14. Monetary Policy and the Term Structure

John B. Taylor, May 17, 2013

Put no-arbitrage affine term structure into a macro model

Derive "term structure of policy rules" for a simple policy rule

$$i_t^{(n)} = a_n + b_n \pi_t,$$

Simple policy rule

$$(3) \quad r_t = \delta \pi_t$$

$$(4) \quad i_t^{(n)} = -n^{-1} \log(P_t^{(n)})$$

No-arbitrage conditions

$$(5) \quad P_t^{(n+1)} = E_t[\quad (m_{t+1} P_{t+1}^{(n)})]$$

$$(6) \quad m_{t+1} = \exp(-r_t - 0.5\lambda_t^2 - \lambda_t \varepsilon_{t+1})$$

$$(7) \quad \lambda_t = -\gamma_0 - \gamma_1 \pi_t$$

Distributed i.i.d. N(0,1)

Inflation equation

$$(8) \quad \pi_t = \pi_{t-1} - \phi(r_{t-1} - \pi_{t-1}) + \sigma \varepsilon_t,$$

$P_{t+1}^{(1)} = \exp(-r_{t+1})$, which can be substituted into equation (5) to

$$P_t^{(2)} = E_t[m_{t+1} \exp(-r_{t+1})].$$

Now, by substituting for m_{t+1} and r_{t+1} from equations (3), (6), and (8) we get

$$\begin{aligned}
P_t^{(2)} &= E_t[\exp(-\delta\pi_t - 0.5\lambda_t^2 - \lambda_t\varepsilon_{t+1} - \delta(\pi_t - \phi(\delta\pi_t - \pi_t) + \sigma\varepsilon_{t+1}))] \\
&= \exp(-\delta\pi_t - 0.5\lambda_t^2 - \delta(\pi_t - \phi(\delta - 1)\pi_t)E_t[\exp(-(\delta\sigma + \lambda_t)\varepsilon_{t+1})] \\
&= \exp(-\delta\pi_t - 0.5\lambda_t^2 - \delta(\pi_t - \phi(\delta - 1)\pi_t) + 0.5\delta^2\sigma^2 + \delta\sigma\lambda_t + 0.5\lambda_t{}^2) \\
&= \exp(-\delta\pi_t - \delta(\pi_t - \phi(\delta - 1)\pi_t) + 0.5\delta^2\sigma^2 - \delta\sigma(\gamma_0 + \gamma_1\pi_t)) \\
&= \exp(0.5\delta^2\sigma^2 - \delta\sigma\gamma_0 - \delta(2 - \phi(\delta - 1) + \sigma\gamma_1)\pi_t),
\end{aligned}$$

$$P_t^{(2)} = \exp(-2i_t^{(2)})$$

$$(11) \quad i_t^{(2)} = 0.5\delta\sigma\gamma_0 - 0.25\delta^2\sigma^2 + 0.5\delta(2 - \phi(\delta - 1) + \sigma\gamma_1)\pi_t,$$

$$(12) \quad b_2 = \frac{\delta(2 - \phi(\delta - 1) + \sigma\gamma_1)}{2}.$$

For z dist $N(\mu,\sigma^2)$
$E(\exp(z)) = \exp(\mu + \sigma^2/2)$

After deriving formula for n=2

$$(12) \quad b_2 = \frac{\delta(2 - \phi(\delta - 1) + \sigma\gamma_1)}{2}.$$

Follow the same approach for case of general n>2 to get

$$(13) \quad b_n = \frac{\delta \sum_{i=0}^{n-1} (1 - \phi(\delta - 1) + \sigma\gamma_1)^i}{n}$$

$$(15) \quad \frac{\partial b_2}{\partial \delta} = \frac{2 + \phi + \sigma\gamma_1}{2} - \delta\phi > 0.$$

$$\frac{\partial b_n}{\partial \delta} = \frac{1}{n}(1 + \sum_{i=0}^{n-2} (1 + \phi + \sigma\gamma_1)(1 - \phi(\delta - 1) + \sigma\gamma_1)^i - \phi\delta \sum_{i=0}^{n-2} (i + 2)(1 - \phi(\delta - 1) + \sigma\gamma_1)^i)$$

You can show that $b_2 < \delta$ if

$$\delta > 1 + \frac{\sigma\gamma_1}{\phi}$$

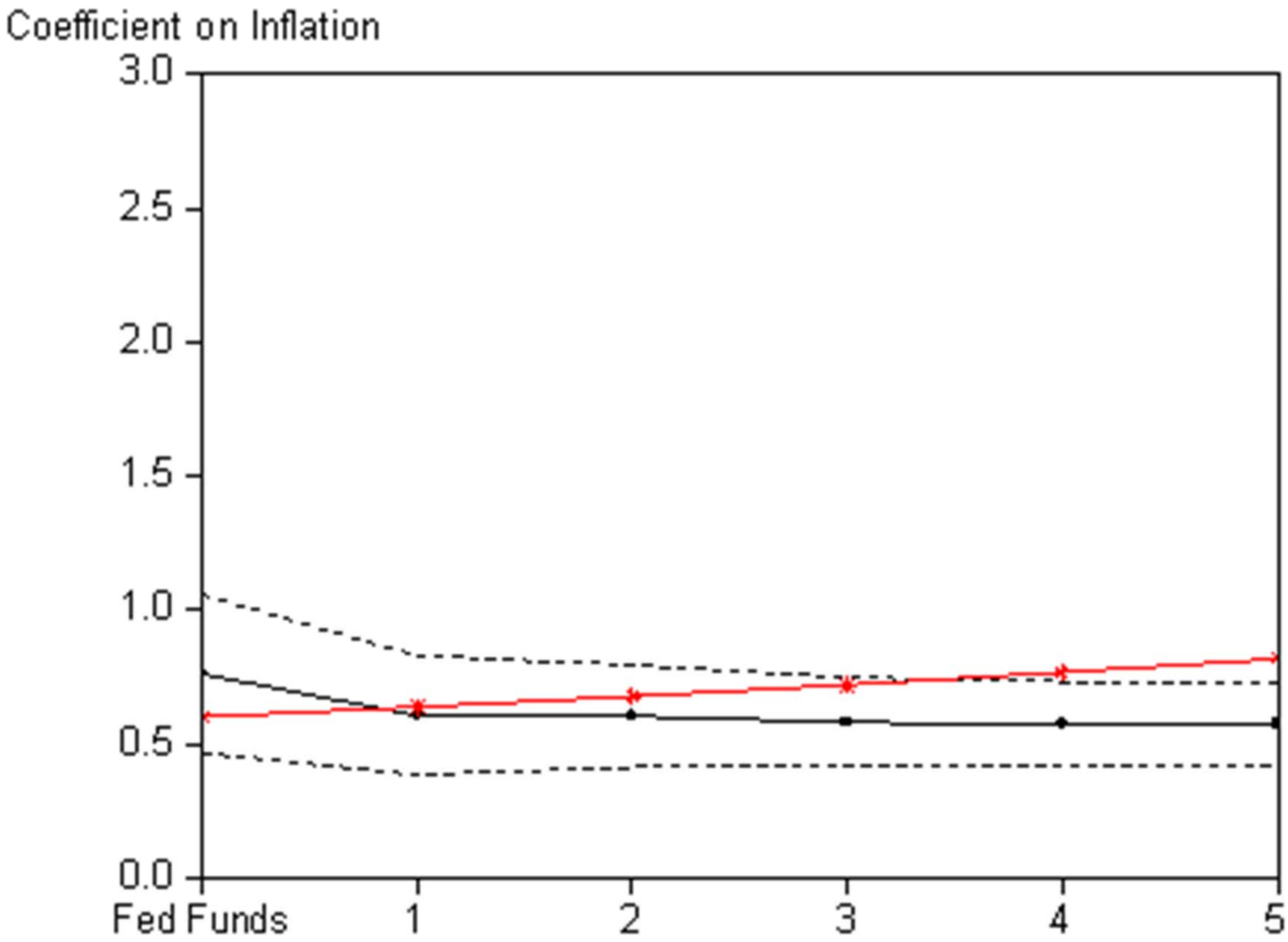
and then the terms in the geometric series in the numerator of b_n are also less that one.

Impacts of Changes in the Monetary Policy Rule on the Term Structure

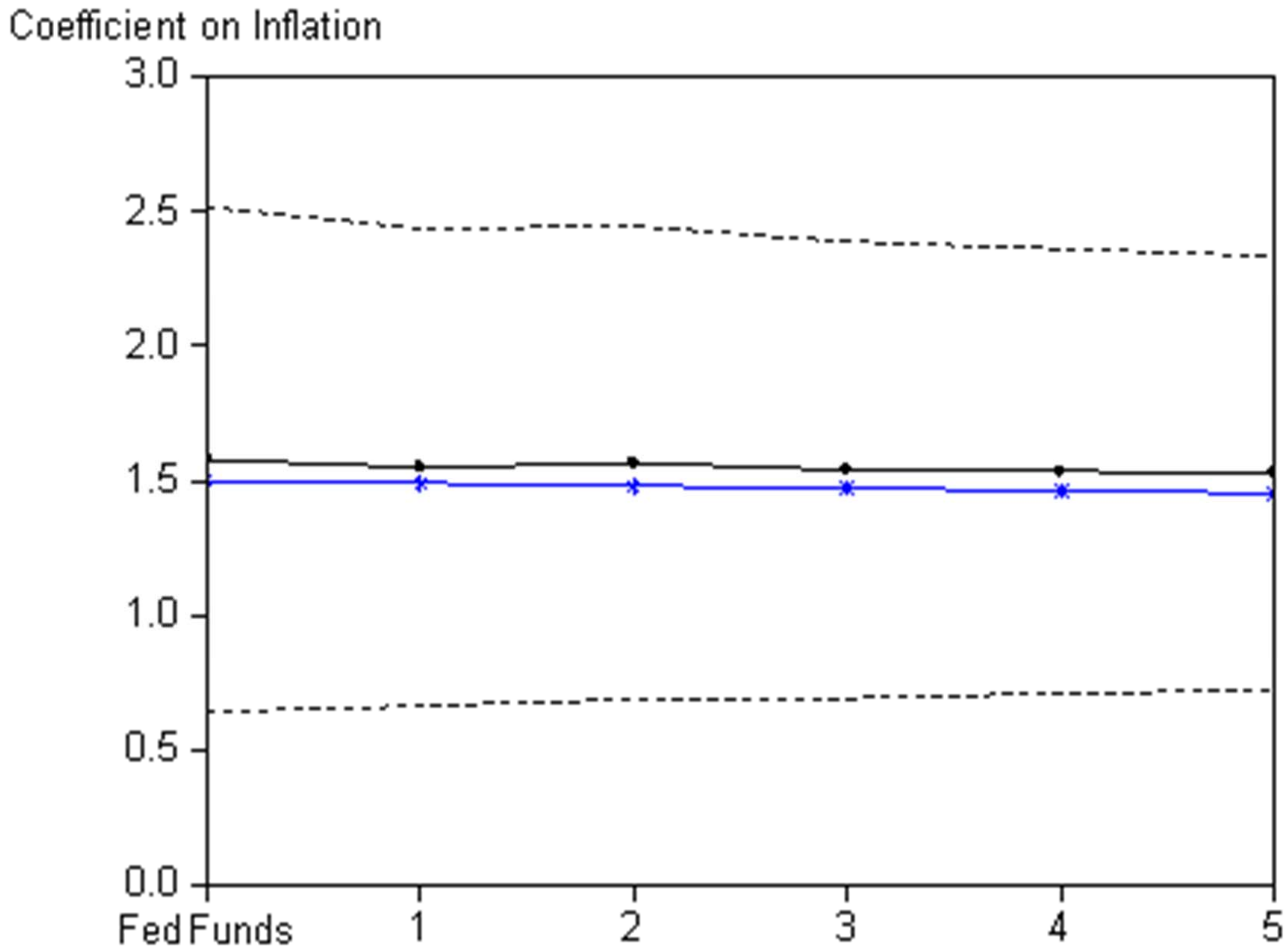
- As one changes the coefficients on the policy rule the term structure coefficients change
  - Can use the formula to trace out the effects
- Now compare the model with the regression estimates

Estimates in the simple case where output is out

Maturity (Years)	1960Q1 – 1979Q4			1984Q1 – 2006Q4		
	a_n	b_n	R^2	a_n	b_n	R^2
Fed Funds	2.234	0.761	0.603	1.312	1.579	0.264
	(0.493)	(0.151)		(1.277)	(0.479)	
1	2.913	0.605	0.681	1.487	1.549	0.279
	(0.388)	(0.113)		(1.201)	(0.453)	
2	2.989	0.603	0.758	1.790	1.566	0.286
	(0.333)	(0.097)		(1.141)	(0.449)	
3	3.188	0.580	0.797	2.114	1.540	0.291
	(0.295)	(0.085)		(1.061)	(0.433)	
4	3.282	0.573	0.816	2.338	1.536	0.297
	(0.28)	(0.081)		(1.001)	(0.422)	
5	3.319	0.573	0.832	2.480	1.528	0.305
	(0.288)	(0.077)		(0.951)	(0.412)	



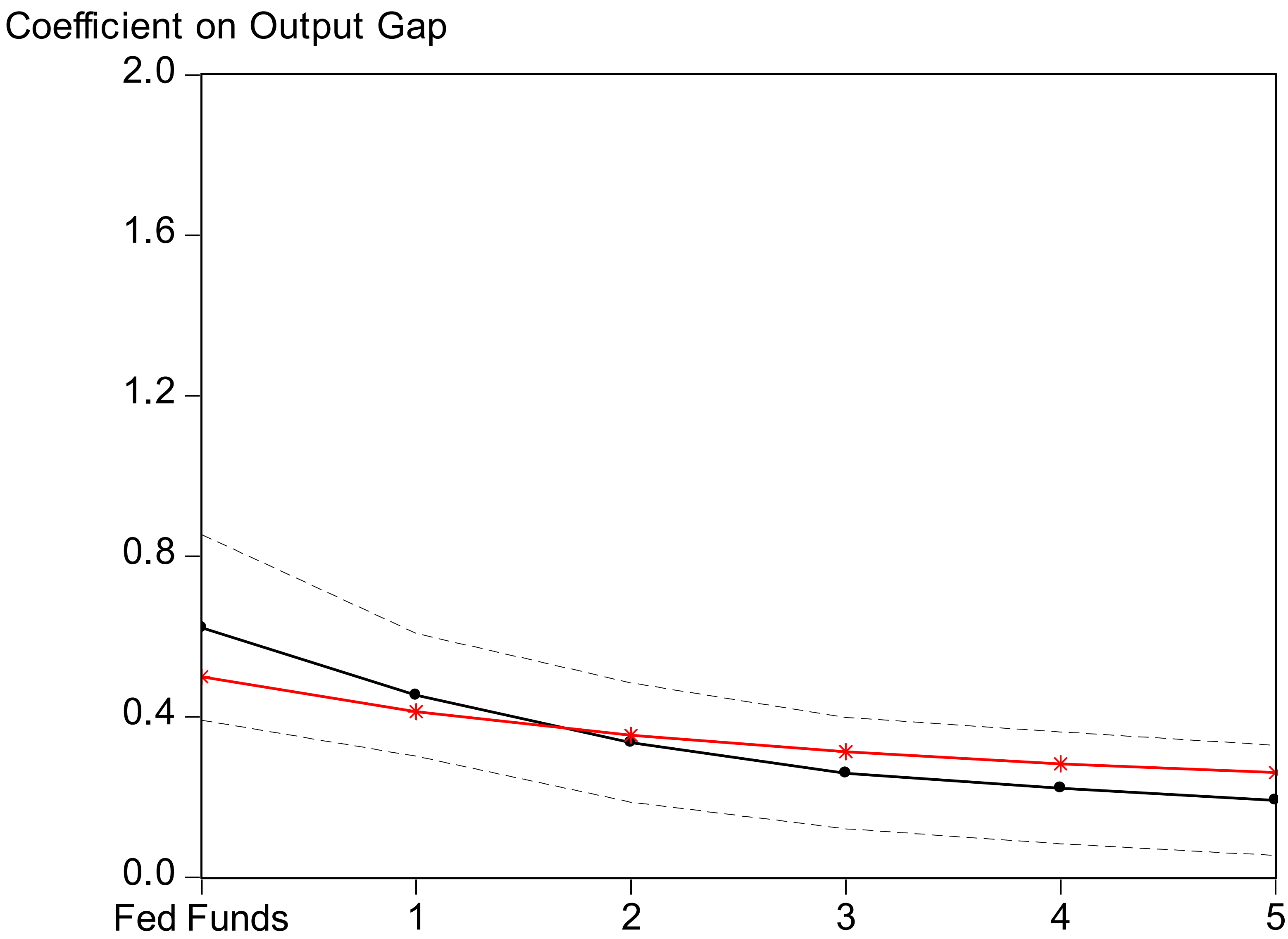
The affine response coefficient curve for inflation derived from the model with inflation only and estimated over the period 1960Q1-1979Q4.



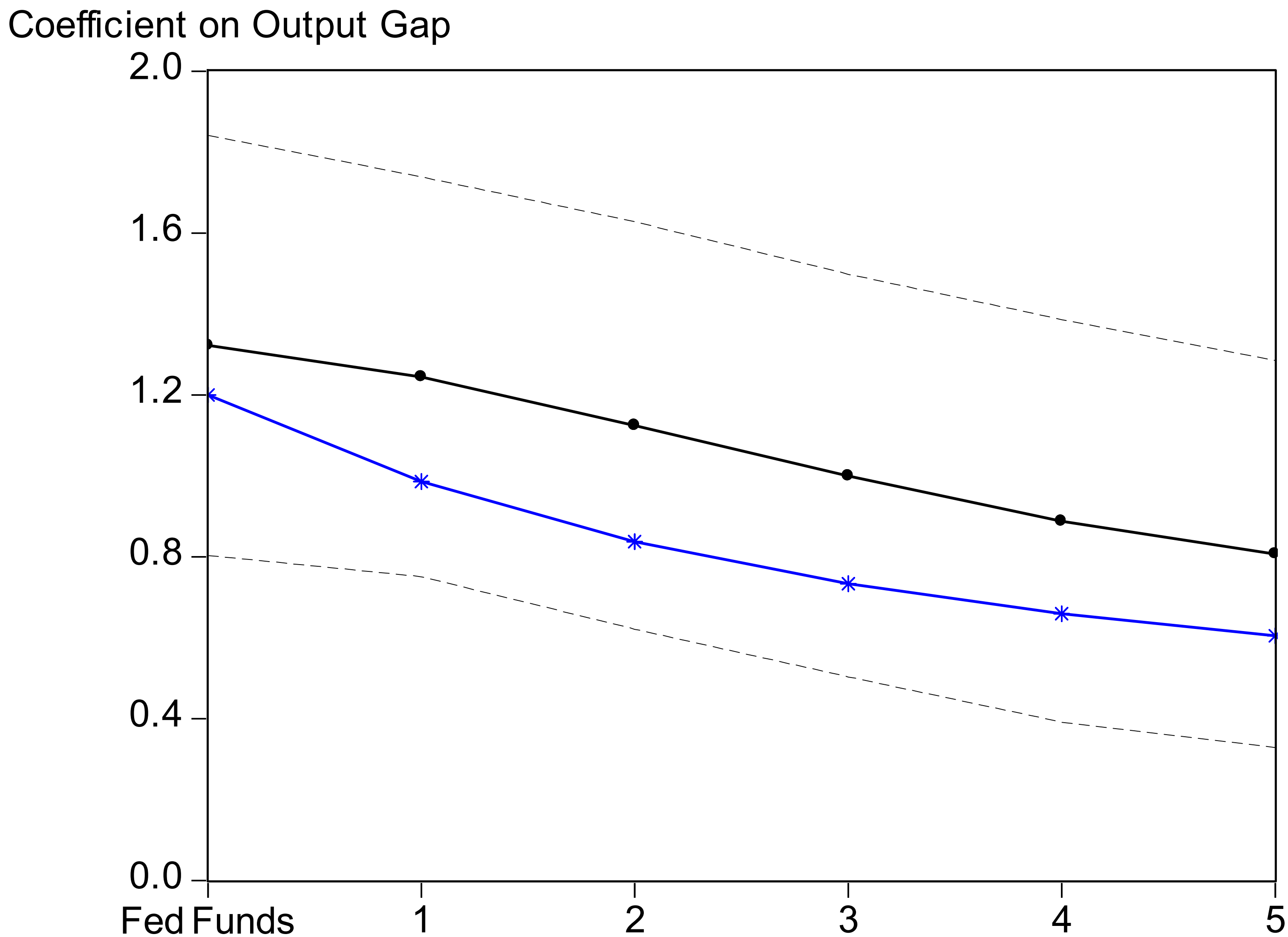
The affine response coefficient curve for inflation, derived from the model and estimated over the period 1984Q1-2006Q4.

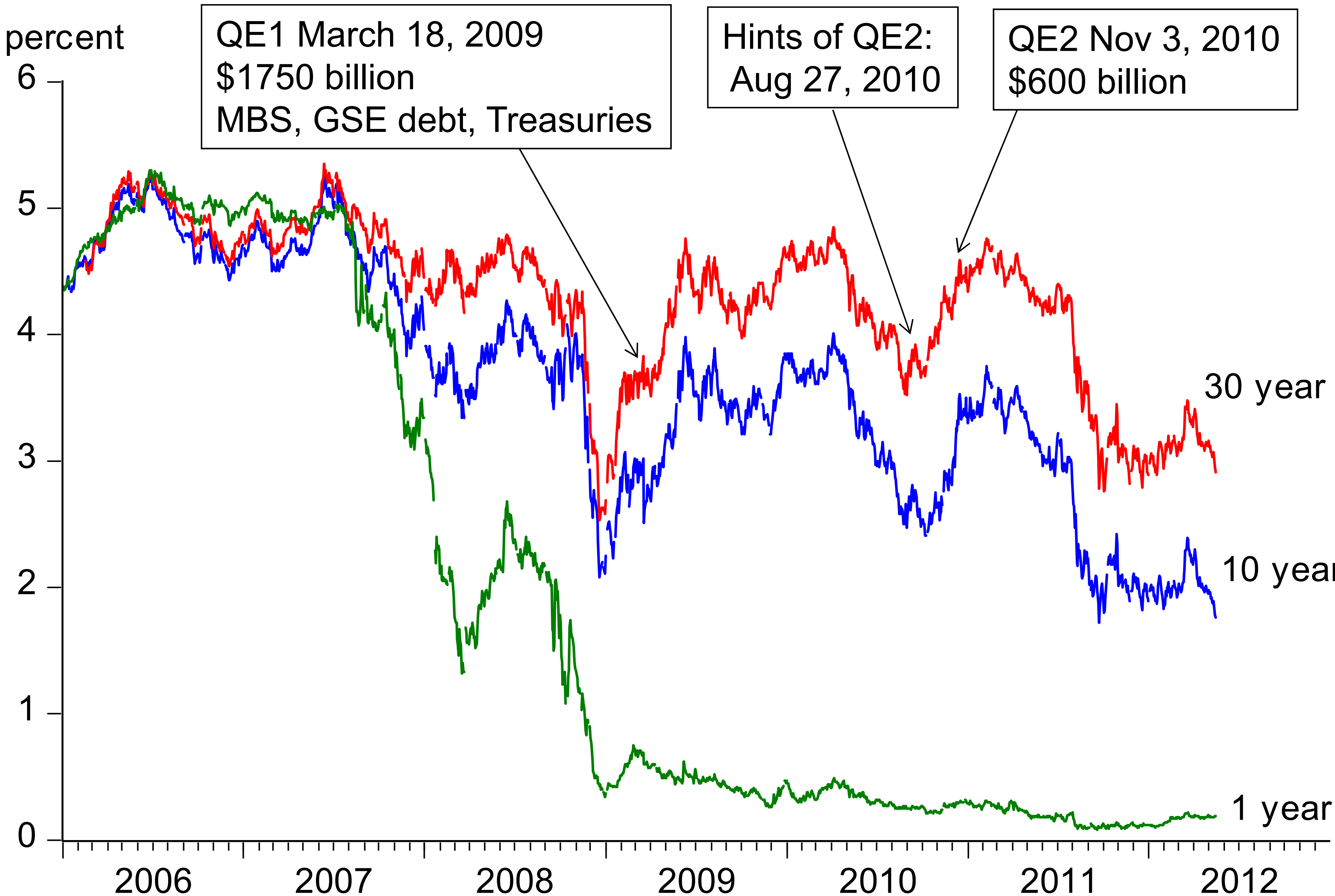
Estimates in the case where output is in

Mat'ity (Years)	1960Q1 – 1979Q4				1984Q1 – 2006Q4			
	a_n	$b_{1,n}$	$b_{2,n}$		a_n	$b_{1,n}$	$b_{2,n}$	
Fed Funds	2.180	0.623	0.760	0.777	2.022	1.322	1.234	0.504
	(0.278)	(0.118)	(0.080)		(0.921)	(0.265)	(0.362)	
1	2.874	0.454	0.604	0.847	2.156	1.244	1.224	0.513
	(0.194)	(0.078)	(0.051)		(0.856)	(0.252)	(0.343)	
2	2.960	0.335	0.602	0.860	2.394	1.124	1.273	0.478
	(0.186)	(0.076)	(0.053)		(0.849)	(0.257)	(0.356)	
3	3.166	0.259	0.580	0.865	2.652	1.000	1.279	0.451
	(0.181)	(0.071)	(0.052)		(0.818)	(0.254)	(0.355)	
4	3.263	0.222	0.572	0.868	2.815	0.888	1.304	0.426
	(0.183)	(0.071)	(0.053)		(0.798)	(0.254)	(0.358)	
5	3.303	0.191	0.573	0.872	2.913	0.806	1.318	0.416
	(0.186)	(0.07)	(0.054)		(0.777)	(0.244)	(0.356)	



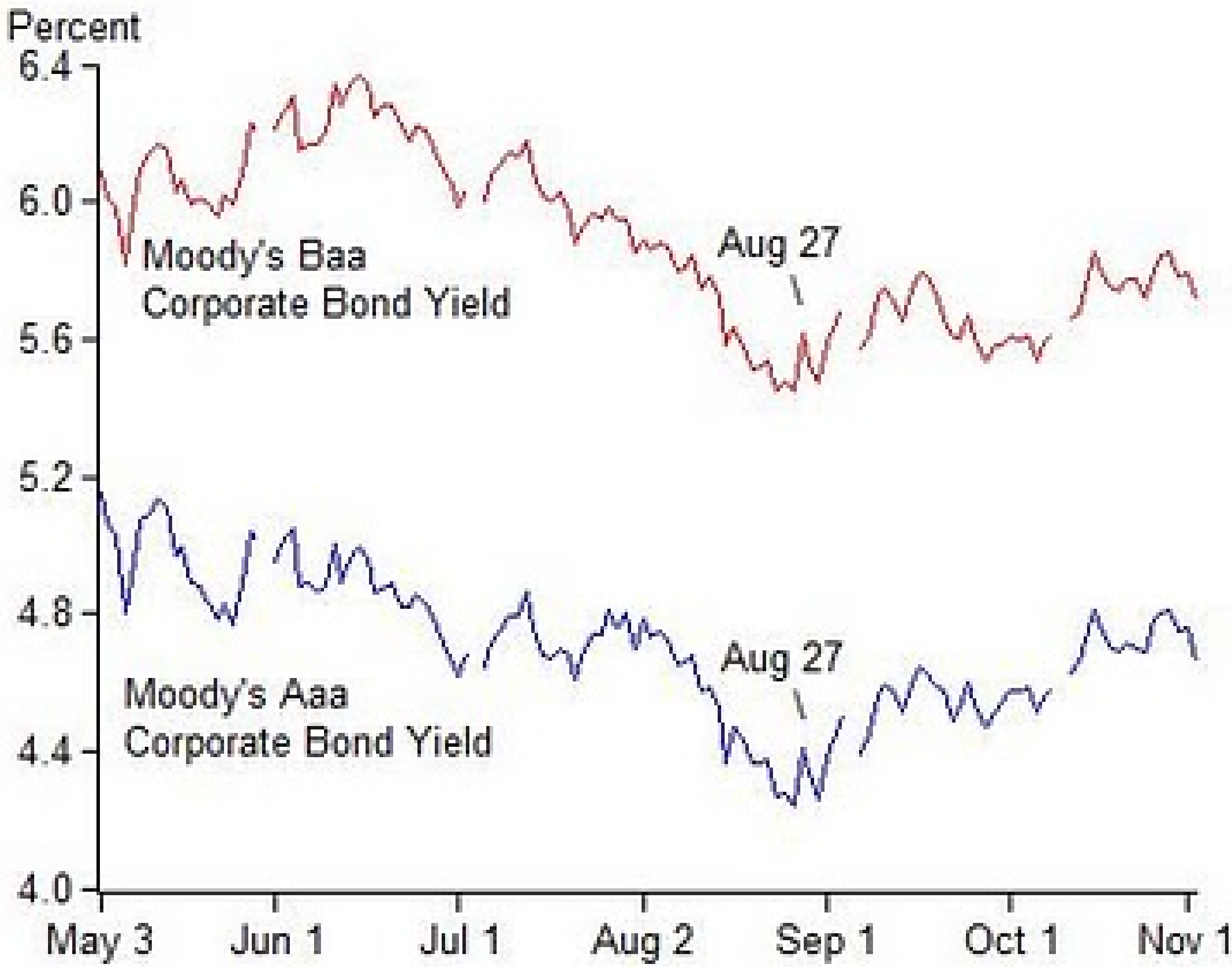
Coefficient on Output Gap
2.0
1.6
1.2
0.8
0.4
0.0
Fed Funds
1
2
3
4
5


Coefficient on Output Gap
2.0
1.6
1.2
0.8
0.4
0.0
Fed Funds
1
2
3
4
5



US Treasury Constant Maturity Yields

Anticipation of QE2 starting August 27, 2010



Percent
6.4
6.0
5.6
5.2
4.8
4.4
4.0
Moody's Baa
Corporate Bond Yield
Aug 27
Moody's Aaa
Corporate Bond Yield
Aug 27
May 3
Jun 1
Jul 1
Aug 2
Sep 1
Oct 1
Nov 1

Empirical Assessments of QE?

- Little effect of size of purchases
  - Work at Stanford with Johannes Stroebel
  - Take a look at broad trends
- Announcement effects appear to be significant
  - Joe Gagnon et al
- But are not lasting (Jonathan Wright)
- May be signaling future interest rate policy
  - Bauer-Rudebusch
  - Back to pure expectations model again!