Submodularity of Energy Storage Placement in Power Network

Junjie Qin, ICME, Stanford
Joint work with Insoon Yang (USC) and Ram Rajagopal (Stanford)
Motivation

US distributed PV installation and storage deployment

Source: Rocky Mountain Institute and GTM Research
Location matters in storage deployment

Question

Where to deploy storage?
Practical Storage Placement

Proposed Storage Placement: $s$

Simulation
- Power Flow
- Reliability
- Uncertainty

Cost
- OperationCost($s$)
- FixedCost($s$)

Key features
- Simulation
- Discrete
- Sequential

System Data
- Network
- Generators
- Loads
Literature: Continuous Optimization

• Formulation

\[
\max_{s \in \mathbb{R}^N_+} V(s)
\]

s.t. \( \text{FixedCost}(s) \leq \text{Budget} \)

• Structural results and simple computation

• Drawbacks
  – Simulation
  – Discrete
  – Sequential

OpCost(0) – OpCost(s)

Sjodin, Gayme & Topcu, ACC12; Bose et al., CDC12; Q & Rajagopal, PES13; Bose & Bitar, CDC14; Wogrin & Gayme, TPS15; Pandzic et al., TPS15; Thrampoulidis, Bose & Hassibi, TAC16; Tang & Low, CDC16...
Practice: Greedy Heuristic

- How to place $\{1\text{ MWh}, 1\text{ MWh}\}$ in network?

![Diagram of a network with buses and devices, showing savings at each bus: 10, 15, 5, 7, 12.]

- Desired properties are satisfied
  - Simulation ✓
  - Discrete ✓
  - Sequential ✓
Performance of Greedy Heuristic

- Greedy placement works well in simulation (Dvijotham, Cherkov & Backhaus, HICSS14)
- Performance benchmark: Discrete optimization

\[ V^* = \max_{s \in S} V(s) \]

s.t. \( \text{FixedCost}(s) \leq \text{Budget} \)

Achievable storage capacities

Theorem (adapted from Nemhauser et al. 78)

If \( V : \mathbb{R}^N \mapsto \mathbb{R} \) is nondecreasing and submodular, then

\[ \frac{V_{\text{greedy}}}{V^*} \geq 1 - \frac{1}{e}. \]
Value of Networked Storage

- \( V(s) = \text{OpCost}(0) - \text{OpCost}(s) \)
- Operation model

\[
\text{OpCost}(s) = \min_{u, g \in \mathbb{R}^{NT}} \sum_{t \in T} \sum_{n \in \mathcal{N}} \text{GenCost}_{n,t}(g_n(t))
\]

\[
\text{s.t. } 1^T(g(t) - d(t) + u(t)) = 0, \quad t \in T,
\]

\[
-\bar{f} \leq H(g(t) - d(t) + u(t)) \leq \bar{f}, \quad t \in T,
\]

\[
u_n \in \mathcal{U}(s_n), \quad n \in \mathcal{N}
\]

Proposition

\( V : \mathbb{R}^N \mapsto \mathbb{R} \) is nondecreasing and concave, and \( \nabla^2 V(s) \) exists almost everywhere.

Also see Bitar et al. 16 and Bose & Bitar 16
Substitutability of Networked Storage

Working Definition

\[ V : \mathbb{R}^N \mapsto \mathbb{R} \text{ is called submodular if wherever } \nabla^2 V(s) \text{ exists,} \]

\[ \frac{\partial}{\partial s_j} \left( \frac{\partial V(s)}{\partial s_i} \right) \leq 0, \quad i, j \in \mathcal{N}. \]

MV of storage at \( i \)

- Submodular if \( s_i \) substitutes \( s_j \)
- Supermodular if \( s_i \) complements \( s_j \)
- \( i = j \): Substitutability holds by concavity
- \( i \neq j \): Intuitive to have substitutability but hard to check
Complementarity of Networked Storage

**Theorem**

\[ V : \mathbb{R}^N \leftrightarrow \mathbb{R} \text{ is not submodular in general.} \]

**Proof idea:** Construct example where \( s_i \) complements \( s_j \)

![Diagram](image)

Complementarity exists under certain congestion patterns!
Theorem

$$V : \mathbb{R}^N \mapsto \mathbb{R}$$ is not submodular in general.

Proof idea: Construct example where $s_i$ complements $s_j$

Complementarity exists under certain congestion patterns!
Complementarity of Networked Storage

Theorem

$V : \mathbb{R}^N \mapsto \mathbb{R}$ is not submodular in general.

Proof idea: Construct example where $s_i$ complements $s_j$

Complementarity exists under certain congestion patterns!
Complementarity of Networked Storage

**Theorem**

\[ V : \mathbb{R}^N \leftrightarrow \mathbb{R} \text{ is not submodular in general.} \]

Proof idea: Construct example where \( s_i \) complements \( s_j \)

Complementarity exists under certain congestion patterns!
Complementarity of Networked Storage

Theorem

\( V : \mathbb{R}^N \mapsto \mathbb{R} \) is not submodular in general.

Proof idea: Construct example where \( s_i \) complements \( s_j \)

Complementarity exists under certain congestion patterns!
Complementarity of Networked Storage

**Theorem**

\[ V : \mathbb{R}^N \leftrightarrow \mathbb{R} \text{ is not submodular in general.} \]

Proof idea: Construct example where \( s_i \) complements \( s_j \)

Complementarity exists under certain congestion patterns!
Certifying Submodularity

Certification Problem

Determine whether $V$ is submodular given problem data and region of interest $[0, \bar{s}^{\max}]^N$.

Idea: Group $s$ according to spatial-temporal congestion pattern

Theorem

(a) Congestion patterns define critical regions (CRs);
(b) Hessian is identical for all $s$ in the same CR.
Hessian Computation

Hessian Decomposition

Whenever $\nabla^2 V(s)$ exists, it can be decomposed into

$$\nabla^2 V(s) = \sum_{t \in \mathcal{T}} SC_t(s) NC_t(s) SC_t^\top(s),$$

where

- $SC_t(s)$ depends only on storage congestion pattern,
- $NC_t(s)$ depends only on problem data and network congestion pattern.

- $SC(s)$ can be computed using only $LMP(s)$
- $NC(s)$ can be computed using only $LMP(s)$ and input problem data if the network is a tree
Small Storage

- **Theory:** The number of CRs can be **large**
- **Practice:** The amount of storage to place is **small**
  - 2015: 221 MW US storage deployment, 467 GW average US generation

**Small Storage Hypothesis**

\[ [0, \bar{s}^{\text{max}}]^N \subset \text{CR}_1, \text{ the CR containing } s = 0. \]

- **Verification:** Solve a simple LP
- **Consequences:**
  - Single CR
  - Determination of submodularity using current operation data
Numerical Example

Figure 3: Boxplots of price and load data.

- (a) Price
- (b) Load

• Small Storage ✓
• Submodularity ✓
• Surprise: greedy finds exact OPT
Summary and Future Work

Summary
• Greedy placement
  – Practically desirable
  – Requires submodularity to have performance guarantee
• Complementarity exits under certain congestion pattern
• Methods & structural result for certifying submodularity

Future work
• Risk-averse placement
• Detection of complementarity and alg. for general case
• Certification with realistic operation models & simulation
• Implications on decentralized storage investments
Thank you for your attention!