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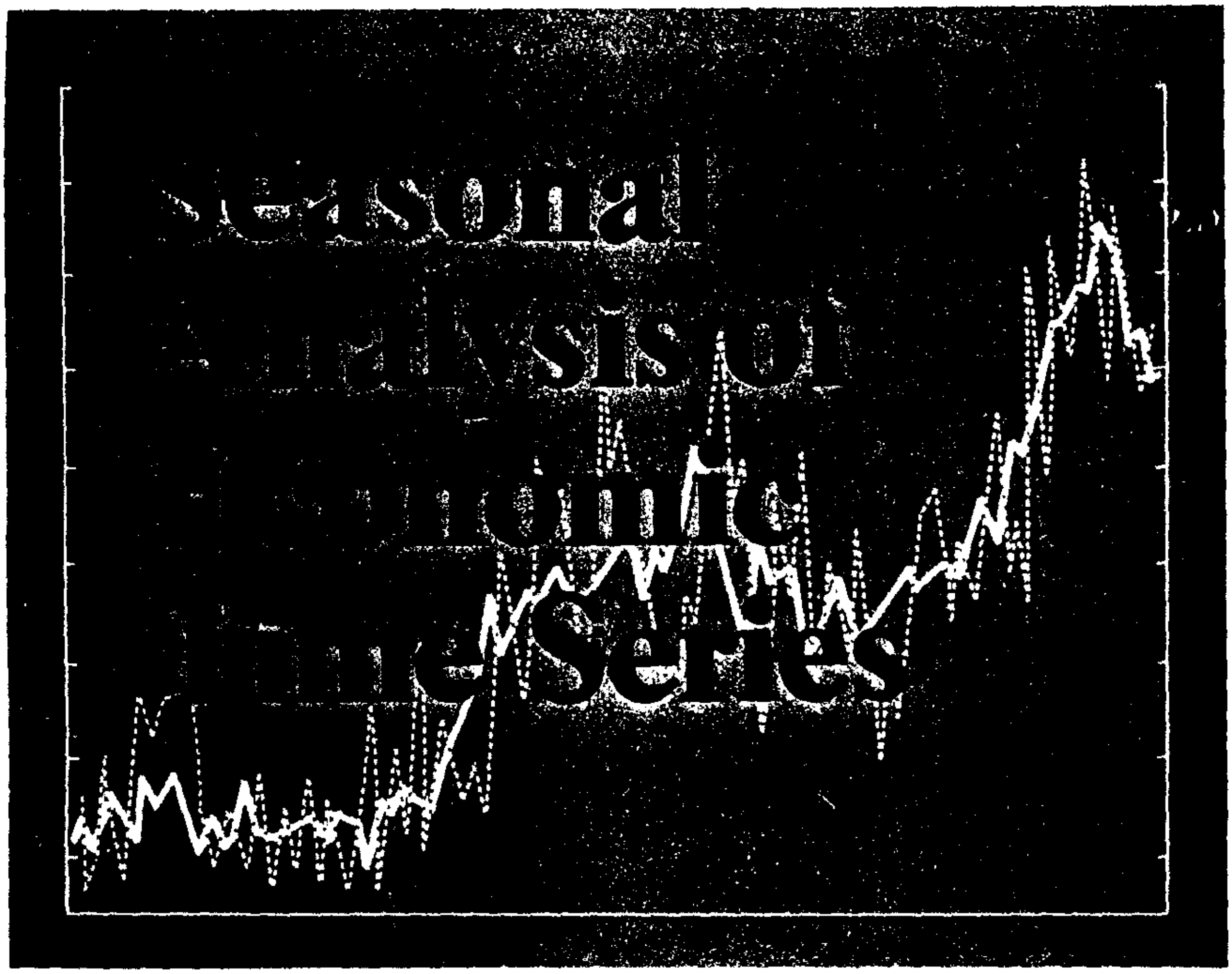

PROCEEDINGS OF THE CONFERENCE ON THE SEASONAL ANALYSIS OF ECONOMIC TIME SERIES

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COMMENTS ON "THE TEMPORAL AND SECTORAL AGGREGATION OF SEASONALLY ADJUSTED TIME SERIES" BY JOHN GEWEKE

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By treating the problem of seasonal adjustment as a general minimum mean-square signal extraction problem, Geweke derives a procedure for the seasonal adjustment of aggregate time series that optimally utilizes the correlation structure among the components of the aggregate series. His paper also contains a very general proof that the procedure is efficient, relative to procedures that either ignore the information provided by the component series or use it incorrectly. A byproduct of this derivation is a formula for the minimized value of the mean-square loss function that he uses to compute the efficiency of the optimal procedure, relative to several suboptimal procedures. The general finding is that the relative efficiency of the optimal procedure depends on the spectral density of the nonseasonal and seasonal components of the aggregate series and, in some cases, may be quite large.

In discussing this paper, it will be convenient to illustrate the problem in a considerably more simple model than used in Geweke's paper. This will allow abstraction from some technical issues that would otherwise significantly lengthen the discussion. Although most of the main points can be demonstrated in this less general setting, actual applications, of course, require the full model of the Geweke paper.

Consider two observable economic variables, x_1 and x_2, which are both additively composed of unobservable nonseasonal and seasonal parts

$$x_1 = x_1^N + x_1^S \qquad (1)$$

$$x_2 = x_2^N + x_2^S \qquad (2)$$

It is assumed that the pair of nonseasonal components (x_1^N, x_2^N) has a zero mean and covariance matrix $\sum^N$, and, similarly, the pair of seasonal components has a zero mean and covariance matrix $\sum^S$. Furthermore, the pair (x_1^N, x_2^N) is uncorrelated with (x_1^S, x_2^S).

The seasonal adjustment of the observable variables x_1 and x_2 can be viewed as a problem of extracting the nonseasonal parts x_1^N and x_2^N. However, suppose that the ultimate objective is not extracting the individual nonseasonal components x_1^N and x_2^N but, instead, the aggregate[1] nonseasonal component $y = x_1^N + x_2^N$. If the loss function is quadratic, then this objective can be formally represented as finding a value $\hat{y}$ to minimize

$$E[(y-\hat{y})^2 | x_1, x_2] \qquad (3)$$

Clearly, the minimizing value of $\hat{y}$ is

$$\hat{y}_A = E(y | x_1, x_2) = E(x_1^N | x_1, x_2) + E(x_2^N | x_1, x_2) \qquad (4)$$

and, in general, the values

$$\hat{y}_B = E(x_1^N | x_1) + E(x_2^N | x_2) \qquad (5)$$

or

$$\hat{y}_C = E(x_1^N + x_2^N | x_1 + x_2) \qquad (6)$$

will not minimize the criterion. Given the obvious optimality of the solution $\hat{y}_A$ in this simple problem, one might wonder why $\hat{y}_B$ and $\hat{y}_C$ would ever be of interest. The answer is that, in more complex practical problems with important data or computing limitations, $\hat{y}_B$ and $\hat{y}_C$ are frequently used as solutions to seasonal adjustment problems. In equation (5), each individual series is adjusted before aggregation without reference to the other series. The seasonally adjusted aggregate is then the sum of these individually adjusted series. This method is frequently used in practice when the series of interest is an aggregate of several other series. For example, the U.S. money supply is adjusted in this way, with the components of currency and demand deposits each adjusted before aggregation into M1. In equation (6), on the other hand, the observable series are first aggregated and then seasonally adjusted. This approach is frequently used in cases of temporal aggregation.

The optimal solution in equation (4) is the one derived by Geweke for general time series problems, and its superiority over the other two approaches is the reason for his statement that "for virtually every conceivable time series ... and a reasonably inclusive class of potential adjustment procedures, minimum mean-square error adjustment implies that seasonal adjustment should always precede temporal or sectoral aggregation." However, as it stands, this statement is misleading. As a comparison of (4) and (5) shows, it is not the preaggregation adjustment that is crucial for optimality, but rather the utilization of all observable components simultaneously. The solution $\hat{y}_B$ seasonally adjusts before aggregation but does not

[1] This sum may represent either a temporal or a sectoral aggregate.

utilize the joint distribution of the components of x_1 and x_2 properly. On the other hand, looking only at the first equality in (4), it is optimal to project the aggregate variable y on x_1 and x_2 to get the seasonally adjusted value.

In deciding whether the optimal solution $\hat{y}_A$ should be used in practice, the relative efficiency of this solution should be compared with the other methods. Geweke computes the relative efficiency of the optimal solution for several time series using his general formula for the mean square error, and this is a very useful and informative part of his analysis. The results indicate that, in cases where the series that are aggregated are very heterogeneous or where the stochastic structure of the nonseasonal and seasonal components are dissimilar, the relative efficiency of the optimal procedure is quite high. However, in cases where the series that are aggregated are homogeneous, the efficiency gains are likely to be small.

Some of the intuition behind these results comes from examining the simple model previously discussed. Suppose that $x_1^N, x_2^N, x_1^S, x_2^S$ has a joint normal distribution. Then, it is easy to show that

$$\hat{y}_A=a_1x_1+a_2x_2$$

$$\hat{y}_B=b_1x_1+b_2x_2$$

$$\hat{y}_C=c(x_1+x_2)$$

where the a, b, and c coefficients depend on the elements of $\sum^N$ and $\sum^S$ in such a way that, if $\sum^S=\sum^N$, then $a_1=a_2=b_1=b_2=c$ so that each procedure is identical. Further, if the covariance between x_1^N and x_2^N, as well as between x_1^S and x_2^S, is zero, then $\hat{y}_A=\hat{y}_B$.

These results provide a useful guide for the selection of seasonality problems that can potentially benefit from the optimal method. However, the results also suggest that there are likely to be relatively few aggregate series in this category. Moreover, those which are in this category should be disaggregated before serious economic analysis. Aggregation theory suggests that one should avoid aggregating heterogeneous components. Consequently, many analyses are likely to avoid aggregate series composed of grossly heterogeneous series. This may limit the potential usefulness of the results presented in this paper, but further experimentation with the methods is required before a definitive answer can be given.