OPTIMAL STABILIZATION RULES IN A STOCHASTIC MODEL OF INVESTMENT WITH GESTATION LAGS

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I. INTRODUCTION

In recent years there has been an extensive amount of economic research devoted to deriving investment demand functions from stochastic dynamic models of firm behavior.[2] Two advantages of such derived demand functions are related to economic policy and have motivated much of this research. First, the parameters of the demand functions depend explicitly on technological properties of the firm's production process and therefore can be assumed to be independent of economic policy which is external to the firm. Second, the investment demand functions show how the firm's decisions depend on expected *future* variables, and thereby permit one to investigate how anticipations of future policy actions might influence the effectiveness of economic policy. Reduced-form functions in

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[2]*See, for example, Lucas and Prescott (1971), Sargent (1979, Ch. 14), and Kydland and Prescott (1980).*

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which investment demand is written as a fixed distributed lag of past variables, regardless of the stochastic process affecting these variables, do not have these advantages.

Although policy questions have been investigated using dynamic models of firm investment behavior, to date there has been little research on the calculation or characterization of *optimal* policy using such models.[3] In this paper we consider the problem of finding optimal control rules to stabilize fluctuations in investment demand using such a model. In the model used here the dynamics of investment are generated by heterogeneous gestation lags between the start and completion of capital projects, rather than by adjustment costs in the installation of capital. Gestation lags permit an analytic calculation of optimal stabilization policy under a wide range of stochastic processes generating firms' desired capital stock, and potentially can be estimated using technological data on capital construction.

The paper is organized as follows. In Section II the dynamic investment model is presented and an investment demand equation is derived. In Section III a procedure for calculating the optimal stabilization policy rules is derived for an arbitrary autoregressive process generating the fluctuations in sales. In Section IV the optimal rules are calculated for the case of a second-order autoregressive business cycle model. In Section V we examine through stochastic simulation the effects of using certain suboptimal policy rules which might

[3] *Policy questions relating to investment in dynamic models have been addressed by Sargent (1979, p. 344), Kydland and Prescott (1980), Summers (1981), Hayashi (1982), and Taylor (1982). Lucas (1976) addresses similar policy issues in a more general setting.*

be employed when there are practical constraints on the design of the optimal rules.

II. AN INVESTMENT MODEL WITH HETEROGENEOUS GESTATION LAGS[4]

Suppose that firms use n different types of capital inputs. Let the stock of capital of type i at the start of time period t be denoted by k_{it}, $i = 1,\ldots,n$. The types of capital differ in their *gestation* times; that is, the time it takes to build a unit of capital. Capital of type i is assumed to take i periods to build. Let s_{it} be the value of capital projects of type i *started* at time t. Then we have

$$k_{it+i} = (1-h_i)k_{it+i-1} + s_{it}\,, \qquad (1)$$

where h_i is a constant proportional depreciation rate for each type of capital. According to equation (1) capital projects of type i started at time t are completed and added to the capital stock at time $t+i$. Depreciation of the amount $h_i k_{it+i-1}$ is subtracted from gross completions to get the net increase in capital.[5]

Investment expenditure, or "value put in place," during the gestation period of each project depends on the technology of construction. Let x_{it} be the value put in place on a capital project of type i during period t. Let w_{ij} be the fraction of the project of type i put in place during

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[4] *This approach to investment demand which emphasizes* *heterogeneous* *gestation lags was applied to a Swedish investment problem in Taylor (1982).*

[5] *Where confusion does not arise, we generally omit a comma between the different indices in the double subscripts. No multiplication of subscript indices appears in this paper.*

the j^{th} period following the start of the project. Then total investment expenditures on projects of type i are given by the distributed lag

$$x_{it} = \sum_{j=1}^{i} w_{ij} s_{it-j+1}, \tag{2}$$

for $i = 1,\ldots,n$. Note that $\sum_{j=1}^{i} w_{ij} = 1$ for each $i = 1,\ldots,n$ and in particular that $w_{11} = 1$. The fractions w_{ij} are determined by the construction technology. In some cases such weights can be obtained in surveys.

In order to obtain an investment demand function we assume that firms decide at each time period τ on a sequence of capital projects of each type in order to minimize the expected value of the intertemporal objective

$$\sum_{t=\tau}^{\infty} \beta^t [.5 \sum_{i=1}^{n} d_i (v_i y_t - k_{it})^2 + \sum_{i=1}^{n} c_{it} x_{it}], \tag{3}$$

where β is a discount factor, v_i and d_i, $i = 1,\ldots,n$ are fixed positive parameters, the c_{it} are the costs of investment goods of type i, and y_t is a measure of sales. The variable y_t is assumed to follow a known univariate stochastic process exogenous to the firm. As will be explained below the variables c_{it}, which are also exogenous to the firm, will be policy determined as a function of y_t. The interpretation of (3) is that a firm's production process calls for capital of each type in a fixed ratio v_i to total sales y_t, and that it is costly for the firm to deviate from that amount of capital in either a positive or a negative direction. This approach is similar to assuming a fixed coefficient production function with capital input coefficients equal to v_i^{-1}, but it permits more flexibility in that the firm can deviate (at some cost) from these input coefficients. Note that we assume that there are no interaction affects in the costs of deviating from these input coefficients for different types of capital: one type of capital deviating from its appropriate level, neither increases nor decreases the costs of another type of capital deviating from its appropriate level. The lack of interaction makes possible a convenient analytical solution of the model, and seems reasonable given the fixed coefficient production interpretation of the objective function.

By substituting equation (1) and (2) into (3) and differentiating with respect to the k_{it}, noting that k_{it+i} or equivalently s_{it} is a decision variable at time t, the following optimal level of starts can be obtained for each time period

$$\begin{aligned} s_{it} &= v_i\hat{y}_{t+i} - (1-h_i)k_{it+i-1} \\ &\quad - \frac{1}{\beta^i d_i}\sum_{j=0}^{i-1} \beta^j w_{ij+1}(\hat{c}_{it+j} - \beta(1-h_i)\hat{c}_{it+j+1}), \end{aligned} \tag{4}$$

where the hat over a variable represents its minimum mean square predictor, or conditional expectation given information through period t. In the case of y_t, for example, $\hat{y}_{t+i} = E(y_{t+i}|y_t, y_{t-1}, \ldots)$. Equation (4) holds for each type of project from $i = 1,\ldots,n$ and can be substituted into (2) in order to obtain the demand for investment. Note that equation (4) indicates that the resulting investment demand function depends explicitly on technological parameters and on expectations of future variables, a general property of demand functions obtained from intertemporal investment models mentioned in the introduction.

In the special case where the depreciation rates $h_i = 0$ and the discount factor is equal to 1, the optimal level of starts depends on a distributed lead in the expected *changes* in the cost of investment goods. In the case where depreciation rates are $h_i = 1$, the distributed lead is in the *level* of the costs of investment goods.

III. OPTIMAL POLICY RULES

The model has been designed so that y_t is a correlated disturbance that causes fluctuations in investment. We view y_t as driven by an exogenous time series process representing, for example, business cycle fluctuations. One objective of policy is to reduce the fluctuations in investment by using investment incentives to offset the influence of this disturbance. Investment incentives affect the actual cost paid by firms for investment goods which we have represented by c_{it} in the model. Hence, the optimal control problem we consider is that of choosing a sequence of policy *instruments* c_{it} so as to minimize the fluctuations in the *target* x_{it}. The optimal choice of c_{it} depends on the stochastic process for y_t. As with most optimal control or regulator problems the effect of the disturbances can be completely offset if there are a sufficient number of instruments. As indicated by (4), the number of instruments needed for complete offset is equal to the number of different types of capital. In principle, therefore, it is necessary to have investment incentives for each type of capital so that each of the c_{it} can be set independently. In practice, tax incentives have differed for capital with different useful lives, but not for capital with different gestation periods.[6]

In order to offset the effects of demand fluctuations on investment it is necessary that the cost variable c_{it} respond to y_t in such a way that the forecasts of future values of c_{it} exactly offset the forecasts of future y_t in equation (4). That is, c_{it} needs to be set so that

$$\beta^i d_i v_i \hat{y}_{t+i} = \sum_{j=0}^{i-1} \beta^j w_{ij+1}(\hat{c}_{it+j} - \beta(1-h_i)\hat{c}_{it+j+1}) . \qquad (5)$$

for $i = 1,\ldots,n$. It is clear from equation (4) that such a choice of c_{it} will eliminate the effect of the disturbance y_t on starts and thereby on investment expenditures. Our objective is to calculate and characterize these optimal c_{it}.

Assume that y_t is determined by the following p^{th} order autoregressive process:

$$y_t = \alpha_1 y_{t-1} + \cdots + \alpha_p y_{t-p} + u_t . \qquad (6)$$

where u_t is an uncorrelated random variable with a zero mean. Equation (6) can be used to generate predictions of the future values of y_t that appear in equation (5) using results from prediction theory. See Anderson (1971, Ch. 5). In order to obtain the optimal rule for the determination of the c_{it} we start with the general linear form

$$c_{it} = g_{i1}y_t + g_{i2}y_{t-1} + \cdots + g_{ip}y_{t-p+1} , \qquad (7)$$

where the coefficients g_{i1} through g_{ip} are as yet undetermined. Predictions of future c_{it} can be obtained using

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[6] *For example, in the United States the investment tax credit depends on the useful life of the capital equipment purchased.*

(7) and the predictions of y_t generated by (6). The problem of finding the optimal rule is thus reduced to the problem of finding the values of the coefficients that satisfy equation (5) for all t. These values can be found by substituting into (5) the forecasts of y_t and c_{it} using (6) and (7), and finding the values of g_{i1} through g_{ip} which bring the coefficients of y_t through y_{t-p+1} to equality on both sides of (5). We now show how this procedure results in a set of linear equations in g_{i1} through g_{ip} which are straightforward to solve, even for fairly large values of n and p. The procedure has some similarities to the feedforward control schemes proposed by Box and Jenkins (1970, Ch. 12) for conventional linear regulator problems.

The forecasts of future y_t are given by

$$\hat{y}_{t+1} = \gamma_{s1} y_t + \gamma_{s2} y_{t-1} + \cdots + \gamma_{sp} y_{t-p+1} \quad \text{for} \quad s \geq 1, \tag{8}$$

where the γ-coefficients can be obtained recursively from the equations

$$\begin{aligned} \gamma_{sj} &= \alpha_j \gamma_{s-1,1} + \gamma_{s-1,j+1}, \qquad j = 1,\ldots,p-1 \\ \gamma_{sp} &= \alpha_p \gamma_{s-1,1}. \end{aligned} \tag{9}$$

The recursion starts at $s = 1$ with $\gamma_{1j} = \alpha_j$, $j = 1,\ldots,p$. See Anderson (1971, p. 168) for a derivation of the recursion relationships in (9). Note also that $\hat{y}_{t+s} = y_{t+s}$ for $s < 1$. The forecasts of future c_{it} are

$$\hat{c}_{it+s} = \sum_{j=1}^{s} g_{ij} \hat{y}_{t+s-j+1} + \sum_{j=s+1}^{p} g_{ij} y_{t+s-j+1}, \tag{10}$$

where the values for $\hat{y}_{t+s-j+1}$ can be obtained from (8).

Starting with the case where $i = 1$ (the single period construction projects) we substitute these forecasting equations into (5) as follows. When $i = 1$ equation (5) becomes:

$$\beta d_1 v_1 \hat{y}_{t+1} = w_{11}(\hat{c}_{1t} - \beta(1-h_1)\hat{c}_{1t+1}) , \tag{11}$$

which can be written as

$$\begin{aligned}\beta d_1 v_1 \hat{y}_{t+1} &= w_{11}(g_{11}y_t + \ldots + g_{1p}y_{t-p+1}) \\ &\quad -\beta(1-h_1)w_{11}(g_{11}\hat{y}_{t+1} + g_{12}y_t + \ldots + g_{1p}y_{t-p+2})w_{11} ,\end{aligned} \tag{12}$$

after substitution of $\hat{c}_{1t}$ and $\hat{c}_{1t+1}$ from (10) with $s = 1$ and $i = 1$. Using equation (8) to substitute for $\hat{y}_{t+1}$ in (12), we obtain

$$\begin{aligned}&\beta d_1 v_1(\gamma_{11}y_t + \ldots + \gamma_{1p}y_{t-p+1}) \\ &= w_{11}(g_{11}y_t + \ldots + g_{1p}y_{t-p+1}) \\ &\quad - \beta(1-h_1)[g_{11}(\gamma_{11}y_t + \ldots + \gamma_{1p}y_{t-p+1}) \\ &\quad + g_{12}y_t + \ldots + g_{1p}y_{t-p+2})]w_{11} .\end{aligned} \tag{13}$$

Equating the coefficients of $y_t, y_{t-1}, \ldots, y_{t-p+1}$ in (13) results in a set of linear equations in g_{11} through g_{1p} which will be useful to write out in detail

$$\begin{aligned}\beta d_1 v_1 \gamma_{11} &= w_{11}(1-\beta(1-h_1)\gamma_{11})g_{11} - w_{11}\beta(1-h_1)g_{12} , \\ \beta d_1 v_1 \gamma_{12} &= -w_{11}\beta(1-h_1)\gamma_{12}g_{11} + w_{11}g_{12} - w_{11}\beta(1-h_1)g_{13} , \\ \beta d_1 v_1 \gamma_{13} &= -w_{11}\beta(1-h_1)\gamma_{13}g_{11} + w_{11}g_{13} - w_{11}\beta(1-h_1)g_{14} , \\ &\vdots \\ \beta d_1 v_1 \gamma_{1p-1} &= -w_{11}\beta(1-h_1)\gamma_{1p-1}g_{11} + w_{11}g_{1p-1} - w_{11}\beta(1-h_1)g_{1p} , \\ \beta d_1 v_1 \gamma_{1p} &= -w_{11}\beta(1-h_1)\gamma_{1p}g_{11} + w_{11}g_{1p} .\end{aligned} \tag{14}$$

Although we have written (14) using the general notation introduced for an arbitrary gestation lag, in this case we have that $\gamma_{1j} = \alpha_j$, $j = 1,\ldots,p$ and $w_{11} = 1$. The p equations in (14) are clearly linear in the p unknowns g_{11} through g_{1p} and can be solved to obtain the optimal control rule for c_{1t}. In the special case of full depreciation ($h = 1$) the off-diagonal terms in the system of equations in (14) are equal to zero, so that the solution is given simply by $g_{1j} = \beta d_1 v_1 \alpha_j$ for $j = 1,\ldots,p$. In this special case the optimal control coefficients are proportional to the coefficients of the difference equation generating the disturbance y_t.

The equations in (14) can alternatively be organized in matrix form. Let $\underline{g}_i = (g_{i1},\ldots,g_{ip})'$ and $\underline{\gamma}_s = (\gamma_{s1},\ldots,\gamma_{sp})'$. The equation system becomes

$$\underline{A}_1\underline{g}_1 = \underline{\gamma}_1\beta d_1 v_1 , \tag{15}$$

where $\underline{A}_1$ is a $p \times p$ matrix. Denoting the representative element of $\underline{A}_1$ by $a^{(1)}_{jm}$ the non-zero elements of the matrix are given by

$$\begin{aligned}
a^{(1)}_{11} &= w_{11}(1 - \beta(1-h_1)\gamma_{11}) , \\
a^{(1)}_{jj} &= w_{11} , && j = 2,\ldots,p , \\
a^{(1)}_{j-1,j} &= -w_{11}\beta(1-h_1) , && j = 2,\ldots,p , \\
a^{(1)}_{j1} &= -w_{11}\beta(1-h_1)\gamma_{1j} , && j = 2,\ldots,p ,
\end{aligned} \tag{16}$$

and all other elements are equal to zero. The optimal values for the control rule coefficients for c_{1t} are then written as

$$\underline{g}_1 = \underline{A}_1^{-1}\underline{\gamma}_1\beta d_1 v_1 . \tag{17}$$

This same procedure can be used to compute the control rule coefficients for the c_{it} variables corresponding to the longer gestation lags. That is, the forecasting equations with values of i from 2 through n can be substituted into (5), and equations in the control rule coefficients can be obtained by equating coefficients of $y_t, y_{t-1},\ldots,y_{t-p+1}$. For each value of i there will be p linear equations in p unknowns. Before considering the results for the general case it is useful to consider the equations for $i = 2$. In this two-period case

$$\underline{g}_2 = \underline{A}_2^{-1}\underline{\gamma}_2\beta^2 v_2 d_2 \,. \tag{18}$$

The non-zero elements of $\underline{A}_2$ are given by

$$\begin{aligned}
a_{11}^{(2)} &= w_{21} + \beta(w_{22} - (1-h_2)w_{21})\gamma_{11} - \beta^2 w_{22}(1-h_2)\gamma_{21} \\
a_{j1}^{(2)} &= \beta(w_{22} - (1-h_2)w_{21})\gamma_{1j} - \beta^2 w_{22}(1-h_2)\gamma_{2j}, \quad j=2,\ldots,p. \\
a_{12}^{(2)} &= \beta(w_{22} - (1-h_2)w_{21}) - \beta^2 w_{22}(1-h_2)\gamma_{11}\,, \\
a_{22}^{(2)} &= w_{21} - \beta^2 w_{22}(1-h_2)\gamma_{12}\,, \\
a_{j2}^{(2)} &= -\beta^2 w_{22}(1-h_2)\gamma_{1j}\,, && j=3,\ldots,p, \\
a_{jj}^{(2)} &= w_{21}\,, && j=3,\ldots,p, \\
a_{j-1,j}^{(2)} &= \beta(w_{22} - (1-h_2)w_{w1})\,, && j=3,\ldots,p, \\
a_{j-2,j}^{(2)} &= -\beta^2 w_{22}(1-h_2)\,, && j=3,\ldots,p.
\end{aligned} \tag{19}$$

The remaining elements of $\underline{A}_2$ are equal to zero. Note that with full depreciation $(h_2 = 1)$ the matrix $\underline{A}_2$ does not become diagonal, unlike in the one period projects. The development of the coefficients of $\underline{A}_i$ as i increases from 1 to 2, continues for i equal 3 and so on, establishing a general formula which can be used for any value of i.

In order to express the solution for $\underline{g}$ in the general case, some additional notation is useful. Define a sequence b_{ij}

$$b_{i0} = w_{i1}$$

$$b_{ij} = \beta^j (w_{i,j+1} - (1-h_i) w_{i,j}) , \qquad j = 1,\ldots,i-1 \text{ (for } i \geq 2)$$

$$b_{ii} = -\beta^i (1-h_i) w_{ii} \tag{20}$$

for each $i = 1,\ldots,n$. The b_{ij} coefficients thus depend on the structural parameters of the model and are easily computed.

The solution in the general case can be written

$$\underline{g}_i = \underline{A}_i^{-1} \underline{\gamma}_i \beta^i v_i d_i , \tag{21}$$

where the non-zero elements of the $p \times p$ matrix $\underline{A}_i$, denoted by $a_{jm}^{(i)}$, are given by the following set of equations for $i = 1,\ldots,n$,

$$a_{jm}^{(i)} = b_{i,m-j} + \sum_{q=m}^{i} b_{iq} \gamma_{q-m+1,j} , \qquad j = 1,\ldots,m , \; m = 1,\ldots,i,$$

$$a_{jm}^{(i)} = \sum_{q=m}^{i} b_{iq} \gamma_{q-m+1,j} , \qquad j = m+1,\ldots,p, \; m = 1,\ldots,i,$$

$$a_{j-i+r,j}^{(i)} = b_{i,i-r} , \qquad r = 0,\ldots,i , \; j = i+1,\ldots,p. \tag{22}$$

Note the equations in (22) are equivalent to the equations in (16) for $i = 1$, and to the equations in (19) for $i = 2$. These equations provide an easily computable way to evaluate the matrix $\underline{A}_i$ for an arbitrary i and p. Hence, the entire set of optimal control coefficients $\underline{g}_i$, $i = 1,\ldots,n$ can be computed. Since the dimension of the matrix $\underline{A}_i$ is equal to the order of the autogressive model generating the disturbances (which will usually be relatively small) and is not influenced by the length of the gestation lag (which could be quite long), computation costs should be low for this procedure.

IV. PROPERTIES OF OPTIMAL POLICY IN A SECOND ORDER CYCLICAL MODEL

In this section we examine the properties of the optimal rules for the case where sales disturbances y_t follow a second order process (p = 2). A second order model permits a fairly close approximation to the stochastic properties of business cycles observed in most countries, if y_t is interpreted as proportional to detrended fluctuations in real GNP or some other measure of the state of aggregate economic activity.

For the second order model the optimal policy rules have the form

$$c_{it} = g_{i1}y_t + g_{i2}y_{t-1}, \qquad i = 1,\dots,n, \tag{23}$$

which is a special case of equation (7). The control coefficients g_{i1} and g_{i2} completely characterize the policy and of course are different for each type of capital i.

The policy coefficients associated with $i = 1$, the single period projects, are obtained by solving equation (15) and are given by

$$g_{11} = \beta v_1 d_1 \left[\frac{\alpha_1 + \alpha_2(1-h_1)\beta}{1 - \beta(1-h_1)(\alpha_1 + \alpha_2(1-h_1)\beta)} \right], \tag{24}$$

$$g_{12} = \beta v_1 d_1 \left[\frac{\alpha_2}{1 - \beta(1-h_1)(\alpha_1 + \alpha_2(1-h_1)\beta)} \right]. \tag{25}$$

If depreciation occurs in one period ($h_1 = 1$) then the policy rules can be characterized easily. In that case the policy coefficients are proportional to the parameters of the autoregressive process α_1 and α_2. For example, if y_t is proportional to real GNP and $\alpha_1 = 1.4$ and $\alpha_2 = -.5$, then the stabilization rules call for an increase in investment costs if real GNP is above normal levels, or if real GNP has been growing. For parameter values $\beta = 1$ and $v_1 d_1 = 1$, (24) and (25) imply

$$\begin{aligned} c_{1t} &= 1.4y_t - .5y_{t-1} \\ &= .9y_t + .5(y_t - y_{t-1}) . \end{aligned} \tag{26}$$

Note that it is never optimal to react only to current y_t unless $\alpha_2 = 0$, in which case the model is first-order. As we show in the next section failure to react to lagged y_t as in (26) can lead to a policy rule which destabilizes output. According to equation (26) investment costs should be raised by an extra amount if real GNP has been growing.

The results are different if depreciation rates are smaller. The proportionality of the g_{1i} and α_i will no longer hold, and the size of the reaction coefficients will be larger. Consider, for example the opposite extreme where $h = 0$. The stabilization rule becomes

$$c_{1t} = 4y_t + 5(y_t - y_{t-1}) . \tag{27}$$

The reaction coefficients are much larger than in (26) and the size of the coefficient on the first difference of y_t is larger relative to the size of the coefficient on the level of y_t.

V. STOCHASTIC SIMULATION RESULTS WITH SUBOPTIMAL POLICIES

The optimal policy rules derived and examined in the previous two sections have several features which are not usually characteristic of investment stabilization policy in practice. First, the policy is *dynamic*: lagged values of y_t influence the optimal policy. In practice only the current level of y_t seems to have been a factor in the determination investment stabilization policy. Second, the policy instruments vary *continuously* with the values of y_t. In practice the policy instruments are likely to be set discretely — they are either on or off depending on the state of the business cycle. Third, the policy instrument must be targetted at the components of investment, distinguishing between different types of capital by gestation time. If the instrument is not targetted to each type of capital, perhaps because of the restriction that $c_{it} = c_{jt}$ for $i \neq j$, then there will be an insufficient number of instruments and a constrained optimation approach is necessary. The methods developed in Chow (1980) might be used in such a situation. In this section of the paper we examine through the use of some simulation experiments what happens when policy is restricted to be suboptimal either because lagged values are omitted or because the instrument settings are limited to discrete values.

A. Omission of Lagged Variables

Consider the case where $n = 1$ and $p = 2$, and it is therefore optimal for g_{12} to be non-zero. Suppose, however, that g_{12} is restricted to be zero. In order to determine the possible impact of such a restricted investment policy on the stability of investment, we performed stochastic simulation for the set of parameter values for the intertemporal model calculated in Taylor (1982). There values are $v_1 = .2$, $d_1 = .07$, $h_1 = .026$, and $\beta = .94$. We also set $\alpha_1 = 1.4$ and

$\alpha_2 = -.5$ as in the previous section. The variance of investment was then calculated by performing 1,000 Monte Carlo simulations of 30 periods each, with the shocks u_t being drawn from a normal distribution with mean 0 and variance 1 and with the path of investment being determined by the model. The simulations were started from $k_{1,0} = 0$. The variance of investment was found to be an *increasing* function of g_{11} for this set of autoregressive parameter values. In the steady state (approximated at t = 30), the variance of x_{1t} was equal to .00069 when $g_{11} = 0$, increased to .00125 at $g_{11} = .002$, and increased further to .00201 when $g_{11} = .004$. Hence, this type of suboptimal policy could actually lead to perverse destabilization of investment.[7] This particular suboptimal policy is worse than no policy at all. Note that for this example the optimal values for g_{11} and g_{12} are .090 and -.048, respectively.

B. Discrete Values for the Instruments

Consider the case where $n = 1$ and $p = 1$. The optimal policy rule then has the form $c_{1t} = g_{11}y_t$. Suppose, however, that only discrete changes in c_{1t} are feasible in practice, and that c_{1t} is therefore set according to the rule

$$c_{1t} = \begin{cases} c^* & \text{if} \quad y_t > 0 \\ 0 & \text{if} \quad y_t = 0 \\ -c^* & \text{if} \quad y_t < 0 \end{cases} . \tag{28}$$

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[7] *Christiano (1982) has shown analytically that such perverse destablization can occur when* y_t *follows an ARMA(1,1) process. Baumol (1961) and Howrey (1966) have investigated similar problems with suboptimal policy rules in models where anticipations of future policy do not affect decisions explicitly.*

For this policy the forecasts of investment costs are not linear functions of y_t as with the forecasting rules used in Section 2. Nevertheless the forecasts of c_{1t+1} conditional on y_t, which is necessary for evaluating the decision rule (4), can be evaluated for the case where u_t is normally distributed. Using this conditional expectation for $\hat{c}_{1t+1}$ and the rule in (28) we stochastically simulated the model with the same parameter values used for the previously described set of stochastic simulations. The results are shown in the first column of Table I. (The other columns in Table I marked by the parameter δ signify a different discrete policy rule described below). The results indicate that while there is some reduction in the variance of investment with the discrete model, it is very small. Moreover when the step size (c^*) increases beyond some small value the variance of investment begins to increase rapidly, indicating the potential for some destablization. The restriction of c_{1t} to a discrete set of values results in a serious deterioration of the performance of the policy.

Table I. The Variance of Investment (Var x_{1t}) for Alternative Discrete Policy Rules[a]

c^* \ δ	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	.10
.000	.33	.33	.33	.33	.33	.33	.33	.33	.33	.33	.33
.001	.31	.17	.12	.13	.24	.26	.25	.29	.30	.30	.47
.002	1.15	.47	.09	.09	.24	.27	.26	.30	.30	.32	.64
.003	2.87	1.24	.25	.23	.31	.38	.36	.35	.35	.38	.84

a) *Each entry of the table is the Var(x_{1t}) × 1000 and is computed by stochastic simulations using the investment rule in equation (29) in the text for different values of c^* and δ. The variance is computed at $t = 30$ which approximates the steady state variance.*

One of the reasons for the poor results with this sub-optimal policy is that c_{1t} moves by a large amount when y_t deviates only slightly from 0. An improvement would therefore be expected if the rule were modified so that

$$c_{1t} = \begin{cases} c^* & \text{if} \quad y_t > \delta \\ 0 & \text{if} \quad |y_t| < \delta \\ -c^* & \text{if} \quad y_t < -\delta \end{cases} \tag{29}$$

With rule (29) small movements in y_t will not trigger a large response in c_{1t}. Clearly equation (29) reduces to equation (28) when $\delta = 0$. The simulation results for this alternative are shown in Table I in the columns marked with different values of δ. As expected there is some reduction in the variance of x_{1t} but not as much as would be possible with the completely continuous optimal rule. Note also that Table I suggests that the best policy of the form (29) has δ between .2 and .3 and c^* near .002. These values depend on the parameters used in the simulation experiment, but they indicate the advantages of choosing the step-size and trigger points optimally even if policy is restricted to a discrete set of values. To the extent that such constraints are important in practice, further research to characterize how the best step-size and trigger values depend on the parameters of the model in this and more complicated examples would be useful.

VI. CONCLUDING REMARKS

This paper has considered the problem of obtaining optimal control rules for stabilizing investment fluctuations in a model where investment demand depends on expected future values of the policy instruments. Simple expressions for evaluating the control rules were derived using results from prediction theory. These expressions were used to characterize some of the main properties of the control rules. In addition, the loss from using certain suboptimal rules was investigated. While suboptimal rules are clearly inferior to optimal rules, and in some cases inferior to no feedback rule at all, practical constraints on economic policy could lead to the use of such rules.

Although the formula for the control rule was derived for a particular dynamic investment model, the prediction theory approach that was employed could be used in other similar problems. The essential characteristic of the control problem studied here is that the target variable depends on forecasts of future values of the control instruments and on future exogenous variables. In the traditional control problem, the target variables depend on current and lagged values of the control instruments and the exogenous variables. This difference indicates why prediction theory is particularly useful for the type of problem studied in this paper.

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