detailed techniques.[4] The circuit which represents the equation of the model is represented in Figure A1.1. The bottom loop of this circuit represents

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[3]For a good description of some applications, see R. H. Strotz, J. F. Calvert, and N. F. Morehouse, "Analoge Computing Techniques Applied to Economics," Transactions in the American Institute of Electrical Engineers I (1951).

[4]See Walter Soroka, Analog Methods in Computation and Simulation (New York: McGraw Hill, 1954).

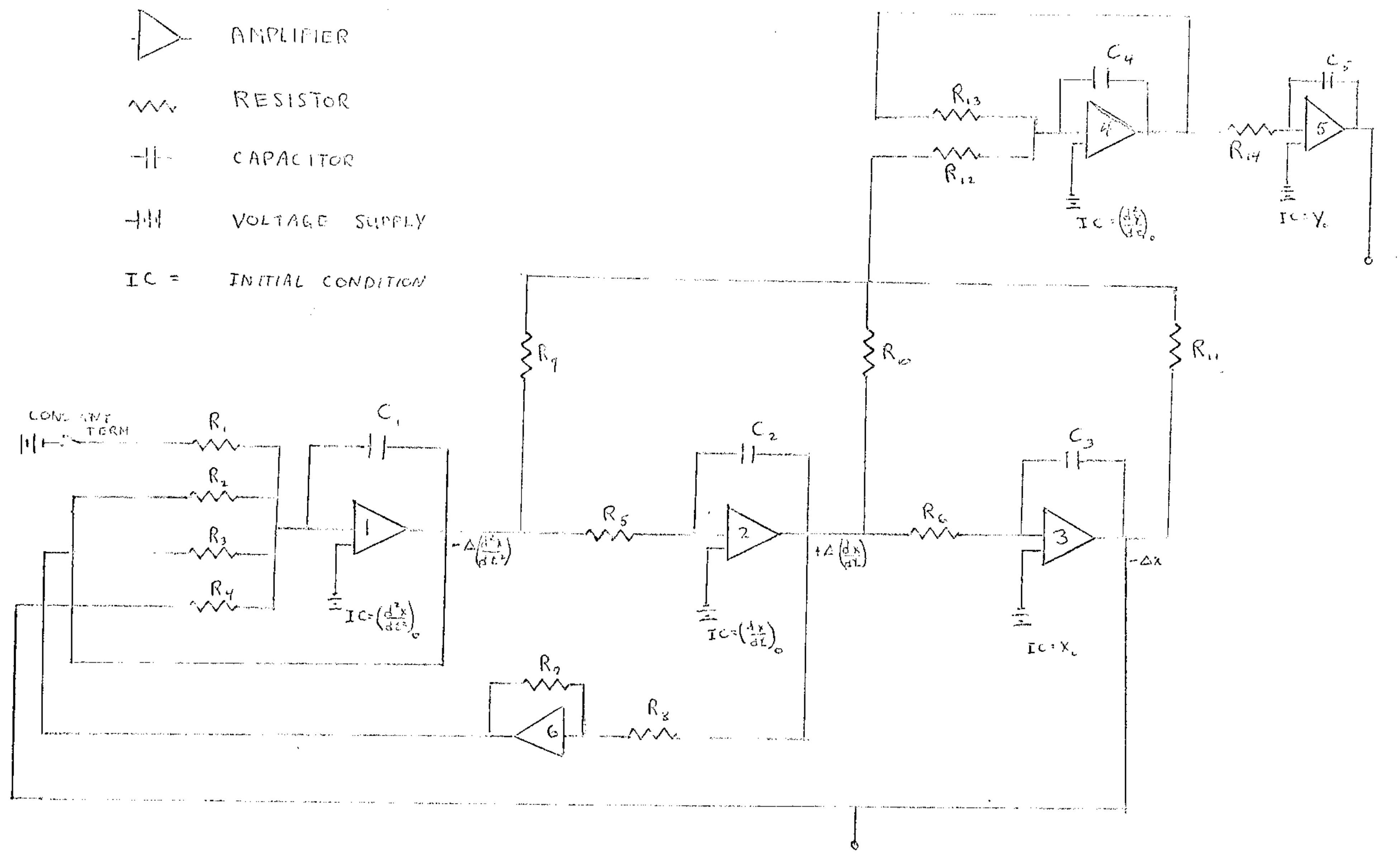

Figure A1.1. Circuit for the Simulation of the Differential Equations of the Regulated Model on an Electro-Analogue Computer the three integrations needed to solve the third order equation for the actual to full capacity ratio x. The upper loop represents the growth equation for y_F which is a function of D^2x, Dx and x. By changing the values of the resistors (this can be done with variable resistors) the parameters of the models can be changed. Two oscilloscopes are then connected to the circuit to observe the path of the variables. Using the TR-20 Analog-Computer this circuit is relatively simple to set up.

An advantage of electro-analogue solutions over numerical solutions is that the integrations are performed continuously rather than in discrete intervals. However, there are several disadvantages. When the parameters of the system are larger the values of the resistors must be very low, and there is a possibility of inaccurate measurement with a potentiometer. Furthermore, the results are not given in numerical form and must be computed with a plotting device. Also, when we wish to examine many different values of the parameters, it is necessary to calculate the values of the resistors in each case.

Because of these disadvantages the numerical methods on the digital computer are an attractive alternative and would be a useful tool in other economic studies.

APPENDIX 2

THE EFFECT OF A POSSIBLE MODIFICATION OF GOVERNMENT FISCAL POLICY: INVESTMENT VERSUS CONSUMPTION SPENDING

In Chapter 5 we mentioned that some of the effects of government policy on the growth rate were due to the definition of government spending. This appendix will demonstrate how that definition assumed that all government spending was on consumption goods, and will investigate the effects of an alternative definition. The analysis is hypothetical, but it does show how different definitions might affect investigations of fiscal policy and growth.

In Chapter 4, we defined G as a budget surplus collection or deficit spending. Now, let us further define how that deficit might be composed. Where G represents deficit spending we will write:

$$G = G_I + G_C \quad . \tag{A2.1}$$

where G_I is defined as government investment expenditures plus private investments included in government projects. Thus G_I will include investment projects which the government contracts to private industry. Today, such contracts make up the larger part of government investment spending. G_C is that part of government consumption which changes with stabilization policy. Neither G_I nor G_C include "necessary" government spending; for example, national defense is not considered as a variable to be adjusted for stabilization.

The size of G will be determined as in Chapter 4 by the three stabilization policies: proportional, integral, and derivative. But now the government will decide how much of G is to be investment and how much consumption. Let us assume that the government decides to make each type of spending a fixed proportion of total spending. We can then write:

$$G_I = b_I G$$
$$G_C = (1-b_I)G \qquad \text{(A2.2)}$$

where b_I and $(1-b_I)$ are the fixed proportions of investment and consumption respectively.

The income equation is now written as

$$Y_A = (1-s)Y_A + I + G_I + G_C$$

so that total investment in the economy is:

$$I + G_I = sY_A - G_C \quad .$$

Substituting from equation (A2.2) this becomes:

$$I + G_I = sY_A - (1-b_I)G \ . \qquad \text{(A2.3)}$$

With the addition of government investment in the model, we will have to redefine the rate of capital accumulation:

$$K = \int_0^t (I + G_I)dt$$

or

$$DK = I + G_I \qquad \text{(A2.4)}$$

Substituting this expression into the production function we have:

$$DY_F = vDK = v(I + G_I)$$

and the proportional rate of growth is

$$y_F = DY_F/Y_F = \frac{v}{Y_F}(I + G_I)$$

Now, if we substitute for $(I + G_I)$ from equation (A2.3) this expression for growth becomes

$$y_F = v[\ sY_A - (1-b_I)G\ ]/Y_F \qquad \text{(A2.5)}$$

This equation can now be compared with equation (5.10):

$$y_F = v[\ sY_A - G\ ]/Y_F$$

There are several important points.

First, if $b_I = 0$ then equation (A2.5) is the same as equation (5.10). Thus we can deduce, as mentioned in Chapter 5, that the previous analysis assumed that $G = G_C$. That is, the entire government deficit consisted of government consumption.

Second, if $b_I = 1$ then the equation for growth becomes $y_F = svx$ which is exactly what it was in the unregulated model where government spending was zero.

As an example, consider proportional fiscal policy without a lag. Before the growth rate was given as:

$$y_F = svx + g_p v(x-1)\ .$$

But when $b_I = 1$ the second term on the right will be zero. Thus the only effect of proportional policy on the growth rate is through its effect on x. This could have implications for the stabilizing aspects of fiscal proportional policy. For example, when $x < 1$, that is $Y_A < Y_F$, then Y_F is permitted to increase by the full amount as determined by the increase in x. This increase in Y_F may be greater than the increase in Y_A so that the eventual effect could be destabilizing. When the second term on the right existed for proportion policy, we found that Y_F was increased by a smaller amount. Therefore, it seems that the inclusion of government investment in the analysis could have interesting effects on stability.

This development could suggest that further studies of fiscal policy in a cyclical growth model might be concerned with such effects.

BIBLIOGRAPHY

Agnew, R. P. Differential Equations. New York: McGraw Hill, 1960.

Alexander, S. S. "The Accelerator as a Generator of Steady Growth," Economic Journal (May, 1949).

Allen, R. G. D. "The Engineers Approach to Economic Models," Economica (May, 1955).

Allen, R. G. D. "The Structure of Macro-Economic Models," Economic Journal (March, 1960).

Allen, R. G. D. Mathematical Economics. New York: Macmillan, 1959.

Allen, R. G. D. Macro-Economic Theory. New York: Macmillan, 1967.

Baumol, W. J. Economic Dynamics. New York: Macmillan, 1959.

Baumol, W. J. "Pitfalls in Contracyclical Policies: Some Tools and Results," Review of Economics and Statistics (February, 1961).

Bergstrom, A. R. "A Model of Technical Progress, the Production Function, and Cyclical Growth." Economica (November, 1962).

Bourneuf, Alice. "Investment, Excess Capacity, and Growth," American Economic Review (September, 1964).

Choudhry, N. K. and Mohabbat, K. A. "Economic Fluctuations and Growth: Notes on Testing the Smithies' Model," Oxford Economic Papers (March, 1965).

Clark, John J. and Cohen, Morris (ed.). Business Fluctuations, Growth, and Economic Stabilization. New York: Random House, 1963.

Cornwall, John. "Three Paths to Full Employment Growth," Quarterly Journal of Economics (February, 1963).

Cundiff, I. T. "On the Automation of the Numerical Solution of Ordinary Differential Equations," A Paper Presented at the ACM Symposium on Interactive Systems for Experimental Applied Mathematics. Washington, D. C. (August, 1967).

Domar, E. D. "Expansion and Employment," American Economic Review (March, 1947).

Duesenberry, J. S. Business Cycles and Economic Growth. New York: McGraw Hill, 1958.

Duesenberry, J. S., Eckstein, O., and Fromm, G. "A Simulation of the United States Economy in Recession," Econometrica (October, 1960).

Fellner, W. I. Trends and Cycles in Economic Activity. New York: Henry Holt and Co., 1956.

Fox, K. A., Sengupta, J. K. and Thornbecke, E. The Theory of Quantitative Economic Policy with Applications to Economic Growth and Stabilization. Chicago: Rand McNally, 1966.

Friedman, Milton. "A Monetary and Fiscal Framework for Economic Stability," American Economic Review (June, 1948).

Friedman, Milton. "The Effects of Full Employment Policy on Economic Stability: A Formal Analysis," in Essays in Positive Economics. Chicago: University of Chicago Press, 1953.

Friedman, Milton. A Program for Monetary Stability. New York: Fordam University Press, 1960.

Gordon, R. A. Business Fluctuations. New York: Harper and Row, 1961.

Gordon, R. A. and Klein, L. R. (ed.). Reading in Business Cycles. Homewood, Illinois: Richard D. Irwin Co., 1965.

Goodwin, R. M. "The Nonlinear Accelerator and the Persistence of Business Cycles," Econometrica (January, 1951).

Goodwin, R. M. "The Problem of Trend and Cycle," Yorkshire Bulletin of Economic and Social Research (1953).

Goodwin, R. M. "A Model of Cyclical Growth," in The Business Cycle in the Postwar World, Proceedings of International Economic Association Conference, edited by E. Lundberg. London: Macmillan, 1955.

Hahn, R. F. and Matthews, R. C. O. "The Theory of Economic Growth: A Survey," Economic Journal (December, 1964).

Hansen, A. H. Fiscal Policy and Business Cycles. New York: Macmillan, 1941.

Harrod, R. F. "An Essay in Dynamic Theory" Economic Journal (March, 1939).

Harrod, R. F. Toward a Dynamic Economics. London: Macmillan, 1948.

Hart, A. G. "Model-Building and Fiscal Policy," American Economic Review (September, 1945).

Henrici, P. Discrete Variable Methods in Ordinary Differential Equations. New York: Wiley, 1962.

Hicks, J. R. "Mr. Harrod's Dynamic Theory," Economica (May, 1949).

Hicks, J. R. A Contribution to the Theory of the Trade Cycle. Oxford: The Oxford University Press, 1950.

Howrey, E. Philip. "Stabilization Policies in Linear Stochastic Systems," Econometric Research Program Research Memorandum No. 83 (September, 1966).

Kaldor, Nicholas. "The Relation of Economic Growth and Cyclical Fluctuations," Economic Journal (March 1954).

Kurihara, K. K. "An Endogenous Model of Cyclical Growth," Oxford Economic Papers (October, 1960).

Matthews, R. C. O. The Business Cycle. Chicago: The University of Chicago Press, 1959.

Matthews, R. C. O. "Duesenberry on Growth and Fluctuations," Economic Journal (December, 1959).

Minsky, Hyman P. "A Linear Model of Cyclical Growth," Review of Economics and Statistics (May, 1959).

Pack, Howard. Formula Flexibility: A Quantitative Analysis. Ph.D. Dissertation: Massachusetts Institute of Technology, 1964.

Phillips, A. W. "Stabilization Policy in a Closed Economy," Economic Journal (June, 1954).

Phillips, A. W. "Stabilization Policy and the Time-Form of Lagged Responses," Economic Journal (June, 1957).

Phillips, A. W. "A Simple Model of Employment, Money and Prices in a Growing Economy," Economica (November 1964).

Prasad, P. H. "Business Cycle Phenomena in the Harrod-Domar Model," International Economic Review (January, 1965).

Samuelson, P. A. "Interactions Between the Multiplier Analysis and the Principle of Acceleration," Review of Economics and Statistics (May, 1939).

Sato, Ryuzo. "Fiscal Policy in a Neo-Classical Growth Model," Review of Economic Studies (February, 1963).

Schumpeter, J. A. Theory of Economic Development. Cambridge, Massachusetts: Harvard University Press, 1934.

Schumpeter, J. A. Business Cycles. New York: Mc Graw Hill, 1939.

Smithies, Arthur. "Economic Fluctuations and Growth," Econometrica (January, 1957).

Soroka, W. A. Analog Methods in Computation and Simulation. New York: McGraw Hill, 1954.

Strotz, R. H., Calvert, J. F. and Morehouse, N. F. "Analog Computing Techniques Applied to Economics," Transactions in the American Institute of Electrical Engineers (1951).

Tinbergen, J. and Bos, H. C. Mathematical Models of Economic Growth. New York: McGraw Hill, 1962.

Tinsley, P. A. Potential GNP and Discretionary Fiscal Policy. Ph.D. Dissertation: Princeton University, 1965.

Tobin, James. "Economic Growth as an Objective of Government Fiscal Policy," American Economic Review (May, 1964).

Tustin, Arnold. The Mechanism of Economic Systems. Melbourne: William Neinemann Co., 1953.