Reprinted from Advances in Economic Theory
edited by Werner Hildenbrand

PART VI

APPLIED MACROECONOMICS

CHAPTER 9

Macroeconomic tradeoffs in an international economy with rational expectations

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The purpose of this chapter is to study the macroeconomic policy tradeoff between output and price stability confronting individual countries in an international economy linked by trade flows and a managed exchange-rate system. A tradeoff between output and price stability arises because of the tendency for wage and price decisions to be staggered over time. This prevents general price and wage levels in each country from adjusting quickly to changes in nominal variables - such as the money supply or the exchange rate - and creates an effect of these variables on real output and trade flows. Within each country, however, these real effects are tempered by expectations of future aggregate demand policy, both at home and abroad, and by exchange-rate policies generally. In this chapter we use the rational expectations approach to describe these expectations effects.

The policy implications of staggered wage setting and rational expectations have been studied in a closed-economy context by Taylor (1980a) and in a small open-economy context by Dornbusch (1980, Ch. 9). This chapter represents a multicountry extension of these earlier studies. Its main aim is the development of an applied econometric framework for

Prepared for the 1980 World Congress of the Econometric Society, Aix-en-Provence, France. I am grateful to Ates Dagli, Robert J. Gordon, Maurice Obstfeld, Edmund Phelps, and Stephen J. Turnovsky for useful comments and discussions. This research was supported by a grant from the National Science Foundation.

evaluating aggregate demand policies and exchange-rate rules under rational expectations.

Within a multicountry framework, it is possible in principle to determine an optimal set of policy rules for the world economy, given a social welfare function that depends on the macroeconomic goals of price and output stability in each country. The specific form of the optimal policy rules will of course be conditioned on parameters that need to be estimated empirically. Under certain conditions, the optimal rules will entail an exchange-rate regime in which each country makes its own decision regarding monetary accommodations although at the same time maintaining external stability. That is, each country chooses a point on its domestic tradeoff between output and price stability according to its own preferences and economic structure and independently of the preferences and economic structures abroad. But under other conditions, some coordination of aggregate demand policies is needed, for it is difficult to achieve isolation of each country from the preferences and economic structure of others. Not surprisingly, these conditions depend on how international price linkages, operating through import costs, affect the wage-price dynamics in each country. Such linkages have been studied by Bruno (1978) and Dornbusch and Krugman (1976) and emphasized in practical discussions of the vicious circle. An optimal aggregate demand and exchange-rate policy will reflect these price linkages. However, the analysis shows that the price linkages in turn depend on the policy rules through the effect of rational expectations. This is another example of the importance of the Lucas (1976) critique of econometric policy evaluations that generally ignore these expectations effects.

The chapter proceeds as follows. Section 1 outlines the major assumptions and the basic structure of the model. Because of the empirical aims of the analysis, we develop the model in terms of a general N-country framework. By introducing appropriate vector and matrix terminology, the N-country model can be described abstractly but simply as a vector generalization of the closed economy model. Many of the technical features of the analysis carry over directly from earlier studies by making matrix generalizations of scalar techniques. For example, matrix polynomial factorization is used in place of scalar polynomial factorization to obtain the rational expectations solution. In Section 2, we describe how the rational expectations solution is obtained and show how optimal aggregate demand and exchange-rate policy can be computed.

In Section 3, the policy problem of obtaining internal and external balance is considered and related to earlier work on the subject, in particular that of Corden (1969), who emphasized Phillips curve constraints. We show that, in the absence of direct price linkages between countries, it is possible for each country to choose a point on its own macroeconomic tradeoff and maintain external stability through variations in the exchange rate.

In Section 4, we show how maximum likelihood estimates of the model can be obtained subject to the rational expectations restrictions. It is important to note that in order to incorporate the rational expectations restrictions, it is necessary to estimate the policy and structural parameters simultaneously for all countries in the analysis. Although feasible and straightforward analytically, this is a computationally difficult task and has not yet been attempted. However, certain unrestricted reduced-form parameter estimates are shown to be useful for comparing policies across countries and for determining the extent of interaction between economies. Estimates of these parameters for 7 countries - the United States, West Germany, Italy, the United Kingdom, Canada, Japan, and France - during the 1970s are presented and compared. The results suggest that there are strong interactions between the policies in these countries. This indicates the potential for improving previous empirical international comparisons (see Taylor [1980b], for example) that have not incorporated this interaction. It also indicates the potential value of a full empirical multicountry analysis along the lines suggested here, despite the heavy econometric demands.

1 The model and assumptions

The structure of the demand side of the model can be summarized in the following equations representing each country $i=1,\ldots,n$ at time t

$$y_{it} = \sum_{j=1}^{n} \alpha_{ij}(m_{jt} - p_{jt}) + \sum_{\substack{j=1 \\ j\neq i}}^{n} \beta_{ij}(p_{jt} + e_{ijt} - p_{it}) + u_{it} \tag{1}$$

$$z_{it} = \sum_{j=1}^{n} \gamma_{ij}(m_{jt} - p_{jt}) + \sum_{\substack{j=1 \\ j\neq i}}^{n} \delta_{ij}(p_{jt} + e_{ijt} - p_{it}) + v_{it} \tag{2}$$

where y_{it} = real output; z_{it} = trade deficit (+); m_{it} = money supply; p_{it} = aggregate price level; and e_{ijt} = exchange rate (currency i price of currency j). All variables except z_{it} are measured as detrended logarithms, thereby representing percentage deviations from a given trend. In order to be able to enforce the constraint that trade deficits sum to zero across countries while at the same time using units comparable to detrended logarithms, we measure z_{it} as the trade deficit of country i divided by the trend in world trade. Since $\Sigma_{i=1}^{n} z_{it}=0$ we can either drop one of the z_{it} equations or enforce the restriction that the coefficients of each explanatory variable sum to zero across the n countries.

Equation (1) is an aggregate demand relationship for the goods and services produced by each country. Aggregate demand is assumed to depend positively on the supply of money relative to the price level in each country and positively on the price of foreign goods $(p_{jt}+e_{ijt})$ relative to home goods p_{it}. That is $\alpha_{ij}>0$ and $\beta_{ij}>0$ for all i and j. Equation (1) reduces to a simple quantity theory equation for each country when $\alpha_{ii}=1$, $\alpha_{ij}=0$, and $\beta_{ij}=0$. Permitting $\alpha_{ii}\neq 1$ is a generalization of such a quantity theory relationship. When $\alpha_{ij}>0$, a more stimulative aggregate demand policy[1] in country j spills over into country i by increasing demand for the exports of country i. When $\beta_{ij}>0$, then substitution effects reduce the demand for the exports of country i when either the price level or the exchange rate of country i rises, or the price level of country j falls. The random shock u_{it} is assumed to be serially uncorrelated with mean zero in the theoretical analysis but may be correlated across countries.

The trade deficit equation, Equation (2), depends on the same variables as Equation (1) and for similar reasons. For example, an increase in real balances, which increases demand in country i, will increase the imports demanded by country i and hence increase the trade deficit of country i $(\gamma_{ii}>0)$ and reduce the trade deficit of all other countries $(\gamma_{ij}<0)$. The trade deficit z_{it} does not feed back into any equations in the model. It is included here as an indicator of external balance to be incorporated in the criterion function for the analysis of optimal policy. The elasticities δ_{ij} should all be negative reflecting conventional substitution effects.

The main focus of the model is on the wage and price dynamics, which are summarized in the following three equations:

$$x_{it} = \phi_{i1}\pi_i(L^{-1})\hat{w}_{it} + \phi_{i2}\pi_i(L^{-1})\hat{p}_{it} + \phi_{i3}\pi_i(L^{-1})\hat{y}_{it} + \epsilon_{it} \tag{3}$$

$$w_{it} = \pi_i(L)x_{it} \tag{4}$$

$$p_{it} = \theta_{ii}w_{it} + \sum_{\substack{j=1\\ j\neq i}}^{n} \theta_{ij}(p_{jt} + e_{ijt}) + \eta_{it} \quad \left(\theta_{ii} = 1 - \sum_{\substack{j=1\\ j\neq i}}^{n} \theta_{ij}\right) \tag{5}$$

where x_{it} is the contract wage and w_{it} is the aggregate wage. The hat over a variable represents its conditional expectation based on information through period $t-1$, and L is the lag operator ($L^s x_{it}=x_{it-s}$ and $L^{-s}\hat{x}_{it}=\hat{x}_{it+s}$). The term $\pi_i(L)$ represents a polynomial in the lag operator so that $\pi_i(L^{-1})\hat{w}_{it}$ represents a distributed lead in the expected aggregate wage.

As discussed earlier, the temporary rigidity in this model is due to the staggering of nominal wage contracts. We assume that there is a distribution of wage contracts by length, which is stable over time. The variable x_{it} is an index of contract wages signed at time t, and the weights in the polynomial $\pi_i(L)$ are determined by the distribution of wage contracts in this index. The average wage w_{it} is then a weighted average of past contract wages as indicated in Equation (3) where w_{it} is a distributed lag in the contract wage x_{it}.

Equation (3) describes how the nominal wage contracts are set. It is assumed that workers and firms setting wage contracts look ahead to the period during which the contract is set. As described in Taylor (1980a) the determinants of these contract wages within each country include the expected average wage w_{it}, which represents an estimate of the going wage during the contract period, and expected future demand conditions y_{it}, which represent labor market tightness. Labor market conditions in other countries do not directly affect wage decisions because of limited labor mobility across countries.[2] If prices tend on average to be set by a fixed markup over wages (as was assumed in the closed economy treatment of Taylor (1980a)), then incorporating future expected prices as well as future expected wages in the contract equation does not change the basic structure of the wage equation. (Essentially $\hat{p}_{it}$ can be replaced by $\hat{w}_{it}$.) However, if prices tend to be set on average by a fixed markup over wages *and* the costs of other factors of production, then clearly incorporating future expected prices in the contract equation does change the basic structure. Because in this analysis we wish to emphasize the direct international price linkage which comes from imported inputs to production, it is necessary to allow for the possibility that wage contracts will reflect future price movements, at least to a degree. Hence, $\hat{p}_{it}$ with weight ϕ_{i2} as well as $\hat{w}_{it}$ with weight ϕ_{i1} is incorporated in Equation (3). The markup pricing behavior is summarized in Equation (5), where the price level p_{it} is a markup over wages and import costs. The terms ϵ_{it} and η_{it} represent random shocks to wages and prices, which are assumed to be uncorrelated over time.

In order to close the model it is necessary to make assumptions about aggregate demand policy and exchange-rate policy. A general set of policy rules for these variables can be represented by[3]

$$m_{it} = \sum_{j=1}^{n} g_{ij} p_{it} \tag{6}$$

and

$$e_{ijt} = c_{it} - c_{jt} \tag{7}$$

where $c_{it}=\Sigma_{j=1}^{n} h_{ij} p_{jt}$. The monetary policy rule in Equation (6) allows for accommodation to domestic price movement g_{ii} as well as to foreign price movement. For a monetarist rule $g_{ij}=0$, the money supply grows at its trend rate in each country. The exchange-rate rules in Equation (7) can be similarly accommodative to price movements. If $h_{ij}=0$ for all i and j, then exchange rates are fixed except for any trend differences in the rates of inflation between countries (recall e_{ijt} is measured relative to trend). In general, however, exchange rates are accommodative to price movements in the sense that increases in the price level in a country lead to a depreciation of that country's exchange rate. Equation (7) with values for the h_{ij} describes a managed exchange-rate system. When obtaining the rational expectations solution of the model it will be necessary to assume that people know how this exchange-rate system works (that is, they know the h_{ij}).

Equations (1) through (7) describe the full model. The analysis of the model and the calculation of the rational expectations solution can be made considerably easier by introducing matrix notation. For this purpose, define the vector of country outputs $y_t=(y_{1t}, y_{2t}, \ldots, y_{nt})^1$ and similarly define the vectors z_t, p_t, m_t, c_t, x_t, w_t, u_t, v_t, ϵ_t, and η_t.

Using the representation in Equation (7), substitute for the exchange rate in Equation (1) to obtain:

$$y_{it} = \sum_{i=1}^{n} \alpha_{ij}(m_{jt}-p_{jt}) - \sum_{\substack{j=1\\ j\neq i}}^{n} \beta_{ij}(c_{jt}-p_{jt}) + \sum_{\substack{j=1\\ j\neq i}}^{n} \beta_{ij}(c_{it}-p_{it}) + u_{it} \tag{8}$$

$$y_t = A(m_t - p_t) + B(c_t - p_t) + u_t \tag{9}$$

where

$$A = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\ \vdots & & & \\ \alpha_{n1} & & & \alpha_{nn} \end{bmatrix}$$

and

$$B = \begin{bmatrix} \Sigma\beta_{ij} & -\beta_{12} & \cdots & -\beta_{1n} \\ -\beta_{21} & \Sigma\beta_{2j} & \cdots & -\beta_{2n} \\ \vdots & & & \\ -\beta_{n1} & & & \Sigma\beta_{nj} \end{bmatrix}$$

where the sum of the ith diagonal is from $j=1,\ldots,$ excluding i. Note that B is singular since the sum of each row is zero.

The trade deficit vector z_t can be represented in a similar way by substituting $e_{ijt}=c_{it}-c_{jt}$ into Equation (2) to obtain

$$z_t = \Gamma(m_t - p_t) + \Delta(c_t - p_t) + v_t \tag{10}$$

where Γ and Δ are composed of the γ_{ii} and δ_{ij} in the same way that A and B are composed of α_{ij} and β_{ij}.

The policy rules in Equations (6) and (7) can be written as

$$m_t = Gp_t \tag{11}$$

$$c_t' = Hp_t \tag{12}$$

where G is the matrix with elements g_{ij} and H is the matrix with elements h_{ij}. Note that if $H=I$ then $e_{ijt}=p_{it}-p_{jt}$ which is a purchasing power parity rule.

The N equations in Equation (3) can be written:

$$x_t = \Phi_1\pi(L^{-1})\hat{w}_t + \Phi_2\pi(L^{-1})\hat{p}_t + \Phi_3\pi(L^{-1})\hat{y}_t + \epsilon_t \tag{13}$$

where Φ_1, Φ_2, and Φ_3 are diagonal matrices with the elements ϕ_{1i}, ϕ_{2i}, and ϕ_{3i} on the diagonal, and where $\pi(L^{-1})$ is a diagonal matrix polynomial with $\pi_i(L^{-1})$ at the ith diagonal element. Note that Equation (13) does not directly incorporate any intercountry effects. All off-diagonal elements in these matrices are zero. Given this definition of $\pi(.)$ Equation (4) can be written simply as

$$w_t = \pi(L)x_t \tag{14}$$

To obtain a matrix expression for Equation (5) substitute for e_{ijt} to obtain

$$p_{it} = \theta_{ii}w_{it} + \sum_{\substack{j=1\\ j\neq i}}^{n} \theta_{ij}(p_{jt} + c_{it} - c_{jt})$$

Define a matrix Θ in terms of the elements θ_{ij} in the same way that B and Δ were defined, and let D be the diagonal matrix consisting of the diagonal elements of Θ. Then

$$p_t = (I - D)w_t + \Theta c_t - (\Theta - D)p_t + \eta_t \tag{15}$$

is a matrix representation for Equation (5).

2 Solution and properties of the model

Equations (9) through (15) represent the full model in vector form. To describe the properties of the model we begin by substituting the policy rules (11) and (12) into Equation (9) for the aggregate demand vector y_t to obtain

$$(16) \qquad y_t = -Qp_t + u_t$$

where $Q = A(I-G) + B(I-H)$.

The matrix Q is a world accommodation matrix. It is a representation of multicountry accommodation to price shocks and incorporates both aggregate demand policy G and exchange-rate policy H. Recall that $G=0$ represents a monetarist aggregate demand policy, while $H=0$ represents fixed exchange rates and $H=I$ (the identity matrix) represents "purchasing power parity" rules. When $Q=0$ the world economy is held at full employment regardless of price developments. This full employment rule can be obtained by setting $G=I$, which means full monetary accommodation in each country (real balances are held constant), and by setting $H=I$, which means full exchange-rate accommodation (a purchasing power parity rule). However, if price stability is desired it will not be optimal for Q to equal zero. As will be discussed below, when price linkages are negligible ($\theta_{ij}=0$ and $\phi_{i2}=0$), the optimal accommodation matrix Q is a diagonal matrix with positive diagonal elements: A price shock in one country should be followed by a slowing of real output growth in that country with no effects on output in other countries. Under a fixed exchange rate system ($H=0$) combined with monetarist money supply rules in each country ($G=0$), all elements of Q will be positive so that a price shock in one country will require the slowing of all other economies. Some unrestricted estimates of Q are reported in Section 4 for seven large countries during the 1970s.

Substituting the policy rules (11) and (12) into the trade deficit equation, (10), results in

$$(17) \qquad z_t = -Rp_t + v_t$$

where $R = \Gamma(I-G) + \Delta(I-H)$. The matrix R is analogous to Q. It is a world *external* accommodation matrix. External balances will be independent of price shocks under full monetary and exchange-rate accommodation. That is, $G=H=I$ implies that $R=0$. Note, however, that other values of G and H can also achieve this.

Equations (16) and (17) describe how internal and external balance is influenced by price movements for a given set of policy rules. We now proceed to derive the dynamics of prices and thereby close the system. According to Equation (13), the contract wage x_t depends on rationally expected wages, prices, and output during the contract period. From Equation (16) a forecast of output in future periods can be obtained in terms of a forecast of prices. Forecasts of prices in terms of wages can in turn be obtained by solving the markup Equation (15) for p_t. That is, by substituting the exchange rate rule into Equation (15) to obtain

$$p_t = (I - D)w_t - (\theta(I - H) - D)p_t + \eta_t \tag{18}$$

Taking expectations in Equation (18) and solving for $\hat{p}_t$ results in

$$\hat{p}_t = K\hat{w}_t \tag{19}$$

where $K = [I - D + \Theta(I - H)]^{-1}(I - D)$ is the world markup matrix. According to Equation (19) expectations of future prices can be written in terms of expectations of future wages. Within each country markups determine prices in terms of wages and import costs. But, given an exchange-rate rule, import costs in turn are determined by prices in other countries and hence by wages in other countries. Therefore, the expected price in each country will depend in general on expected wages both at home and abroad. This line of reasoning depends heavily on the rational expectations assumption: An expected wage increase abroad will raise (if people understand the model and the exchange-rate system) expected prices abroad and thereby raise import costs and prices at home unless a purchasing power parity exchange-rate rule is in effect. The increase in prices at home will then raise prices abroad by a bit more by the same mechanism, and so on. Equation (19) represents the net effect of these interactions.[4]

Under certain conditions the world markup matrix K will be diagonal and direct wage-price linkages between countries will not appear. For example, if import costs do not get incorporated in the general price level (that is if $\theta_{ii} = 1$ and $\theta_{ij} = 0$ for each country i), then Θ is diagonal and $K = I$. In that case $\hat{p}_{it} = \hat{w}_{it}$ in each country. However, exchange-rate policy can also interfere with the wage-price linkages. In particular, the purchasing power parity exchange rate rule $(H = I)$ also results in $K = I$ and hence $\hat{p}_{it} = \hat{w}_{it}$ in each country i. In this case, any expected boom in wages in one country is offset by an expected depreciation of the currency, which leaves prices unchanged in other countries and the expected price level in the wage boom country marked up by the full extent of the expected wage boom.

Given the world markup equation, (19), the wage and price dynamics of the system can be determined as follows. Take expectations in Equation (16) and use (19) to obtain

$$\begin{aligned} \hat{y}_t &= -Q\hat{p}_t \\ &= -QK\hat{w}_t \end{aligned} \tag{20}$$

Substituting Equations (19) and (20) into the contract wage Equation (13) we have

(21) $$x_t = [\pi(L^{-1})(\Phi_1 + (\Phi_2 - \Phi_3 Q)K)]\hat{w}_t + \epsilon_t$$

which shows how contract wages set in each country will depend on expected average wages in all other countries during the contract period. Although π and Φ have been assumed to be diagonal to reflect the lack of labor mobility across countries, the world accommodation and markup matrices (Q and K) will not in general be diagonal as discussed above. If the off diagonal elements of Q are positive and large enough, then increases in expected wages in one country will have a depressing influence on contract wage negotiations in other countries, as workers and firms in those countries figure out that the restrictive policy response will cause a reduction in labor market tightness.

Taking expectations in Equation (21) and substituting for the average wage $\hat{w}_t$ in terms of the expected contract wages $\hat{x}_t$ using Equation (14) results in

(22) $$[I - \pi(L^{-1})(\Phi_1 + (\Phi_2 - \Phi_3 Q)K)\pi(L)]\hat{x}_t = 0$$

which is a two-sided vector difference equation in the contract wage vector. Solving this set of equations involves eliminating the expected future values of the expected contract wage, which can be accomplished by factoring the matrix polynomial on the left-hand side of Equation (22) into two polynomials, one which involves negative powers of L and one which involves positive powers of L. The solution for $\hat{x}_t$ in terms of past values of x_t can then be obtained by eliminating the polynomial in the negative powers of L. That is, Equation (22) can be written as

(23) $$\psi(L^{-1})\Omega\psi(L)\hat{x}_t = 0 \quad (\psi_o = I),$$

where the polynomial $\psi(.)$ and the normalization matrix Ω depend on the polynomial π and the matrices Φ_1, Φ_2, Φ_3, Q, and K. Premultiplication of Equation (23) by $\Omega^{-1}\psi^{-1}(L^{-1})$ results in[5]

(24) $$\psi(L)\hat{x}_t = 0$$

and since $x_t - \hat{x}_t = \epsilon_t$ we obtain a reduced-form stochastic vector difference equation in the contract wages

(25) $$\psi(L)x_t = \epsilon_t$$

From the definition of the average wage we can now obtain a vector ARMA model for the average wage vector w_t; that is,

(26) $$\psi(L)w_t = \pi(L)\epsilon_t$$

The behavior of the price vector p_t is obtained from

(27) $p_t = Kw_t + [I - D + \theta(I - H)]^{-1}\eta_t$

and the behavior of the output vector and the trade balance vector follows directly from Equations (16) and (17).

Equations (26), (27), (16), and (17) describe the behavior of all variables in the multicountry model in terms of the structural parameters of each country and in terms of the policy parameter of the money supply and exchange-rate rules. These equations can be used for policy evaluation purposes simply by observing how the behavior of the system changes when the parameters of the policy rule changes. The equations can also be used for optimal policy calculation by maximizing a social welfare function with respect to the policy parameters. For example, suppose that high variability of output, aggregate prices, and the trade deficit reduce social welfare in each country. Then the optimal policy could be calculated by minimizing

(28) $E[y_t'\Lambda_1 y_t + p_t'\Lambda_2 p_t + z_t'\Lambda_3 z_t]$

with respect to G and H, where Λ_1 and Λ_2 are diagonal weighting matrices whose diagonal elements contain the relative weights on price and output fluctuations in each country's utility function. The dimensons of Λ_1, and Λ_2 are n. The elements of the diagonal matrix Λ_3 measure the importance of external stability. Clearly, the rank of Λ_3 should be $n-1$ because there are only $n-1$ degrees of freedom for z_t in the world economy. Without loss of generality, we assume that the first $n-1$ diagonal elements of n_3 are nonzero, whereas the nth element is zero.

In previous closed-economy analyses of models with contracts and rational expectations, the criterion of optimal policy for country i would simply be the minimization of $E[\lambda_i y_{it}^2 + (1-\lambda_i)p_{it}^2]$. By minimizing this quantity with respect to the policy parameters for various values of $0 \leqslant \lambda_i \leqslant 1$, a tradeoff between fluctuation in output (measured by Var y_{it}) is traced out. A tradeoff curve can then be represented in Var y_{it} and Var p_{it} space. An optimal policy is given by a point on this tradeoff curve. In the following section we examine how international considerations influence each country's attempts to maintain optimal policy as defined by a point on this tradeoff.

3 Internal and external stability[6]

One of the arguments in favor of flexible exchange rates - whether managed or not - is the freedom they give every country in an international economy to simultaneously achieve internal and external balance. In terms of the model in this chapter, external balance has the obvious interpretation as a situation where all elements of the trade balance vector z_t are constantly set to zero except for the unavoidable random shocks v_t. Internal balance is more difficult to define, however, because of the macroeconomic tradeoff that exists between output and price stability. One possible interpretation of internal balance is a full-employment policy where y_t is constantly held at zero except for unavoidable random shocks u_t. This corresponds to the notion of internal balance used in the early discussion by Corden (1960).

In order to achieve internal balance in this sense it is necessary to choose policies which generate $Q=0$ and $R=0$, thereby bringing y_t to full employment and z_t into balance except for the random shocks. It is of course possible to set Q and R to zero simultaneously, because the number of targets are equal to the number of instruments: The targets y_t and z_t have $2n-1$ independent elements, and the instruments m_t and c_t have $2n-1$ independent elements. The values of the policy parameters that achieve this are $G=I$ and $H=I$, fully accommodative monetary and exchange-rate policy. Note that both internal and external balance can be achieved in this case, whether or not there are direct price linkages across countries. When $H=I$, the world markup matrix K is equal to I so that price and wage developments in each country are independent of developments elsewhere. This is because $\Phi_1+(\Phi_2-\Phi_3 Q)K$ in the contract wage Equation (21) is diagonal when $Q=0$ and $K=I$.

There is a serious problem with this interpretation of internal balance, however. It focuses entirely on employment stability and ignores price stability. In fact, this fully accommodative monetary and exchange-rate strategy will lead to complete price instability. Equation (25) for the contract wage will be unstable.[7] Both output and price stability should be incorporated in the definition of internal balance when there is a tradeoff between the two as in this chapter. For this reason we interpret internal balance as a situation where each country is able to achieve whatever combination of output and price stabilty it prefers without regard to the preferences of those in other countries. This definition corresponds more closely with that used in more recent discussions of the question by Corden (1969) and Johnson (1969). Using this interpretation, internal balance is achieved by setting R equal to zero but setting Q equal to a diagonal matrix with the diagonal elements chosen to minimize

$$E[y_t'\Lambda_1 y_t + p_t'\Lambda_2 p_t] \tag{29}$$

Under such a policy, each country is isolated from economic developments and policy choices in other countries and is therefore able to determine its own point on the tradeoff between output and price stability. It is clearly possible to make Q diagonal and $R=0$ using the $2n-1$ instruments available.

The required mixture of exchange-rate and aggregate demand policies will have certain important features. To see these, suppose that there are only two countries, one with a fully accommodative policy $g_{11}=1$ and the other with a less accommodative policy $g_{22}<1$. Then for external balance, country 1 will have to expand its money supply when there is a price shock in country 2 in order to offset the contraction that the policy rule of country 2 requires. In addition the exchange rate (currency 1 price of currency 2) between the two countries should depreciate more when there is a price shock in country 1 than it appreciates when there is a price shock in country 2.

Note that, unlike the fully accommodative case, the isolation of the two countries disappears when there are price linkages between them. Policies that achieve an internal balance through a combination of price stability and output stability do not have the property that $H=I$. Hence, if there are price linkages $(\Theta \neq 0)$, then the world markup matrix will not be diagonal. This has two implications: (1) in a world where $\Theta \neq 0$ it is not possible for each country to choose a point on its internal tradeoff between output and price stability and be independent of policy choices elsewhere; and (2) the optimal policy which minimizes the criterion function in Equation (28) will not have the property of perfect external stability $(R=0)$. The optimal policy will entail a response of trade balances to purely nominal swings in prices caused, for example, by wage shocks or aggregate demand policy errors. In this sense the case where $\Theta \neq 0$ requires policy coordination between countries. Of course the policy rules which underlie this coordination will depend on the quantitative value of Θ as well as of the other parameters in the structural model. Moreover, the policy rules will not in general entail fixed exchange rates $(H=0)$.

4 Estimation

Econometric estimation of this model can be approached using constrained maximum likelihood techniques to enforce the rational expectations restrictions. Under the assumption that the vector $(u_t', v_t', \epsilon_t', \eta_t')'$ is normally distributed, the likelihood function of the observations on y_t, p_t, w_t, and z_t can be computed from the vector ARMA equations: (26), (27), (16), and (17). This likelihood function can then be maximized with respect to the structural parameters of the model using standard nonlinear techniques.[8] Note that in order to perform this constrained estimation it is necessary to estimate the structural equations and policy parameters for all countries simultaneously. Clearly, there are severe practical problems if such estimation is attempted without a number of approximations in addition to those already made. One possibility would be to consider a small number of large economies and aggregate the smaller economies into groups. Another possibility would be to use several two-step procedures as described in the previous note. Since degrees of freedom are likely to be in short supply the constraints provided by the rational expectations should be useful. Recall that it is necessary to have relatively stable monetary policies and exchange-rate rules during the estimation period if the approach is to be accurate.

In this section we report estimates of a simplified version of the model, which is illustrative of the basic approach and provides information useful for making international comparisons and determining the importance of interactions between the countries. We focus on seven large industrial economies: the United States, Germany, Italy, and United Kingdom, Canada, Japan, and France. The vector y_t consists of the linearly detrended log of output (real GNP or GDP) for these countries for quarterly time periods from 1970:1 through 1979:4. The vector p_t consists of the linearly detrended log of the output deflator.

Our main interest is in estimating the world accommodation matrix Q (under the approximation that these seven countries constitute the entire set of interactions in the world economy). Recall that the diagonal elements of Q indicate how accommodative each country is to its own price movements, and the off-diagonal elements indicate cross accommodation. Cross accommodation consists both of exchange-rate accommodation and of aggregate demand accommodation, but we do not attempt to disentangle these two forms of accommodation here.

To illustrate how Q might be estimated, assume that price linkages between countries are small so that $K=I$ and $\Phi_2=0$, and also that price movements parallel wage movements closely over the cycle. Then for the case where $\pi(L)$ is first order, we have from Section 2 that

$$p_t = \psi p_{t-1} + 0.5(\epsilon_t + \epsilon_{t-1}) \tag{30}$$

where ψ is related to the other parameter of the model by the constraint

$$4[2\psi + I + \psi^2]^{-1} = I - \Phi_3 Q \tag{31}$$

where Φ_3 is the diagonal matrix that describes how labor market pressures influence wage behavior in each country. If ϵ_t is uncorrelated with u_t in Equation (16), then the system consisting of Equations (30) and (16) is block recursive so that Q can be consistently estimated by ordinary least squares. Such estimates, however, will not in general be efficient, because they ignore the constraint, (31), across the equations,

Table 9.1. *Estimates of the accommodation matrix Q*

	US	Germany	Italy	UK	Canada	Japan	France
Estimates of Q constrained to be diagonal							
US	0.98						
Germany		0.38					
Italy			0.16				
UK				0.31			
Canada					0.31		
Japan						0.23	
France							0.65
Estimates of Q unconstrained							
US	0.80	0.78	0.01	0.12	−0.02	0.01	0.01
Germany	0.03	0.60	0.07	0.14	−0.44	0.14	0.55
Italy	0.29	0.44	−0.01	0.50	−1.18	0.18	0.51
UK	0.29	−0.19	−0.27	0.40	−0.52	0.15	0.79
Canada	0.53	−0.28	0.09	0.08	−0.42	−0.18	0.43
Japan	0.42	0.06	0.00	0.21	−0.75	0.33	0.86
France	0.38	0.05	0.23	0.05	−0.38	0.05	0.19

which is due to the rational expectations assumption. Note that in the special case where Q is diagonal, the model is just identified. If so, then the lack of correlation between ϵ_t and $u_{t'}$ along with the recursive nature of the model implies that (16) and (30) can be estimated separately. This is an important simplification, because joint estimation of Equations (16) and (30) requires dealing with a 14-dimensional constrained ARMA model. Even with the low orders considered here this is not an easy estimation problem.

Table 9.1 reports estimates of Q both unconstrained and constrained to be diagonal. In the diagonally constrained case all elements are positive but less than 1, thereby indicating partial accommodation. Note that these estimates indicate that Italy had the most accommodative policy during the 1970s whereas the United States had the least accommodative policy. Germany and France were more accommodative than the U.S. but less accommodative than the United Kingdom, Canada, and Japan.

However, the constraint that Q is diagonal can easily be rejected by the data. Hence, it is not appropriate to use this condition to estimate Φ_3 without jointly estimating Equations (30) and (16) together. Examining the unconstrained estimate of Q it is clear that these economies have not been achieving internal balance independently of economic developments elsewhere. Either suboptimal policy rules are keeping Q from becoming diagonal, as is required for independent internal balance, or price linkages are large enough that it is not optimal for Q to be diagonal.

The diagonal elements of the unconstrained Q indicate that, with respect to internal price movements, the United States and Germany have been the least accommodative, whereas Italy and Canada have been the most accommodative. Some of the cross-accommodation terms are quite large. The United States responds almost as strongly to German price shocks as it does to its own. On the other hand, Germany is almost completely isolated from U.S. price movements. All countries apply at least some restriction to their economies when U.S. inflation increases; not surprisingly Canada and Japan apply the most restrictive pressures whereas Germany applies the least. The most notable reverse responses in the Q matrix are those describing the reaction of countries to price increases in Canada.

5 Concluding remarks

This chapter has examined aggregate demand policy and exchange-rate policy rules in an international economy in which each country faces a tradeoff between the goals of output and price stability. An analytically convenient procedure for generalizing earlier closed-economy studies of staggered wage contracting with rational expectations was developed in order to provide a framework for this international study. Within this framework, policy evaluation, optimality calculations, and estimation – all subject to the rational expectations restrictions – can be performed. The problems of achieving internal and external stability and the potential needs for policy coordination across countries were examined within this framework.

Data from seven large industrial economies during the 1970s were used to estimate a world accommodation matrix that summarizes the aggregate demand policies and exchange-rate policies for these countries and how they respond to price shocks at home and abroad. Strong cross-accommodation effects were found in this accommodation matrix, which indicates that previous closed-economy or small open-economy frameworks would benefit from international generalizations of this kind. Such an international framework is apparently necessary for evaluating the effects of aggregate demand policy even if one is interested solely in the effects of policy on internal stability.

NOTES

1 A relationship between real balances and aggregate demand can formally be derived from a standard IS–LM framework; however, because expenditures depend on the real rate of interest, the reduced-form impact of real balances

on demand will depend on the expected rate of inflation. If market interest rates are thought to be the only channel through which real money influences demand, then our framework also requires an imperfect capital mobility assumption. An important extension of this model would be a full development of a capital market so that alternative assumptions about capital mobility could be analyzed. Fair (1979) has developed a quantitative model that includes international capital markets, but without rational expectations.

2 As will be clear in what follows, it is possible to modify the labor immobility assumption within this framework by allowing for cross effects in the contract wage equation.

3 The c_{it} notation is introduced for analytic convenience and does not restrict the class of exchange-rate policies. Note that $e_{ijt} = e_{ikt} + e_{kjt} = c_{it} - c_{kt} + (c_{kt} - c_{jt}) = c_{it} - c_{jt}$ as one should expect.

4 In effect, the price component of the "expected" vicious circle completes its rounds within the time period. The impact on wages occurs more slowly, however, because of the staggering of wage contracts.

5 Noting the analogy with the closed-economy case considered by Taylor (1980a) is useful at this point. In the closed-economy model the scalar contract wage x_t satisfied a two-sided scalar difference equation. A scalar polynomial in the lag operator was factored to obtain the constrained reduced-form differences equation in x_t. The same procedure is being used here except that we are working with matrix polynomials rather than vector polynomials.

6 Because there is only one instrument of aggregate demand policy in this model we have abstracted from the Mundell (1962) type of policy mix.

7 This follows from the closed-economy model, because with the fully accommodative policies the countries are isolated from each other.

8 Clearly, the policy rules could be estimated jointly with the structural parameters by adding stochastic terms to Equations (11) and (12) and joining these equations to (26), (27), (16), and (17). Alternatively, one can use a two-step procedure, estimating the policy rules first and taking these as given when estimating Equations (26), (27), (16), and (17).

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