FORECASTING WITH RATIONAL EXPECTATIONS MODELS

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A great deal of effort has been made by many individuals in recent years to use new macroeconometric models with rational expectations to analyze practical policy problems. These efforts have focussed on estimating the conditional impact of a policy change on the economy relative to a given future baseline path for the economy. For example, the Congressional Budget Office (CBO) used such models to estimate the effects of the budget deficit reduction plan of 1993. The CBO examined simulations from different rational expectations models to see what change in the interest rate or the exchange rate might occur if policy changed toward more future budget deficit reduction in a way that was credible. Similar simulations were performed at the time of the budget reduction plan of 1990 at the Council of Economic Advisers (CEA). A Brookings Institution study conducted by Bryant, Hooper and Mann (1993) reported the impact of different rules for monetary policy from stochastic simulations of seven different rational expectations models; in each case the performance of the economy was measured relative to an assumed baseline path for each of the models.

These new models have advantages over the traditional Keynesian models. In particular, they can make distinctions between anticipated and unanticipated policy changes, as well as between temporary and permanent policy changes. Because they are structural models, they also can take account of changes in certain reduced form parameters that may occur when a policy rule changes. They are therefore useful for addressing many policy issues which traditional Keynesian models cannot address. This probably explains the increased use of these models in comparison with traditional Keynesian models for this type of policy analysis. A dozen years ago there were only a few prototype estimated rational expectations models (see Taylor (1979), for example). By 1986, 1/4 of the models surveyed in the first Brookings model comparison exercise were rational expectations models (see Bryant (1988)). By 1993, 1/2 of the models used by the CBO for its budget deficit simulations were rational expectations models. And 3/4 of the models in the most recent Brookings comparison were rational expectations models (see Bryant, Hooper and Mann (1983)).

Despite the advantages, the new rational expectations models have not been used more frequently in comparison with the traditional models in another, perhaps more common, mode of operation: forecasting. The new models are not being used to make forecasts of where the economy is headed in the future--to predict the future baseline path itself. It appears that none of the over 50 forecasters surveyed by the Blue Chip report on forecasts uses a rational expectations model to make forecasts. Although there is little doubt that conditional policy analysis represents a useful contribution from the new models, the fact that they are not widely used for practical forecasting is troublesome for several reasons.

First, forecasting is an important part of practical policymaking. When the Federal Open Market Committee (FOMC) meets to deliberate about policy, the members look over the staff forecasts of the economy for the next several quarters. Similarly, the CBO and the CEA make forecasts of the economy which influence policy decisions. If rational expectation models are to be useful in this part of policy analysis, they will have to be used for forecasting.

Second, forecasting frequently can improve a model by providing tests of the model's structure using events unknown when the model was formulated and estimated. Poor forecasts can indicate structural errors that may be hidden if the model is only used for conditional simulations and policy analysis. While estimating a model with real world data certainly provides useful test statistics of the model's accuracy, practical forecasting goes well beyond such tests.

Third, there may be some advantages to forecasting with the help of rational expectations models. The alleged advantage of using structural economic models for forecasting is that they allow the forecaster to bring economic theory into forecasting in a systematic and objective way (see Klein (1971)). Since rational expectations models have structures that are different from traditional models, they may improve on our ability to forecast. But if the models are never used regularly for forecasting, we will never know.

For all these reasons, finding ways to use the new rational expectation models for forecasting seems like a fruitful area for future research. The purpose of this paper is to examine some of the issues that arise when using estimated rational expectations models for forecasting. We show that there are some serious practical problems that now may be standing in the way of using estimated rational expectations models for forecasting. Many of these problems are due to peculiar "jumps" in the variables which are common to rational expectations models and make it difficult to adjust the constants in rational expectations models, a practice common in traditional macroeconomic forecasting; other problems relate to forecasting "exogenous" variables. We propose some solutions to these problems and illustrate these solutions in some simple forecasting applications.

1. A Rudimentary Example

To illustrate these forecasting problems, we begin with a simple macroeconomic model of the type used for forecasting but which can be solved analytically. Most existing econometric models with rational expectations are nonlinear and much more complex than this rudimentary model.

Suppose that the central bank conducts open-market operations in such a way that the interest rate follows the interest rate rule:

$$(1) \qquad i_t = (1 + \alpha)\pi_t + \beta + e_t$$

where i_t is the short-term interest rate, π_t is the inflation rate, e_t is a serially uncorrelated random variable with zero mean, and α and β are positive parameters.

Suppose that there is a negative relationship between real GDP and the real interest rate given by

$$(2) \qquad y_t = - \gamma (r_t - \delta) + v_t$$

where r_t is the real interest rate, y_t is real GDP measured so that its average value is zero, v_t is a serially uncorrelated random variable with zero mean, and γ and δ are positive parameters.

Finally, suppose that the "production function" for output is simply a first order autoregression:

$$(3) \qquad y_t = \rho\, y_{t-1} + u_t\,, \quad |\rho| < 1$$

where u_t is a serially uncorrelated random variable with mean zero.

We define the real interest rate in two ways depending on whether the model is a rational expectations model or a traditional econometric model. For the rational expectations model the real interest rate r_t is

$$(4) \qquad r_t = i_t - E_t\, \pi_{t+1}$$

where E_t is the conditional expectations operator conditional on information at time t. In the case of the traditional econometric model without rational expectations, the real interest rate is given by

$$(5) \qquad r_t = i_t - \pi_t$$

Here we simply set the expectation of inflation over the next period equal to the most recent inflation rate.

By substituting either real interest rate definition with equation (2), the rudimentary model in its structural form consists of three equations in these variables: y_t, π_t and i_t. For both expectations assumptions, a reduced form of the model is easily determined with the inflation rate, the interest rate, and output functions of the shocks and lagged variables. This reduced form is simply a constrained vector autoregression.

Minimum Mean Square Prediction

The standard prediction problem is to find a predictor which minimizes the mean square prediction error. (See Anderson (1971) or Harvey (1991).) For example, if z_{t+s} is the variable being forecast, then the minimum mean square predictor $\hat{z}_{t+s}$ is chosen so that it minimizes

$$(6) \qquad E_t\,(z_{t+s} - \hat{z}_{t+s})^2$$

for s = 1,2,.... The minimum mean square predictor is the conditional mean of z_{t+s} given all information available through time t. In what follows we assume that only variables dated t or earlier are known at time t. The reduced form for the model can be used to compute these conditional expectations.

Forecasting and Constant Adjustment

In the case of the expectations assumption in equation 5 the reduced form of the model, after solving for i_t and r_t, is given by:

$$(7) \qquad \pi_t = \frac{\delta - \beta}{\alpha} - \frac{\rho}{\alpha\gamma} y_{t-1} + \frac{1}{\alpha\gamma}(v_t - u_t - \gamma e_t)$$

with y_t given by equation (2) and i_t given by equation (1) with (7) inserted. Thus, if the parameters were known, the minimum mean square predictor of π_{t+s} would simply be:

(8) $$\hat{\pi}_{t+s} = \frac{\delta - \beta}{\alpha} - \frac{\rho^s y_t}{\alpha\gamma}$$

for s = 1,2,... . The one-step-ahead forecast is when s = 1.

However, few forecasters who use traditional models actually make forecasts in this way. Instead, the forecasters adjust the constants of the equations: "Such adjustments start the prediction calculations with the system approximately on track for the initial period," as explained by Lawrence Klein (1971). In practice, constant adjustment is not done in a mechanical or objective way; it usually reflects the forecasters' judgment about the current state of the economy in comparison with what the model is saying about the economy. There are various rationales for constant adjustment--the models equations may be drifting off, coefficients may be changing or more recent data within the quarter may be available which would help in the forecast. There have been numerous attempts over the years to formalize this constant adjustment procedure--time-varying coefficient models are one important example--but for the most part, practitioners make the adjustments in a more subjective way.

In this example, a constant adjustment would add a term to any equation which did not fit during the most recent period. The addition has the affect of putting one step ahead forecast close to the current period.

In the case of the rational expectations version of the model, we have:

$$\pi_t = \frac{1}{\gamma(1+\alpha)} E_t \sum_{j=0}^{\infty} (1+\alpha)^{-j} \{\gamma(\delta - \beta - e_{t+j}) + v_{t+j} - y_{t+j}\}$$

from which we can obtain the reduced form

$$(9) \qquad \pi_t = \frac{\delta - \beta}{\alpha} - \frac{\rho}{\gamma(1 + \alpha - \rho)} y_{t-1} + \frac{1}{\gamma(1 + \alpha)}(v_t - u_t - \gamma e_t)$$

As in the traditional model, this reduced form can be used to obtain the minimum mean error forecast.

$$(10) \qquad \hat{\pi}_{t+s} = \frac{\delta - \beta}{\alpha} - \frac{\rho^s y_t}{\gamma(1 + \alpha - \rho)}$$

for s = 1,2,... . Although such models are not used for forecasting, it is likely that forecasters would need to adjust the constant in this model as well as the traditional model.

Consider the case where the most recent observation of the short-term interest rate is different from what is implied by the model parameters and the existing inflation rate in equation (1). This would imply a need for a constant adjustment to the interest rate equation. In principle it is possible to make the same adjustment to the other equations based on observed values of the model, but we focus on the interest rate equation in this example.

The possible ways of adjusting the constant to the equation reflect the forecasters' views of the nature of the error in the equation. First, a constant could be added to the equation to put it exactly on track in the most recent period; then the constant could be phased out gradually. Second, the constant could be phased-in gradually to a level that would have put it on track in the most recent period.

Let A be the actual residual for equation (1) in the most recent period; that is

$$A = i_t - (1 + \alpha)\pi_t - \beta$$

where t is the last period before the forecast. The two alternatives are then (i) add $\lambda^{s-t}A$ to equation (1) and (ii) add $(1 - \lambda^{s-t})A$ to equation (1) where $0 \leq \lambda \leq 1$. How do these two alternatives affect the forecast?

The change in forecasts of inflation from equations (8) and (10) is shown in the following table:

Table 1

	phase out	phase in
traditional model	$\frac{\lambda^s A}{\alpha}$ (1)	$\frac{(1-\lambda^s)A}{\alpha}$ (1)
rational expectations model	$\frac{\lambda^s A}{\alpha + (1-\lambda)}$ (.5)	$\frac{A}{\alpha} - \frac{\lambda^s A}{\alpha + (1-\lambda)}$ (1.5)

The table shows that the phase-out method has a big effect on the traditional model forecasts but a small effect on the rational expectation model. (The numerical examples in the table one for $A = 1$, $\alpha = .5$, $\lambda = .5$ and $s = 1$.) On the other hand, the rankings are reversed for the phase-out approach. The comparison shows that a constant adjustment procedure that might appear sensible in a traditional model can have much different and perhaps counterintuitive effects in a rational expectations model.

2. An Estimated Rational Expectations Model of the U.S. Economy

The example in the previous section indicates how different forecasting with traditional and rational expectations models can be. In this section, we present an econometrically estimated rational expectations macroeconomic model, similar in structure to many now used to form conditional policy analysis, analyze the forecasts it generates, and show how the potential problems illustrated in the rudimentary example exist in this more realistic model. By using the estimated model to do some actual forecasts, we show how these problems might be dealt with in practice.

The model we use for this purpose is a modified and simplified single-country (United States) version of the rational expectations multicountry model used by Taylor (1993). The model consists of nine equations and one identity, describing the behavior of interest rates, aggregate demand components, and nominal wages and prices. The parameters of the model are estimated using quarterly data over 1970-1992. Key features of the model include: staggered-wage setting which generates sticky nominal wages and prices; demand shocks which affect aggregate output in the short run but not in the long run; efficient capital markets across countries; forward-looking behavior in consumption wage-setting and the determination of long- term bond rates, and rational expectations.

Description of the Estimated Equations

All price and wage variables are nominal quantities. Real GDP and its components are measured in 1987 dollars. Except where otherwise noted, each equation error term, u_t, is assumed to be a serially uncorrelated mean zero innovation.

Interest Rates

The short-term interest rate (federal funds rate) is determined by an interest rate reaction rule relating the nominal short-term rate, i_t, to a constant plus a weighted average of the percent deviation of output from potential, y_t, and the deviation of the current inflation rate, $\pi_t = p_t - p_{t-4}$, from a constant target, π^*_t. We focus on this equation for illustrating the problems with practical forecasting when a given equation goes off track. The parameter values are consistent with actual policy over the period 1987-1991 are used, but the equation is currently off. The policy rule is given by:

$$(11) \quad i_t = 0.02 + \pi_t + 0.5*y_t + 0.5(\pi_t - 0.02) + u_{1,t}, \quad \pi_t = p_t - p_{t-4}$$

implying a target inflation rate of 2 percent. If the equilibrium real interest is 2 percent, then the interest rate will converge to 4 percent. The long-term interest rate (government bond rate) is related to present and future short-term rates by the following linear term structure relationship:

$$(12) \quad i_t^l = \underset{(0.002)}{0.000} + \frac{1-.784}{1-(.784)^{17}} E_t \sum_{j=0}^{16} \underset{(0.054)}{(0.784)^j} \, i_{t+j} + u_{2,t}$$

This equation was estimated using generalized method of moments, the resulting parameter estimates are inserted into the equation, and standard error estimates appear in parentheses beneath the respective point estimates. The set of future short-term rates is truncated at four years; parameter estimates were insensitive to minor reductions or extensions of this frontier. Note that the estimated term premium is almost exactly zero; this result is discussed in the following section.

The real long-term interest rate, relevant for consumption and investment demand, is assumed to be given by $r_t = i^l_t - E_t(p_{t+4} - p_t)$.

Nominal Wages and Prices

Nominal wages are assumed to be determined according to a staggered wage structure. This is described in detail in Taylor (1993). Parameter estimates are taken from this source. Each period, a subset of wage contracts is negotiated; the resulting contract wage is influenced by expected aggregate wages, w_{t+i}, i=0,1,2,3, and labor market tightness, as represented by the deviation of output from potential, over the period of the contract. This yields an equation for the contract wage, z_t, of the following form:

(13) $$z_t = E_t(0.327\ w_t + 0.274\ w_{t+1} + 0.200\ w_{t+2} + 0.200\ w_{t+3})$$

$$+\ \underset{(0.011)}{0.030}\ E_t(\underset{(0.015)}{0.327}\ y_t + \underset{(0.013)}{0.274}\ y_{t+1} + 0.200\ y_{t+2} + 0.200\ y_{t+3}) + u_{3,t}.$$

The aggregate nominal wage (w_t) is identically equal to a weighted average of past and present contract wages:

(14) $w_t = 0.327\ z_t + 0.274\ z_{t-1} + 0.200\ z_{t-2} + 0.200\ z_{t-3}$.

Note that, in the steady state, any constant inflation rate is consistent with output equaling potential, and accelerating wage inflation results from a level of output greater than potential.

The nominal aggregate price level is assumed to be a mark-up over the aggregate nominal wage level and foreign prices (a trade-weighted average of GDP deflators, converted to dollars by the relevant exchange rate, in the six other G7 countries), p^W_t:

(15) $p_t = \underset{(0.003)}{0.006} + \underset{(0.079)}{0.899}\ p_{t-1} + \underset{(0.074)}{0.093}\ w_t + \underset{(0.005)}{0.008}\ p^W_t + v_t, \quad v_t = \underset{(0.191)}{0.367}\ v_{t-1} + u_{4,t}$.

The autocorrelation of the residuals is assumed to follow an AR1 error structure. We impose homogeneity on the price equation by constraining the coefficients on nominal variables to sum to unity. Thus, in the long run, a 1 percent increase in the wages and foreign prices is consistent with a 1 percent increase in the price level.

Aggregate Demand Components

We disaggregate aggregate output, Y_t, into consumption, C_t, fixed investment, F_t, inventory investment, N_t, net exports, X_t, and government purchases, G_t. We <u>define</u> potential (or trend) output, Y_t^T be to equal a linear trend growing at an annual rate of 2.45 percent, with base level normalized such that the percent deviation from potential of -2.4 percent in 1991 (this is based on an OECD estimate and is comparable to many other current estimates). For purposes of estimation and model solution, we are interested in converting demand components into stationary variables. We choose to do this by normalizing each component series by the potential output series; thus, the resulting normalized series represent ratios of demand components relative to potential output. Normalized demand components are denoted by lower case letters, e.g., $c_t = C_t/Y^T_t$. Note that the sum of normalized demand components equals aggregate output divided by potential output, which is equal to $1 + y_t$. For each demand component, we approximate the behavior of the variable by a dynamic linear equation.

The equation for normalized consumption is based on a standard rational expectations forward-looking model of consumption, represented by including the expected real long-term interest rate and a measure of permanent income. We approximate permanent income by a discounted average of deviations of output from potential over the present and eight future quarters:

$$(16) \quad y_t^P = \frac{1-0.8}{1-(0.8)^9} E_t \sum_{j=0}^{8} (0.8)^j \, y_{t+j}$$

Inspection of the data shows that the consumption ratio series is highly upward trended over the full sample period (1970-1992). This may be due to structural changes in the consumption relationship or changes in tax policies not captured by this model. Such a secular trend is inconsistent with the steady state structure of the model as formulated here. The consumption ratio appears to be stationary during the latter half of the sample, so this sample was used in estimation, using GMM, of the consumption equation:

$$(17) \quad c_t = \underset{(0.023)}{0.224} + \underset{(0.033)}{0.669}\, c_{t-1} + \underset{(0.025)}{0.269}\, y^P_t - \underset{(0.024)}{0.040}\, r_t + u_{5,t}.$$

The equations for fixed and inventory investment are of the accelerator type: changes in sales (as represented by output) generate changes in investment, and were estimated over the entire sample using GMM. For fixed investment, inclusion of two lags of the dependent variable was found to be preferable:

$$(18) \quad f_t = \underset{(0.004)}{0.013} + \underset{(0.097)}{1.191}\, f_{t-1} - \underset{(0.090)}{0.271}\, f_{t-2} + \underset{(0.045)}{0.273}\, y_t - \underset{(0.042)}{0.266}\, y_{t-1} - \underset{(0.007)}{0.009}\, r_t + u_{6,t}.$$

Inventory investment is given by:

$$(19) \quad n_t = \underset{(0.001)}{0.003} + \underset{(0.062)}{0.575}\, n_{t-1} + \underset{(0.084)}{0.352}\, y_t - \underset{(0.080)}{0.316}\, y_{t-1} - \underset{(0.014)}{0.011}\, r_t + u_{7,t}$$

Net exports depend on domestic income, foreign income, y_t^W, and the real exchange rate, e_t:

(20) $x_t = 0.000 + 0.898\, x_{t-1} - 0.037\, y_t + 0.102\, y^W_t - 0.011\, e_t + u_{8,t}$

(0.001) (0.045) (0.035) (0.050) (0.013)

Foreign income is defined to be a trade-weighted average of real income (GDP or GNP) in the other G7 countries. Similarly, the real exchange rate is the trade-weighted average of the bilateral real exchange rates (in terms of posted exchange rates and GDP deflator).

Government purchases are exogenous. The paths of government purchases used in the simulations are described in detail below. The long-run steady state government purchases share is set at .185.

Steady State Solution and Calibration

In forward-looking models where many variables are not predetermined, the approach to the steady state can be very rapid. Thus, the steady state properties of the model often have large direct effects on the forecasts generated by it, even for relatively short forecast horizons. In this section we discuss the determinants of various steady state values and describe how the estimated model is calibrated to assure reasonable steady state behavior.

In the steady state, the spending shares of GDP, the real interest rate, inflation rate, and real exchange rate are constant. From the wage-setting equations (13-14), it is seen that a constant inflation rate is only consistent with a zero steady state deviation of output from potential. Thus, in the steady state, output equals potential. Note that the wage-setting equations place no restrictions on the steady state wage inflation rate. In the steady state, each demand component share is a linear function of the real interest rate and the real exchange rate; aggregate demand is equal to the sum of these equations and equals potential. This yields a single equation in the real interest and exchange rates. In a two-country model, the second country provides an additional demand equation and the steady state real interest and exchange rates are determined jointly by these two equations (capital mobility implies equal real interest rates in the two countries, up to a constant risk and term premium differential). In the single- country model, we treat the real exchange rate as an exogenous variable (and set it equal to a constant). Given the ad hoc nature of the treatment of net exports in a single-country model and the elimination of the real exchange rate as an explanatory variable, we adjust the constant term in the net export equations such that the steady state net export share equals -0.01.

Using the estimated coefficients and a government share of .185, the implied steady state real interest rate is found to equal 7 percent. This seems unreasonably large. Due to the imprecision of the estimated coefficients, the implied imprecision for this estimate of the steady state interest rate is very large. Hence, we recalibrate the model so that the steady state real interest rate equals 4 percent. Although this can be done in a number of ways, we simply lower the constant term in the consumption equation by a small amount. This shift is less than one-half of a standard error.

Figure 1 illustrates how this calibration is done. The objective is to make the sum of the shares of consumption, investment, and net exports in GDP equal to what is left after the government takes its 18.5 percent share. Consumption and investment shares depend on the real interest rate in the steady state (net export is exogenous in this model); the sum of the three shares on the right equals 81.5 at an interest rate of 7 percent. Shifting in the consumption share reduces the interest rate to 4 percent.

In the steady state, the long-run and short-run interest rates differ only by a constant term premium. The estimated term premium is nearly zero. For the purpose of forecasting, this coefficient is assumed to be 0.02. The interest rate rule implies a positive linear relation between the steady state inflation and real interest rate. A constant real exchange rate is only consistent with a rate of inflation of foreign prices (converted to dollars) equal to the domestic inflation rate. For the price adjustment equation (15) to be consistent with any steady state inflation rate, the constant term must equal zero; in the simulations discussed below, this adjustment is made.

These calibrations are made to assure reasonable steady state behavior of the model which is very important for a rational expectations model used in forecasting. These calibrations are conceptually separate from the issue of constant adjustment. While such modifications seem essential for forecasting with estimated rational expectation models, even in a model where no such calibrations are necessary, the practical issue of constant adjustments arises.

3. Forecasting

The estimated and calibrated model can be solved to obtain a reduced form in which the endogenous variables depend on logged endogenous variables, linear combinations of the shocks to the equations and the single exogenous variable, government purchases. The model is linear in the logs of the wages and prices, the shares of GDP, and the interest rates. Hence, the reduced form is linear. The reduced form has many cross-equation constraints due to rational expectations and other parts of the model's structure. This reduced form can be used to minimize mean square error forecast just as in the rudimentary example.

To illustrate this procedure, we consider an example forecast of the major components of GDP along with the interest rates and the inflation rate for the period starting in the third quarter of 1993 and going through the forth quarter of 1994. The example forecast was made given information through the second quarter of 1993. Government purchases are assumed to grow to bring their share to 18.5 percent of GDP.

The example forecast without any constant adjustment is shown in Table 2. The forecast shows a rebound from slow growth in the first half of 1993 and then a return to potential GDP growth. This rebound did materialize but not at such a high rate, perhaps because government purchases grew much less than assumed; a further rebound may occur in the fourth quarter. But our focus here is on the interest rate forecast which better illustrates the problems of forecasting with rational expectations.

The forecast predicts that the federal funds rate would rise well above the current level of 3 percent. The reason is that the interest rate equation was off by about this amount in the second quarter. Hence, by not adjusting the constant of the interest rate (policy rule) equation, there is a jump in the interest rate. A graphical illustration of this jump is shown in Figure 2. The model equation is off by about 1 percent and the one-step-ahead forecast is up by 1.6 percent.

In this situation, most traditional economic forecasters would adjust the constant of this equation; typically the adjustment would be a permanent shift, perhaps by the amount designated A in the rudimentary example with $\lambda = 1$ (no phase out or phase in). Such an adjustment might be based on an analysis of what the Fed policymakers are likely to do.

The effects of such a permanent constant adjustment on the forecast is shown in Table 3 and Figure 2. Again we focus on the interest rate forecast, where the effects of the constant adjustment raise several issues that the federal fund rate does not increase as much as in the first case. There is still an increase because of the rebound in the economy, but the adjustment of the interest rate equation has been successful in its aim of bringing the interest rate down to a more "reasonable" level. But observe what happened to the forecast of long-term interest rates. Rather than declining slightly, the long-term rate jumps to over 7 percent. Thus, the jump in the short-term rate has been replaced by an even larger jump in the long-term rate. It is this type of jump that has reduced the appeal of rational expectations model in a forecasting environment. While the ability of rational expectations models to adjust rapidly to new information is an attractive feature, optimal forecasts are likely to have more continuity.

What is causing the jump in long-term interest rates? The permanent adjustment of the policy rule leads to a higher inflation rate. This is seen by comparing the inflation forecast in Table 2 with Table 3. The inflation rate increases by about 2 percentage points because the adjustment of Fed's policy rule has generated more inflation. Effectively, this adjustment has led to an increase in the Fed's target of inflation. With a rational expectations model this higher inflation rate raises expectations of inflation immediately and as a result expectations of future interest rates rise. Because the bond market is forward-looking, this causes a rise in the long- term rate immediately. Figure 2 shows how the downward adjustment of the policy rule leads to higher interest rates in the long run. While such a jump might not be appealing to a forecaster it is a sensible implication of what would happen if the Fed did permanently adjust its policy rule.

An alternative constant adjustment would be to phase out the error in the equation in the current period. This would imply a view that the Fed would gradually come back to the model equation. The effect of such a forecast is shown in Table 4. In this case there is no rise in long-term interest rates; and the fact that inflation does not pick up means that there is even less of an increase in the short-term interest rate compared to the case of the permanent constant adjustment. While perhaps counter-intuitive, a phased-out constant adjustment has a larger effect on the variable described by the equation being adjusted than a permanent one.

Clearly, there are alternative adjustments possible, and it would be possible to consider adjusting some of the other equations. However, as more equations are adjusted and the pattern of the adjustments gets more complex, the process of constant adjustment could come to dominate the model itself. We see a tradeoff between adjusting the model too much and losing too much of the economic content of the model. In these examples we still probably have more to gain from further adjustment than we have to lose by putting less reliance on the model structure. We have only considered one equation and our adjustments of that equation have been based on economic consideration concerning Fed policy.

4. Projecting Exogenous Variables

The forecast values depend on the underlying assumptions regarding the time path of government purchases, which we treat as exogenous and known to everyone at the time of the forecast. Alternative assumptions alter the results substantially.

As an example of the effect of different assumptions regarding the future path of government purchases on forecast values, we consider forecasts from the vantage point of 1990:II based on two different paths of government purchases. Actual values of the government purchases as a share of trend output, along with the paths used for the two forecast simulations, are shown in Figure 3.

The first forecast simulation, labelled Sim I, is based on the assumption that the future path of government purchases (including our choice of values for 1993-96) was known at the time of the forecast, the second quarter of 1990. Solving the model with this assumed government purchases path yields the forecast series for output deviations from potential, labelled Sim I in Figure 4. This figure also portrays the actual series of output deviations. The forecast series is similar to the actual series, although the magnitude of the output deviations is on average about half as large. The second forecast, labelled Sim II, is based on the assumption that the government purchases share of trend output gradually falls from its high level in 1990 (over 0.190) to the steady state value of 0.185 in 1995:IV. This generates a forecast of output falling smoothly to potential over three years.

Due to the forward-looking structure of the model, the expected future path of government purchases has first order effects on the forecast values. Note that in the case of Sim I the fall in government purchases began in earnest at the end of 1991, yet forecasted values are immediately affected. In a traditional model, the future path of variables does not have such an effect. Thus, the rational expectations model is capable of forecasting turning points such as the 1990-91 recession. However, this requires good estimates of the path of future exogenous variables and the belief that people forecast that path accurately.

5. Conclusion

We have chosen to focus this discussion of forecasting with rational expectations models around an example. This example is of considerable practical importance for forecasting: what is the likely policy rule for the central bank during the forecasting period and beyond? This policy rule has big effects on the forecast, especially on long-term interest rates. If the policy rule is off track at the start of the forecast, the forecaster must make a decision about whether to put it back on track, and, if so, how fast. The decision involves an analysis of the likely decisions of the policymakers at the central bank and is therefore likely to involve some judgment. But when this judgment is combined with the more formal theory and more quantitative instruments of the rational expectations model, better forecasts may evolve. One of the purposes of this paper is to see whether this is the case in practice.

Table 2 Example Forecast with no Policy Rule Adjustment

Growth Rates of GDP Components	93:1	93:2	93:3	93:4	94:1	94:2	94:3	1993	1994
Real GDP	0.8	1.8	4.9	4.4	3.8	3.3	2.9	3.0	3.1
Consumption	0.8	3.2	2.3	2.8	2.9	2.9	2.8	2.3	2.8
Fixed Investment	10.2	7.5	8.8	7.8	6.2	4.6	3.3	8.6	4.2
Government Purchases	-6.6	4.2	3.2	3.2	3.2	3.2	3.2	1.0	3.2
GDP Deflator	3.6	2.3	2.0	2.0	2.0	2.0	2.0	2.5	2.0
Interest Rates and Ratios									
Federal Funds Rate	3.0	3.0	4.6	4.3	3.9	3.9	4.0	3.7	4.0
Long-term bond rate	6.9	6.6	6.2	6.1	6.0	6.0	6.0	6.4	6.0
Consumption	.659	.660	.660	.661	.662	.663	.664	.660	.663
Fixed Investment	.150	.152	.154	.156	.158	.159	.159	.153	.159
Inventory Investment	.006	.003	.005	.007	.007	.007	.007	.005	.006
Government Purchases	.180	.181	.181	.182	.182	.183	.183	.181	.183
Net Exports	-.012	-.014	-.013	-.013	-.012	-.012	-.012	-.013	-.012
GDP Gap	-.018	-.019	-.012	-.007	-.004	-.001	.000	-.014	-.001

Table 3 Example Forecast with Permanent Policy Rule Adjustment

Growth Rates of GDP Components	93:1	93:2	93:3	93:4	94:1	94:2	94:3	1993	1994
Real GDP	0.8	1.8	5.6	4.5	3.4	2.8	2.6	3.2	2.8
Consumption	0.8	3.2	2.7	2.8	2.8	2.7	2.6	2.4	2.6
Fixed Investment	10.2	7.5	10.3	8.4	5.7	3.6	2.5	9.1	3.5
Government Purchases	-6.6	4.2	3.2	3.2	3.2	3.2	3.2	1.0	3.2
GDP Deflator	3.6	2.3	3.5	4.0	4.1	4.2	4.2	3.3	4.2
Interest Rates and Rations									
Federal Funds Rate	3.0	3.0	4.2	4.7	5.1	5.9	6.1	3.8	5.8
Long-term bond rate	6.9	6.6	7.3	7.7	7.9	8.2	8.2	7.1	8.1
Consumption	.659	.660	.661	.661	.662	.663	.663	.660	.663
Fixed Investment	.150	.152	.155	.157	.158	.159	.159	.153	.159
Inventory Investment	.006	.003	.006	.007	.007	.007	.006	.005	.006
Government Purchases	.180	.181	.181	.182	.182	.183	.183	.181	.183
Net Exports	-.014	-.012	-.013	-.013	-.012	-.012	-.012	-.013	-.012
GDP Gap	-.018	-.019	-.011	-.005	-.002	-.001	.000	-.013	-.001

Table 4 Example Forecast with Temporary Policy Rule Adjustment

Growth Rates of GDP Components	93:1	93:2	93:3	93:4	94:1	94:2	94:3	1993	1994
Real GDP	0.8	1.8	5.2	4.4	3.7	3.1	2.8	3.0	3.0
Consumption	0.8	3.2	2.5	2.8	2.9	2.8	2.7	2.3	2.7
Fixed Investment	10.2	7.5	9.4	8.1	6.0	4.2	3.1	8.8	3.9
Government Purchases	-6.6	4.2	3.2	3.2	3.2	3.2	3.2	1.0	3.2
GDP Deflator	3.6	2.3	2.0	2.0	2.0	2.0	2.0	2.5	2.0
Interest Rates and Ratios									
Federal Funds Rate	3.0	3.0	3.6	3.7	3.6	3.9	4.0	3.3	3.9
Long-term bond rate	6.9	6.6	5.8	5.9	5.9	6.0	6.0	6.3	6.0
Consumption	.659	.660	.660	.661	.662	.663	.663	.660	.663
Fixed Investment	.150	.152	.154	.157	.158	.159	.159	.153	.159
Inventory Investment	.006	.003	.006	.007	.007	.007	.006	.005	.007
Government Purchases	.180	.181	.181	.182	.182	.183	.183	.181	.183
Net Exports	-.012	-.014	-.013	-.013	-.012	-.012	-.012	-.013	-.012
GDP Gap	-.018	-.019	-.012	-.006	-.003	-.001	.000	-.014	-.001

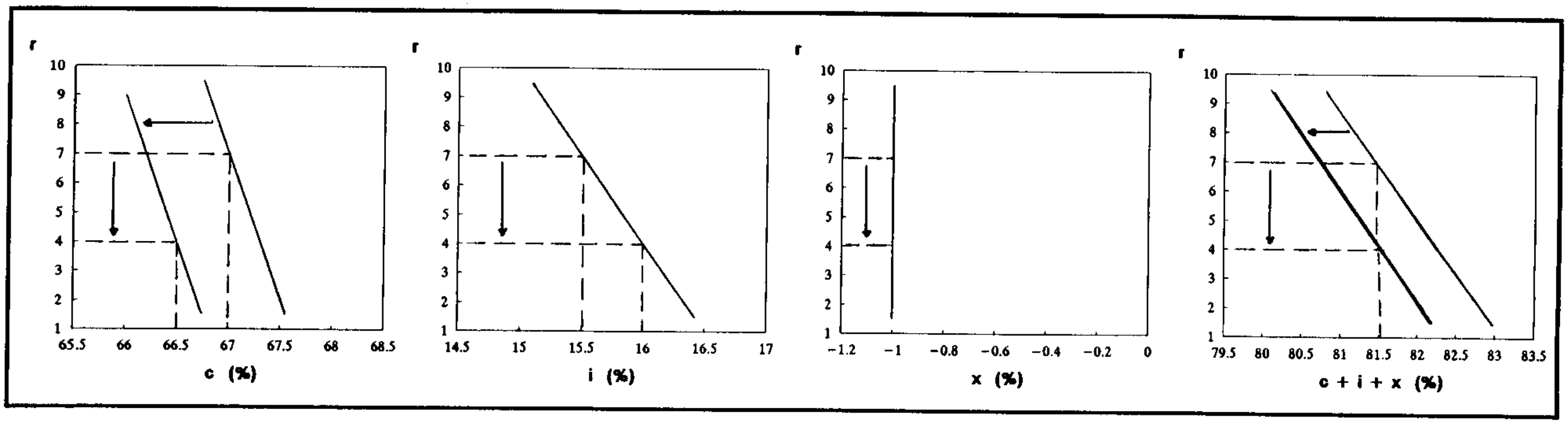

Figure 1. Illustration of the Calibration

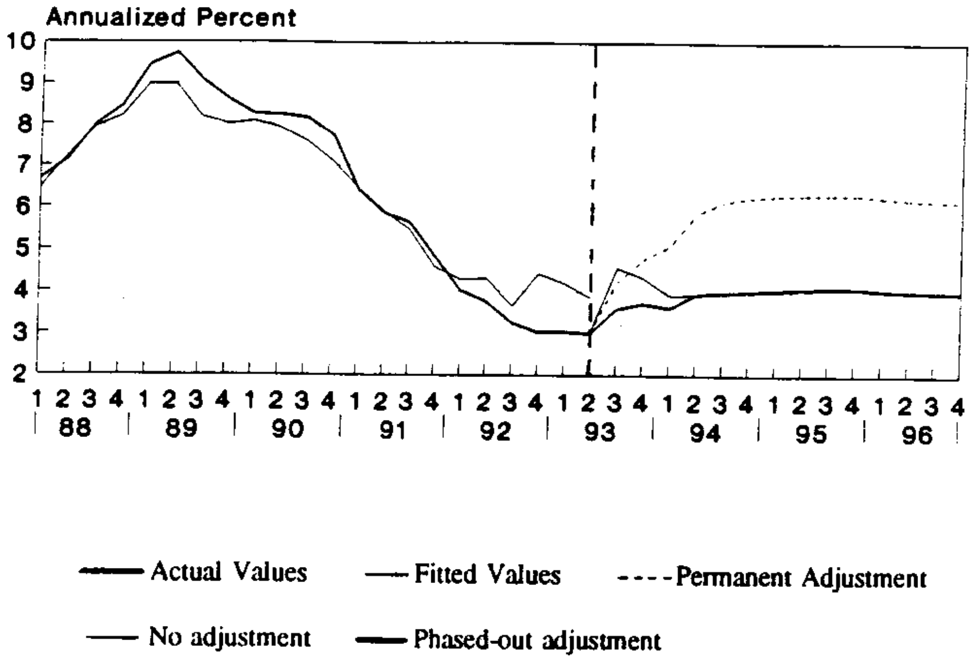

Figure 2. Alternative Forecasts of Federal Funds Rate

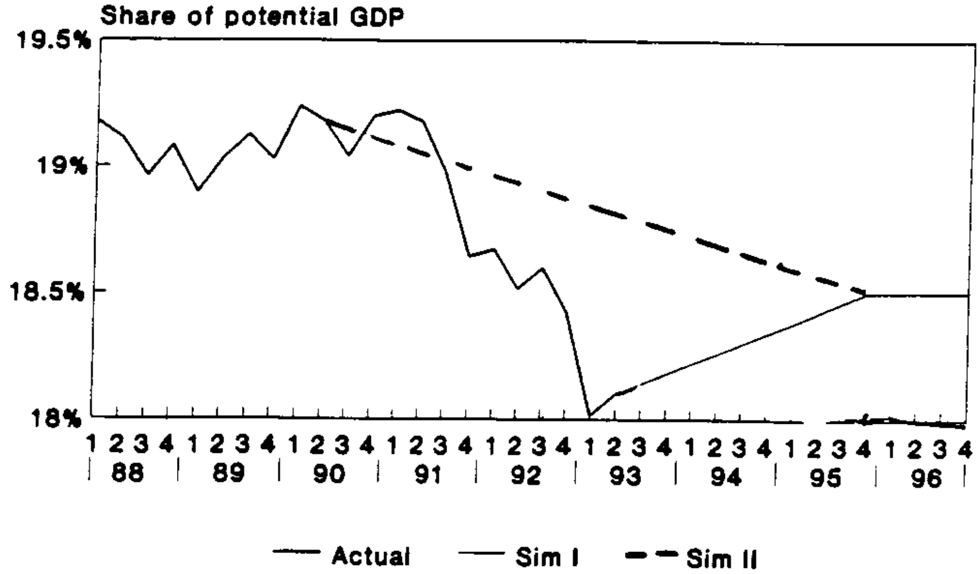

Figure 3. Alternative Assumptions about Government Purchase

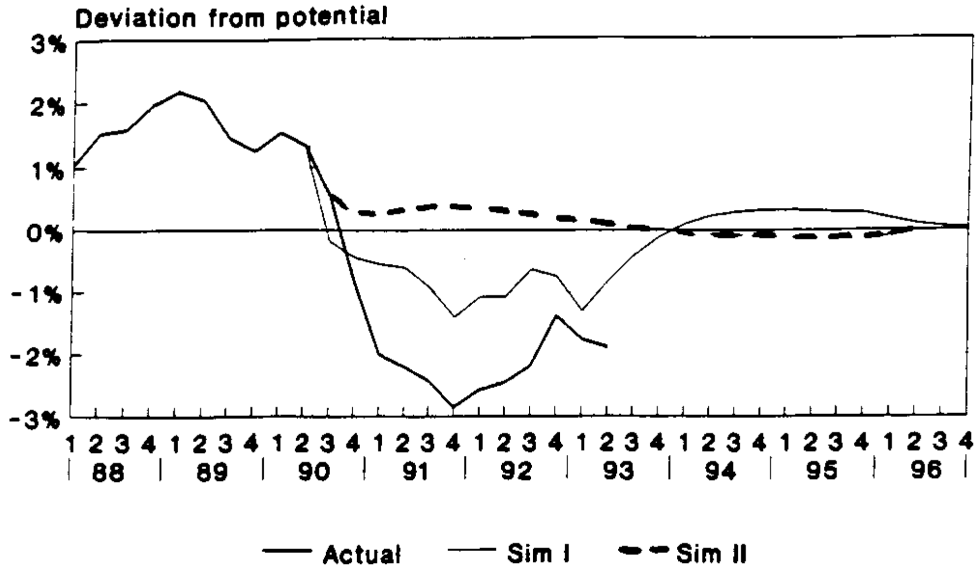

Figure 4. Forecasts with Alternatives Assumptions about Future Government Spending

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