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An Econometric Business Cycle Model with Rational Expectations: Some Estimation Results

by

John B. Taylor

Columbia University

The purpose of this paper is to develop and estimate a quarterly model of wage and price behavior which captures important expectational influences without neglecting the impact of wage contracts on the inflationary dynamics. While the work focuses on the wage-price sector we use the label "business cycle model" for two reasons: (1) The wage and price dynamics of the model produce cyclical swings in output, employment, and inflation which closely resemble business cycle fluctuations in the U.S. and other countries. In a typical cyclical pattern, inflation accelerates, government policy becomes restrictive, recession ensues, policy eases, a renewed inflationary boom begins, and so on. (2) The criterion we use for evaluating alternative policy proposals is their estimated ability to hold these recurrent swings of inflation and employment within tolerable limits. An anti-inflation policy, for example, would be judged not only by its forecasted success at bringing down inflation from an historically given high level, but also by its ability to prevent renewed cyclical inflationary surges. Similarly an antirecession policy would be judged not only by its success at stimulating the economy out of a particular recession, but also by the attention it pays to expectations and the cyclical workings of the economy to prevent a renewed recession shortly thereafter.

The premise upon which the model is based is that forecasts of future inflation rates and business conditions, which figure into wage negotiations, can be represented approximately by the forecasts of the model itself. This is, of course, the rational expectations technique. As a technique it is useful if it works better than available alternative techniques. Practical alternative expectations techniques now in use in econometric models include adaptive expectations mechanisms or subjective "constant adjustment" of expectations equations to make them look more reasonable or perhaps consistent with forecasters' expectations. The view presented here is that the rational expectations technique is potentially more useful than these alternatives. Of course, future research may discover alternative techniques (perhaps with learning behavior incorporated explicitly) which are superior to those currently available.

This paper describes the structure of the model, develops an estimation technique, and reports some estimation results. A follow-up paper will describe the policy evaluation procedures.[1]

1. The Structure of the Model

The macroeconomic model contains 5 endogenous variables and is constructed with quarterly U.S. data. Though small by some standards the model appears rich enough to serve as a framework for examining the policy questions mentioned above without straining computational resources. As will be shown, the full system is represented as a 5-dimensional vector simultaneous autoregressive moving average (ARMA) model with cross equation and cross error constraints. It is estimated by nonlinear maximum likelihood techniques. Because the constraints generally have no explicit analytic representation, even this simple model requires rather involved computations to obtain the maximum likelihood estimates. Limited information techniques might be useful for obtaining initial parameter estimates, but the essence of the rational expectations approach is that the cross equation constraints be fully specified and the system estimated jointly, so that the impact of changes in each behavioral equation on expectation formation in other equations is adequately accounted for.

In Sections (1.1), (1.2), and (1.3) which follow, we describe the structural equations of the model along with their stochastic elements. In Section (1.4) we specify the interaction between shocks to individual equations, and consider the model as a whole.

1.1 Contract Duration and the Determination of Wages and Prices

A difficulty with trying to incorporate wage contracts into a macroeconometric model is the reduction of the intricate detail of real world labor market contracts into a manageable framework suitable for empirical work without losing those details which make contracting important. The problem is to find a method of aggregating across contracts which are written in the same period with the same duration, and then determining the behavior of these "contract aggregates". If the method is to capture the interaction between contracts negotiated at different dates, then aggregation across contracts written at different points in time can only occur at the last stage of analysis after this interaction has been modelled. The problem becomes more difficult when the contract wages are unobservable.

Although very little information is available on implicit or explicit wage contracts in the U.S., information on the approximately 10 percent of U.S. workers in major collective bargaining gives perspective to the problem. Table 1 shows the number of workers in this group negotiating explicit contracts of different lengths each quarter during the last several years. Obviously it would be too gross an approximation to assume that contracts in the U.S. economy are all of the same length. Even if we ignore the 90 percent of all workers not represented in Table 1, and who probably work under implicit or explicit contracts averaging about 1 year in duration, the range of contract length is quite wide. On the other hand, abstracting from seasonal influences, the distribution of workers by contract length does not show any systematic pattern over time (with the exception of the well-known 3 year cycle in the longer term contracts). In particular, there does not appear to be a marked tendency for the distribution of contracts to change over the business cycle. Hence, if one is ultimately interested in describing the behavior of seasonally adjusted data, and if the one-year and two-year contracts in Table 1 are more representative of the economy as a whole, then a first approximation would be to assume that the distribution of workers by contract length is homogeneous overtime. This approximation results in a major simplification for the aggregation procedures, and will be made in the analysis which follows. We will refer to this as approximation (1).

To complete the aggregation procedure, three additional approximations will be made: (2) the variation in average contract wages across contract classes of different lengths is negligible relative to the variation in contract wages over time; (3) all wage adjustments occur during the quarter in which the contract is negotiated[2]; (4) any indexing which changes the wage contract at regular intervals during the contract period can be represented as a series of short term contracts, rather than as one long term contract. The third approximation is partially a matter of definition, and will tend to make our estimated distribution of contract lengths shorter than what a literal reading of Table 1 would indicate. Most indexing in the U.S. economy is found in multiyear contracts. The import of approximation (3) is that these indexed contracts are comparable to shorter contracts with lengths equal to the indexing review period. It is an approximation because contract wage adjustments are influenced by a wider range of factors when they are adjusted by renegotiation than by indexing.

Using these approximations we now proceed to develop a wage determination equation which takes account of the interaction of contracts of different duration signed at different points in time. Let

x_{jt} = average contract wage set in quarter t in contracts which are j quarters in length (j=1,...,J)

n_{jt} = number of workers affected by contract wage changes in quarter t in contracts which are j quarters in length (j=1,...,J)

w_t = average wage in the economy in quarter t

Then, by definition of w_t we have

$$(1.1) \qquad w_t = \frac{\sum_{j=1}^{J} \sum_{s=0}^{j-1} n_{jt-s} x_{jt-s}}{\sum_{j=1}^{J} \sum_{s=0}^{j-1} n_{jt-s}}$$

If the distribution of workers by contract length is homogenous over time ($n_{jt} = n_j$), and if the variation of average contract wages over contracts of different length is negligible ($x_{jt} = x_t$), then (1.1) reduces to

$$(1.2) \qquad w_t = \frac{\sum_{j=1}^{J} \sum_{s=0}^{j-1} n_j x_{t-s}}{\sum_{j=1}^{J} \sum_{s=0}^{j-1} n_j}$$

$$= \frac{\sum_{s=0}^{J-1} \sum_{j=s+1}^{J} n_j x_{t-s}}{\sum_{j=1}^{J} j n_j}$$

$$= \sum_{s=0}^{J-1} \pi_s x_{t-s}$$

$$= \pi(L) x_t$$

where the π_s are defined as

$$\pi_s = \left(\sum_{j=s}^{J-1} n_{j+1}\right)\left(\sum_{j=1}^{J} j n_j\right)^{-1} \tag{1.3}$$

and where $\pi(L)$ is a J-1 order polynomial in the lag operator L with the π_s as coefficients. Note that the π-weights sum to 1 and are time invariant. Hence the aggregate wage w_t is a fixed coefficient moving average of the "index" of contract wages x_t set in the recent past. An important presumption behind the empirical work which follows is that these weights are structural--they constitute the "contract technology" and are relatively insensitive to changes in economic policy.[3]

Some examples are useful for illustrating how the π weights depend on the distribution of workers across contracts of different lengths. If all contracts are the same length, say 4 quarters, then $n_1 = n_2 = n_3 = 0$, and $\pi_0 = \pi_1 = \pi_2 = \pi_3 = .25$. This is the type of contract distribution used in the theoretical examination of staggered contracts presented in Taylor (1979b). If the distribution of workers across contracts of different lengths is uniform up to 4 quarter contracts, then $n_1 = n_2 = n_3 = n_4$ and the π weights decline linearly:

$\pi_0 = .4$, $\pi_1 = .3$, $\pi_2 = .2$, and $\pi_3 = .1$. Note that the distribution of workers across contracts can be recovered from the π weights through the identity:

$$(1.5) \qquad (\pi_{i-1}-\pi_i)\pi_0^{-1} = n_i\Big(\sum_{j=1}^{J} n_j\Big)^{-1} \qquad i = 1,2,\ldots,J\ , \quad (\pi_J = 0).$$

The π-weights and hence this distribution of workers will be part of the economic structure to be estimated along with the other parameters of the model.

Equation (1.2) describes how the aggregate wage w_t evolves from the index of contract wages x_t. Since the contract wages which constitute this index will prevail for several quarters, workers and firms negotiating a contract wage will be concerned with the labor market conditions expected to prevail during the upcoming contract period. For example, those setting 4 quarter contracts will be concerned with the going wage and the availability of workers during the next 4 quarters, while those setting 8 quarter contracts must forecast these variables 8 quarters ahead. Moreover, in the process of forecasting future wages, these firms and workers will take account of contracts negotiated in the recent past since these will be part of the relative wage structure during part of the contract period.[4]

A behavioral equation for the determination of the contract wage index which takes account of these factors is given by

$$(1.6) \qquad x_t = \pi(L^{-1})\hat{w}_t + h_1\pi(L^{-1})\hat{e}_t$$

where h_1 is an adjustment parameter, e_t is an index of excess demand in the labor market, and p_t is the aggregate price level. The hat represents conditional expectations given information at time t-1, and the operator L^{-1} holds this viewpoint date constant while advancing the variable to be forecast one period ($L^{-1}\hat{w}_t = \hat{w}_{t+1}$). The polynomial $\pi(\cdot)$ is the same as that described in equation (1.3). Future average wages and demand conditions are weighted by these factors because of the distribution of contracts extending into future periods. For example, π_1 of all contracts signed in period t will last through the end of period t+1, and π_4 of all contracts will last through the end of period t+4. Therefore, $\hat{w}_{t+1}$ and $\hat{e}_{t+1}$ should be weighted by π_I while $\hat{w}_{t+4}$ and $\hat{e}_{t+4}$ should be weighted by π_4. The forward expectations operator in (1.6) is a compact way to represent this weighting scheme.

Some of the important questions about wage and price dynamics can be cast in terms of the parameters in equation (1.6). The parameter h_1 should be positive and of significant magnitude if aggregate demand management is to be effective in stabilizing wage inflation. Whether h_1 is large or small is relevant for determining how accommodative policy should be toward price or supply shocks. However, lagged price shocks could enter (1.6) directly to portray catch-up effects. This latter possibility will be considered below when we introduce a stochastic structure to the behavioral equations.

In what follows it will be convenient to express the equations of the model in log-linear form. For this reason we will interpret w_t,

x_t and p_t as logarithms of the average wage, the contract wage index, and the price level. Hence, the aggregation of the contract wages into the average wage should be interpreted as a geometric averaging procedure. The excess labor demand variable e will be measured as the negative of the unemployment rate, measured as a fraction of the labor force rather than as a percentage. Hence, we do not take a logarithmic transformation of e_t. In equation (1.6) and in the equations which follow the constant terms and any trend factors will be omitted since we will be working with detrended data when the model is estimated.[5]

Given the aggregate wage w_t as determined from the contract wage index x_t, we will assume that prices are determined on the basis of wage and other costs, that is

$$(1.7) \qquad p_t = \hat{w}_t + \theta_p(L)u_{pt}$$

where $\theta_p(L)$ is a lag polynomial and u_{pt} is a serially uncorrelated shock. The term $\theta_p(L)u_{pt}$ is a measure of other factors affecting pricing decisions. Our assumption is that the prices which underlie the index p_t are relatively free to vary so that no additional dynamics in the model enter explicitly through staggered price contracts. However, we do model the influence of wages on price decisions as operating with a one period lag; firms forecast their wage costs $\hat{w}_t$ during the current period and set p_t accordingly. In the empirical work which follows the error term will be a general stochastic process, so that exogenous serial correlation in the detrended real wage $w_t - p_t$ will be part of the model. Some of the other factors which may affect p_t relative to

$\hat{w}_t$ might be demand conditions, raw material costs, and temporary fluctuations in productivity about trend.

1.2 Aggregate Demand and Employment

The main reason for modelling the aggregate demand side of the economy is to close the model so that expectations of future wages and employment can be forecast rationally. To keep the analysis simple we will assume that

$$(1.8) \qquad y_t = \alpha_1(m_t - p_t) + \theta_y(L)u_{yt}$$

where y_t is the log of (detrended) real output, and m_t is the log of the (detrended) money supply. As in equation (1.7) $\theta_y(L)$ is a lag polynomial and u_{yt} is a serially uncorrelated error. The main variable missing from equation (1.8) is a measure of the real interest rate which would link this equation explicitly with investment and consumption decisions and with fiscal policy. Our approach at this stage is to model these factors as part of a general stochastic structure; an alternative procedure would be to add lagged values of y_t, m_t and p_t to equation (1.8), but sorting out these lags from the serial correlation structure is quite difficult. A natural extension of this model would be to introduce real interest rate factors or measures of expected inflation in equation (1.8).

To link the aggregate demand variable y_t to our measure of labor market tightness e_t, we will utilize an Okun's law type relationship with serial correlation to approximate temporary discrepancies or lags. That is, (1.9) $$e_t = \alpha_2 y_t + \theta_e(L)u_{et}$$

where $\theta_e(L)$ is a polynomial in the lag operator and u_{et} is a serially uncorrelated error. Since y_t is the log deviation of output about trend it will behave like the negative of the percentage output gap. With e_t defined as the negative of the unemployment rate, α_2 should approximately equal the inverse of the Okun's law multiplier.

1.3 The Government Policy Reaction Function

Since fiscal policy is assumed to be incorporated in the error structure of the aggregate demand equation (1.8), the only tool of aggregate demand management which we model explicitly is monetary policy. We will consider feedback reaction functions of the following form:

(1.10) $$m_t = g_1\hat{p}_t + g_2\hat{w}_t + g_3\hat{y}_t + \theta_m(L)u_{mt}$$

where $\theta_m(L)$ is a polynomial in the lag operator and u_{mt} is serially uncorrelated. Equation (1.10) is a feedback rule because all the variables on the right hand side are predetermined; they are forecasts of conditions in period t given information through the previous period. The coefficient g_3 represents attempts at counter cyclical monetary policy; we would expect that g_3 is negative. The coefficients g_1 and g_2 and their sum are measures of how accommodative monetary policy is to price shocks or wage shocks. If $g_1 = g_2 = 0$, then policy is not accommodative at all, while if $g_1 < g_2$ then policy is less accommodative to price shocks than to wage shocks. An important policy question is whether it is appropriate to accommodate prices, but not wages: the answer depends in part on whether prices enter the wage equation. In order to explore possible variations in g_1 and g_2 it is necessary to estimate these parameters jointly with the rest of the model. It should be emphasized that the form of equation (1.10) is not derived from a policy optimization procedure. In general we would expect an optimal feedback rule to depend more explicitly on lagged values of the endogenous variables or on the shocks to the other equations.[6]

1.4 Summary of the Equations and the Stochastic Structure

Gathering the above equations together we have:

$$
\begin{aligned}
y_t &= \alpha_1(m_t - p_t) + \theta_y(L)u_{yt} \\
p_t &= \hat{w}_t + \theta_p(L)u_{pt} \\
m_t &= g_1\hat{p}_t + g_2\hat{w}_t + g_3\hat{y}_t + \theta_m(L)u_{mt} \qquad (1.11) \\
e_t &= \alpha_2 y_t + \theta_e(L)u_{et} \\
w_t &= \pi(L)x_t \\
x_t &= \pi(L^{-1})\hat{w}_t + h_1\pi(L^{-1})e_t + \theta_{xp}(L)u_{pt} + \theta_x(L)u_{xt}
\end{aligned}
$$

Note that in the contract wage index equation we have added a serially correlated error structure $\theta_x(L)u_{xt}$ and more importantly a cross error term $\theta_{xp}(L)u_{pt}$ which captures catchup effects from past price shocks to wages.

We will assume that the vector $(u_{yt}, u_{pt}, u_{mt}, u_{et}, u_{xt})$ is serially uncorrelated with zero mean and covariance matrix Σ. This correlation assumption does put restrictions on the model despite the fact that we are considering fairly general error processes in each equation via the θ parameters. We are assuming that there is only one cross-effect in the errors (θ_{xp}); the omission of other cross-effects is a constraint.

The parameters of the model are α_1, α_2, h_1, g_1, g_2, g_3, Σ and the coefficients of the polynomials $\pi(L)$, $\theta_y(L)$, $\theta_p(L)$, $\theta_m(L)$, $\theta_e(L)$, $\theta_{xp}(L)$, $\theta_x(L)$. Hence the number of parameters depends on the length of the longest contract considered (which determines the order of π), and the extent of serial correlation. The model has two simultaneous equations where more than one current endogenous variable appear. Most of the equations contain one-period ahead rational forecasts of the endogenous variables, but only the contract wage equation contains multiperiod forecasts. Extensions of the model might add multiperiod forecasts in other equations--in particular the aggregate demand equation and the price equation. In the next section we show how the model can be manipulated to obtain a form which can be estimated.

2. Solution and Estimation Techniques

The contract wage equation for x_t involves forecasts of the wage rate w_t and labor market demand e_t as far into the future as the length of the longest contract. These forecasts are conditional on all information available through the end of period t-1, and can be written as functions of the past shocks to each equation of the model. One solution technique is therefore to solve the equation of the system for $\hat{w}_{t+1}$ and $\hat{e}_{t+1}$--the rational forecasts--substitute these into the contract wage equation and finally determine a reduced form for the contract wage. The solutions of the model for e_t and w_t are

$$(2.1)\qquad \hat{e}_t = -\alpha_2(1 - \alpha_1 g_3)^{-1}[\beta\alpha_1\pi(L)\hat{x}_t - \theta_y(L)\hat{u}_{yt} + \beta_1\alpha_1\theta_p(L)\hat{u}_{pt} - \alpha_1\theta_m(L)\hat{u}_{mt}] + \theta_e(L)\hat{u}_{et}$$

$$(2.2)\qquad \hat{w}_t = \pi(L)\hat{x}_t$$

where $\beta = 1 - g_1 - g_2$ and $\beta_1 = 1 - g_1$ and where it should be noted that $\hat{u}_{ys} = 0$ for $s > t - 1$ and $\hat{u}_{ys} = u_{ys}$ for $s \leq t - 1$ and similarly for the other random shocks.

Substituting (2.1) and (2.2) into the equation for the contract wage x_t and taking expectations results in

$$(2.3)\qquad [1 - (1 - h_1\gamma\beta\alpha_1)\pi(L^{-1})\pi(L)]\hat{x}_t = h_1\gamma\pi(L^{-1})\theta_y(L)\hat{u}_{yt} - h_1\gamma\beta_1\alpha_1\pi(L^{-1})\theta_p(L)\hat{u}_p + h_1\gamma\alpha_1\pi(L^{-1})\theta_m(L)\hat{u}_{mt} + h_1\pi(L^{-1})\theta_e(L)\hat{u}_{et} + \theta_{xp}(L)\hat{u}_{pt} + \theta_x(L)\hat{u}_{xt}$$

where $\gamma = \alpha_2(1 - \alpha_1 g_3)^{-1}$. Equation (2.3) is a difference equation in the forecast of the contract wage $\hat{x}_{t+s}$ conditional on information through period t-1 with various combinations of the past shocks as forcing variables. To solve the equation we note its symmetry since the coefficients of L^s and L^{-s} in $\pi(L^{-1})\pi(L)$ are the same. Hence, the lag operator in brackets on the left hand side of (2.3) can be factored into a form $\lambda A(L)A(L^{-1})$. Imposing stability on the system requires that we chose A(L) so that its roots are outside or on the unit circle and since, by symmetry, half the roots are outside or on the unit circle the factorization is unique (see Taylor (1979b)). Multiplying both sides of (2.3) through by $[\lambda A(L^{-1})]^{-1}$ and adding u_{xt} to the equation gives

$$(2.4)\qquad A(L)x_t = h_1\gamma\lambda^{-1}H_y(L)u_{yt} + [H_{xp}(L) - h_1\gamma\beta_1\alpha_1H_p(L)]u_{pt} + h_1\gamma\alpha_1\lambda^{-1}H_m(L)u_{mt} + h_1\lambda^{-1}H_e(L)u_{et} + H_x(L)u_{xt} + u_{xt}$$

where

$$(2.5)\qquad \begin{aligned} H_y(L) &= [(A(L^{-1}))^{-1}\pi(L^{-1})\theta_y(L)]_+ \\ H_p(L) &= [(A(L^{-1}))^{-1}\pi(L^{-1})\theta_p(L)]_+ \\ H_m(L) &= [(A(L^{-1}))^{-1}\pi(L^{-1})\theta_m(L)]_+ \\ H_e(L) &= [(A(L^{-1}))^{-1}\pi(L^{-1})\theta_e(L)]_+ \\ H_x(L) &= [(A(L^{-1}))^{-1}\theta_x(L)]_+ \\ H_{xp}(L) &= [(A(L^{-1}))^{-1}\theta_{xp}(L)]_+ \end{aligned}$$

and the notation $[\cdot]_+$ means that only the positive powers of L in the polynomial products are retained (see Whittle (1963) or Hansen and Sargent (1979)).

Equation (2.4) is autoregressive in the contract wage with moving average errors entering from all the equations of the model. Past shocks to aggregate demand enter the equation with positive coefficients (if these shocks are positively correlated), because aggregate demand shocks are an indicator of low unemployment in the near future which tends to bid up contract wages ($h_1 > 0$). For similar reasons monetary surprises u_{mt} and employment surprises u_{et} enter the wage equation with positive coefficients. The impact of past price shocks on wage determination is ambiguous, however. We would expect the sum of the coefficients of $H_{xp}(L)$ to be positive, but this catchup effect may be offset in the reduced form by $h_1\gamma\beta_1\alpha_1 H_p(L)$ which captures the "anti-inflation" reaction of the monetary authorities to price shocks. If $g_1 = 1$, so that price movements are completely accommodated, then $\beta_1 = 0$ and the "anti-inflation" effect drops out. But if g_1 is less than one, then price shocks may appear to have a negative effect on wages.

The sensitivity of wages to excess demand h_1 enters into the reduced form wage equation in several ways. It is one of the determinants of the autoregressive coefficients in A(L) because it appears in the symmetric lag polynomial on the right hand side of equation (2.3). Higher values of h_1 will tend to reduce the coefficients of A(L) and make wage changes less persistent. However, h_1 also enters into the serial correlation and cross serial correlation coefficients in the x_t equation. In these serial correlation expressions higher values of h_1 will raise the impact of all these shocks on wage behavior. This effect represents the interaction between the forecasts of future labor market demand (via extrapolations from the model using recent observations) and the impact of demand on wage behavior.

The policy parameters g_1, g_2, and g_3 also enter the equation in several ways. The sum of $g_1 + g_2$ represents the combined accommodation of monetary policy to price and wage shocks. This sum enters the autoregressive coefficients through the parameter β. Larger values of $g_1 + g_2$ imply larger autoregressive coefficients and more persistence of wage changes. As mentioned above, g_1 has an effect which g_2 does not have; the price accommodation parameter tends to affect the feedback of prices onto wage determination. Hence, if g_1 is small then price shocks will not have as large an impulse effect on the wage-price dynamics. But g_1 does not have any unique ability to change the propagation of these price shocks once they are into the dynamics. Both accommodation parameters are equally powerful at changing the propagation properties (i.e. the autoregressive weights). Proposals for policies which are very accommodative toward price shocks, but not toward wage shocks, evidently place emphasis on reducing the propagation effects while not regarding large impulse effects. This rather technical discussion is of course closely related to the question of whether price shocks get incorporated into the underlying inflation rate.

To complete the solution of the model we need to substitute the reduced form contract equation back into the structural equations. First we compute the average wage w_t which simply requires us to pass x_t through the moving average operator $\pi(L)$. This results in

(2.6) $w_t = -A_1(L)w_t + G_y(L)u_{yt} + G_p(L)u_{pt} + G_m(L)u_{mt} + G_e(L)u_{et} + G_x(L)u_{xt} + u_x$

where $G_y(L) = \pi(L)h_1\gamma\lambda^{-1}H_y(L)$

$G_p(L) = \pi(L)[H_{xp}(L) - h_1\gamma\beta_1\alpha_1 H_p(L)]$

$G_m(L) = \pi(L)[h_1\gamma\alpha_1\lambda^{-1}H_m(L)]$

$G_e(L) = \pi(L)[h_1\lambda^{-1}H_e(L)]$

$G_x(L) = \pi(L)H_x(L)$

$A_1(L) = [A(L)]_+$

Equation (2.6) along with the equations for y_t, p_t, m_t, and e_t constitute a system in which only one period ahead forecasts of the endogenous variables appear, all with viewpoint date t-1. Using matrix notation we can write this system as

(2.7) $y_t = C_o y_t + C_1\hat{y}_t + C(L)y_t + D(L)u_t + u_t$

where the C and D matrices are relatively sparse:

$$C_o = \begin{bmatrix} 0 & -\alpha_1 & \alpha_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \alpha_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ g_3 & g_1 & 0 & 0 & g_2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C(L) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -A_1(L) \end{bmatrix}$$

$$D(L) = \begin{bmatrix} \theta_y(L) & 0 & 0 & 0 & 0 \\ 0 & \theta_p(L) & 0 & 0 & 0 \\ 0 & 0 & \theta_m(L) & 0 & 0 \\ 0 & 0 & 0 & \theta_e(L) & 0 \\ G_y(L) & G_p(L) & G_m(L) & G_e(L) & G_x(L) \end{bmatrix} ,$$

and where

$$y_t = (y_t, p_t, m_t, e_t, w_t)'$$

$$u_t = (u_{yt}, u_{pt}, u_{mt}, u_{et}, u_{wt})' .$$

Substituting for $\hat{y}_t$ (which involves no factorization) results in

$$(2.8) \qquad y_t = C_0 y_t + (I - C_0)(I - C_0 - C_1)^{-1} C(L) y_t + (I - C_0)(I - C_0 - C_1)^{-1} D(L) u_t + u_t$$

or

$$(2.9) \qquad A_0 y_t = A(L) y_t + B(L) u_t$$

where the matrix A_0 and the matrix polynomials $A(L)$ and $B(L)$ are defined accordingly. This equation system is simultaneous ($C_0 \neq 0$) with an autoregressive moving average structure. The structure is very heavily constrained both because $C(L)$ and $D(L)$ are constrained as discussed above, and because C_0 and C_1 contain many of the same elements which are in C(L) and D(L).

We can estimate the system (2.8) using maximum likelihood techniques using the working assumption that u_t is normally distributed. The concentrated log likelihood function (given a set of initial conditions) can be written as

$$(2.10) \quad -\frac{T}{2} \log \left| \sum_{t=1}^{T} u_t u_t' \right| + T \log |I - C_0|$$

excluding the constant term. This function can be evaluated numerically in terms of the fundamental structural and stochastic parameters introduced in Section 1, and hence can be maximized using numerical techniques. Tests of the model can be developed using likelihood ratio tests, and standard errors can be estimated from the matrix of second derivatives of the likelihood function. Since the factorization technique requires finding the roots of polynomials with orders as high as 8, we cannot represent the constrained likelihood function analytically. Hence derivatives and second derivatives must be computed numerically. We had most success using the Davidon-Fletcher-Powell technique, computing numerical first derivatives during each iteration. The matrix of second derivatives was computed at the last iteration for the purposes of statistical inference.

3. Empirical Implementation

Specific empirical measures for the 5 endogenous variables used for estimation were: real GNP for y, the GNP deflator for p, compensation per manhour in the private sector for w, the M1 definition of the money supply for m, and the (inversely scaled) unemployment rate for males between the ages of 25 and 54 for e. Seasonally adjusted data was used in each case. For all the variables except e, a logarithmic transformation was used, and the transformed data was detrended linearly over the sample period 1960:1 through 1977:4.

In order to limit the number of parameters to be estimated in the stochastic processes describing the shocks to each equation, the general moving average representation for these shocks was restricted. The restrictions were of two types: first, the moving average was truncated after a certain number of lags, and second, the coefficients of the resulting truncated lag were constrained to be functions of a smaller number of parameters than the length of the lag. More specifically the following parametric forms were assumed for the θ-polynomials describing the stochastic part of the model:

$$\theta_y(L) = [1 - \rho_{y1}L - \rho_{y2}L^2]^{-1} \tag{3.1}$$

$$\theta_p(L) = [1 - \rho_p L]^{-1} \tag{3.2}$$

$$\theta_m(L) = [1 - \rho_{m1}L - \rho_{m2}L^2]^{-1} \tag{3.3}$$

$$\theta_e(L) = [1 - \rho_{e1}L]^{-1} \tag{3.4}$$

(3.5) $$\theta_{xp}(L) = [1 - \rho_{xp}L]^{-1}$$

(3.6) $$\theta_x(L) = 1 + \theta_{x1}L + \theta_{x2}L^2 + \theta_{x3}L^3 + \theta_{x4}L^4$$

where the subscripted ρ and θ parameters were treated as unconstrained. The infinite power series in the lag operator in equations (3.1) through (3.5) were truncated at the 4th order. We found that some truncation of these polynomials was necessary to keep the order of moving average parts of the vector model from growing too large. The G polynomials in equation (2.6) have orders equal to the maximum contract length plus the order of the corresponding θ polynomials. (Exploiting the simple form of the inverse of the nontruncated θ polynomials was not useful in reducing the moving average length as is typical in ARMA modelling). In choosing the parametric form and the truncations in (3.1) through (3.6), the serial correlation matrices of the residuals were examined; when the serial correlation was too high, the restrictions were loosened.

In addition to restricting the θ polynomials, we also put constraints on the π polynomial which describes the distribution of contracts in the economy by length. For the results reported below we truncated π at the 7th lag, thereby permitting a maximum contract length of 8 quarters. The shape of π was also constrained to decline very slowly for short lags and never to take on negative values. (Recall that neither negative nor increasing π weights make any economic sense from the point of view of contract distributions). Operationally, these constraints were imposed by assuming that π_j/π_0 is equal to $K_1 \exp(-j^2/\delta)$ with K_1 chosen to make the first value equal to 1. This is simply the right hand side of a normal density, and while it is convenient and has the slope we would expect for the π weights, further work will be required to see whether the constraint is statistically acceptable. The hypothesis could be tested by estimating the model with fully unconstrained π weights and comparing the value of the likelihood function with that in the constrained case.

With these specifications the system (2.9) becomes a 5-dimensional vector ARMA (7,11) model with simultaneous relationships among the dependent variables. The 90 elements of the autoregressive and moving average matrices are functions of 18 fundamental parameters. The computational steps for evaluating the likelihood function in terms of these parameters are summarized as follows: (1) evaluate $\pi(L)\pi(L^{-1})$ and hence the 14th order symmetric polynomial on the left hand side of equation (2.3); (2) factor this polynomial to obtain $A(L)$ in equation (2.4) and use the inverse of $A(L)$ to evaluate the truncated polynomials in (2.5) and thereby obtain the basic G-polynomials in the wage equation (2.6); (3) evaluate $(I - C_0)(I - C_0 - C_1)^{-1}$ in equation (2.8); (4) compute a time series of vectors u_t corresponding to these parameters using equation (2.8) and from these compute the log likelihood function (2.10). Each function evaluation requires this same sequence of computations and for the model estimated here takes about .1 second of CPU time on an IBM 360/91. These function evaluations were used for computing gradients during the iterations of the Davidon-Fletcher-Powell algorithm and for computing numerical second order derivatives for estimating the variance-covariance matrix of the estimated coefficients.

4. Estimation Results

The estimates of the structural and stochastic shock parameters are given in Table 2 along with the ratio of these coefficients to their standard errors as computed from the inverse of the second derivative matrix of the likelihood function. All the structural coefficients have signs and magnitudes which are reasonable. The elasticity of real GNP with respect to real money balances α_1 corresponds to an income elasticity of money demand of about 2/3. The estimated elasticity of unemployment with respect to the output gap is .4, which corresponds to an Okun's law multiplier of 2.5. The responsiveness of contract wages to excess demand h_1 is .11, but is only marginally significantly different from zero. Since the policy evaluation procedures, which we will report subsequently, are very dependent on h_1, it will be important to establish whether h_1 is sensitive to changes in functional forms or stochastic assumptions.

The policy parameters g_1, g_2, and g_3 indicate that monetary policy was significantly accommodative during the sample period. The sum of g_1 and g_2 which represents the combined accommodation to wage and price shocks is .53 with a standard error of .18. (The estimated covariance between the estimates of g_1 and g_2 is -.019). However, the individual accommodation coefficients suggest that it is important to distinguish wages from prices when estimating reaction functions. According to these estimates, policy is almost fully accommodative to wages, but not at all accommodative to prices. In fact price shocks seem to generate a restrictive monetary policy, after taking account of the accommodation to wages. Whether this is optimal or not depends on how price shocks enter into the inflationary dynamics--both through expectations and contract effects--and is an issue which we hope to be able to address with this type of a model. Finally, the parameter g_3 indicates a countercyclical reaction of monetary policy. When the economy is expected to move below full employment, monetary policy becomes more stimulative.

The estimate of δ, which constrains the contract distribution weights, is most easily interpreted in terms of the π-weights or the implied distribution of contract lengths. These are given in Table 4. According to these estimates, contract lengths in the 3 to 4 quarter range appear to predominate. This corresponds to the general view that most implicit contracts are about 1 year in length.

The parameters of the stochastic processes which describe the stochastic shocks are generally very significant with the exception of the last three unconstrained parameters of the wage shock. The serial correlation matrices presented in Table 5 suggest that some serial correlation remains in the estimated residual vectors so that further experimentation with the stochastic processes would be useful.[7]

The cross-serial correlation parameter between price shocks and wage shocks is very significant and indicates that past price shocks do feed back into the wage formation process. If this impulse effect of price shocks on the inflationary propagation process is to be offset, then price shocks should not be accommodated as much as wage shocks. Attempts to reduce this impulse effect may explain the big difference between the price and wage accommodation parameters g_1 and g_2 during the sample period.

The implied autoregressive and moving average coefficients for the model are given in Table 3, where the constraints we have imposed are fairly evident. Note that these coefficients represent the simultaneous form of the model. The reduced form would be obtained by multiplying through by A_0^{-1} and would have fewer elements constrained to equal zero. For example, some of the reduced form dynamics in y_t are due to past movements in m_t and p_t. These would be evident in the reduced form, but are only implicit in the simultaneous form of the model.

The constraints which the model and the expectations assumptions put on the wage and price dynamics are evident in the second and fifth rows of the matrices A and B given in Table 5. The second row corresponds to the price dynamics and the fifth row to the wage dynamics. With one important exception these dynamics are the same. The exception is that $B(2,2) \neq B(5,2)$. The B(2,2) coefficients are partially determined by the impact of non-wage shocks on the pricing process; that is, the influence of the other components of unit costs, such as productivity shifts. The B(5,2) coefficients reflect the impact of these same price shocks on wages.

Note, however, that both B(2,2) and B(5,2) as well as most of the other elements of the matrices A and B depend on the policy parameters g_1, g_2, and g_3. As these coefficients change, the coefficients of A and B will change in a predictable way. It is this impact of the policy parameters on the dynamics of the model which will form the basis of the policy evaluation procedure.

5. Concluding Remarks

The aim of this paper has been to develop a quarterly empirical model of wage and price dynamics and to embed these dynamics in a simple model of the U.S. economy. The main features of the model are (1) the use of rational expectations to describe the impact of future inflation rates, business conditions, and economic policy on current wage determination, and (2) the use of a wage contracting process to explain the serial persistence of wages which is not captured by pure expectations effects. A discussion of econometric and computational techniques showed how the parameters of the model could be estimated in practice, and some estimation results were reported for a specific form of the model. The econometric techniques seem to work well and the estimation results seem reasonable. While alternative functional forms of the equations and the stochastic structure might be investigated, the general framework of the model appears to be an adequate basis for evaluating alternative policy proposals.

Footnotes

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1. The framework presented here rests heavily on extensive research during the last several years. In particular: the critique by Lucas (1976); the rational expectations modelling techniques studied by Barro (1976), Fischer (1977), Phelps and Taylor (1977), Sargent and Wallace (1975), and Taylor (1979b); the empirical work by Sargent (1978) and Taylor (1979a, 1979c) ; and the econometric techniques studied by Hansen and Sargent (1979), Ravenkar (1979), and Wallis (1979).

2. This "full front-end loading" approximation is the easiest of the four to modify, and if deferred wage increases turn out to be important empirically (see Table 1a for an indication), it should be modified.

3. Note that approximation (2) could be generalized to permit a time-invariant dispersion in the contract wages across different contract lengths while still retaining the simplification of invariant π_s. This would alter the interpretation of π_s which follows, however.

4. See Taylor (1979b) for a further discussion of these issues.

5. It can be shown that if all variables in the model are measured as deviations from trend then x_t can also be measured as a deviation from trend.

6. An optimal control calculation is one of the aims of the model estimated in Taylor (1979a). Future work might be concerned with estimating a policy function of a more general type so that optimal control can be considered in this model via parametric changes in a given functional form.

7. We tried extending the truncation period for the stochastic processes out as far as 8 quarters, but only for a model with contract lengths equal to 4. This did appear to reduce the serial correlation. However, with 8 quarter contracts this would give rise to an ARMA(7,15) model which we did not try to estimate.

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TABLE 1

Number of Workers in Major Collective
Bargaining Situations by Contract Duration*

1974:III - 1978:IV

(Thousands)

	1-Year Contracts	2-Year Contracts	3-Year Contracts
1974:III	233	269	1477
IV	131	116	612
1975:I	67	86	395
II	172	326	264
III	325	215	529
IV	77	84	231
1976:I	29	67	158
II	109	259	1044
III	163	159	673
IV	82	78	1104
1977:I	43	98	226
II	215	138	950
III	125	121	1325
IV	52	60	400
1978:I	19	29	338
II	104	195	380
III	70	238	599
IV	56	83	344

*A contract is classified as an N year contract if its duration is N years plus or minus 6 months.

Source: <u>Current Wage Developments</u>; figures refer to situations with 1000 or more workers.

TABLE 1a

Negotiated Wage Adjustments in Major Collective Bargaining Situations by Contract Length

1974:III - 1978:IV

(Percent Change at Annual Rate)

	1-year Contracts	2-Year Contracts		3-Year Contracts		
	First Year	First Year	Second Year	First Year	Second Year	Third Year
1974:III	10.5	10.4	5.9	10.7	5.0	4.5
IV	11.4	11.2	6.4	10.4	5.7	4.9
1975:I	6.8	13.8	9.6	13.2	4.0	4.3
II	6.5	10.9	8.4	10.9	6.0	5.3
III	7.8	8.9	6.6	10.7	6.0	5.8
IV	7.3	10.6	7.1	9.8	5.3	4.5
1976:I	6.2	8.0	6.9	9.3	8.1	6.1
II	5.8	7.0	5.2	9.0	6.1	5.8
III	5.5	8.7	6.5	10.7	6.7	5.5
IV	6.2	7.0	5.2	7.2	4.1	3.3
1977:I	3.4	9.2	8.5	7.7	5.6	4.9
II	5.6	7.5	5.8	8.8	5.1	3.9
III	6.1	7.0	6.3	7.8	4.3	3.9
IV	3.8	9.2	6.7	8.2	5.4	5.5
1978:I	6.4	5.5	4.6	10.6	6.4	5.5
II	5.2	7.7	7.1	6.8	5.4	4.9
III	6.5	7.4	6.0	7.5	5.7	5.6
IV	7.2	8.7	5.9	7.6	5.2	5.5

Source: Current Wage Developments; figures exclude cost of benefits and refer to situations with 1000 or more workers.

TABLE 2

Maximum Likelihood Estimates
of the Structural Parameters

Parameter	Estimate	Asymptotic "t-ratio"
α_1	1.48	5.5
α_2	.40	11.6
h_1	.11	1.5
g_1	- .46	3.5
g_2	.99	6.2
g_3	- .11	2.1
δ	2.55	5.4
ρ_{y1}	1.30	14.9
ρ_{y2}	- .50	4.9
ρ_p	.78	3.5
ρ_{m1}	1.37	11.7
ρ_{m2}	- .56	6.2
ρ_e	.66	2.1
θ_{x1}	- .58	4.7
θ_{x2}	.10	1.2
θ_{x3}	.17	1.9
θ_{x4}	.06	.7
θ_{xp}	.69	6.1

Maximum value of the log likelihood: 1284.20
Sample period: 1961.4-1977.4
Correlation between actual values and sample period simulations:
y: .972; p: .995; m: .975; e: .977; w: .991

TABLE 3

Constrained Simultaneous Vector ARMA Model

$$A_0 y_t = A(L) y_t + B(L) u_t$$

	lag 1	2	3	4	5	6	7	8	9	10	11
A(2,5)	.359	.230	.131	.066	.028	.010	.003				
A(3,5)	.216	.138	.079	.034	.017	.006	.002				
A(5,5)	.359	.230	.131	.066	.028	.010	.003				
B(1,1)	1.230	1.189	.897	.572							
B(2,1)	.016	.024	.026	.022	.015	.009	.005	.002	.001	.000	.000
B(2,2)	1.108	1.123	1.041	.883	.362	.227	.125	.061	.024	.007	.002
B(2,3)	.025	.040	.043	.036	.025	.015	.008	.004	.001	.000	.000
B(2,4)	.017	.026	.027	.022	.015	.009	.004	.002	.001	.000	.000
B(2,5)	.376	.445	.535	.470	.337	.219	.127	.057	.031	.010	.002
B(3,1)	-.115	-.099	-.007	-.004	-.009	.005	.003	.001	.000	.000	.000
B(3,2)	.001	.002	.218	.212	.217	.136	.075	.037	.014	.005	.001
B(3,3)	1.188	1.142	.891	.574	.015	.009	.005	.002	.001	.000	.000
B(3,4)	.010	.016	.016	.013	.009	.005	.003	.001	.000	.000	.000
B(3,5)	.226	.267	.321	.282	.202	.131	.076	.034	.019	.006	.001
B(4,4)	.656	.430	.282	.185							
B(5,1)	.016	.024	.026	.022	.015	.009	.005	.002	.001	.000	.000
B(5,2)	.326	.511	.562	.508	.362	.227	.125	.061	.036	.007	.002
B(5,3)	.025	.040	.043	.036	.025	.015	.008	.004	.001	.000	.000
B(5,4)	.017	.026	.027	.022	.015	.009	.005	.002	.001	.000	.000
B(5,5)	.376	.445	.535	.470	.337	.219	.127	.057	.031	.010	.002

Note: Each column represents a matrix coefficient stacked by rows from the matrix polynomial A(L) or B(L). If a component is not listed then the coefficients corresponding to that component are constrained to zero for all lags. If there is no entry for a listed component, then that coefficient is constrained to zero.

TABLE 4

Estimated Distribution of Workers by Contract Length

Contract Length in Quarters	Fraction of Workers	Cumulative (π-weights)
1	.074	1.000
2	.190	.925
3	.234	.735
4	.208	.501
5	.146	.293
6	.084	.147
7	.040	.063
8	.023	.023

TABLE 5

First through Fourth Order
Serial Correlation Matrices

$$\Gamma_s = \hat{E}(\hat{u}_t, \hat{u}_{t-s})$$

Γ_1 =

.13	-.20	.11	.01	.13
-.11	.33	-.07	-.14	-.47
.23	-.06	.19	.19	.05
.02	-.02	-.05	.15	.18
-.01	.10	.13	.09	.29

Γ_2 =

.08	-.12	-.22	-.21	.04
-.14	.24	.17	-.06	-.11
-.10	-.22	.10	-.22	.03
.34	-.11	-.09	.10	.09
.15	-.06	.16	.07	.11

Γ_3 =

.28	-.19	.03	.00	-.04
.16	.31	.09	-.04	.12
.02	-.10	.17	-.15	.13
.47	-.18	.16	.22	.10
.19	-.21	.18	.07	.06

Γ_4 =

.12	.04	-.10	.00	-.07
-.19	.41	.17	-.02	.17
.27	-.20	.19	-.00	.18
.18	-.10	-.08	-.01	.06
.25	.01	.27	.13	.17