

FISCAL AND MONETARY STABILIZATION POLICIES  
IN A MODEL OF ENDOGENOUS CYCLICAL GROWTH

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## FISCAL AND MONETARY STABILIZATION POLICIES

### IN A MODEL OF ENDOGENOUS CYCLICAL GROWTH

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In recent years various articles and papers have been written on the effectiveness of government fiscal and monetary contracyclical policies.<sup>1</sup> These studies have been mainly concerned with the effects of such policies on short run fluctuations in national income with little mention of effects on growth. They have been limited to economic models which are valid only in short period analysis - that is, models which display oscillations about a stationary level of income. Thus there has been a gap in the theoretical examination of government regulation, which leaves the effects of these policies on growth largely unexplored. This paper attempts to fill that gap by applying fiscal and monetary policies to a model which produces endogenously both growth and oscillations. In this way we can examine the influence of government regulation on economic growth as well as any additional stability considerations which arise as a consequence of cyclical growth.

We shall first present the model, briefly describe the product and money markets as developed by Phillips, and derive the government policies which will regulate the model. Then, after explaining the differential equations of the system, we will investigate the various policies both analytically and by numerical simulation.

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<sup>1</sup>See A. W. Phillips "Stabilization Policy in a Closed Economy," Economic Journal (June, 1954), W. J. Baumol "Pitfalls in Contracyclical Policies: Some Tools and Results," Review of Economics and Statistics (February, 1961) and E. Philip Howrey "Stabilization Policy in Linear Stochastic Systems," Review of Economics and Statistics, 49, (1967).

## I. The Model

The endogenous cyclical growth model which we shall use in this analysis is a modification of that developed by Phillips.<sup>2</sup> The advantage of this model is that it can display both growth and fluctuations without resorting to exogenous trends, which most previous models could not avoid. The model is demand oriented and includes both product and money markets. It reduces to a system of linear differential equations whose variables are the ratios of the real variables of the model.

An important feature of the model is the concept of full-capacity output. Full-capacity output  $Y_F$  is distinguished from actual output  $Y_A$  and can be defined as the output achieved under regular working conditions with no overtime or extra shifts. Notice that with this definition, actual output can be greater or less than full-capacity output. In fact, we would expect actual output to fluctuate around the full-capacity level.

We will find it useful to consider the ratio of actual to full-capacity output in this model; that is  $Y_A/Y_F = x$ . Values of  $x$  less than unity represent depressed levels of output, and values greater than unity represent over-utilization or inflation. The value  $x-1$  could indicate a proportional gap.

The model does not explicitly distinguish between full-capacity output and full-employment output. However, it is likely that the rate of employment will fluctuate in much the same manner as  $x$ . We would expect a low level of employment when actual output is less than full-capacity output. As the economy

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<sup>2</sup>A. W. Phillips, "A Simple Model of Employment, Money, and Prices in a Growing Economy," Economica (Nov. 1961).

pushes past full-capacity the level of employment should increase until bounded by structural restraints and resulting inflation. Studies have shown that the fluctuations of the two are highly correlated.<sup>3</sup>

The proportional growth rate of full-capacity output is defined as

$y_F = DY_F/Y_F$  where  $D = d/dt$ . The final differential equations of the model are expressed in terms of the variables  $x$  and  $y_F$ .

The model, consisting of six equations, is listed below with definitions of variables and parameters:

$$(1) \quad Y_F = vK$$

$$(2) \quad Y_A = (1-s)Y_A + I + G$$

$$(3) \quad K = \int_0^t Idt \quad \text{or} \quad DK = I$$

$$(4) \quad I = \frac{1}{T_I D + 1} K [E + \gamma(x-1) + \rho(c-r)]$$

$$(5) \quad r = k' + \mu(\log P + \log Y_A - \log M)$$

$$(6) \quad DP/P = \beta(x-1) - DY_F/Y_F + \delta$$

Variables:

$Y_A$  = actual level of income (output)

$Y_F$  = full capacity level of income (output)

$K$  = stock of capital

$I$  = net investment

$r$  = rate of interest

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<sup>3</sup>ibid. p. 361.

$P$  = price level

$G$  = government demand

$M$  = money supply

$x$  =  $Y_A/Y_F$

$y_F$  =  $DY_F/Y_F$

Parameters:

$v$  = output-capital ratio

$s$  = marginal propensity to save

$\gamma$  = speed of entrepreneurs investment adjustment to reduce excess capacity or increase insufficient capacity

$c$  = marginal efficiency of capital

$\rho$  = influence of interest rate on investment

$\mu$  = influence of price level, actual income, and money supply on interest rate

$\beta$  = influence of over or under capacity output on prices

$T_I$  = time constant of investment lag

$\delta$  = constant price trend

The first three equations represent a constant output-capital ratio production function, the familiar income identity, and a definition of the accumulation of capital through net investment. Since we assume that there is no output-income lag, we can refer to  $Y_A$  as either actual output or actual income.

The fundamental disequilibrium condition of the model is the investment function described in equation (4). Investment depends on the size of the capital stock in combination with two other determinants: first, the reaction of entrepreneurs to greater than or less than full capacity operation; and second, the deviation between the market rate of interest and the marginal

efficiency of capital.<sup>4</sup> Thus both the product and money markets are relevant in determining the level of net investment. Investment is then lagged on these variables; the lag being of exponentially distributed form with time constant  $T_I$  and speed of response  $1/T_I$ .

The money market, in the absence of contracyclical monetary policy, is then defined in equations (5) and (6). In equation (5) the interest rate is a function of the price level, actual income, and the money supply. Such a relationship represents an economy where the demand for cash balances is a function of money income ( $PY_A$ ) and the interest rate. In equation (6) the rate of change in prices depends on the proportional gap between actual and full capacity income and on changes in productivity, of which the rate of growth is a close proxy. The positive constant term represents the tendency for prices to drift steadily upward over time.

## II. Government Stabilization Policies

Equations (7) and (8) below represent the government fiscal and monetary stabilization policies which we will use to regulate the model of cyclical growth.<sup>5</sup> These policies are analogous to feedback methods of control which engineers have used in stabilizing electrical systems.

$$(7) \quad G = \frac{1}{T_G D + 1} \left[ -g_p Y_F (x-1) - g_i Y_F \int_0^t (x-1) dt - g_D Y_F D x \right]$$

$$(8) \quad \log M = \log M_F - g_{mp} (x-1) - g_{mi} \int_0^t (x-1) dt - g_{mD} D x$$

<sup>4</sup>Phillips also includes a term reflecting the influence of business expectations on investment. However, through the bulk of his analysis he assumes that this is a constant, in which case the term does not affect the solution.

<sup>5</sup>These policies are modified versions of the types of fiscal policies first suggested by A. W. Phillips op.cit. (1954).

Government Stabilization Policy Parameters:

- $g_p$  = proportional fiscal policy
- $g_i$  = integral fiscal policy
- $g_D$  = derivative fiscal policy
- $T_G$  = time constant of fiscal lag
- $g_{mp}$  = monetary proportional policy
- $g_{mi}$  = monetary integral policy
- $g_{mD}$  = monetary derivative policy
- $M_f$  = that part of the money supply which increases at a constant rate
- $m_f$  = constant rate of increase in  $M_f$

Fiscal Policies. By fiscal stabilization policies we shall mean policies through which the government creates an addition (or subtraction) to the total demand of the economy by adjusting expenditures, tax rates, or transfer payments. Such policies have the aim of moving the economy closer to the full-capacity level of income and offsetting undesirable income trends so as to keep the economy stabilized around this desirable full-capacity level. Such policies reduce the size and frequency of fluctuations and as a consequence, it is frequently claimed, increase the rate of growth of the economy. This last claim rests on the assumption that economic fluctuations are not necessary for a high rate of economic growth, and in fact that they tend to lower the growth rate. This analysis will help to examine that last assumption.

In the following representations of fiscal policy there will be no distinction among the actual methods used by the government to create demand. Instead expenditures, taxes, and transfer payments will be lumped under one concept:

budget surplus (+) or deficit (-). The policies discussed are of a continuous type; that is, we assume that the government can continuously adjust the budget according to present levels and trends in income.

The three terms in equation (7) represent three forms of continuous fiscal policies: proportional, integral, and derivative. Using the proportional policy the government will spend in proportion to the difference between the desired level of income and the actual level. We let the desired level be full-capacity, so that the government demand will be  $-g_p(Y_A - Y_F) = -g_p Y_F (x-1)$ .

The most important change between this policy and that of Phillips is that  $Y_F$  is now a variable. Thus government policy demand may change even though the level of actual income remains steady. An example of such a situation could be a technological advance suddenly introduced throughout the economy. This would cause  $Y_F$  to increase and the government policy would call for increased spending even though  $Y_A$  might remain constant. In previous stabilization studies such a development would not be possible because  $Y_F$  was assumed to be constant. We will find that the more realistic assumption of a variable  $Y_F$  has important consequences for stability.

The proportional policy alone cannot bring the economy completely to the desired level of output. This is because the proportional spending gets smaller as we approach the desired income level. The level will be approached but never reached. Therefore we introduce a second type of policy which is designed to eliminate the gap completely. The integral policy is applied to the cumulative gaps in income over time; government demand is proportional to the integrated sum of differences between  $x$  and unity where  $g_i$  is the factor of proportionality. This integral is then multiplied by  $Y_F$ , so that the government



spending will be proportional to the size of the economy. Stabilization would require more spending in a larger economy where  $Y_F$  is larger, than in a smaller economy.

The purpose of derivative fiscal policy is to offset income trends by deficit spending when  $x$  is falling (i.e. full capacity is increasing faster than actual income) and by surplus collection in the opposite case. The policy coefficient is  $g_D$  and the term is further multiplied by  $Y_F$  to insure that the government action is proportional to the size of the economy.

Together these three policies represent the potential policy demand. If there is no time lag then the potential policy demand will equal actual policy demand. However, it is realistic to incorporate an exponentially distributed lag between potential and actual policy. The time constant of this lag is  $T_G$ .

Monetary Policies. Since the model of cyclical growth includes a money market, it seems appropriate to introduce some methods of monetary stabilization.<sup>6</sup> We will assume that the central bank of the economy can increase or decrease the supply of money through actions on the open market. By selling securities the authority can decrease the money supply and conversely by buying securities the authority can increase the money supply. We will assume that these operations on the open market can be carried on continuously, and that the correct action is calculated by observation of economic indicators.

The structure of the monetary policies in equation (8) is analogous to that of the fiscal policies already mentioned. The monetary authority will adjust the money supply depending on three variable terms: proportional, integral, and derivative with factors of proportionality  $g_{mp}, g_{mi}, g_{mD}$ .<sup>6</sup> The

<sup>6</sup>Phillips op.cit. (1961) considers derivative monetary policy. We will consider all three policies.

authority will be adjusting the rate of growth of the money supply around some constant equilibrium rate. Logarithmic notation is used so that the expression is in proportional terms.

The variable  $M_F$  can be interpreted as the money supply when there is no monetary policy or when the effect of the monetary policy is zero. We will assume that the central bank increases  $M_F$  at a constant rate so that  $D \log M_F = \frac{DM_F}{M_F} = m_F$  which is a constant. A constant  $m_F$  in the absence of monetary contracyclical policy is the kind of monetary action that Friedman has suggested.<sup>7</sup> Thus we are able to separate such a proposal from the contracyclical policies.

An appropriate lag could be incorporated in this monetary policy similar to the fiscal lag. However, such a lag greatly complicates the analysis and necessitates further assumptions for rigorous mathematical analysis. We will assume that there is no lag. This can be partially justified by assuming the monetary authority is less subject to political restraints and delays than are the fiscal authorities.

The stabilization equations (7) and (8) are added to equations (1) through (6) to complete the government regulated model. The schematic block diagram in Figure 1 represents the interrelationships of the model. The boxes indicate algebraic operations and the lines are either flow conditions or definitional relations between variables. The broken lines indicate the fiscal and monetary policies. It can be seen that the direct effect of fiscal policy is to add or subtract from  $Y_A$  while monetary policy affects the interest rate  $r$ .

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<sup>7</sup>Milton Friedman, "A Monetary and Fiscal Framework for Economic Stability," American Economic Review (June 1948), reprinted in Readings in Macroeconomics, M. G. Mueller (Ed.) p. 352.

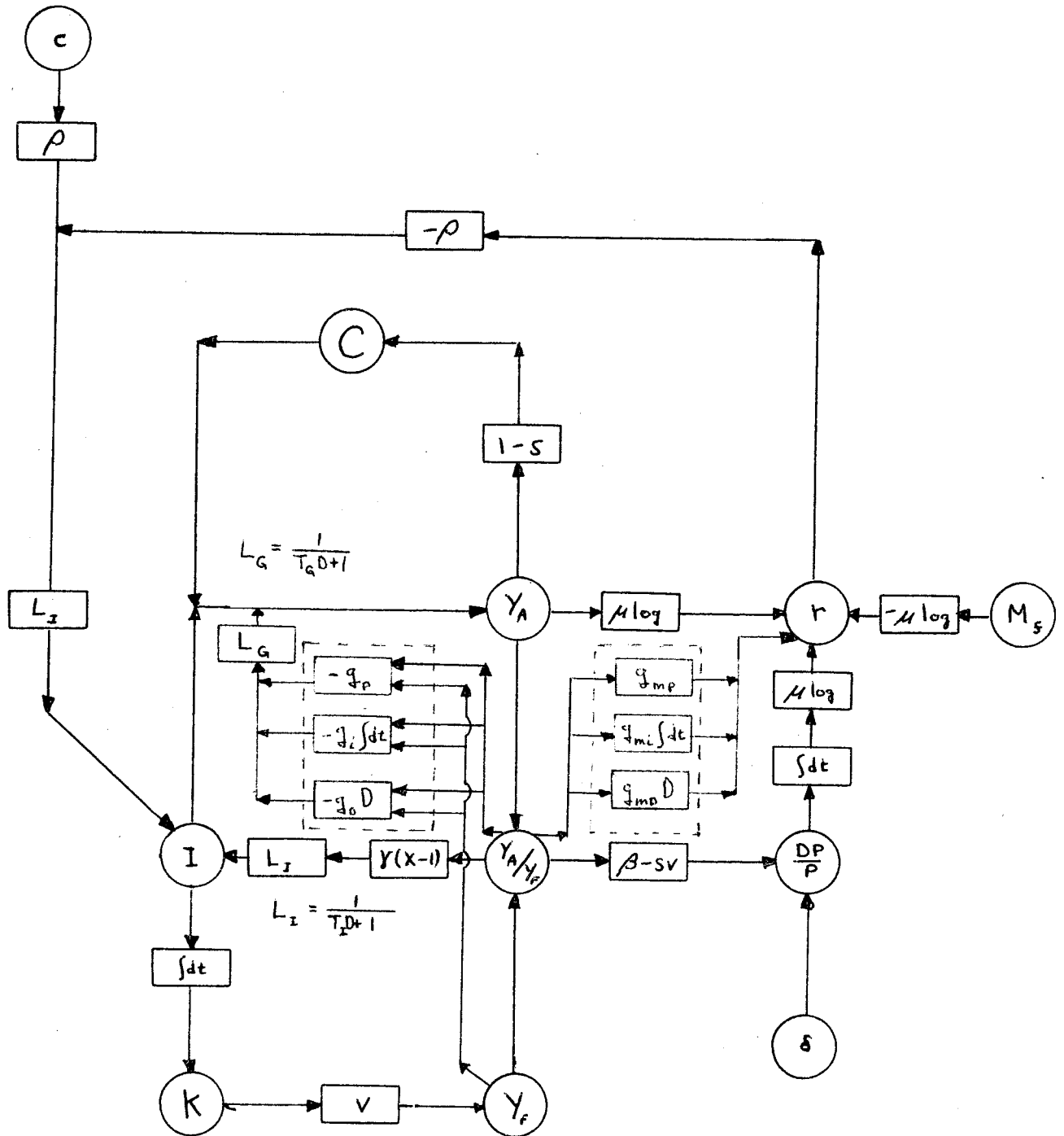


Figure 1. Block Diagram of the Cyclical Growth Model when Regulated by Fiscal and Monetary Policies

III. The Differential Equation of the Model.

From these eight equations we can derive the linear differential equation which will describe the entire system. From equation (2) and from the definition of  $x$  we can write:

$$I = sxY_F - G \quad (9)$$

Substituting for  $G$  from equation (7) and for  $Y_F$  from equation (1)

$$I = svKx + \frac{vK}{T_G^{D+1}} [g_p(x-1) + g_i \int_0^t (x-1)dt + g_D Dx] \quad (10)$$

Equation (10) can now be combined with the investment function in equation (4) which, after cancelling  $K$  and differentiating, becomes a third order differential equation in terms of  $x$  and  $r$ :

$$\begin{aligned} & (svT_I T_G + T_I g_D v) D^3 x + [sv(T_I + T_G) + T_I g_p v + g_D v - T_G \gamma] D^2 x \\ & + (sv + T_I g_i v + g_p v - \gamma) Dx + g_i vx + T_G \rho D^2 r + \rho Dr - v g_i = 0. \end{aligned} \quad (11)$$

In order to obtain the final differential equation of the model, we must substitute appropriate expressions for  $D^2 r$  and  $Dr$ . The money market with the proposed monetary policy must now be considered. We begin by substituting for  $\log M$  in equation (5) and differentiating with respect to time:

$$Dr = \mu(D \log P + D \log Y_A - D \log M_F + g_{mp} Dx + g_{mi}(x-1) + g_{mD} D^2 x). \quad (12)$$

By substituting for  $D \log P$  and  $D \log Y_A$  and collecting terms we arrive at the

<sup>8</sup>To find an expression for  $D \log Y_A$  an approximation is used:

$$\begin{aligned} \log Y_A & \approx \log Y_F + (x-1) \text{ which is valid for values of } x \text{ near unity since} \\ \log Y_A - \log Y_F & = \log [1-(x-1)] = (x-1) + \frac{1}{2}(x-1)^2 + \dots \end{aligned}$$

following equation for  $Dr$  in terms of  $x$  :

$$Dr = \mu[g_{mD}D^2x + (g_{mp} + 1)Dx + (\beta + g_{mi})x - \beta + \delta - m_f - g_{mi}] \quad (13)$$

Now by differentiating (13) we can get a similar expression for  $D^2r$  which along with  $Dr$  can be substituted into equation (11)

$$\begin{aligned} & [svT_I T_G + T_I g_D v + T_G \rho \mu g_{mD}] D^3x \\ & + [sv(T_I + T_G) + T_I g_p v + g_D v - T_G \gamma + T_G \rho \mu (g_{mp} + 1) + \rho \mu g_{mD}] D^2x \\ & + [sv + T_I g_i v + g_p v - \gamma + T_G \rho \mu (\beta + g_{mi}) + \rho \mu (g_{mp} + 1)] Dx \\ & + [g_i v + \rho \mu (\beta + g_{mi})] x = v g_i + \rho \mu [\beta + m_f - \delta + g_{mi}] \quad (14) \end{aligned}$$

In order to complete the equations of the model we must show the relationship between  $x$  and  $y_F$ . From equations (1), (2), and (3) we know that:

$$y_F = \frac{DY_F}{Y_F} = v(sY_A - G)/Y_F \quad (15)$$

Substituting for  $G$  from equation (7) and differentiating we get:

$$T_G D^2 y_F - D y_F = (T_G sv + g_D v) D^2 x + (sv + g_p v) Dx + g_i x - g_i \quad (16)$$

This equation is of second order in  $y_F$ . When combined with equation (14) the entire system becomes a linear fifth order differential system. By examining the behavior of equations (14) and (16) over time we can determine the various effects of the government policies.

IV. Investigation and Results.

If we ignore fiscal lag and assume that there is no fiscal derivative policy ( $T_G = g_D = 0$ ) then equation (14) reduces to a second order linear differential equation:

$$D^2x + \frac{sv + T_I g_i v + g_p v - \gamma + \rho\mu(g_{mp} + 1)}{T_I v(s + g_p) + \rho\mu g_{mD}} Dx + \frac{g_i + \rho\mu(\beta + g_{mi})}{T_I v(s + g_p) + \rho\mu g_{mD}} x = \frac{v g_i + \rho\mu(\beta + m_f - \delta + g_{mi})}{g_i v + \rho\mu(\beta + g_{mi})} \quad (17)$$

By examining the roots of equation (17) we can determine analytically the effects of all policies without lag except fiscal derivative. The fifth order system which persists when fiscal lags and derivative policy are considered does not readily lend itself to analytic investigation. Numerical simulation is used to examine the behavior of the model in these cases.

We will first consider each policy by itself without lags. In each case we examine the damping factor  $\alpha$  as represented by the real part of the roots of the characteristic form of the second order equation (17). The condition for stability in each case is that  $\alpha < 0$ . When  $\alpha > 0$  the system is explosive. The greater the value of  $\alpha$  the more unstable is the model. By taking the partial derivative of this damping factor with respect to the government policy parameter in question, we can determine the effect on stability. When  $\frac{\partial \alpha}{\partial g}$  is negative then the policy increases stability by decreasing  $\alpha$ , and when  $\frac{\partial \alpha}{\partial g}$  is positive then the policy decreases stability.

The effect on growth can then be determined from equation (16). Table 1 summarizes the results of this method of analytic investigation.

No Regulation. As a point of reference we can examine the behavior of the unregulated model by setting all policy parameters equal to zero. The model will display cyclical growth when the roots of equation (17) are complex; that is, when  $\beta > \frac{1}{4} (sv - \gamma + \rho\mu)^2 / sv \rho\mu T_I$ . The necessary and sufficient condition for stability in this case is that  $(sv - \gamma + \rho\mu) > 0$ . Notice that if  $sv > \gamma$ , the system will necessarily be damped. On the other hand if  $sv < \gamma$ , then the model will be damped or explosive depending on the value of  $\rho\mu$ . The parameter  $\rho\mu$  can be interpreted as the strength of monetary influences on net investment, so that stronger monetary influences tend toward stability.

Proportional Fiscal Policy. Examining  $\frac{\partial \alpha}{\partial g_p}$  in Table 1, we see that this proportional policy will increase damping by decreasing the magnitude of  $\alpha$  if  $\gamma > \rho\mu$ , and conversely will decrease damping if  $\gamma < \rho\mu$ . To understand the significance of this, consider the unregulated model. If the unregulated model is unstable then  $\gamma > sv + \rho\mu$  which implies that  $\gamma > \rho\mu$ . Therefore, in the unstable unregulated system the initiation of proportional fiscal policy will increase stability. If the unregulated system was already stable, then the proportional policy will either increase or decrease damping depending on whether  $\gamma$  is greater or less than  $\rho\mu$ . If  $\gamma = \rho\mu$  then the policy will not change the stability of the system, and  $\alpha$  will remain equal to  $(-\frac{1}{2} T_I)$  no matter how much we increased  $g_p$ . If  $\gamma > \rho\mu$  then  $\alpha > (-\frac{1}{2} T_I)$  and the policy will move  $\alpha$  toward  $(-\frac{1}{2} T_I)$  and thus increase damping. If  $\gamma < \rho\mu$  the policy will decrease damping. Since  $(-\frac{1}{2} T_I) < 0$  this policy used alone will never make the system unstable, but it could reduce damping. In previous studies where  $Y_F$  was a constant, such conditional results were not found.<sup>9</sup> There proportional fiscal policy always increased stability.

<sup>9</sup>See Phillips op.cit. (1954) and Baumol op.cit. (1961).

The effect of proportional policy on the growth rate  $y_F$  is also shown in Table 1. In the unregulated model,  $y_F = svx$  so that proportional policy has the effect of adding an additional term to the growth rate equation. However, the impact of the policy cannot be determined simply by comparing these two expressions for growth, because the value of  $x$  will be different in each case. To see the impact on growth, consider a situation where  $x$  is less than unity in the unregulated system; that is  $Y_A < Y_F$ . In this case the proportional policy will increase demand by deficit spending; thus total demand  $Y_A$  will be increased. This will result in higher value for  $x$  which will in turn induce investment causing a higher rate of growth. This effect is represented by the term  $svx$ . But we must also consider the second term. If  $x$  remains less than unity (although now greater than before) the term  $g_p v(x-1)$  will have the effect of not letting the growth rate increase quite as much. This is due to the fact that government spending, as we have defined it, does not add directly to investment. In this model, government expenditure is devoted entirely to the purchase of consumption goods; the addition to investment comes indirectly through the increase in the term  $x$  of the investment function.<sup>10</sup> If the increase in  $Y_A$  had been accomplished in the private sector then investment would have increased by the normal amount and the growth rate would be  $svx$ . The total effect of this government policy is to increase the rate of growth by increasing  $x$ , but not by the same amount as a similar increase in demand in the private sector. The argument is similar when  $x > 1$ ; in that case the government decreases the rate of growth by decreasing  $x$ , but not by the same amount as an equivalent decrease in demand in the private sector.

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<sup>10</sup>It can be shown (see Appendix 2 of the authors thesis of the same title) that if  $G$  is entirely investment spending then there is no effect on the growth rate. However, in this case the policy will have no stability effects.



	No Regulation	Fiscal Proportional $-g_p Y_F (x-1)$	Fiscal Integral $-g_i Y_F \int (x-1) dt$	Fiscal Derivative $-g_D Y_F Dx$
Damping Factor $\alpha$	$\frac{sv-\gamma+\rho\mu}{-2svT_I}$	$\frac{sv-\gamma+\rho\mu+g_p v}{-2(svT_I+T_I g_p v)}$	$\frac{sv-\gamma+\rho\mu+T_I g_i v}{-2T_I vs}$	
$\frac{\partial \alpha}{\partial g}$		$\frac{\gamma-\rho\mu}{-2T_I v (s+g_p)^2}$	$-\frac{1}{2sv}$	
Stability Increased if:		$\gamma > \rho\mu$	always	
Growth Rate $y_F$	$svx$	$svx+vg_p (x-1)$	$svx+vg_i \int_0^t (x-1) dt$	$svx+vg_D Dx$
Steady State Ratio $x_s$	$1 + \frac{m_f - \delta}{\beta}$	$1 + \frac{m_f - \delta}{\beta}$	$1 + \frac{\rho\mu(m_f - \delta)}{\rho\mu\beta + vg_i}$	$1 + \frac{(m_f - \delta)}{\beta}$

TABLE 1: The Effects of Various Policies with Zero Lag: Analytical Results.

Monetary Proportional $-g_{mp}(x-1)$	Monetary Integral $-g_{mi} \int (x-1) dt$	Monetary Derivative $-g_{mD} Dx$
$\frac{sv-\gamma+\rho\mu+\rho\mu(g_{mp}+1)}{-2T_I vs}$	$\frac{sv-\gamma+\rho\mu}{-2T_I vs}$	$\frac{sv-\gamma+\rho\mu}{-2(T_I vs + \rho\mu g_{mD})}$
$-\frac{\rho\mu}{2T_I vs}$	$0$	$\frac{T_I vs + \rho\mu g_{mD} - (sv-\gamma+\rho\mu)\rho\mu}{-2(T_I vs + \rho\mu g_{mD})^2}$
always	no effect	$\alpha < 0$
SVX	SVX	SVX
$1 + \frac{m_f - \delta}{\beta}$	$1 + \frac{m_f - \delta}{\beta + g_{mi}}$	$1 + \frac{m_f - \delta}{\beta}$

Now let us look at the goal of full capacity output in light of this effect on growth. Full-capacity output is defined by  $x = Y_A/Y_F = 1$ . Following the same procedure as above, if  $x < 1$  then government policy increases  $Y_A$ . But if  $Y_F$  increases by the same amount then the gap is not decreased in absolute terms. However, the government policy does not let  $Y_F$  increase by the same amount as  $Y_A$ , because the term  $vg_p(x-1)$  has a negative effect on  $y_F$ . We now see that the effect of  $g_p$  on the growth equation is necessary if the stability of the system is to be increased by proportional policy.

This proportional policy has no effect on the steady state value of  $x$ . If we want the steady state to be consistent with the desired level  $x = 1$ , then we must adjust  $m_f$ , the rate of increase in the money supply. Thus we might say that the target of  $x = 1$  is established by adjusting  $m_f$ , and proportional policy can be used to get us there faster.

Integral Fiscal Policy. Integral policy without lag will always increase the stability of the system by decreasing  $\alpha$ . If the system is already stable then the initiation of integral fiscal policy will bring the economy to full capacity output more quickly than it otherwise would.

To evaluate the impact of integral fiscal policy on growth we must compute the integral term in table 1. This is not possible unless we know the exact path of  $x$ , but we can consider some cases which help to explain its behavior. Let us assume that  $x < 1$  and that there is a steady increase in  $x$  towards 1 as the integral policy is applied. As in the proportional case there is an increase in the term  $svx$ . But now the negative effect of the integral policy is larger than the positive effect of  $svx$ . It is possible that the integral policy could drastically reduce the growth rate if it is initiated during a period

of depression ( $x < 1$ ). An explanation for this behavior can be found by examining the operation of the system after the steady state has been reached. If we assume that the steady state value of  $x$  is 1, then the value of the integral will approach a constant. That is,  $G$  will be a positive constant in the income equation. But  $I$  in the income equation will not be increased by  $G$  at all as can be seen in Figure 1. The value of  $x$  is 1 so that entrepreneurs will keep investment at a constant level.<sup>11</sup> The result is that government policy is having no positive effect on  $DY_F$ . But  $Y_F$  is now larger because  $Y_A$  is larger and  $Y_A = Y_F$ . Therefore, the proportional rate of growth  $y_F$  is smaller. Numerical simulation supports this conclusion as can be seen in Figure 2. Of course we would have the opposite chain of events when  $x > 1$  and the result would be a higher rate of growth. However, this seems less likely in most modern economies where unemployment is the more prevalent situation.

This negative effect indicates a trade-off consideration between stability and growth when using integral policy during a depression. The policy might be effective in bringing  $Y_A$  into equality with  $Y_F$ , but the economy would be growing at an extremely slow rate. Again this is a development not discovered in earlier studies because the growth rate was not considered.

Integral policy has a desirable effect on the steady state ratio of  $x$ . The higher the value of  $g_i$  the closer the economy will be to full capacity in the steady state. However, this fact must also be evaluated in light of the possible negative effects on growth.

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<sup>11</sup>The effect of interest rate would make investment even lower, because  $\gamma$  is greater with a higher value of  $Y_A$ .

Fiscal Derivative Policy. When this form of fiscal policy is included in the analysis, equation (14) does not reduce to second order. Therefore, an analytic investigation of stability including all possibilities is not feasible in simple mathematical terms. Numerical simulation shows that in all cases the derivative fiscal policy used alone with no lags was destabilizing. Figure 2 with  $g_D = .5$  shows the effect of this policy on the model.

Monetary Policies. Proportional monetary policy will always increase stability, and the growth relationship  $y_F = svx$  still holds as in the unregulated case. Thus the effect of proportional policy on growth is dependent on its effect on  $x$ . When  $x < 1$  the monetary authority will increase the money supply which will cause interest rates to fall and thus stimulate investment expenditure. The result is an increase in the rate of growth. In our model this increase is represented by an increase in  $x$ , because an increase in investment causes an increase in  $Y_A$  relative to  $Y_F$ .

Since there is no term  $g_{mi}$  in the damping factor  $\alpha$ , the integral monetary policy has no effect on the stability of the system. However, integral monetary policy does have the advantage of bringing the steady state value of  $x$  closer to unity.

The partial derivative of  $\alpha$  with respect to  $g_{mD}$  is negative if  $\gamma < 0$  and positive if  $\gamma > 0$ . Thus if the system is explosive the introduction of derivative policy will make the system less explosive but it will never stabilize the system.

If the economy is stable already, then the initiation of the derivative policy will decrease stability.

Lags in Fiscal Policy. When fiscal lags are included in the model, equation (14) cannot be reduced to second order and together with equation (16) forms a fifth order system. Numerical simulation was therefore used to examine the effects of lags.<sup>12</sup> Parameter values were chosen so that the unregulated economy had a zero damping factor with fluctuations of constant amplitude. Since the aim of the stabilization policies is to reduce these fluctuations without reducing growth, we can test their effectiveness by comparing unregulated and regulated simulations of the model. Figure 3 shows some selected results of these simulations with a lag of two years. Both the actual to full capacity ratio and the growth rate are plotted.

Fiscal proportional policy is still stabilizing with lags, but the longer the lag the longer it takes to reach full capacity. The effects of lags on growth are similar in that the steady state growth rate is approached more slowly as the lag is increased.

As lags are introduced in fiscal integral policy the desirable stabilizing effects are lost and pronounced fluctuations are generated. With a lag of two years the system becomes completely explosive. In addition the growth rate is reduced even further when lags are introduced. In both respects this integral policy used alone with lags has very undesirable consequences.

Where lags had undesirable effects on the other fiscal policies, similar lags have surprisingly desirable effects on fiscal derivative policy. With no lags we found that derivative policy was destabilizing, but as lags are introduced this policy can stabilize the system. The fluctuations of the growth rate are also stabilized.

<sup>12</sup>The numerical solution of these equations was performed at the Princeton University Computer Center. For a discussion of the programming methods involved see I T. Cundiff, "On the Numerical Solution of Ordinary Differential Equations," (Washington, D.C. August, 1967). Numerical parameter values are chosen such that the time unit is one year. They are:  $s=0.1$ ,  $v=0.25$ ,  $T_I = 1.0$ ,  $\gamma = 0.1$ ,  $\delta=0.03$ . The other parameters can be altered to generate various cases of the models behavior.

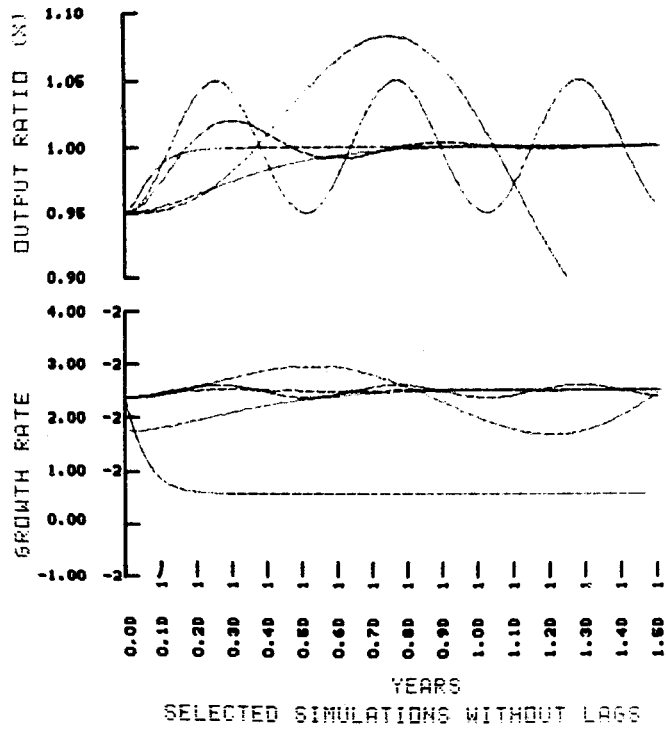


Figure 2

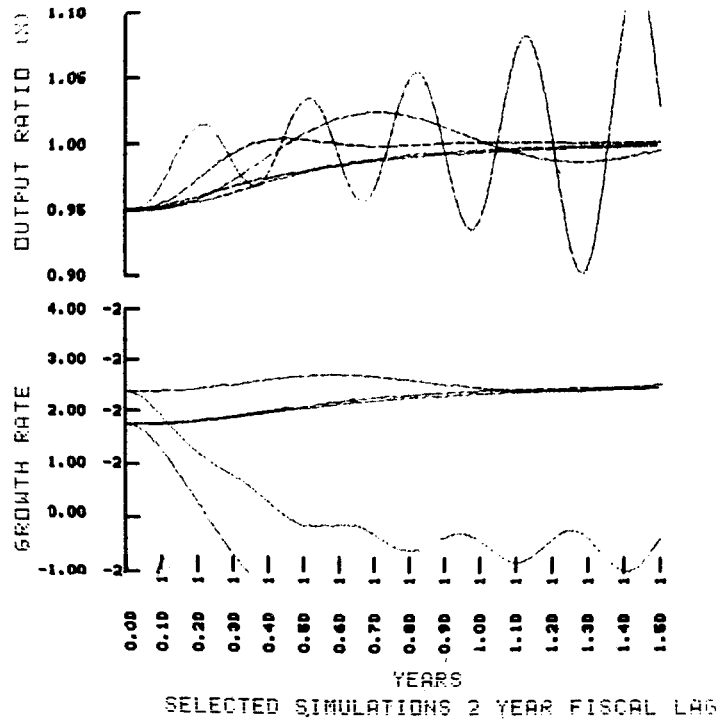


Figure 3