2. Observing Monetary Phenomena
Some Monetary Phenomena Are Easy to Observe

From Dwyer and Hafer FRB Atlanta, *Economic Review*, 2Q 1999
And Others Are Really Obvious To Everyone!
But more subtle facts are not so easy to uncover

- Time series analysis is needed
  - Stationarity, detrending

- Here we use
  - Multivariate time series
    - VARs, Impulse Responses, Granger Causality
    - Focus on relations between variables
    - Such as inflation, output or unemployment, and interest rate:

\[
\mathbf{y}_t = \begin{pmatrix} \pi_t \\ u_t \\ r_t \end{pmatrix}
\]
Recall alternative detrending methods to achieve stationarity

- First differencing
- Hodrick-Prescott filter

\[
\sum_{t=1}^{T} (y_t - s_t)^2 + \lambda \sum_{t=2}^{T-1} ((s_{t+1} - s_t) - (s_t - s_{t-1}))^2
\]

- Use economics to estimate trend (CBO)
Real GDP (in billions of dollars)
Real GDP

billions of dollars
First difference detrending

Growth rate of Real GDP

Great Moderation

Great Recession
Billions of dollars

CBO estimate of Potential GDP

Real GDP
Potential GDP
Note big differences between HP filter and CBO
Close correlation between unemployment rate and output gap

Output gap (CBO)

\[ Y_{gap} = 2.3(5.6 - u) \]

Note: \( u = 5.6 - 0.44y_{gap} \)
VARs and impulse response functions

\[ VAR(1) \quad y_t = A_1 y_{t-1} + u_t \quad Eu_t = 0, Eu_t' = \Sigma, \quad Eu_t' = 0 \text{ for } t \neq s \]

\[ VAR(p) \quad y_t = A_1 y_{t-1} + A_2 y_{t-2} + \ldots + A_p y_{t-p} + u_t \]

Infinite MA (or impulse response function):

\[ y_t = u_t + \Theta_1 u_{t-1} + \Theta_2 u_{t-2} + \ldots \]

where \( \Theta_i \) are functions of the \( A \)'s

Example for \( VAR(1) \) : \( \Theta_i = A_1^i \)

for \( VAR(2) \) : \( \Theta_1 = A_1, \Theta_2 = A_1 \Theta_1 + A_2, \Theta_i = A_1 \Theta_{i-1} + A_2 \Theta_{i-2}, i = 3,4,\ldots \)

\( A \)'s can be estimated with OLS, and \( \Theta \)'s can then be calculated
Granger-causality

Consider two variables: $\pi_t$ and $y_t$. Then $y$ is said to Granger – cause $\pi$ if

$$\sigma^2_{\text{prediction}}(\pi_t \mid \pi_{t-1}, \pi_{t-2}, \ldots, y_{t-1}, y_{t-2}, \ldots) < \sigma^2_{\text{prediction}}(\pi_t \mid \pi_{t-1}, \pi_{t-2}, \ldots)$$

To show how to construct a test, consider a VAR

$$\pi_t = a_{11}\pi_{t-1} + a_{12}\pi_{t-2} + b_{11}y_{t-1} + b_{12}y_{t-2} + u_{1t}$$
$$y_t = a_{21}\pi_{t-1} + a_{22}\pi_{t-2} + b_{21}y_{t-1} + b_{22}y_{t-2} + u_{2t}$$

and the null hypotheses:

$H_0 : b_{11} = b_{12} = 0 \iff y$ does not Granger cause $\pi$

$H_0 : a_{21} = a_{22} = 0 \iff \pi$ does not Granger cause $y$

which can be tested with F – statistics. Note that both can be rejected indicating that both variables Granger - cause the other.
<table>
<thead>
<tr>
<th></th>
<th>PI</th>
<th>U</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI(-1)</td>
<td>1.493709</td>
<td>0.069717</td>
<td>0.257393</td>
</tr>
<tr>
<td></td>
<td>[25.8660]</td>
<td>[1.42398]</td>
<td>[1.67973]</td>
</tr>
<tr>
<td>PI(-2)</td>
<td>-0.509598</td>
<td>-0.067106</td>
<td>-0.140108</td>
</tr>
<tr>
<td></td>
<td>[-8.71516]</td>
<td>[-1.35366]</td>
<td>[-0.90300]</td>
</tr>
<tr>
<td>U(-1)</td>
<td>-0.197355</td>
<td>1.590920</td>
<td>-0.865766</td>
</tr>
<tr>
<td></td>
<td>[-2.78772]</td>
<td>[26.5063]</td>
<td>[-4.60872]</td>
</tr>
<tr>
<td>U(-2)</td>
<td>0.170520</td>
<td>-0.633061</td>
<td>0.845529</td>
</tr>
<tr>
<td></td>
<td>[2.45850]</td>
<td>[-10.766]</td>
<td>[4.59412]</td>
</tr>
<tr>
<td>R(-1)</td>
<td>0.036667</td>
<td>-0.001178</td>
<td>1.013047</td>
</tr>
<tr>
<td></td>
<td>[1.30057]</td>
<td>[-0.04930]</td>
<td>[13.5415]</td>
</tr>
<tr>
<td>R(-2)</td>
<td>-0.035442</td>
<td>0.017206</td>
<td>-0.103890</td>
</tr>
<tr>
<td></td>
<td>[-1.27631]</td>
<td>[0.73085]</td>
<td>[-1.40992]</td>
</tr>
<tr>
<td>Const</td>
<td>0.212963</td>
<td>0.160966</td>
<td>0.211128</td>
</tr>
<tr>
<td></td>
<td>[2.30893]</td>
<td>[2.05845]</td>
<td>[0.86264]</td>
</tr>
</tbody>
</table>
Impulse response functions (Eviews)
Response to Nonfactorized One Unit Innovations ± 2 S.E.

Response of PI to PI

Response of PI to U

Response of PI to R

Response of U to PI

Response of U to U

Response of U to R

Response of R to PI

Response of R to U

Response of R to R

Response to Nonfactorized One Unit Innovations ± 2 S.E.
Granger Causality Tests; Sample: 1955Q1 - 2011Q4

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>U does not Granger Cause PI</td>
<td>0.0002</td>
</tr>
<tr>
<td>PI does not Granger Cause U</td>
<td>0.0175</td>
</tr>
<tr>
<td>R does not Granger Cause PI</td>
<td>0.0093</td>
</tr>
<tr>
<td>PI does not Granger Cause R</td>
<td>0.0939</td>
</tr>
<tr>
<td>R does not Granger Cause U</td>
<td>0.0061</td>
</tr>
<tr>
<td>U does not Granger Cause R</td>
<td>0.0010</td>
</tr>
</tbody>
</table>
Impulse Responses in the Inflation-Unemployment-Interest Rate Recursive VAR

Source: Stock and Watson (2001)
Granger Causality

<table>
<thead>
<tr>
<th>Regressor</th>
<th>$\pi$</th>
<th>$u$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>0.00</td>
<td>0.31</td>
<td>0.00</td>
</tr>
<tr>
<td>$u$</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$R$</td>
<td>0.27</td>
<td>0.01</td>
<td>0.00</td>
</tr>
</tbody>
</table>

$p$ - values are shown in the table

Source: Stock and Watson (2001)
Key Monetary Facts Revealed

- Strong dynamic correlation of each series
  - Diagonal elements of impulse response function
- Unemployment impacts inflation negatively
- Inflation impacts unemployment positively
  - Because unemployment and output gap move inversely (and relatively contemporaneously), the signs in second and third points above should reverse if you use the output gap.
- Unemployment impacts interest rate negatively
- Inflation impacts interest rate positively
- Interest rate impacts unemployment positively
Thinking About What Might Explain the Facts

\[ \pi_t = \pi_{t-1} + .3y_t + v_t \]  \hspace{1cm} \text{(slow price adjustment)}

\[ y_t = .9y_{t-1} - .2(i_t - \pi_t) + e_t \]  \hspace{1cm} \text{(inter-temporal substitution)}

\[ i_t = 1.5 \pi_t + .5y_t + w_t \]  \hspace{1cm} \text{(monetary policy rule)}

\[ u_t = -.4y_t \]  \hspace{1cm} \text{(Okun’s Law)}

where
- \( u_t \) is the unemployment rate (deviation from mean)
- \( y_t \) is real output as a percentage deviation from trend
- \( i_t \) is the nominal interest rate (deviation from mean)
- \( \pi_t \) is the inflation rate (target rate assumed to be zero)
- \( e_t, v_t, w_t \) are serially uncorrelated zero mean shocks

Question: Would implied three equation VAR result in IRF and Granger-causality as in the data?