Welfare Analysis of Transfer Programs with Jumps in Reported Income: Evidence from the Brazilian *Bolsa Família**

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Abstract

Transfer programs based on income often generate non-convex kinks in budget sets, particularly in their phase-out regions. In such settings, optimizing agents may respond to changes in the schedule by "jumping" from one bracket of a tax and transfer schedule to another, a behavior that is ruled out by the widely used "first-order" approach in optimal tax theory. This paper presents evidence that such jumps are empirically important using administrative data on reported income that spans a reform of the Brazilian anti-poverty program *Bolsa Família*. I develop a theoretical framework that allows for such jumping behavior and show that an additional set of "jumper shares" coupled with standard parameters yield sufficient statistics for welfare analysis. Estimating these shares using the Brazilian data, I document that for every marginal *real* (R$) transferred by the reform, 12 cents were lost due to the efficiency costs of jumping behavior. Simulations suggest that "jumping" behavior substantially affects the welfare analysis of more general reforms.

JEL:H21, I38, J22

Keywords: Taxation Efficiency, Effects of Welfare Programs, Labor Supply

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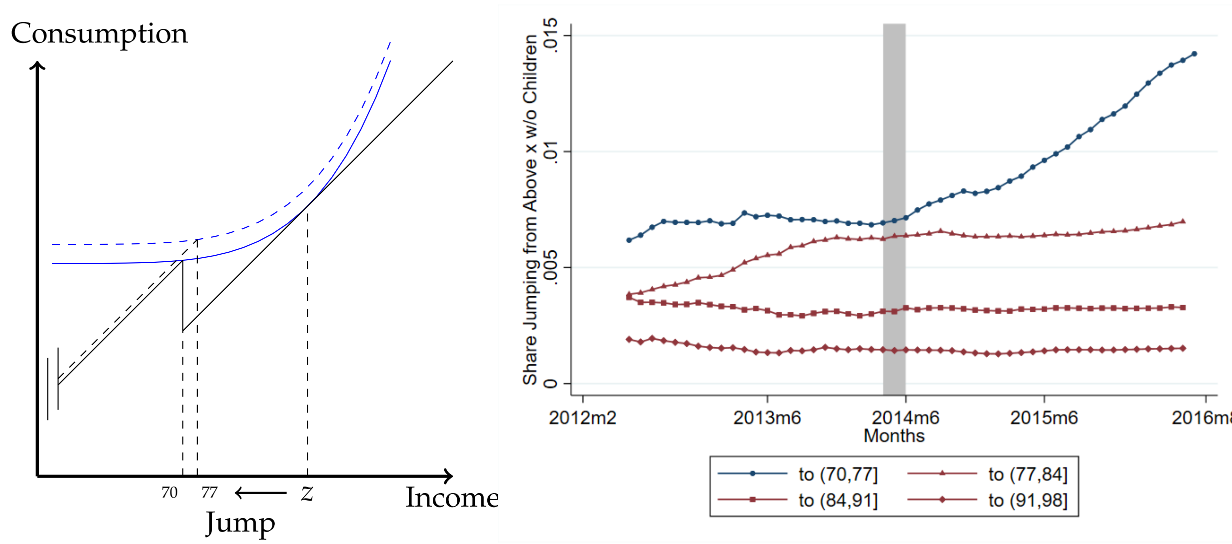
1 Introduction

Transfer programs based on income often generate non-convex kinks in budget sets – i.e., points at which marginal tax rates fall when income rises. For instance, for the Earned Income Tax Credit (EITC) in the US, marginal tax rates are higher in the phase-out region of the program than at higher income levels. In such settings, agents with neoclassical preferences can be indifferent between two tax/transfer brackets. These indifferent applicants could respond to small reforms of the schedule by "jumping" from one bracket to the other. Such behavior has not received attention from the most common approach in optimal tax theory: the reduced-form sufficient statistics approach (Diamond (1998) and Saez (2001)). This paper presents evidence of jumping in a large anti-poverty program and develops a theoretical framework that takes this behavior into account in the welfare analysis of transfer/tax reforms.

The empirical setting is the Brazilian cash transfer program *Bolsa Família* (BF), "the largest conditional cash transfer program in the developing world" (Lindert et al., 2007).[1] Household per capita income determines eligibility for the program. Below the eligibility threshold, the magnitude of the transfer depends only on household composition, i.e., the marginal transfer (along the income dimension) is zero above and below this endpoint. In Figure 1a, the solid black line represents the budget set faced by households without children around the limit of eligibility. In April 2014, the Brazilian government announced a reform that would increase both transfers and the eligibility criteria by 10%. The dashed black line in the same figure plots the budget set of the same households after this reform.

To understand the key idea of this article, note that households could be indifferent between joining the program or not. The solid indifference curve in Figure 1a represents the preferences of one of these applicants before the reform. This household breaks its indifference by choosing the income level above the eligibility threshold. The indifferent agent should respond to the infra-marginal BF reform by jumping to the new threshold (R$77),[2] as indicated in the figure. Note, however, that there is no change in the slope (marginal after-tax/transfer income) or intercept (virtual income) of the linearized schedule around its initial income level z. Therefore, the usual sufficient statistics in the first-order approach (income elasticity with respect to the marginal after-tax income and the virtual income) do not capture such behavior. Furthermore, this jump does not correspond to a participation (extensive margin) response — a behavior also addressed by previous extensions of the

[1] BF had more than 42 million beneficiaries as of March 2015.
[2] The Brazilian currency (*real* or plural *reais*) is denoted by R$.



(a) *Bolsa Família* Schedule

(b) Share of Jumpers up to Each Month

Figure 1: *Bolsa Família* Reform for Families without Children

Note: Panel (a) displays the BF reform effect on earnings choices of a noneligible applicant indifferent between being at income level z and at the eligibility threshold. Panel (b) depicts the cumulative shares of households above R$x that moved to the $(x-7, x]$ interval up to each month in time, out of all households initially above R$x and that updated up to the same month. The blue line with circles and the red lines with triangles, squares, and diamonds plot the shares for $x = 77, 84, 91$, and 98, respectively. The gray vertical bar indicates the months between the announcement and the enactment of the reform.

standard approach (see, for instance, Saez (2002), Jacquet et al. (2010), and Scheuer and Werning (2016)).

In this environment, I conduct three exercises. First, I present evidence of jumps in reported income as a response to the reform. Second, I develop a theoretical framework that accommodates this behavior and conduct a welfare analysis of the change in the schedule. Third, I illustrate the importance of jumps in simulations of different reforms.

The first part of the paper documents this jumping behavior using BF administrative data from December 2011 to September 2016. Figure 1b's line marked with circles plots the share of households that jumped from some income level above R$77 to the $(70, 77]$ interval among those that updated from above R$77 up to each month in time. This segment only became attractive after the reform, providing incentives for jumping. The gray area indicates the months between the announcement and enactment of the reform (June 2014). There is a sharp increase in the share of jumpers, which starts around these months and continues for the two following years. A counterfactual series is necessary to investigate whether the reform caused the increase in the share of jumpers. I plot three alternative trends in the same figure: symmetric shares of households jumping from above 84, 91, and 98 to the 7 *reais* interval right below these numbers. None of these intervals was affected by the reform. Under the identifying assumption that the trends in shares of jumpers to the affected and alternative regions would remain parallel after the reform, the increase in the first share corresponds to the causal effect of the reform on the share of jumpers. This evidence indicates that applicants to the BF program changed their reported income in response to the infra-marginal change in the schedule. This result is robust to alternative exercises that explore the different impacts of the reform on households with and without children, as well as alternative placebo intervals used as control groups. I find that 0.6% of households without children with income above R$77 jumped to the $(70, 77]$ interval.

The second part of the paper presents a theoretical framework that accommodates this jumping behavior. I show that the share (more precisely, density) of households jumping to the new threshold along the reported income dimension is the sufficient statistic for welfare analysis of the BF reform. The benchmark framework consists of a labor supply model for simplicity. I consider an economy in which agents are not only heterogeneous in ability as in Mirrlees (1971), but also in elasticity. In this setting, there are different types located at each income level, in contrast to the unidimensional case. Hence, for any small infra-marginal reform, such as the one discussed above, some agents located at each income level would jump while others would not. This replicates the pattern seen in the data. The share of jumpers captures the behavioral responses along this margin. The reform could also generate income effects on households that were below the threshold before the reform. However, these responses do not affect the government's budget, because the marginal transfer is zero below the threshold. Since the reform does not change the marginal transfer, there are no distortions in the intensive margin. Finally, the envelope theorem guarantees that the effect of the behavioral responses on the utility of the household is second order.

Note that these responses to the reform could come either from changes in misreporting or labor supply behavior. However, I show that Feldstein's (1999) argument that the taxable income (analogous to reported income in the present setting) elasticity is the sufficient statistic for the welfare analysis extends naturally in the case of discrete jumps. To see this, note that the reform will affect welfare through the utility of applicants and the budget of the government. As mentioned above, the effect of the jumping responses on the first term is second order. Intuitively, every jumper (such as the one depicted in Figure 1a) is initially indifferent between their initial income level and the old threshold. For a marginal reform, the welfare gains are infinitesimal for these households, regardless of whether the jump is a result of labor supply or misreporting response. Since there is an infinitesimal number of jumpers, this effect is second order on welfare. On the other hand, the effect on the second term of the welfare (budget of the government) is first order, because the government pays an additional amount proportional to the entire transfer for each jumper. This effect is determined by the share of jumpers and does not depend on the nature of the reported income response in the absence of fiscal externalities of misreporting, i.e., as long as misreporting only affects the budget of the government through the reported income.[3]

A feature of the data is that there are some agents in the dominated area. This is contrary to standard models of choice. To accommodate the data, one must therefore employ a model with some nonstandard features. My theory attributes the dominated choices to imperfect attention, allowing agents to differ also in attention types. A fraction of these inattentive households is located right above the eligibility threshold so that they mechanically become eligible with the reform. Even though the number of such applicants affected by a marginal reform is infinitesimal, each one of them increases their consumption by the amount of the entire transfer. Hence the effect of a change of the threshold on welfare is first order once I account for inattention. This effect is also empirically relevant.

The analysis indicates that for every marginal *real* transferred to the poor with the reform, 66 cents were given to inframarginal households that were eligible even before the reform; 22 cents were transferred to the inattentive households that mechanically became eligible for the increase in the threshold; and 12 cents were transferred to jumpers, thereby accounting for pure efficiency costs. All of this efficiency cost arises due to a jumping response, given that the reform does not alter marginal transfers.

Since the BF reform does not affect the incentives of applicants to respond locally in the intensive margin, one cannot quantify the importance of jumping behavior compared to the usual response in the data. To do so, the third part of the paper simulates an economy with parameters that match empirical estimates for the taxable income elasticity with respect to the marginal after-tax income from the literature. I consider a simple negative income tax (NIT) schedule, i.e., a transfer given to the unemployed phased out with a constant marginal tax rate. Notches are absent in these settings. I compute the efficiency costs of different reforms in the phase-out region. In these simulations, jumping effects account for 6% to 36% of the efficiency cost of the reforms.

Related Literature: This paper relates to the literature on the estimation of labor supply and taxable income elasticities. One approach to estimating these elasticities consists of specifying a structural model for the utility of agents (see, for instance, Hoynes (1993) and

[3] See Chetty (2009), Piketty et al. (2014), and Huang and Rios (2016) for examples in which these fiscal externalities are important. In all these cases, the real income elasticities are also necessary for the welfare analysis and for the optimal policy.

Friedberg (2000)). Another strategy exploits variations in tax/transfer schedules throughout time (e.g., Feldstein (1995) and Gruber and Saez (2002)) or within a cross-section (e.g., Saez (2010) and Kleven and Waseem (2013)) and measures the resulting response in taxable income in a reduced-form manner. Saez et al. (2012) provide a summary of this literature. While the structural approach has the advantage of allowing taxpayers to respond to the nonlinearities of the tax schedule, the strength of the reduced-form approach is to avoid imposing strong restrictions on preferences. In this paper, I propose a method to recover the relevant parameters for the welfare analysis, allowing agents to respond to infra-marginal changes in their budget sets within a reduced-form framework.

In a contemporaneous paper, Lockwood (2016) observes that the sufficient statistic method does not account for this jumping behavior (bunching behavior in his terminology) in the case of notches. This is perhaps the closest work to my article. He proposes a correction for the effect of a tax reform on the excess burden of the tax and calibrates it to the UK VAT tax system. My work extends his study in two ways. First, I present direct evidence that this behavior is relevant in the context of an important transfer program. Second, his correction relies on a specific functional form of agents utility. I propose a method to conduct the welfare analysis from reduced form estimates without imposing full structure on preferences. I also show how to account for this jumping behavior in general non-convex schedules, even in the absence of notches. My approach extends naturally to the characterization of optimal tax schedules.

This paper also speaks to the literature on the empirical implementation of optimal income tax formulas. Diamond (1998) and Saez (2001) rewrite the Mirrlees (1971) formula for the welfare maximizing income tax schedule in terms of labor supply elasticities and moments of the income distribution. A part of this literature addresses a particular type of jump: extensive margin responses. Saez (2002), Jacquet et al. (2010), and Scheuer and Werning (2016) show that if households are allowed to respond to changes in the tax schedule by entering or exiting the labor force, the labor participation elasticity with respect to the average tax rate is an additional sufficient statistic for the optimal tax. I show that the shares of jumpers (which coincides with the participation elasticity in the case of extensive margin jumps) are also in the characterization of the optimum when general jumps are allowed.

Even though jumping behavior does not occur under an optimal tax schedule in a unidimension economy (Mirrlees, 1971), a part of the theoretical optimal tax literature has observed that agents may jump in more general settings. Slemrod et al. (1994) found that taxpayers would jump under the optimum if the planner is restricted to use a two-bracket piecewise linear schedule. In fact, they show that once we account for this jumping behavior the optimal two-bracket schedule presents decreasing marginal tax rates in contrast to Sheshinski (1989)'s result. Dodds (2017) showed that jumping behavior also matters for optimal policy characterization if there is more than one dimension of heterogeneity in the economy. Finally, jumps are expected to occur in non-convex schedules away from the optimum. This might be the most relevant case for empirical researchers since real-world tax schedules are unlikely to be optimal. As far as I know, the present work is the first paper to rewrite the jumping effect that arises in these optimal tax and welfare analysis formulas in terms of shares of jumpers, which are related to the parameters I estimate in this paper.

The remainder of the paper is organized as follows. Section 2 presents the context of the application and Section 3 the reduced-form evidence of jumping responses. I introduce the theoretical framework in Section 4 and discuss the estimation of the relevant parameters in Section 5. Section 6 contains the welfare analysis while Section 7 presents simulations of alternative transfer programs. Section 8 concludes. I leave all formal proofs, the model extension with misreporting, some additional counterfactual analyses, and the optimal tax characterization to the Appendix.

2 The *Bolsa Família* Program

This section describes the context for the empirical application. Section 2.1 describes the *Bolsa Família* program. I then present the data sources in Section 2.2, the characteristics of the BF population in Section 2.3 and the reforms of the schedule which provide the identification in Section 2.4.

2.1 The *Bolsa Família* Program

The Brazilian anti-poverty program *Bolsa Família* was implemented by the Provisional Measure 132 in October 2003. It targets poor households on their per capita income reported to *Cadastro Único* agencies, which are the program offices spread across Brazil's 5,570 municipalities. The social development ministry (*Ministério do Desenvolvimento Social* or MDS) administers the program.

Applicants to the program report information to interviewers at program offices in any weekday. Beneficiaries are required to report their information once every two years in order to keep their benefits. This information includes their income, assets, and socioeconomic demographics. Interviewers input all the information to the *Cadastro Único* system.

Figure 2 shows the entries on the questionnaire used to calculate the per capita income. During the interview, the applicant reports the value for each of the seven income categories for each member in the household. The computer calculates the household per capita income in three steps. First, it gets the minimum between the average monthly income in the last 12 months and the last month income for each individual. Then, it sums this minimum with all other income categories to get the individual monthly income. Finally, it sums this individual monthly income across all members and divides it by the number of members of the household. Once this final per capita income is displayed on the interviewer's computer screen, the interviewer can no longer change the per capita income.

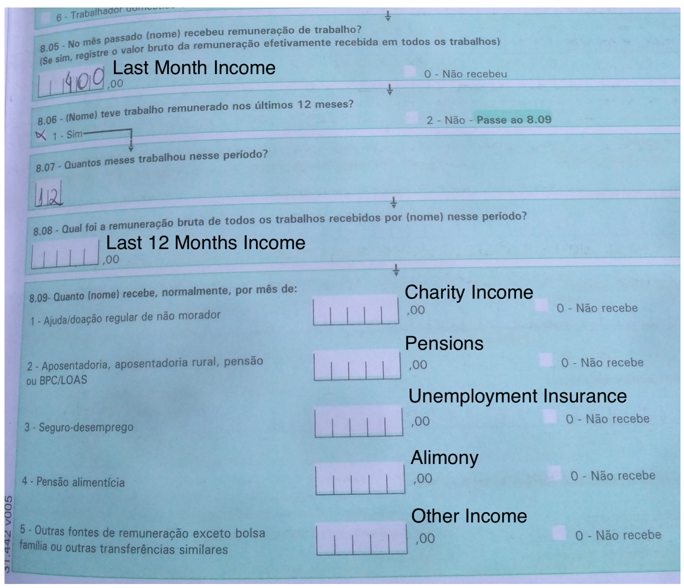


Figure 2: Income Report

Note: The figure depicts the income categories reported by applicants for each member of the household. Each category is translated to English in the picture. This is a print out of the screen seen by the interviewers in their computer when filling in the applicants' information.

The government transfers the money to the potential beneficiary, as long as they fulfill three conditionalities: (1) children must maintain a minimum of 85% of school attendance between ages 6 and 15 and 75% between 16 and 17; (2) households must keep track of their children's vaccines and of nursing mother prenatal visits to the doctor; (3) parents must maintain at least 85% of social-education attendance, if the household has violated child labor laws in the past. These conditionalities were held constant during the period of the analysis.

To enroll in the program, the head of an applicant household must present a government issued ID for himself and each member of the household. Therefore, registering nonexisting members is possible but unlikely.

The MDS has two main enforcing mechanisms to prevent income misreporting. First, the income questions come at the end of the questionnaire, so that the assets and social demographic questions help the interviewer assess the veracity of the income report. Second, the MDS conducts audits. Citizens' complaints and cross-checking of programs data with data on formal employment and the Brazilian social security system can generate these audits. In both cases, government employees may either visit families to update their information and/or require applicants to update their information in the office. The large informal sector in the Brazilian economy leaves scope for misreporting, which could be an important margin of responses to the schedule.

2.2 Data Sources

I have access to the *Cadastro Único* individual and household registry database, which define the eligibility of households for BF and other social security programs discussed in Appendix A.1. the database contains each applicant's characteristics, such as age, gender, race, marital status, schooling, employment status, occupation, income, and disability status. It also has information at the household level, such as per capita expenditures, ownership of durable goods, and per capita income which determines the benefits to which each household is entitled.

Figure 3 presents the timeline of the program and of data extractions. Each extraction contains the information for the last update for each household up to the extraction date. The final data set is constructed by appending eight extractions of the program's administrative records: one in December of each year from 2011 until 2015, and in April and August 2015 and September 2016. For instance, if a household updated its information in August of 2011 and September 2013, its information will appear as August of 2011 in the 2011 and 2012 extractions and as of September 2013 in the 2013, 2014, 2015 and 2016 extractions. The reform, which provides the variation for the analysis, occurred in the middle of the period (June 2014). This is helpful for testing the identification as I discuss in Section 3.3.

I also use municipal population data from the *Instituto Brasileiro de Geografia e Estatística* (IBGE) to compute the share of applicants per municipality.

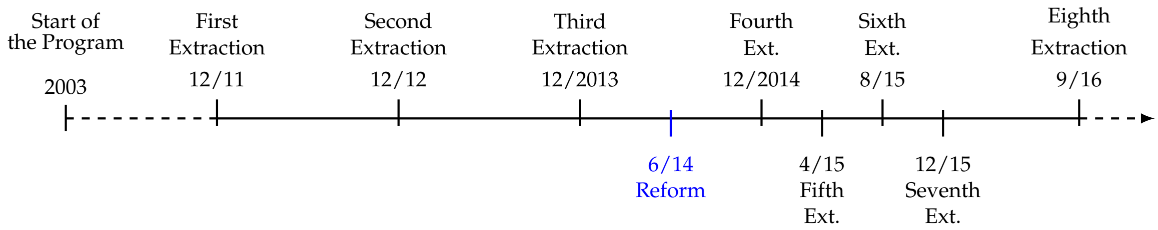


Figure 3: Timeline

Note: The figure describes the timeline of the program and the data. BF started in 2003 and the reform I studied occurred in June 2014. The data is constructed from 8 extractions from December 2011 until September of 2016. Each extraction contains the last information of each household up to the extraction date.

2.3 Sample Description

This section describes *Bolsa Família* applicant characteristics. All results come from a 5% random sample of the data for computational speed. Table 1 displays summary statistics for all households in the sample.

Table 1: Descriptive Statistics

Variables	Mean	Median
Per Capita Income	234.12	161.49
	(3463.32)	
Number of Members	2.94	3.00
	(1.44)	
Children up to 15 yo	1.15	1.00
	(1.11)	
Teenagers	0.20	0.00
	(0.34)	
Households	1,376,383	

Note: The descriptive statistics are calculated at the household level. I first calculate the average across updates for each household and then compute the mean and median in the 5% sample. The per capita income is inflated to June 2014 prices according to INPC.

The average per capita monthly income (R$ 234.12 or US$119.02)[4] is significantly larger than the median (R$ 161.49 or US$82.10) because of outliers. Applicant households have on average 2.94 members, 1.15 children 15 years old or younger and 0.20 teenagers. There are in total 81,404,307 applicants in 27,745,078 households. The 5% sample leaves me with 1,387,254 households or 4,038,784 applicants.

The northeast and north of Brazil are the country's poorest regions. This is reflected in the demand for BF, as shown in Figure 4. This figure displays the spatial variation in

[4] All conversions were made using the power of purchase parity ratio of 1.967 for 2016, according to the OECD.

the share of the population that has applied to the program across the 5,570 Brazilian municipalities. Higher shares of applicants are represented by darker shades in the map. Each color corresponds to a decile of this share distribution. There is substantial variation in the map. As expected, the largest shares are concentrated in the poorest areas of the country.[5]

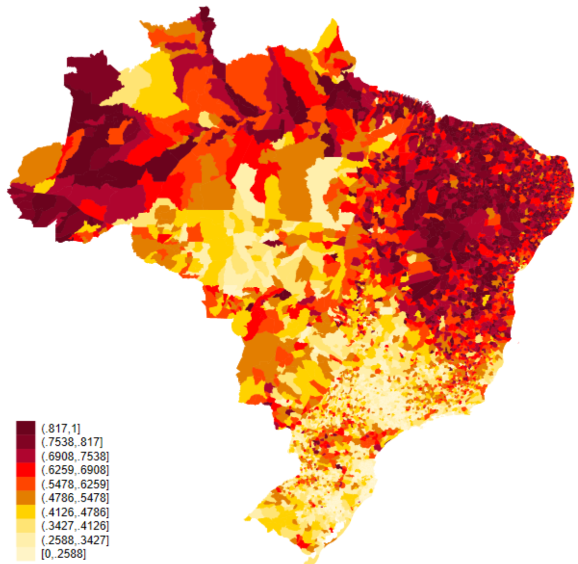


Figure 4: Density of Applicants per Municipality

Note: This figure plots density of applicants by municipality in 2016. Applicants density is defined as the number of individuals in households that ever applied to BF divided by the 2016 municipal population. We divide the observations into deciles within the sample. Each decile is assigned a different color on the map, with darker shades representing higher densities.

2.4 The Transfer Schedule and the June 2014 Reform

Since the beginning of the period of the analysis, the program defines two thresholds in the per capita monthly income distribution: the extreme poverty line (R$70) and the poverty line (R$140). Households with per capita income below the extreme poverty line are eligible for a constant basic benefit, a variable benefit proportional to the number of family members between 0 and 15 years of age, and a benefit proportional to the number of teenagers (individuals of 16 or 17 years of age). Households with per capita income between the extreme poverty and the poverty thresholds only get the variable and the

[5]In 37 municipalities, the share is larger than one for two reasons. First, the population is an estimate based on the 2010 census. Second, I consider any applicants in each municipality since the start of the program. Some of these applicants may have moved and no longer be part of the current municipal population. I re-coded all these observations to have shares equal to one.

teenager benefits. Households with per capita income above the second threshold are not eligible for any cash transfer.

In June of 2014, the government increased the extreme poverty line from 70 to 77 *reais* and the poverty line from 140 to 154 *reais*. The basic benefit was raised from 70 to 77, the benefit per child from 32 to 35, and the benefit per teenager from 38 to 42 *reais*. This reform was announced on national television by the president in April 2014, even though the thresholds were not mentioned.[6] Although transfers are heterogeneous according household composition, threshold values are the same. Table 2 summarizes these aspects of the schedule before (first column) and after (second) the reform. The last two rows display the average transfer for households without and with children.

Table 2: Schedule Details

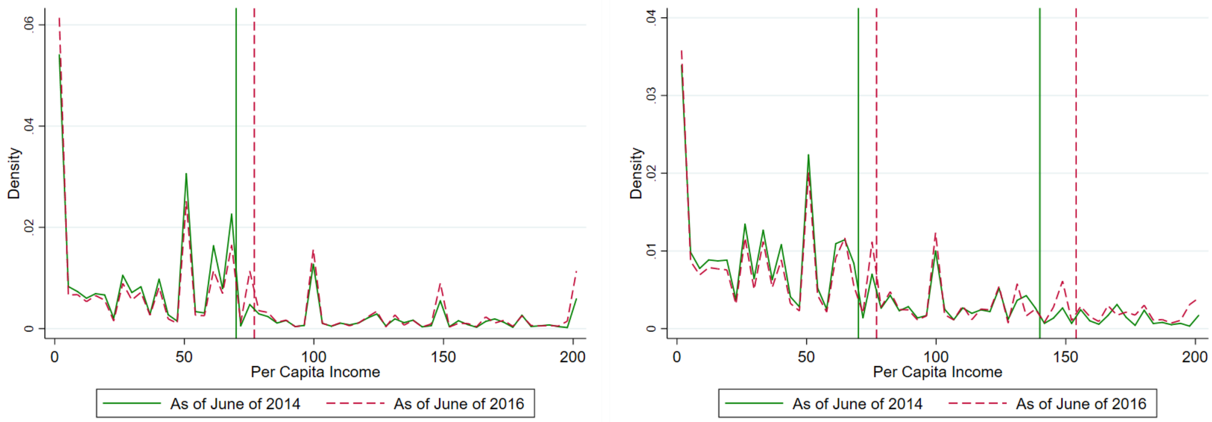
	Before	After
First Threshold	70	77
Second Threshold	140	154
Income Below 1st Threshold	70	77
Per Child 15 or younger (max 5)	32	35
Per Teen 16-18 (max 2)	38	42
Avg. Transfer in 1st Thr. (w/o Kids)	34.16	37.76
Avg. Transfer in 2nd Thr. (with Kids)	16.31	19.89

Note: The first two rows correspond to the threshold for the extreme poverty and poverty line, respectively. The third, fourth and fifth rows display the benefits given to households below the first threshold, households below the second threshold with children and with teenagers, respectively. The average transfers per capita are in the last two rows.

In the period of the analysis, there were four other reforms: in June and November 2012, February 2013, and June 2016. Since these reforms are too close to the beginning of the data (January 2012) and its end (September 2016), I do not use them in the analysis. The first two reforms did not affect the threshold or the transfer around these thresholds, but the last reform did. Hence, I focus on the effects up to June 2016 in the empirical analysis. All of the other reforms are discussed in Appendix A.2.

Figure 5a plots the per capita income distribution as of April 2014 with the solid green line and as of June 2016 with the red dashed line. Vertical lines indicate the eligibility threshold for these households before (green solid line) and after (red dashed line) the reform. Even though there is large bunching in round numbers, bunching below the threshold is visible before and after the reform. Note that there are households in dominated areas of the schedule (right above the threshold) before the reform.

[6]The president stated only the program would be adjusted by 10%.



(a) Households without Children

(b) Households with Children

Figure 5: Per Capita Income Distribution

These figures plot the empirical distributions of reported income for applicants without (Panel (a)) and with children (Panel (b)). The solid green lines and the red dashed lines plot the distributions as of June 2014 as of September 2016, respectively. The green solid vertical lines indicate the extreme poverty (first threshold) and the poverty line (second threshold) before the reform, while the red dashed lines plot the same aspects after the reform.

Figure 5b displays the analogous distributions for households with children as of April 2014 (solid green line) and as of September 2016 (red dashed line). Since these households were affected at both the extreme poverty and poverty thresholds, I depict each with solid green vertical lines before the reform and with red dashed lines after. The same patterns arise here, although bunching is less pronounced at the second threshold (which determines lower transfers). The presence of households in dominated areas is more evident for these distributions.

3 Reduced-Form Evidence of Jumping Effects

This section presents the reduced-form evidence of jumping. I lay out a simple test to asses the existence of jumps in Section 3.1. Sections 3.2 and 3.3 present evidence of the reform's effect on the timing of updates and on the share of jumpers, respectively. Section 3.4 shows the heterogeneity of jumps across different initial income levels, which is an important basis for the theoretical framework introduced later.

3.1 A Simple Test for Jumps

Consider households[7] that choose income y and consumption c in order to maximize their utility $u(c, y)$. Implicitly, households are producing income by supplying labor which is costly.[8] They face a budget constraint that allows them to consume no more than their after-transfer income. Consider a simple anti-poverty program that transfers I to households with income below t (as in the empirical setting). The household problem can be written as:

$$\max_{c,y} u(c, y) \; s.t. \; c \leq y + I * 1(y \leq t).$$

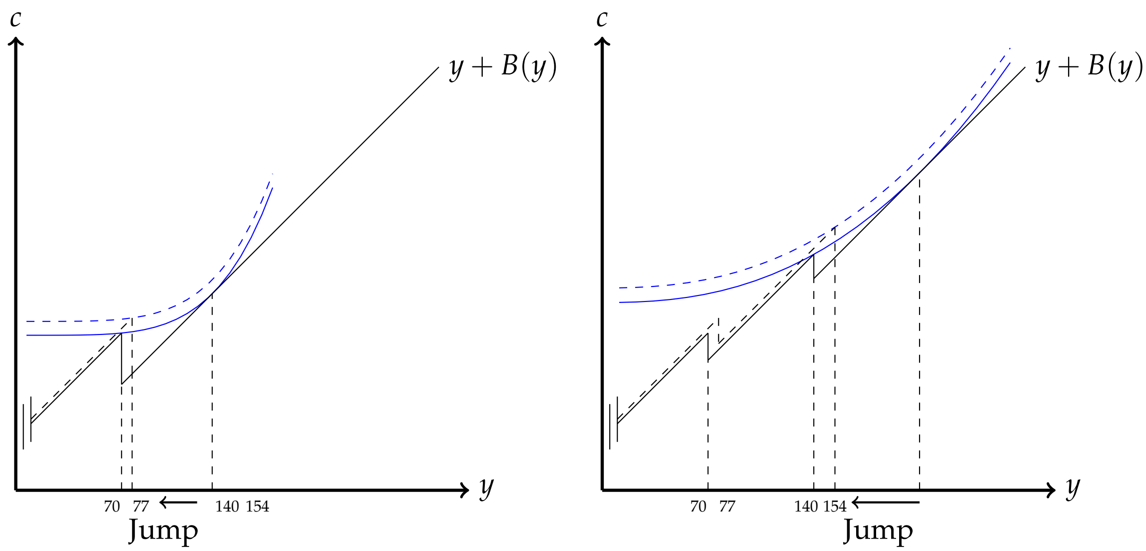
As discussed in Section 2.4, households without children faced an increase in their threshold of eligibility t from 70 to 77 *reais* and also an increase in their transfer. Figure 6a's solid black line illustrates the budget set of these households before the reform and, in dashed black, the corresponding set after the reform. Note that the 45-degree line represents the budget set in the absence of transfers. Since there are transfers for households with income below R$70, the budget set is nonlinear.

In the same figure, the blue indifference curves represent the preferences of a particular household. The preferences are such that utility is increasing in consumption and decreasing in income (labor supply), so that utility increases to the northwest of the graph. Before the reform, the household is indifferent between being in or outside the anti-poverty program, but chooses to be out of the transfer program in such a situation (solid curve). The reform does not affect this household's marginal transfer (slope around its initial income level) or virtual income (intercept of the linearized schedule around its initial income level). Therefore, the reform should not affect this household through local income or substitution effects. However, if households perceive the nonlinearities of the schedule, they could jump to the new threshold in response to the reform (dashed curve). This corresponds to the jumping behavior.

The goal is to test whether households that are initially above the new threshold of eligibility moved to the new threshold (R$77) because of the reform. In practice, I consider any movement to the interval between the old and new threshold $(70, 77]$, since this interval

[7]Since the eligibility of the program is based on the household income per capita, the relevant level of the analysis in the empirical application is the household.

[8]An important part of the responses corresponds to misreporting rather than labor supply behavior. However, in the absence of fiscal externalisties of misreporting, elasticities of the reported income are still the sufficient statistics for the welfare analysis, even in the presence of misreporting responses (Feldstein, 1999). Hence, I use a model of labor for simplicity.



(a) Households without Children

(b) Households with Children

Figure 6: Household Problem Before and After the Reform

Note: Panel (a) displays the BF reform effect on earnings choices of a noneligible household without children indifferent between being outside of the program or at the first threshold. Panel depicts (b) effect of the same reform on a noneligible household with children indifferent between being outside of BF or on the second threshold.

only became attractive after the reform. Let NEA be the number of noneligible applicants that were registered before the reform above 77, and JA the number of such households that jumped to the $(70, 77]$ interval because of the reform (jumping applicants). I denote $share(\Delta t, \Delta I)$ as the share of applicants that jumped to the threshold from an income level above the eligibility threshold, i.e.,

$$share(\Delta t, \Delta I) = \frac{JA}{NEA}. \tag{1}$$

Formally, I test the following hypothesis:

$$\begin{aligned} H_0 &: share(\Delta t, \Delta I) = 0 \; vs. \\ H_a &: share(\Delta t, \Delta I) > 0. \end{aligned}$$

Under the null, households do not respond to the infra-marginal reform. Note that the alternative hypothesis corresponds to a jump to a positive income level that cannot be interpreted as an extensive margin response, as in Saez (2002) or Jacquet et al. (2010).

The threshold of eligibility for households with children increased from 140 to 154 *reais* and their per capita transfer rose as discussed in the previous section. Figure 6b plots in solid and dashed black lines the budget set of these households before and after the reform, respectively. Once again, the blue indifference curves represent the preferences of a household that was out of the program but indifferent to locating at the notch before the reform (solid curve), and that jumps to the new notch after (dashed curve).

Even though these households also faced incentives to move to the new first threshold (77), the budget line they faced between this threshold and the last one (154) also changes. It is possible that responses in this region have to do with that change (income effects), rather than with a change at a more distant part of the budget constraint (jumping effects). Such a possibility pollutes the test for this group. Therefore, I focus on jumps to the second threshold. In this case, NEA are households with children with per capita income above 154 before the reform, and JA is the subset of households that moved to $(140, 154]$ after the reform.

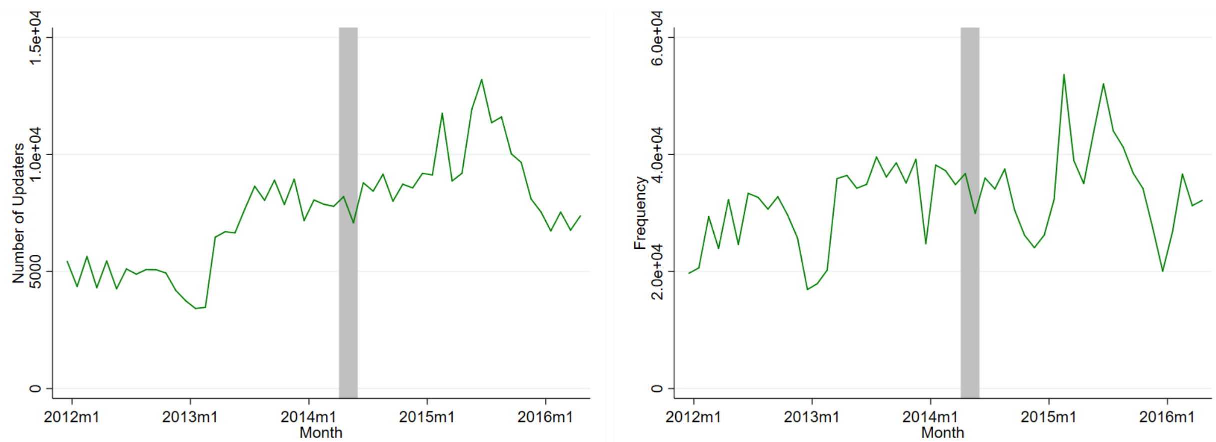
Households change their reported income for many reasons unrelated to the changes in the schedule in the data. Note that this test requires the identification of the part of these movements caused by the reform. Next, I present the research design for this identification and the results of the test.

3.2 Effect of the Reform on the Timing of the Update

Since BF allows applicants to report their information on any day the programs' offices are open, the reform could have affected both the timing of updates as well as the reported per capita income. To investigate the first of these two channels, Figure 7a and 7b plot the distributions of the months of the updates for households without and with children, respectively. The gray area indicates the months between the announcement (April 2014) and the enactment (June 2014) of the reform.

It is possible that the reform could have drawn the applicants attention to the program and increased the number of updates. This would generate a spike in the number of updates after the announcement of the reform. There is no such spike in either of the panels in Figure 22. This suggests that the reform did not substantially affect the timing of the updates. I can not rule this possibility entirely because of the structure of the data. As described in Section 2, I only observe for each household the last update up to each extraction. Mechanically, this generates more updates right before extraction dates. For instance, in 2015 there were three extractions (April, August, and December) explaining the larger number of updates in that year.

The next section documents the second, and most important, effect of the reform: Condi-



(a) Households without Children

(b) Households with Children

Figure 7: Date of Updates Distributions

These figures plot the empirical distributions of the months of updates for households without (Panel (a)) and with children (Panel (b)).

tional on updating, potential jumpers changed their reported income to the areas of the schedule that became more attractive with the reform.

3.3 Main Evidence

I start by performing the test described in Section 3.1 among households without children. Let $share^{no\ Kids}_{77,m}$ be the share of households without children that ever updated their per capita income from above 77 *reais* to the $(70, 77]$ interval up to month m:

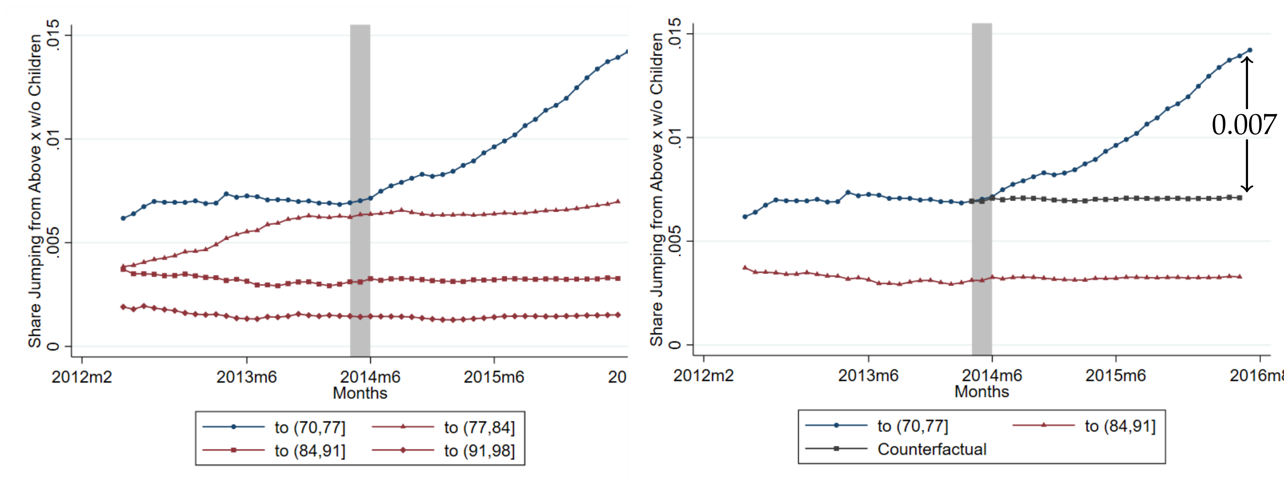
$$share^{no\ Kids}_{77,m} \equiv \frac{\text{N. of hhlds. w/o Kids updating from above 77 to } (70,77] \text{ up to month } m}{\text{N. of hhlds. w/o Kids updating from above 77 up to month } m}.$$

The numerator is the number of households that updated their income from some level above 77 to the relevant interval $(70, 77]$ up to each month. This number includes households that updated to this interval because of the reform but also for unrelated reasons. Since the reform does not seem to have affected the timing of updates, I focus on frequencies of updating to the given intervals conditional on updating. Therefore the denominator is the number of households above 77 that updated their income up to each month.[9]

[9]Perhaps the most natural way to define the numerator would be the number of households with income above R$ 77 in the previous period. However, in this case, the shares are not comparable before the reform because there were more households updating to regions closer to the threshold, probably due to misoptimization. To see this, Appendix A.3 replicates the results of this section with an alternative share definition in which the number of households that were ever above the relevant threshold is the measure of

In Appendix A.4, I show that the reform did not affect the timing of the updates differently across comparison groups. Therefore, the comparison conditional on updating is capturing all the effect of the reform on these shares.

The treatment effect of the reform on $share_{77,m}^{no\ Kids}$ corresponds to $share(\Delta t, \Delta I)$, i.e., the share of households jumping to the new notch because of the reform. Figure 8a plots with a solid blue line $share_{77,m}^{no\ Kids}$ from May 2012 until June 2016. The shaded area in gray corresponds to the months between the announcement of the reform (April 2014) and when it was actually enacted (June 2014).



(a) Evidence of Jumps

(b) Estimating $share(\Delta t, \Delta I)$

Figure 8: Share of Households without Children Jumping from Above 77

Note: Panel (a) depicts the cumulative shares of households without children above R$x that moved to the $(x-7, x]$ interval up to each month in time, out of all households initially above R$x and that update. The blue line with circles and the red lines with triangles, squares, and diamonds plot the shares for $x = 77, 84, 91$, and 98, respectively. Panel (b) replicates the series for $x = 77$ and 91 and draws the counterfactual distribution for the share above R$77, under the assumption that its trend would remain parallel to the trends in the shares above R$91 after the reform (gray line marked with squares). The gray vertical bars indicate the months between the announcement and the enactment of the reform.

There is a sharp increase around the months of the reform and its announcement. This break could still arise from some event around the month of the reform (e.g., an economic crisis) that pushed households to report lower levels of income. To construct a counterfactual series, let $share_{x,m}^{no\ Kids}$ be the share of households without children that jump from above x to $(x-7, x]$ for $x = 84, 91, 98$:

$$share_{x,m}^{no\ Kids} \equiv \frac{\text{N. of hhlds. w/o Kids updating from above } x \text{ to } (x-7, x] \text{ up to month } m}{\text{N. of hhlds. w/o Kids updating from above } x \text{ up to month } m}.$$

The red lines marked with triangles, squares, and diamond in the same figure plot these

NEA. There is still clear evidence of jumps, but the pre-trends are not parallel.

shares; they correspond to $x = 84, 91$, and 98, respectively.[10] None of these intervals became more attractive after the reform. Reassuringly, these series are smooth around June 2014. I interpret this as evidence that households jumped to the new notch because of the reform, i.e., $share(\Delta t, \Delta I) > 0$.

I estimate the share of pre-reform applicants that jumped because of the reform $share(\Delta t, \Delta I)$ with the following differences-in-differences specification.

$$\hat{share}_{77}(\Delta t, \Delta I) = share^{no\ Kids}_{77,6/16} - share^{no\ Kids}_{77,4/14} - \left(share^{no\ Kids}_{91,6/16} - share^{no\ Kids}_{91,4/14} \right). \tag{2}$$

Under the identifying assumption that $share^{no\ Kids}_{77,m}$ and $share^{no\ Kids}_{91,m}$[11] trends would have remained parallel in the absence of the reform, this calculation measures the treatment effect of the reform on the share of jumpers: $share(\Delta t, \Delta I)$. Although this is not directly testable, the trends in shares that jump to $(70, 77]$ and to $(84, 91]$ are parallel before the reform. Figure 8b illustrates this calculation, indicating that $\hat{share}(\Delta t, \Delta I) = 0.007$ — i.e., 0.7% of the households without children with per capita income above R\$77 update their income to the new threshold because of the reform. This corresponds to a 100% increase with respect to the pre-reform share. Appendix A.5 conducts a formal inference test on the parallel trends assumption and on this estimate. The parallel trends assumption cannot be rejected at a 5% significance level and, even in the 5% random sample, the estimate is significant at a 1% level with a *t-statistic* of 13.35.[12]

The eligibility threshold for households with children increased from 140 to 154 *reais*, and their transfers were adjusted as described in Section 2.4. Since households without children were out of the BF at income level 70 or 77, this change did not affect their incentives to move to the $(140, 154]$ interval. Therefore, these households are a useful control group for the analysis around this second threshold. Consider the following share definitions:

$$share^{Kids}_{154,m} \equiv \frac{\text{N. of hhlds. with Children updating from above 154 to } (140, 154] \text{ up to month } m}{\text{N. of hhlds. with Children updating from above 154 up to month } m},$$
$$share^{No\ Kids}_{154,m} \equiv \frac{\text{N. of hhlds. w/o Children updating from above 154 to } (140, 154] \text{ up to month } m}{\text{N. of hhlds. w/o Children updating from above 154 up to month } m}.$$

[10] Notice that the share jumping to $(77, 84]$ increases at the beginning of 2013. This is likely a result of the change in the minimum wage to 678 in that period, which means that that interval included one eighth of the 2013 minimum wage.

[11] Even though the trends in $share^{no\ Kids}_{77,m}$ and $share^{no\ Kids}_{98,m}$ are also parallel, I chose $share^{no\ Kids}_{91,m}$ as the counterfactual group, as this share's levels are closer to the levels of $share^{no\ Kids}_{77,m}$ before the reform.

[12] All inference is based on robust standard errors. These standard errors shrink when I cluster at the household or household-composition level.

Figure 9's blue line with circles plots $share^{Kids}_{154,m}$, and its red line with triangles plots $share^{No\ Kids}_{154,m}$. Once again, there is a sharp increase in the share of jumpers to the interval that became

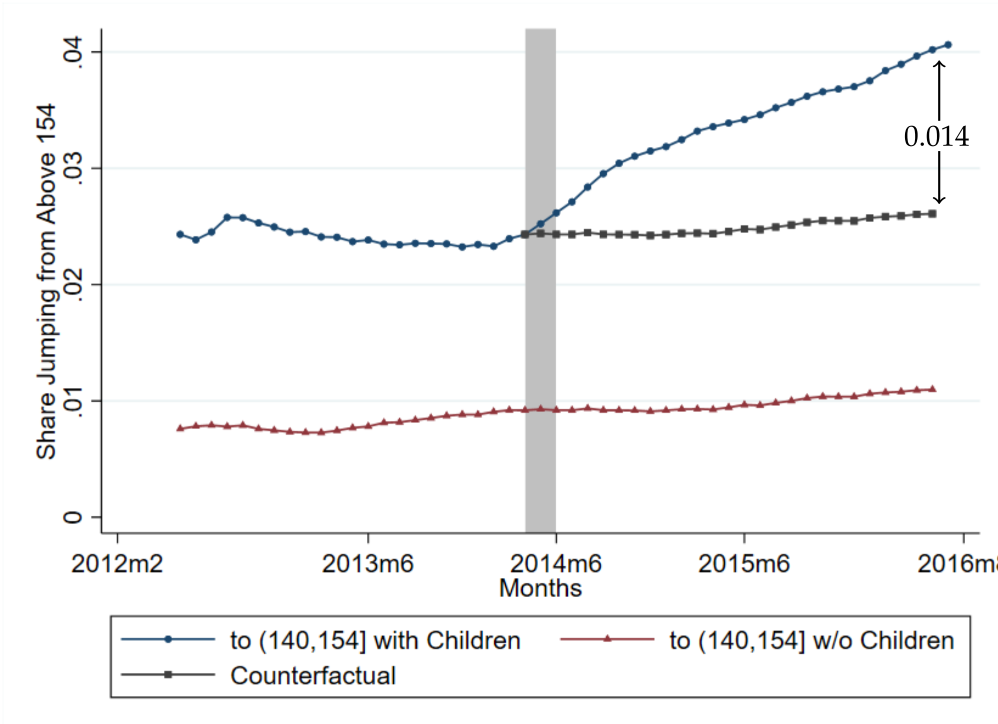


Figure 9: Share of Households Jumping from Above 154

Note: The blue line with circles and the red lines with triangles plot the cumulative shares of households with and without children above R$154 that moved to the $(140, 154]$ interval up to each month in time, respectively. These shares are computed out of all households with and without children that updated. The gray line marked with squares draws the counterfactual distribution for the share of households with children, under the assumption that its trend would remain parallel to the trends in the shares without children after the reform. The gray vertical bar indicates the months between the announcement and the enactment of the reform.

attractive after the reform $(140, 154]$. Furthermore, the same share among households without children does not present the same sharp increase. This figure is evidence of jumping behavior among households with children. I calculate $share^{154}(\Delta t, \Delta I)$ using a similar specification as before:

$$\hat{share}_{154}(\Delta t, \Delta I) = share^{Kids}_{154,6/16} - share^{no\ Kids}_{154,4/14} - \left(share^{no\ Kids}_{154,6/16} - share^{no\ Kids}_{154,4/14}\right). \quad (3)$$

As indicated in Figure 9, $\hat{share}_{154}(\Delta t, \Delta I) = 0.014$, which means that 1.4% of the households with children above 154 jumped to the $(140, 154]$ interval because of the reform (58% increase with respect to the pre-reform share).[13] The differences-in-differences regression

[13] An alternative analysis using households jumping to neighboring intervals is presented in Appendix A.6. These intervals were affected by changes in the minimum wage that pollute the analysis, and for this reason I chose households without children as the control group. The effects with this alternative control groups are, if anything, larger than the ones in reported in this section.

analysis presented in Appendix A.5 finds the same effect, which is significant at a 1% level with an associated *t-statistic* of 15.71.

Appendix A.6 presents (1) graphs with the number of jumpers in each month instead of cumulative shares; (2) graphs that focus only on households that did not change their composition; and (3) some alternative placebo tests. These evidence indicates that jumping effects are robust to different specifications and control groups.

Two aspects of the Brazilian economy might raise concerns with the interpretation of this result. First, Brazil was going through a crisis around that period. Therefore, the increase in the share of jumpers could be arise from households that were previously hit by the crisis, but decided to update only after the reform. Second, the Brazilian economy presented an average inflation rate of 6% during this period. For this reason, a jump in 2016 represents a smaller jump in real in terms than the exact same jump one year earlier. Note, however, that under both interpretations households are still reacting to non-local (with respect to their initial reported income level) changes in the schedule. This is in contrast to the usual responses captured by the intensive margin elasticities and is still evidence of jumping behavior. In section 5, I come back to these interpretations to discuss how they could affect the welfare analysis of the Bolsa Famíia reform.

Figure 10 displays the share of jumpers across Brazilian municipalities. These shares are defined as total number of jumpers divided by the number of households in non-convex areas that are susceptible to jumps after any reform between January 2012 and September 2016.[14]

The distribution of shares of jumpers in the figure above is similar to the distribution of density of applicants in Figure 4. The unconditional correlation between the density of applicants and share of jumpers is 0.467 . This pattern indicates that most jumpers are located in municipalities in which a larger share of the population applies to the program. Such geographical distribution of jumpers suggests that knowledge about the program (which should be higher in municipalities with more applicants) is an important determinant of the jumping responses.

[14] This definition includes also households that jumped because of reforms that modified the first bracket. Even though I do not use them in the main analysis, if we focused only in the jumps to the threshold of 2014, the share would be zero for a large part of the municipalities in the 5% sample because of the small number of jumpers.

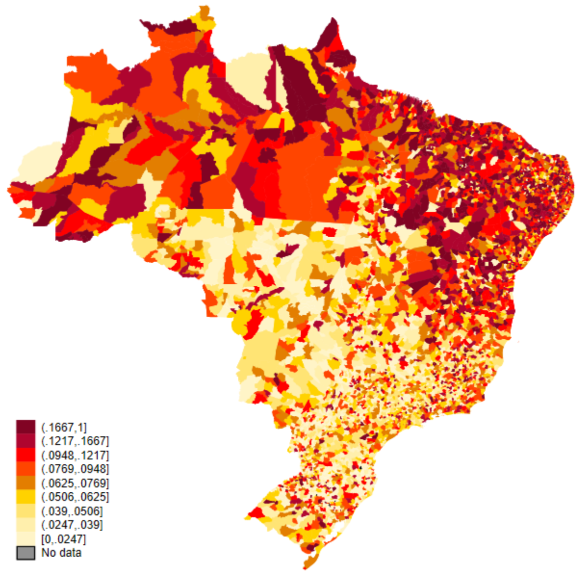


Figure 10: Share of Jumpers per Municipality

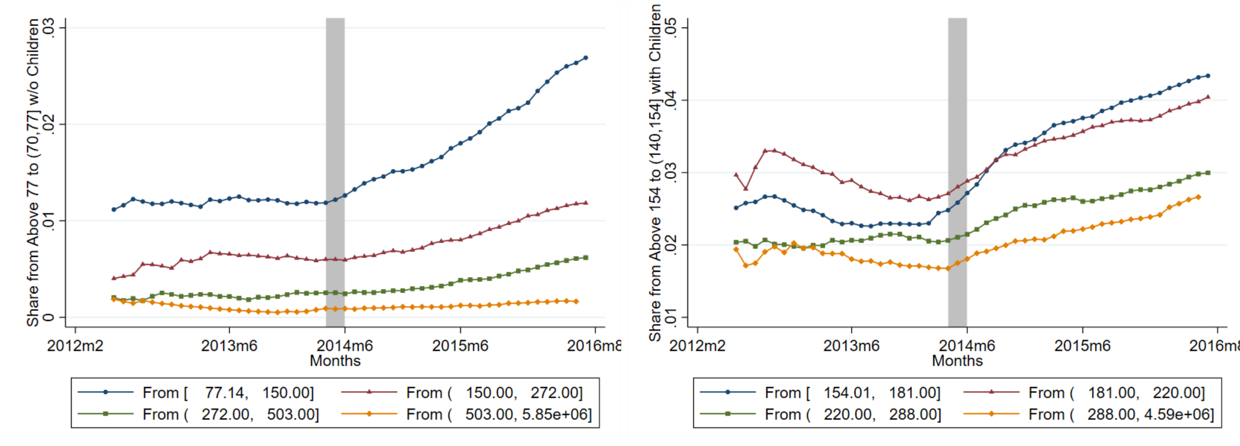
This figure plots share of jumpers in each municipality. This share is defined as the number of households that ever responded to inframarginal changes in the schedule divided by the number of potential jumpers, i.e. households in non-convex areas of the schedule susceptible to jumps. We divide the observations into deciles within the sample. Each decile is assigned a different color on the map, with darker shades representing higher densities.

3.4 Heterogeneity: Jumping from Different Income Levels

Figure 11a presents evidence that the jumps come from different parts of the income distribution of noneligible applicants above the first threshold among households without children. The blue solid line plots the share of households that jumped from the first quartile above 77 *reais* to the $(70, 77]$ interval up to each month in time. The red long-dashed, green dashed, and yellow short-dashed lines plot the share of households jumping from the second, third, and fourth quartile, respectively.

Even though the share of households jumping from the first quartile has a more definitive increase after the reform, shares from the second and third quartiles were also affected.

Figure 11b plots the same shares among households with children that were initially above 154 *reais*. Again, the effect of the reform is larger among households in the first quartile above the threshold. Among this second group of households, the jumps persist even in the fourth quartile. These graphs show that jumpers come from different income levels above the eligibility threshold. Note that the per capita income distribution is more concentrated for households with children above R\$154 (the 75th percentile is R\$288) than for households without children above R\$77 (the 75th percentile is R\$505). This could explain the larger top quartile effect among households with children jumping to the second



(a) Households without Children above 77 (b) Households with Children above 154

Figure 11: Share of Households Jumping from each Quartile above the New Threshold

Note: Panel (a) depicts the cumulative shares of households without children in each quartile above R\$77 that moved to the $(70, 77]$ interval up to each month in time, out of all households initially above R\$x and that update. The blue line with circles, red line with triangles, green line with squares, and yellow line with diamonds plots the shares from the first, second, third and fourth quartile, respectively. Panel (b) displays the same analysis for the shares of households with children initially above R\$154 and that jumped to the $(140, 154]$ reais interval. The gray vertical bars indicate the months between the announcement and the enactment of the reform.

threshold.

A potential concern with this interpretation is that households jumping from the fourth quartile could have moved to lower quartiles before the jump. This mean-reversion process of income is usual in tax records (see, for instance, Gruber and Saez (2002)). One cannot observe all pre-jump income adjustments, because of the unbalanced nature of the panel. For example, a household with income in the fourth quartile of the income distribution above the threshold in 2012 might have changed its per capita income to the first quartile in 2013 without updating this change in the program. I investigate this hypothesis by analyzing income movements across quartiles before the reform in Appendix A.7. A small share of households moves from the fourth to lower quartiles among households with children with income above 154 *reais*. This suggests that mean reversion does not explain jumps from larger income levels.

4 Welfare Framework

In this section, I present a model that takes into account three aspects seen in the data: (1) households jumping to the threshold as a response to some small infra-marginal reform; (2) jumps coming from across the income distribution above the new eligibility threshold;

and (3) the presence of households in dominated areas of the schedule. To accommodate (1) and (2), the model allows households to differ in ability and elasticity types, as described in Section 4.1. This allows me to define the share of jumpers from each income level in Section 4.2. Section 4.3 incorporates inattention in the model to address aspect (3). Section 4.4 introduces the welfare function and derives the welfare effect of a small reform, which is the basis for my empirical analysis.

4.1 Preferences

Households choose consumption c and income y to maximize their utilities. They differ in their income productivity n. Applicants produce income by reducing leisure, and those with higher ability can do this at a more favorable rate. These households are also heterogeneous in their elasticity type m, which determines the convexity of the indifference curves in the the (y, c) plane. While n orders preferences according to the first derivative of the indifference curves in the (y, c) plane, m orders these preferences according to the second derivative. Intuitively, higher elasticity types are more responsive to small changes in the schedule. I discuss this point more formally in what follows.

Let $(n, m) \sim F(\cdot, \cdot)$ and $B(\cdot)$ be the transfer schedule as a function of income. The household problem is:

$$\begin{aligned} &\max_{c,y} u(c, y; n, m) \; s.t. \\ &c \leq y + B(y). \end{aligned} \tag{4}$$

There are three assumptions on preferences in this economy.

Assumption 1. *For any elasticity type m, consumption c, and income level y, the marginal rate of substitution between consumption and income is decreasing with the ability type, i.e.,* $-\frac{\partial}{\partial n} \frac{u_y(c,y;n,m)}{u_c(c,y;n,m)} < 0$.

Assumption 1 ensures that the single-crossing condition holds for any realization of m so that $y(n, m)$ is monotone in n for any transfer schedule $B(\cdot)$.

Assumption 2. *For any type (n, m), the marginal rate of substitution between consumption and income is increasing with income (or decreasing with leisure).*

The following lemma shows that $y(n, m; c, MRS)$ is monotonic in ability n for any elasticity m, consumptio level c and marginal rate of substitution MRS.

Lemma 1. *Under Assumptions 1 and 2, $y(n, m; c, MRS)$ is increasing in n.*

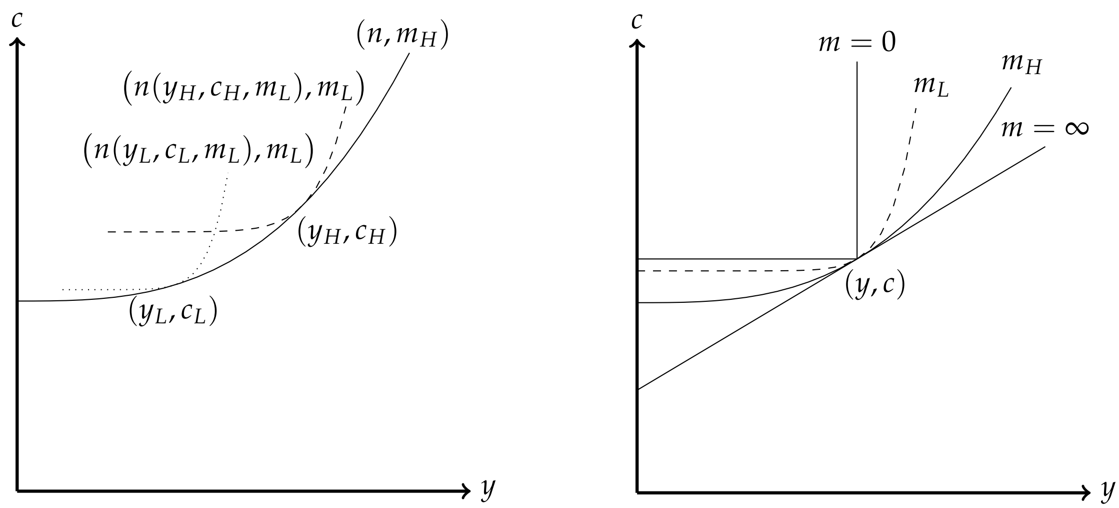
Proof. See Appendix A.8 □

Lemma 1 implies that $y(n, m; c, MRS)$ is invertible with respect to the first argument. Let $n(y, c; m, MRS)$ be the ability type at allocation $(y, c) \in \mathbb{R}^2_+$, with elasticity type m and marginal rate of substitution $MRS \geq 0$.

Definition 1. *The convexity of the indifference curve of agents at an allocation (y, c) with marginal rate of substitution MRS with elasticity types m is:*[15]

$$Convex(m; c, y, MRS) \equiv -\frac{u_{yy}\Big(c, y; n(y, c; m, MRS), m\Big)u_c(\cdot) - 2u_{cy}(\cdot)u_c(\cdot) + \frac{u_{cc}(\cdot)}{u_c(\cdot)}u_y(\cdot)^2}{u_c(\cdot)^2}.$$

This function describes how the convexity of the indifference curves varies with the elasticity type, for a given allocation (y, c) and slope of their indifference curves MRS. Note that as one varies m, the ability type $n(c, y, m, MRS)$ is adjusted so that the new type would still present marginal rates of substitution MRS at the allocation (y, c).



(a) Convexity Degrees at Different Allocations (b) Preferences Satisfying Assumption 3

Figure 12: Preferences under Different Convexity Degrees m

Note: Panel (a) illustrate preferences of three different types. Type (n, m_H) is indifferent between allocations (y_L, c_L) and (y_H, c_H)). Type $(n(y_L, c_L, m_L), m_L)$ has a lower elasticity $m_L < m_H$ and the same MRS of type (n, m_H) at allocation (y_L, c_L). Type $(n(y_H, c_H, m_L), m_L)$ has also a lower elasticity $m_L < m_H$ and the same MRS of type (n, m_H) at allocation (y_H, c_H). None of the ability types need to coincide. Panel (b) presents four different types with the same MRS at the allocation (c, l) with elasticity types varying from 0 (Leontief preferences) to ∞ (linear preferences).

The next assumption orders preferences with respect to m according to the convexity of their indifference curves.

[15] I suppress the arguments of all but the first marginal utility terms, as they are the same.

Assumption 3. *For any allocation $(y, c) \in \mathbb{R}^2_+$, $Convex(m; c, y, MRS)$ is decreasing and continuous in m and:*

$$\underset{m \to 0}{lim}\, Convex(m; c, y, MRS) = \infty \textit{ while } \underset{m \to \infty}{lim}\, Convex(m; c, y, MRS) = 0.$$

Figure 12a shows how this convexity changes as the elasticity type increases from m_L to m_H, for two different allocations ((c_L, y_L) and (c_H, y_H)) and marginal rates of substitution. Assumption 3 is satisfied by preferences represented by the isoelastic utility function, with m indexing the income elasticity as shown in Appendix A.9.

Figure 12b illustrates preferences that satisfy this assumption at a particular allocation and with a particular marginal rate of substitution. The next section shows that in such an economy, the share of households jumping across brackets is well defined in contrast to the unidimensional Mirrleesian framework.

4.2 Share Concepts

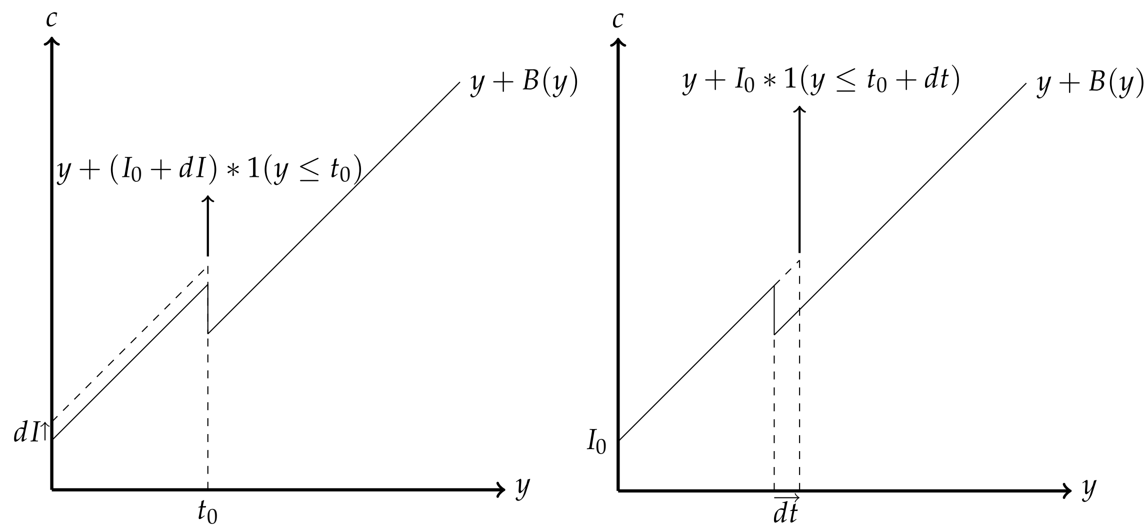
This section defines the "share of jumpers" concepts that are the sufficient statistic for the welfare analysis of the BF reform. I focus on a special case of problem (4), in which $B(y) = I_0 * 1(y \leq t_0)$. Here t_0 and I_0 correspond to the initial threshold of eligibility and transfer to eligible households, respectively. Let t and I define perturbations in these two aspects of the schedule. Figure 13 illustrates these perturbation concepts, which are analogous to the ones observed in the empirical setting.

The income chosen by type (n, m) before $y_b(n, m)$ and after $y_a(n, m)$ the perturbation are defined as:[16]

$$y_b = \arg\max_y u\Big(y + I_0 * 1(y \leq t_0), y; n, m\Big) \text{ and}$$
$$y_a = \arg\max_y u\Big(y + (I_0 + I) * 1(y \leq t_0 + t), y; n, m\Big).$$

For any income level above the dominated interval of the transfer schedule $y \geq I_0 + t_0$, the share of jumpers as a response to a 1% change in the after-transfer income $share^I(y)$

[16] I suppress the type (n, m) from now on to simplify notation.



(a) Transfer Perturbation dI (b) Threshold Perturbation dt

Figure 13: Pertubation Definitions

Note: Panel (a) illustrates a perturbation in the transfer given to the eligible poor, while panel (b) depicts a perturbation in the threshold of eligibility.

and in threshold $share^t(y)$ are:

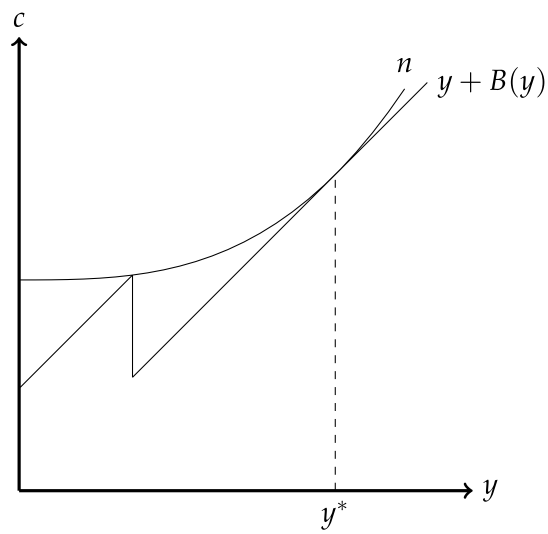
$$share^I(y) = \frac{\partial P(y_a = t_0 | y_b = y)}{\partial I}(t_0 + I_0) \ for\ y > t_0,\ and \qquad (5)$$
$$share^t(y) = \frac{\partial P(y_a = t_0 | y_b = y)}{\partial t} t_0.$$

The first parameter $share^I(y)$ measures the increase in the probability of jumping to the threshold, given a 1% increase in the after-transfer income at the threshold, among households located at y before the perturbation. This is the additional sufficient statistic in the optimal tax formula with jumps, as I show in Appendices A.12 and A.13.[17] The second measures the increase in the probability of jumping to the threshold, given a 1% increase in the threshold t among the same group of households. Average shares across income levels above t_0 are the sufficient statistics for the counterfactuals computed in Appendix A.15.

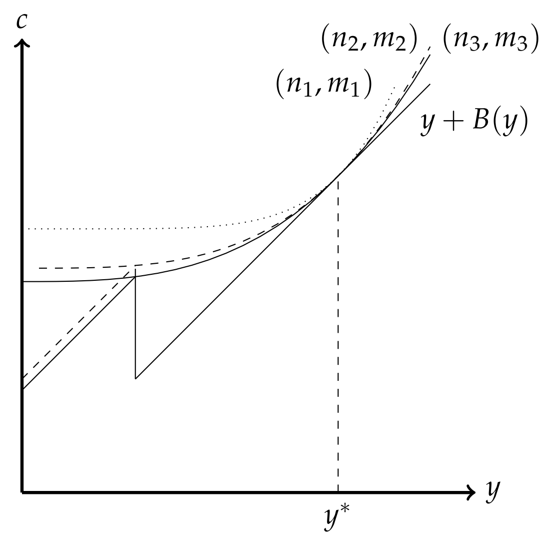
To see that these derivatives are only well defined in a model with multidimensional heterogeneity, consider Figure 14. Panel A displays the only type n that would choose income y^* in an economy with heterogeneity only in ability. Under the single-crossing condition,

[17]In the optimal tax formula, the planner needs this share with respect to a perturbation at any point in the non-convex areas of the schedule and not only at the eligibility threshold.

this is the only type located at y^*. The derivative $\frac{\partial P(y_a=t_0|y_b=y)}{\partial I}$ is not well defined, as this probability jumps from zero to one for any positive perturbation $I > 0$. Panel B illustrates three different types that will choose income y^* in an economy with preference heterogeneity along elasticity types m. In this case, a small perturbation in the schedule $dI > 0$ would make a set of types originally in y^* jump (in the figure, this set would correspond to all the types $(n(y^*, m)m)$ for m between m_2 and m_3). Some other types (consider (n_1, m_1), for instance), however, would not jump due to this perturbation. In this case, the derivative of the probability is well defined.



(a) One-dimension Heterogeneity



(b) Two-dimension Heterogeneity

Figure 14: Types Initially at y^*

Note: Panel (a) shows the preference of a household indifferent between income level y^* and the threshold of eligibility in an economy in which households only differ in ability types. Panel (b) depicts preferences of different types choosing y^* before the reform in an economy with two dimensions of heterogeneity.

Finally, let the share of jumpers at income level z as a response to a marginal reform in the threshold and transfer be defined as

$$share^J(dt, dI; z) = \frac{\partial P(y_a = t_0 | y_b = z)}{\partial I} dI + \frac{\partial P(y_a = t_0 | y_b = z)}{\partial t} dt.$$

I show below that the sufficient statistic for the welfare analysis of a (dt, dI) reform is the average share of jumpers in response to the reform across income levels above t_0. Even though this average share would be well defined in a unidimensional economy, such a model would imply that all jumpers would come from a particular interval of the income distribution, which contradicts the evidence presented in Section 3.4.

4.3 Incorporating Inattention

There is a considerable fraction of households in the data that locate themselves in dominated areas of the schedule. This could be explained, for instance, by mis-optimization or frictions, as discussed by Kleven and Waseem (2013). It is important to take this into account in the welfare analysis, because a small increase in the threshold could impact the utility of households located in the dominated area right above the initial threshold. In this section, I address this issue by allowing households to differ in a third dimension of heterogeneity: attention r.

Attention $r \in \mathbb{R}_+$ is the degree to which households perceive the schedule when choosing their labor supply. The household problem for attention type r is:

$$y(n,m,r) \equiv arg\max_{y} u(y + rB(y), y; n, m). \tag{6}$$

Attentive households $r = 1$ perceive the schedule as it is and maximize their utility. Inattentive households $r = 0$, on the other hand, choose income to maximize utility and ignore the presence of the anti-poverty program. Their labor supply function $y(n,m,0) = arg\max_{y} u(y,y;n,m)$ is independent of the schedule. Households may also under-perceive $r < 1$ or over-perceive $r > 1$ the schedule. Households with low attention $r \approx 0$ could be located right above the eligibility limit t. The demand for consumption can be computed using the actual household budget $c(n,m,r) = y(n,m,r) + B\big(y(n,m,r)\big)$.

To incorporate inattention, $y_b(n,m,r)$ and $y_a(n,m,r)$ are redefined as:

$$y_b = \arg\max_{y} u\Big(y + rI_0 * 1(y \leq t_0), y; n, m\Big) \text{ and}$$
$$y_a = \arg\max_{y} u\Big(y + r(I_0 + I) * 1(y \leq t_0 + t), y; n, m\Big).$$

All of the share concepts remain the same. Here, Assumption 1 ensures that for any elasticity and attention type (m,r) and transfer schedule $B(\cdot)$, $y(n,m,r)$ is increasing in ability n. Hence $n(y,m,r)$ can be defined as the inverse of $y(n,m,r)$ with respect to the first argument, i.e., the ability type that would locate in income level y with elasticity m and attention r under the schedule $B(\cdot)$

Finally, let $(n,m,r) \sim F_{NMR}(\cdot,\cdot,\cdot)$. The following assumption ensures that preferences and attention to the schedule have a joint smooth distribution, so that the welfare function is differentiable.

Assumption 4. *The joint distribution of types (n,m,r) is smooth and has full support in $\mathbb{R}^3_+$.*

Assumptions 1 to 4, together with the attention r definition, characterize preferences in this economy.

4.4 Welfare

Let $H(\cdot)$ be the income distribution of applicants under the observed schedule. Welfare under the schedule $(t_0 + t, I_0 + I)$ is a function of the perturbations t and I:

$$W(t_0+t, I_0+I) = \int_0^\infty \int\int G\Big(u\big(z+B(z), z; n(z,m,r), m\big)\Big) dF_{MR|Y}(m,r|z)dH(z) \\ -\lambda \int_0^{t_0+t} (I_0+I)dH(z), \quad (7)$$

where $B(z) = (I_0 + I) * 1(z \leq t_0 + t)$ are the transfers given to households with income z, $G(\cdot)$ is an increasing and concave function that captures the redistributive motives of the planner, $F_{MR|Y}(\cdot)$ is the joint distribution of elasticity and attention types conditional on income, and λ is the marginal cost of public funds. It represents how much the government values R$1 of revenue relative to a R$1 given to the average applicant in the margin. If the government cares a lot about the poor, λ converges to zero. If the government is indifferent between giving a *real* to the poor and spending it elsewhere, λ approaches 1.

Let $g(z) \equiv \frac{1}{\lambda}\int\int G'(u)\left(u_c + (u_c+u_y)\frac{\partial y}{\partial I}\right) dF(m,r|z)$ be the average social marginal value of consumption for taxpayers with income z expressed in terms of the marginal value of public funds. Note that compared to the case in which households are fully informed about the schedule (see Saez (2001), for instance), the $(u_c+u_y)\frac{\partial y}{\partial I}$ term must be included as the envelope theorem no longer applies. This term corresponds to the behavioral wedge in Farhi and Gabaix (2015). For attentive households $r = 1$, the first-order condition of the household problem $u_c(y + B(y), y) = -u_y(y + B(y), y)$[18] ensures that there is no behavioral wedge. This corresponds to the standard neoclassical case.

Let $\bar{g} = \frac{\int_0^{t_0} g(z)dH(z)}{H(t_0)}$ be the average social marginal value of consumption among the eligible poor; $\bar{g}(t_0) = \int_{t_0}^{t_0+I_0}\int_0^\infty \frac{G'(u(c,t_0,n(t_0,m,0),m,0)u_c(\cdot)}{\lambda I_0} dF_{M|RY}(m|0,t_0)dc$ be the average social marginal value of consumption for households at (t_0, t_0+dt) between consumption levels t_0 and $t_0 + I_0$; and $share^J(dt,dI) = \frac{\int_{t_0}^\infty share^J(dt,dI;z)dH(z)}{1-H(t_0)}$ be the average share of jumpers across all income levels beyond the threshold. The following proposition characterizes

[18]Note that for $r \neq 1$, this first order condition becomes $u_c(y + rB(y), y) = -u_y(y + rB(y), y)$ so that $u_c(y + B(y), y) \neq -u_y(y + B(y), y)$.

the welfare effect for a reform that perturbs the threshold and the transfer by infinitesimal amounts (dt, dI).

Proposition 1. *Under assumptions 1, 2, 3, and 4, an infinitesimal reform that changes the transfer given to the poor by dI and the eligibility threshold by dt impacts welfare by:*

$$dW = \lambda\Big((\bar{g}-1)H(t_0)dI + \big(\bar{g}(t_0)-1\big)I_0 h(t_0)dt - I_0\big(1-H(t_0)\big)share^J(dt,dI)\Big).$$

Heuristic proof. Consider the effect of the transfer perturbation from the initial schedule $B(\cdot)$ depicted in Figure 15. Transfers to households with income between 0 and t_0 are increased by dI and the threshold of eligibility by dt. This reform has three effects on welfare: (1) a transfer effect through the increased benefits dI given to households with income between 0 and t_0; (2) a threshold effect through the mechanical inclusion of households with income between t_0 and $t_0 + dt$ in the transfer program; and (3) a jumping effect on the government's budget coming from households with income above t_0.

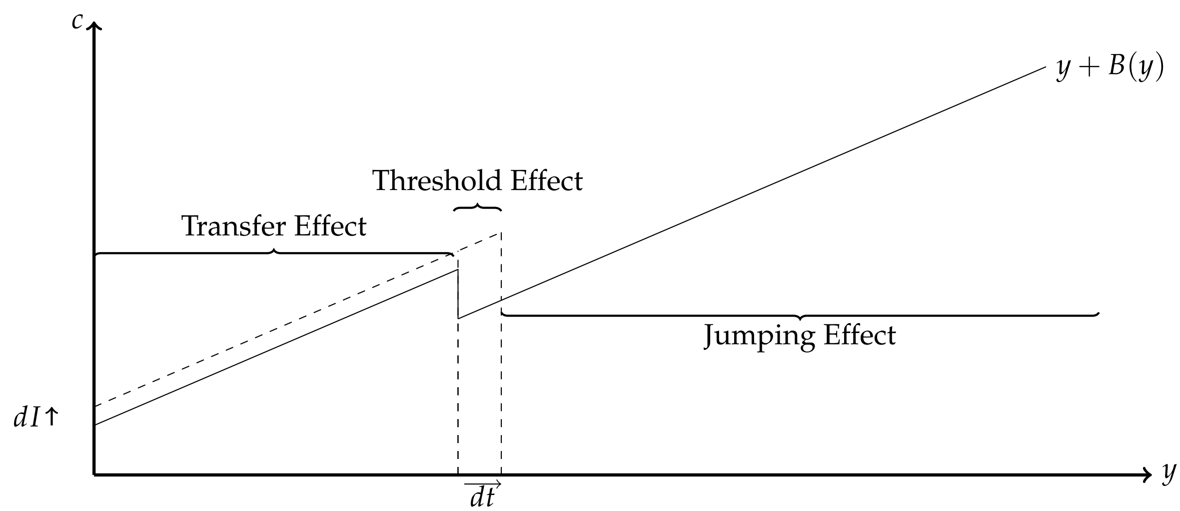


Figure 15: Perturbation from the Observed Schedule

Note: The figure illustrates perturbations in the transfer given to the eligible poor and in the threshold of eligibility as in the BF reform. The picture indicates the effects affecting each interval of the income distribution.

Transfer Effect: Every household with income below the eligibility threshold t_0 will get dI extra reais, which is valued on average by $\lambda\bar{g}dI$ by the government. On the other hand, the costs of such transfers is λdI. Since there are $H(t_0)$ such households, the transfer effect is equal to:

$$\text{Transfer Effect} = \lambda(\bar{g}-1)H(t_0)dI.$$

Threshold Effect: Every household with income between t_0 and $t_0 + dt$ will get I_0 extra reais. The net of costs value of such transfers by the government is $\lambda(\bar{g}(t_0) - 1)I_0$. Since there are $h(t_0)dt$[19] such households, the threshold effect is equal to:[20]

$$\text{Threshold Effect} = \lambda(\bar{g}(t_0) - 1)I_0 h(t_0)dt.$$

Jumping Effect: A fraction $share^J(dt, dI)$ of households initially above t_0 will jump to the threshold. This fraction is small for a small reform, and since these households are initially indifferent, the effect on their utility is also small. Therefore, by the envelope theorem, the effect of these jumps on welfare through jumpers' utility is second order. However, the government loses I_0 *reais* with each jump valued at λ. The fiscal externality is the same, no matter where households are jumping from. For this reason, the average share across income levels $share^J(dt, dI)$, rather than the shares at each income level $share^J(dt, dI; z)$, is sufficient to describe the jumping effect. Since there are $1 - H(t_0)$ potential jumpers, the total jumping effect is:

$$\text{Jumping Effect} = -\lambda I_0 (1 - H(t_0)) share^J(dt, dI).$$

The formula in Proposition 1 follows from the sum of these three effects. □

Appendix A.10 contains the formal proof while Appendix A.11 presents an extension of the model with misreporting. In this case, the share of jumpers along the reported income distribution is the relevant parameter. This share is precisely the one recovered from the data in what follows.

5 Estimating the Share of Jumpers Because of the Reform

The share of potential jumpers that jumped because of the reform $share^J(dt, dI)$ is the sufficient statistic for the welfare analysis. For discrete reforms, this share can be approximated by $share^J(\Delta t, \Delta I)$, i.e., the share of potential jumpers that jumped because of the discrete reform. This set of potential jumpers includes not only households registered in

[19]The precise number is $h(t_0 + dt)dt$.

[20] Technically, there should also be a term accounting for the utility gain of households initially bunching at t_0 and that move to $t_0 + dt$ with the reform. Even though this response would yield an infinitesimal increase on the household utility, there is a large number of such households. Hence this would be a first order effect. However, only 3% of the households originally bunching at the old threshold move to the new threshold. Therefore in practice this effect should be small, and I refrain from including it in the analysis.

the program with per capita income above the eligibility threshold, but also households that entered the program after the reform. This last group was left out of the empirical analysis in Section 3.3, because their pre-reform per capita income is not empirically observable. Hence, the parameter of interest $share^J(\Delta t, \Delta I)$ differs from $share(\Delta t, \Delta I)$, i.e., the share of applicants that updated their income from above the eligibility threshold to the new notch because of the reform. The goal of this section is to recover $share^J$[21] from empirical estimates.

Section 5.1 describes a simple empirical framework that provides bounds and estimates of this share from the treatment effect of the policy on the share of noneligible pre-reform applicants that jumped (estimated in Section 3.3) and on the share of entrants at the new notch (which is also recoverable). A potential concern with the estimates of jumpers is that it includes households that would have jumped to the old threshold in the absence of the reform. In this case, the jumpers would have a much smaller budgetary effect. Section 5.2 presents evidence that rules out this concern. I then present estimates of the effect of the reform on entrants in Section 5.3 and the bounds and estimates for the parameter of interest in Section 5.4.

5.1 Empirical Framework

Per capita income is only observable from the moment at which households apply to BF onward. Let the number of households that entered the program below the new threshold $(t_0, t_0 + \Delta t]$ because of the reform be denoted by E_t. Even though this number is recoverable from the data, one cannot identify which of these households jumped from a higher income level and which were in that interval all along. I denote the first group as jumping entrants JE and the second as non-moving entrants NME so that $E_t = JE + NME$.

Remember from equation (1) that the share of pre-reform ineligible applicants that jumped, denoted by $share$, is the ratio between jumping applicants (JA) and ineligible applicants (NEA) estimated in Section 3.3. To define the share of jumpers $share^J$, note that the total set of jumpers J is given by jumping applicants JA and jumping entrants JE. Potential jumpers PJ include the ineligible pre-reform applicants above $t_0 + \Delta t$ (NEA) and entrants that were not in the $(t_0, t_0 + \Delta t]$ interval before they entered the program ($E - NME$).

[21] Throughout this section, I suppress the empirical share argument $(\Delta t, \Delta I)$ to save notation. I come back to the approximation of $share(dt, dI)$ with $share(\Delta t, \Delta I)$ in Section 6.

Therefore, the share of households jumping to the notch because of the reform $(\Delta I, \Delta t)$ is

$$share^J = \frac{J}{PJ} = \frac{JA + JE}{NEA + E - NME}. \tag{8}$$

This share differs from the share of households that jumped to the new threshold from above the eligibility threshold because of entrants into the program.

There are two natural bounds for the number of non-moving entrants NME: It cannot be less than zero or greater than the number of entrants below the new threshold E_t. Mechanically, this provides bounds on JE too. Substituting $NME = 0$ in the above equation provides an upper bound to the share of jumpers since in this case, all entrants at the new threshold would also be jumpers. Symmetrically, substituting $NME = E_t$ would provide a lower bound.

To compute a point estimate for *share*^{J} from the data, I make the following assumption:

Assumption 5. *The share of entrants below the new threshold that are jumpers is the same as the share of pre-reform applicants below the new threshold after the reform that are jumpers, i.e.,*

$$\frac{JE}{JE + NME} = \frac{JA}{JA + NEA_t},$$

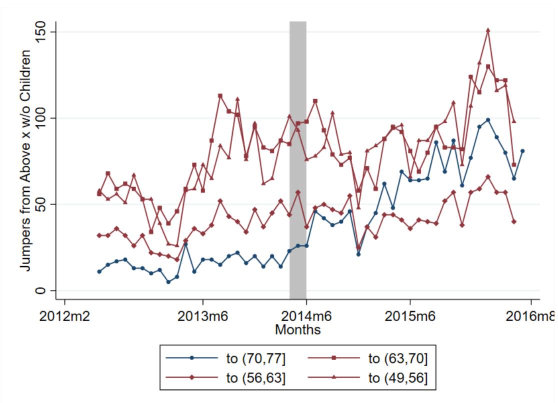
where NEA_t is the number of noneligible pre-reform applicants in $(t_0, t_0 + \Delta t]$ before the reform.

Intuitively, this assumption states that distribution of pre-reform incomes above and below the new threshold conditional on being between the new and the old threshold after the reform is the same for pre-reform applicants and entrants. Note that under this assumption, $NME = \frac{NEA_t E_t}{JA + NEA_t}$.

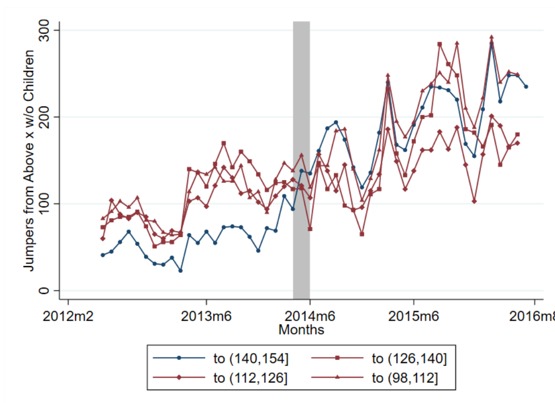
The next sections present estimates of JA and E_t. Since NEA and E are observable in the data, those estimates provide bounds for *share*^{J}. Under Assumption 5, they also provide a point estimate for the parameter of interest.

5.2 Pre-reform Jumping Applicants

In principle, the number of pre-reform ineligible applicants that jumped because of the reform JA is the product between the share of households recovered in Section 3.3 *share* and the total number of households that ever updated from above the new threshold up to June 2016. However, households jumping to the new threshold could have jumped to below the old threshold in the absence of the reform. The budgetary effect would be smaller in this case than if the households would not have jumped in the absence of the reform. It is hard to rule out this possibility with the analysis in Section 3.3 because as the share of households jumping to below the new threshold increases, the share updating elsewhere mechanically decreases. Figure 16 investigates this hypothesis directly. The line marked with circles plots the number of households jumping to the income region that became attractive after the reform in each month. Around the months of the reform, the number of jumpers starts increasing. The lines marked with squares, diamonds, and triangles plot the number of households moving to intervals below the old threshold and are roughly stable during the period. Panels a and b present the analysis for households without and with children respectively.



(a) Households without Children



(b) Households with Children

Figure 16: Number of Households Jumping to Each Interval

Note: Panel (a) depicts the number of households without children updating their per capita income from above x to the $(x-7, x]$ interval up to each month in time. The blue line with circles and the red lines with squares, diamonds, and triangles plot these numbers for $x = 77, 70, 63$, and 56, respectively. Panel (b) replicates the same exercise for households with children updating from above x to $(x-14, x]$ for $x = 154, 140, 126$ and 112. The gray vertical bar indicates the months between the announcement and the enactment of the reform.

The fact that the number of jumpers to the intervals below the old threshold does not decrease indicates that the jumpers are not the households that would have jumped to right below the old threshold anyway. For this reason, I interpret these jumpers as households that jumped because of the reform and would remain ineligible to BF in its absence.

In section 3.3, I discussed two concerns with the interpretations of the jumps of pre-reform applicants. The first was the possibility that jumpers were actually households that faced negative income shocks before the reform and only decided to update their income after the reform. Note that negative shock brought these households close to the new interval (say from 150 to 100 reais per month), but the reform created the extra incentives to make them jump to 77, the behavior would still enter the analysis just like a jumping effect.

However, if the negative shock brought them to the relevant interval (say to R$75) and the reform only made them report their actual income because it made them mechanically eligible, the effect of the reform would be similar to the effect of the reform on the inattentive households at the notch before the reform. I cannot rule out this hypothesis with the data. Under this interpretation, my welfare analysis will provide an upper bound for the efficiency costs of the reform. However, this interpretation would require very high utility costs of under reporting (otherwise households at R$75 would have misreported to R$70 before the reform and collected the transfers) and a very low utility cost of over-reporting (otherwise this household would have updated its income to R$75 before the reform). For this reason, I will interpret pre-reform applicants jumps as behavioral responses that could come from either labor supply or misreporting responses because of the reform from now on.

The second concern was the Brazilian inflation throughout the period. The inflation implies that the size of jumps in real terms varies with the timing of the jump. However, as shown in Section 6, the size of the jumps do not affect welfare to a first order. Therefore, this should not be a concern for the purposes of the welfare analysis of the BF reform.

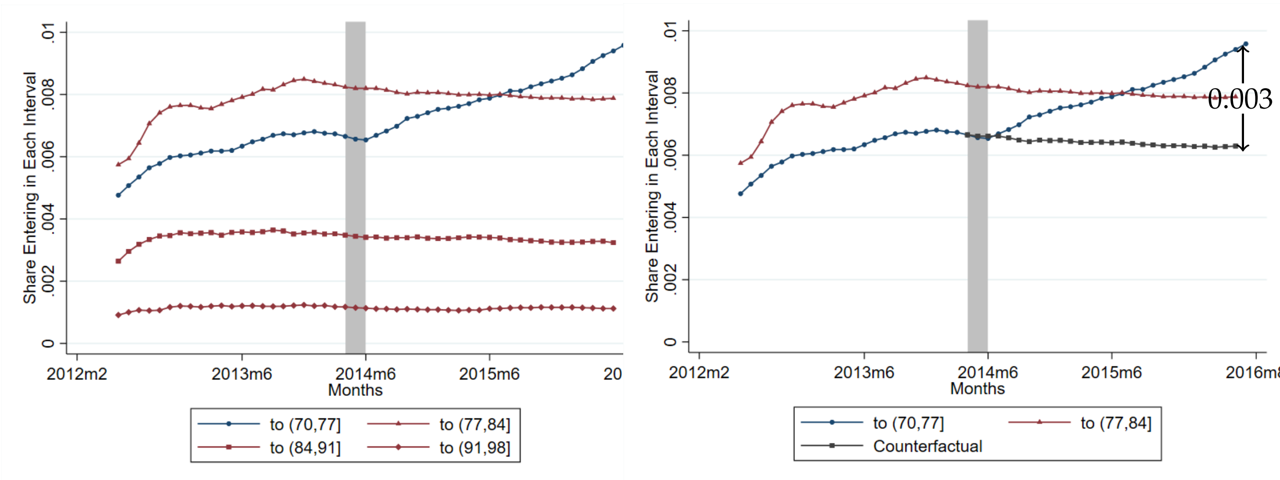
5.3 The Effect of the Reform on Entrants

To recover the number of households that entered the program at the new threshold because of the reform E_t, remember that $(70, 77]$ is the part of the schedule that became attractive after the reform for households without children. I start by presenting evidence in terms of shares of entrants in the relevant interval because of the reform. The number of entrants at the new threshold can be backed out from these shares as I describe in Section 5.4.

Let $share_{77,m,E}^{no\ Kids}$ be the share of households entering the program in the $(70, 77]$ interval since January 2012 (after the first extraction) up to month m among those that entered the program in the same period:

$$share_{77,m}^{no\ Kids,E} = \frac{\text{N. of hhlds. w/o Children entering at } (70,77] \text{ from Jan. 2012 to m}}{\text{N. of hhlds. w/o Children entering from Jan. 2012 to m}}.$$

Figure 17's blue solid line plots $share_{77,m}^{no\ Kids,E}$ for every month from May 2012 to September 2016. Once again, the gray shaded area indicates the months between the announcement and the enactment of the reform. Even though this series trend is already increasing before the reform, there is sharp acceleration after June 2014, which suggests that the reform affected the share of entrants below the new notch.



(a) Evidence of the Effect on Entrants (b) Estimating $share^E$

Figure 17: Share of Households Entering with Income in $(70, 77]$

Note: Panel (a) depicts the cumulative shares of households without children entering at $(x-7, x]$ interval up to each month in time, out of all households entering BF since January 2012. The blue line with circles and the red lines with triangles, squares, and diamonds plot the shares for $x = 77, 84, 91$, and 98, respectively. Panel (b) replicates the series for $x = 77$ and 84 and draws the counterfactual distribution for the share with $x = 77$, under the assumption that its trend would remain parallel to the trends in the shares with $x = 84$ (gray line marked with squares). The gray vertical bar indicates the months between the announcement and the enactment of the reform.

It is important to perform placebo tests to rule out the hypothesis that this was a general trend in the economy. The same figure presents the trends for three placebo series. They correspond to the share of entrants in intervals right above the new threshold. Formally, the red lines marked with triangles, squares, and diamonds plot

$$share^{no\ Kids,E}_{x,m} = \frac{\text{N. of hhlds. w/o Children entering at } (x-7, x] \text{ from Jan. 2012 to m}}{\text{N. of hhlds. w/o Children entering from Jan. 2012 to m}},$$

for $x = 84, 91$, and 98, respectively. All these series are smooth around the reform, suggesting that the increase in the share entering $(70, 77]$ was indeed a consequence of the reform.

To compute $share^E$ for households without children, I use a differences-in-differences specification analogous to (2):

$$\hat{share}^E_{77} = share^{no\ Kids,E}_{77,6/16} - share^{no\ Kids,E}_{77,4/14} - \left(share^{no\ Kids,E}_{84,6/16} - share^{no\ Kids,E}_{84,4/14}\right). \tag{9}$$

I chose here $share^{no\ Kids,E}_{84,m}$ as the control group since it is the closest in levels to $share^{no\ Kids,E}_{77,m}$.

Under the identifying assumption that these trends, $share^{no\ Kids,E}_{77,m}$ and $share^{no\ Kids,E}_{84,m}$, would remain parallel after the reform, $\hat{share}^{E}_{77}$ measures $share^{E}$ among households without children. Even though this assumption is not directly testable, the fact that these trends are parallel before the reform suggests that it holds in the BF setting. As indicated in Figure 17b, I find $\hat{share}^{E}_{77} = 0.003$. The reform increased the share of households without children entering in the $(70, 77]$ by 0.3 percentage points (50% of the pre-reform share). To conduct a formal inference test, Appendix A.14 presents the analogous regression specification. The estimate of the regression is the same and significant at the 1% level with a *t-statistic* of 7.82.

Figure 18 presents a similar analysis for households with children. For these households, the interval $(140, 154]$ became attractive with the reform, although it did not for households without children.

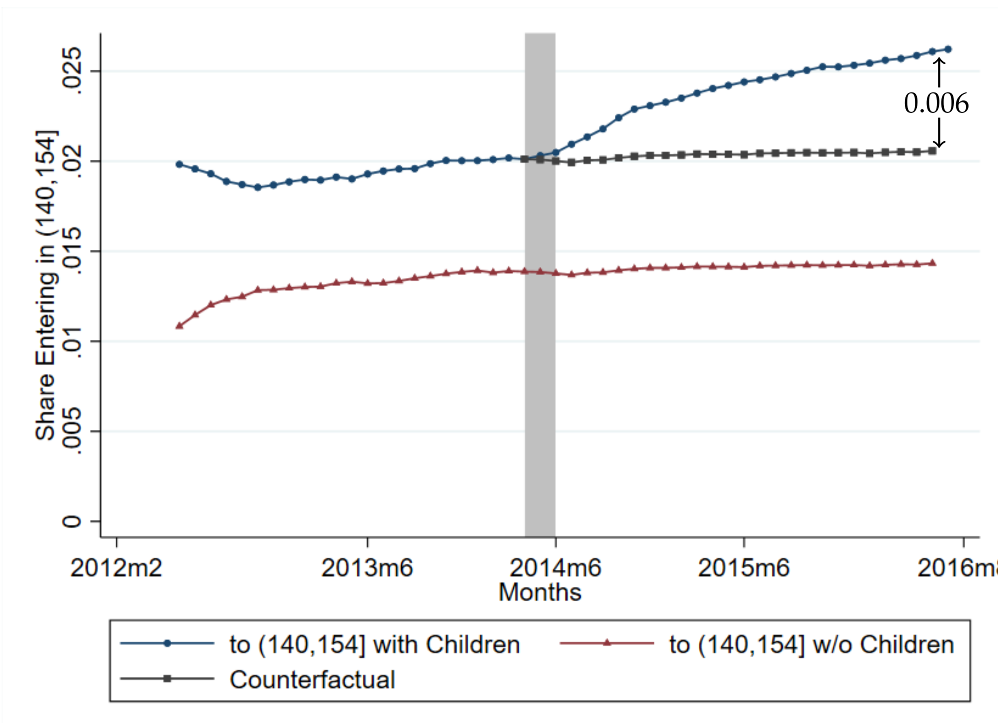


Figure 18: Share of Households Entering with Income in $(140, 154]$

Note: The blue line marked with circles and the red line marked with triangles plot the cumulative shares of households without and with children entering at $(140, 154]$ reais interval up to each month in time, out of all households with and without children entering BF since January 2012, respectively. The gray line marked with squares draws the counterfactual distribution for the share with children, under the assumption that its trend would remain parallel to the trends in the shares of households without children (gray line marked with squares). The gray vertical bar indicates the months between the announcement and the enactment of the reform.

Consider the following share definitions:

$$share^{Kids,E}_{154,m} = \frac{\text{N. of hhlds. with Children entering at } (140, 154] \text{ from Jan. 2012 to m}}{\text{N. of hhlds. with Children entering from Jan. 2012 to m}} \text{ and}$$

$$share^{no\ Kids,E}_{154,m} = \frac{\text{N. of hhlds. w/o Children entering at } (140, 154] \text{ from Jan. 2012 to m}}{\text{N. of hhlds. w/o Children entering from Jan. 2012 to m}}.$$

The blue line marked with circles and the red line marked with triangles plot $share^{Kids,E}_{154,m}$ and $share^{No\ Kids,E}_{154,m}$, respectively. Once again, households affected by the reform (with children) start entering disproportionately more around the months of the reform. The same is not true for households not affected (without children). Under the assumption that these trends would remained parallel after the reform, the following calculation recovers a consistent estimate for $share^{E}$ among households with children:

$$\hat{share}^{E}_{154} = share^{Kids,E}_{154,6/16} - share^{Kids,E}_{154,4/14} - \left(share^{no\ Kids,E}_{154,6/16} - share^{no\ Kids,E}_{154,4/14}\right). \tag{10}$$

This calculation (depicted in the figure) yields $\hat{share}^{E}_{154} = 0.006$. The reform increased the share of households with children entering in the $(140, 154]$ interval by 0.6 percentage points (30% of the pre-reform share). The formal inference test in Appendix A.14 reports the same effect, which is significant at the 1% level with a *t-statistic* of 10.88.

5.4 Bounds and Estimates for *share*^{J}

To compute the bounds and estimates for $share^J$, I measure NEA as the number of households that were in the program with income above $t_0 + \Delta t$ in their last update before the reform (June 2014); E as the number of households that applied to the program for the first time after the reform; NEA_t as the number of households with income in the $(t_0, t_0 + \Delta t]$ interval right before the reform; E_t as the product of the number of households that applied for the first time between January 2012 and June 2016 and $\hat{share}^{E}$; and JA as the product of $\hat{share}$ and the number of households that updated from above the threshold up to June 2016. The resulting numbers for households without children affected by the reform of the first threshold ($t_0 = 70$) and households with children affected by the reform ($t_0 = 140$) are presented in the first five rows of Table 3.

The relation (8) evaluated at the lower (upper) bound for JE (NME) provides a lower bound for the parameter of interest. Conversely, the same relation evaluated at its upper (lower) bound provides an upper bound for $share^J$. These bounds are presented in the

Table 3: Calculations for $share^J$

	Hhlds. without Children ($t_0 = 70$)	Hhlds. with Children ($t_0 = 140$)
NEA	$72,402$	$87,155$
E	$129,972$	$160,153$
NEA_t	$1,691$	$10,311$
E_t	769	$1,782$
JA	963	$2,287$
$share^J_{LB}$	0.005	0.009
$share^J_{UB}$	0.009	0.016
$share^J$	0.006	0.011

Note: The first five rows correspond to the number of non-eligible applicants, entrants, non-eligible applicants between the old and the new threshold, entrants between the old and new threshold and jumping pre-reform applicant, respectively. The last three rows display the implied shares of jumpers' lower and upper bound and the point estimate under assumption 5. The first and second columns show the numbers for households without and with children.

sixth and seventh rows in Table 3, respectively. Finally, using the JE and NME implied by Assumption 5, one can recover point estimates for the sufficient statistic. Estimates from this specification are displayed in the last row. This last share is closer to the lower bound because most of the pre-reform applicants located between the old and the new threshold after the reform were in this same region before the reform. This implies, under Assumption 5, that most of the entrants below the new threshold are non-moving entrants rather than jumping entrants.

6 Welfare Analysis of the Reform

This section presents the welfare effects of the BF reform. I compute the welfare effect of the discrete reform observed in the data $(\Delta t, \Delta I)$ using a linear approximation of the relation described in Proposition 1. In particular, this approximation does not take into account second-order terms that could affect the total effect of the reform. Since the reform is small, this is a reasonable approximation.

$$\frac{dW(\Delta t, \Delta I)}{\lambda} \approx \underbrace{(\bar{g}-1)H(t_0)\Delta I}_{\text{Transfer Effect}} + \underbrace{(\bar{g}(t_0)-1)I_0[H(t_0+\Delta t)-H(t_0)]}_{\text{Threshold Effect}} \underbrace{-I_0 share^J(\Delta t, \Delta I)}_{\text{Jumping Effect}}.$$

The welfare weights ($\bar{g}$ and $\bar{g}(t_0)$) and the marginal cost of public funds (λ) depend on the planner's preferences for redistribution; their estimation is beyond the scope of this paper. Remaining inputs are computed from the data. The income distribution $H(\cdot)$ is recoverable,[22] and Δt and ΔI are given by the schedule reform.[23] The share of jumpers is recovered as described in the previous section. Table 4 presents the inputs for welfare effects of the reform in terms of the marginal cost of public funds $\frac{dW}{\lambda}$.

Table 4: Inputs for the Welfare Analysis of the Reform

Group	$H(t_0)\Delta I$	$I_0[H(t_0+\Delta t)-H(t_0)]$		Jumping Effect	
		Pref. Spec.	Bounds	Pref. Spec.	Bounds
t=70	0.914	0.301	$(0.233, 0.340)$	-0.171	$(-0.239, -0.133)$
t=140	2.366	0.308	$(0.270, 0.317)$	-0.068	$(-0.107, -0.060)$

Note: The first and second rows display the inputs for the welfare analysis for households without ($t = 70$) and with children ($t = 140$).

To interpret these results, it is useful to normalize the sum of the three effects to one. For every marginal *real* transferred by the reform to households without children (with children), 66 (86) cents are transferred to households that were eligible before the reform (transfer effect); 22 (11) cents are transferred to households that became mechanically eligible because of the increase in the eligibility criteria (threshold effect); and 12 (2) cents are transferred to households that jumped to the threshold in response to the reform (jumping effect). While the first two parts of the marginal *real* correspond to a first-order effect on the utility of beneficiaries, jumping effects only generate a second-order increase on welfare. These households were intially indifferent between being in or out of the program, so that the effect of the reform on their utility is small (even though the effect on the budget is not, since the government needs to transfer $I_0 + dI$ for each jumper). Hence, only 12%(2%)[24] of the marginal *real* transferred to households without children (with children) was lost in efficiency cost of applicants jumping into the program. Households with children could also have jumped to the first threshold, which also affected their transfer. For this reason, the total efficiency cost of the marginal transfer for this group was larger than 2%.

This analysis implies that a welfare-maximizing government should increase the generosity of the program if it values a 88 cents increase in consumption for the eligible poor by more than R$1 in its budget on the margin. Appendix A.15 performs a similar welfare analysis of counterfactual reforms.

[22] I use the per capita income distribution before the reform. Although this distribution is not observable for entrants, their income was above t_0; otherwise, they would be in the program. This is enough to compute $H(t_0)$ without ambiguity. To compute $H(t_0 + \Delta t)$, I use the three definitions of NME discussed in the previous section.

[23] I use the average I_0 and ΔI to conduct this analysis.

[24] The lower and upper bounds for these efficiency costs are 10%(2%) and 16%(4%), respectively.

7 Simulation

This section illustrates the imporance of jumping effects more generally. In the case of BF reform, jumping is the only source of efficiency cost, since the marginal transfer does not change. This is not a general aspect of income-based transfer program reforms, which could also distort incentives in the intensive margin through changes in the marginal tax/transfer. To consider such reforms, I simulate an economy and then analyze the relative importance of jumping effects and the usual effects for different reforms.

Households have preferences that can be represented by an isolastic utility function:

$$u(c, y; n, m) = c - \frac{1}{1+\frac{1}{m}} \left(\frac{y}{n}\right)^{1+\frac{1}{m}},$$

where and $n \sim logNormal(2.757, 0.5611)$[25] and $m \sim U(0, 1)$[26] are assumed to be independent. When solving the household problem (4), applicants face a piece-wise linear schedule $B(\cdot)$ of the form:

$$B(y) = \begin{cases} I + \xi y & if\, y \leq t \\ 0 & if\, y > t. \end{cases}$$

Note that ξ corresponds to the marginal after-transfer income so that $m = \frac{1+B'}{y} \frac{\partial y}{\partial \xi}$ is the income elasticity.

Consider the following decomposition effect of the effect of an arbitrary reform on the budget of the government:

$$\underbrace{B_a(y_a) - B_b(y_b)}_{\text{Total Effect}} = \underbrace{B_a(y_b) - B_b(y_b)}_{\text{Transfer Effect}} + \underbrace{B_a(y_a) - B_a(y_b)}_{\text{Behavioral Effect}}.$$

The transfer effect denotes the desirable effect of the policy: transferring income without behavioral responses. The behavioral effect corresponds to the efficiency cost, i.e., the additional transfers that comes from movements of individuals and yield a second-order effect on their utilities with respect to the change in the transfer. This last effect can be

[25]According to Mankiw et al. (2009), this fits the US 2007 wage distribution.

[26]Most empirical estimates of taxable income elasticities are in this range.

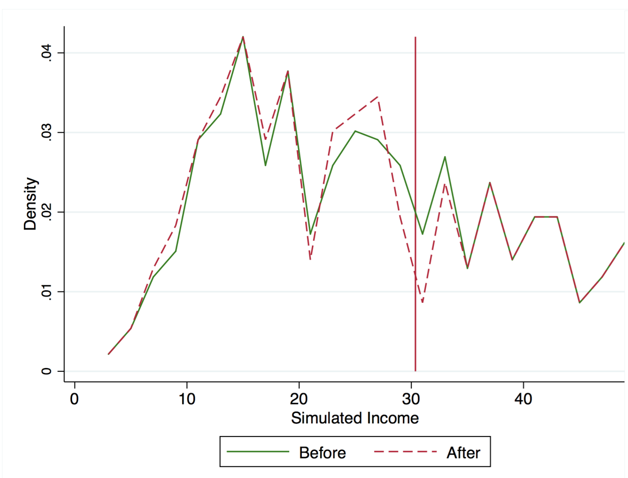
decomposed as:

$$\underbrace{B_a(y_a) - B_a(y_b)}_{\text{Behavioral Effect}} = \underbrace{\Big(B_a(y_a) - B_a(y_b)\Big)1(y_b < t)}_{\text{Elasticity Effect}} + \underbrace{B_a(y_a)1(y_b > t)}_{\text{Jumping Effect}}. \tag{11}$$

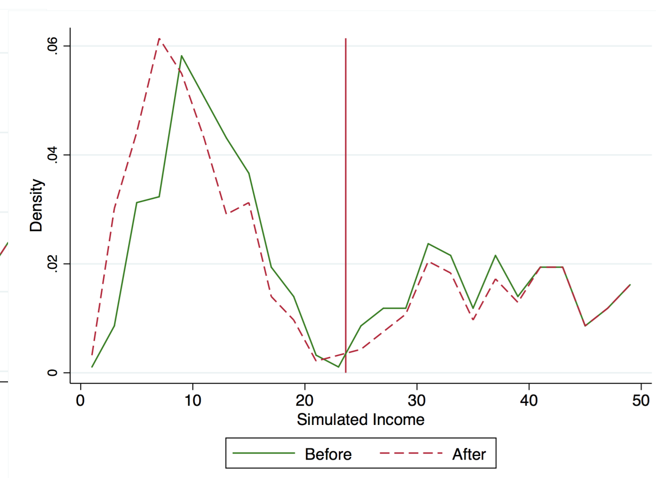
The elasticity effect corresponds to the usual intensive margin response of beneficiaries that are in the anti-poverty program even after the reform. The jumping effect corresponds to the effect on the budget of households that join the program as a response to a change in the schedule.

I consider two reforms. The first changes the marginal after transfer income from .9 to .8, while the second reform changes it from .2 to .1 In both, the pre-reform intercept I_b and threshold t were chosen to match the proportion of beneficiaries to 20.4% of the sample, which is the proportion of the Brazilian population getting transfer from BF. The post-reform intercept I_a was chosen so that the eligibility threshold is unchanged.

Figures 19a and 19b plot the income distribution before (green solid line) and after (red dashed line) the first and second reforms, respectively. The vertical red line indicates the eligibility threshold of the program t for each reform.



(a) First Reform



(b) Second Reform

Figure 19: Simulated Income Distribution

These figures plot the distributions of income for applicants before (solid green line) and after the reform (dashed red line) in the simulated economies. Panel (a) illustrates a reform that decreases the marginal after-tax income from .9 to .8, while panel (b) shows the distributions around a reform that decreases the marginal after-tax income from .2 to .1.

The area around t is dominated because of the non-convex kink. As expected, both distributions present a missing mass around the kink. The reform has two effects on the distribution. The first corresponds to the usual elasticity effect visualized by the movement within the first bracket ($y \leq t$). Households facing a lower marginal after-transfer income supply less labor and reduce their equilibrium earnings. The second corresponds to the jumping effects, and is visualized by the movement from the second to the first bracket. Intuitively, households that were initially at the second bracket and close to indifferent to the first bracket will jump as this first bracket becomes more attractive.

Table 5 presents the shares of the efficiency cost of each reform coming from the elasticity effect and the jumping effect, as described by equation (11).

Table 5: Welfare Analysis in Simulated Economy

Group	Elasticity Effect	Jumping Effect
First Reform	94%	6%
Second Reform	64%	36%

Note: The first and second rows display the inputs for the efficiency cost analysis for the first and second reforms, respectively.

Taking jumping effects into account increases the efficiency cost of the first reform by 6%. The limited effect in this case is due to the low marginal tax rates in the phase-out region. Since the schedule is close to flat around the non-convex kink, only very elastic households would jump after a small reform (1.5% in the simulation). The efficiency cost of the second reform is 56% larger once jumping effects are taken into account. Its kink is more acute and generates jumps for a larger proportion of the population (2.8 % in the sample). These jumps also have larger impacts on the government's budget. This analysis illustrates that jumping effects could affect welfare analysis of reforms of tax schedules, even in the absence of notches.

8 Conclusion

This paper computed the welfare effect of a reform in the *Bolsa Família* program, one of the largest transfer programs in the world. To do so, I present evidence that jumping effects are important in this context and provide a theoretical framework that accommodates jumping effects in the welfare analysis. The application of this framework to the Brazilian context indicates that jumping effects account for the entire efficiency cost of the reform, but these costs account for only 12% of the total cost of the policy change.

Simulations of alternative reforms suggest that this jumping behavior affects the welfare analysis of transfer programs, even when the policy also distorts incentives in the intensive margin. This behavior also affects characterization of the optimum income tax when the planner is restricted to use two linear brackets (Slemrod et al., 1994) or in economies with more than one dimension of heterogeneity (Dodds, 2017). The optimal policy formulas presented in the Appendix may be a fruitful starting point for incorporating this behavior when thinking about the design of these nonlinear policies.

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A Appendices

A.1 Other Social Security Programs Based on the Cadastro Único

This appendix describes other programs that set their eligibility based on the Cadastro Único.

Benefício de Pretação Continuada (BPC): This benefit targets the elderly (above 65 years of age) and the disabled. It gives a minimum wage to all households with such individuals with per capita income up to a quarter of the minimum wage. Table 6 reports the minimum wage and the BPC threshold across all years of the analysis. The Brazilian Social Security System administers administers its own exam in order to define eligibility to this program.

Table 6: Minimum Wage and BPC Eligibility Thresholds

Year	Minimum Wage	BPC Threshold
2011	545.00	136.25
2012	622.00	155.50
2013	678.00	169.50
2014	724.00	181.00
2015	788.00	197.00
2016	880.00	220.00

Carteira do Idoso: This "Elderly Card" guarantees to all individuals 60 years of age or older and with up to 2 minimum wages at least 50% discount in any interstate trip by road, rail or waterway.

Créditos Instalação do Programa Nacional de Reforma Agrária: Households with per capita income up to three minimum wages and that are leaving in camping grounds get points in a system which selects beneficiaries to be settled through the Brazilian land reform.

Facultativo de Baixa Renda: It is a option to contribute to social security with a lower rate (5% of the minimum wage). The individual cannot have any income and the household income needs to be below two minimum wages.

Identidade Jovem (ID Jovem): Discounts for cultural events and trips by road, rail or waterway for individuals between 15 and 29 years of age living in a household with up to 2 minimum wages.

Isenção de taxas de inscrição em concursos públicos: Since 2008, households with per capita income up to half of the minimum wage or total income of up to three minimum wages are exempt from public tender registration payment.

Política Nacional Assistência Técnica Rural - PNATER Brasil Sem Miséria: Technical assistance for households working on activities for the consumption their own in rural areas.

Programa Água para Todos - Programa Nacional de Universalização do Acesso e Uso da Água: Since July 2011, the government has installed cistern to ensure the access to clean water to all Brazilians, with priority with those that fulfill the same criteria of the *Bolsa Família* program.

Bolsa Estiagem: It is a benefit of at least 80 *reais* per month to household with up total income up to two minimum wages that live in areas hit by natural disasters.

Programa Bolsa Verde - Programa de Apoio à Conservação Ambiental: Since October 2011, this program transfer 300 *reais* every three months to households in extreme poverty (first threshold of PBF) and that follow the requirements of the use of natural resources.

Programa Cisternas: This program aims to provide cisterns to low income families registered in the *Cadastro Único*.

Programa de Erradicação do Trabalho Infantil: This program transfer benefits similar to the BF (25 and 40 *reais* per child in municipalities with less and more than 250 thousand inhabitants) that are not eligible to the BF (per capita income above 170 *reais*) with working children (up to 16 years of age).

Programa de Fomento às Atividades Produtivas Rurais: Since 2012, the government makes a one-time transfer of around 2,400 *reais* to families that are eligible for the BF program and work on agricultural activies or belong to native or traditional communities.

Programa Minha Casa Minha Vida: Household with total monthly income up to 1,416.67 *reais* have access to a subsidized credit line to purchase their house.

Programa Nacional de Crédito Fundiário: Household with total monthly income up to 2,500 *reais* have access to a subsidized credit line to purchase their land for production.

Serviços Socioassistenciais: MDS offers these social services to poor individuals and have suffered any type of violence or disregard.

Sistema de Seleção Unificada -Sisu/Lei de Cotas: Since 2016, all the Federal Universities in the country reserve some seats to students coming from families with per capita income up to 1.5 minimum wages.

Tarifa Social de Energia Elétrica: Households with monthly per capita income of up to a half of the minimum wage have access to a favorable electricity price.

Tefone Popular - Acesso Individual Classe Especial: The government offers a landline with lower prices for individuals registered in the *Cadastro Único* database.

Distribuição de Conversores de TV Digital: Since the September of 2015, MDS start offering digital converters to beneficiaries of the program. This help them with the transition of the open TV to the new system.

A.2 Other Reforms

I follow households in the data from the December 2011 until September 2016. During this period the program's schedule went through five reforms. The schedule at the beginning of the period analyzed entitled households with less than R$70 reported per capita income to a R$70 benefit (basic benefit), R$32 for each child up to 15 years of age (variable benefit) and R$38 for each member between 16 and 17 years of age (teenager benefit). The total variable benefit could not exceed R$160 (5 children per household) nor the teenager benefit could exceed R$76 (two teenagers per household).

In the period of analysis, there are three reforms which changed the minimum per capita income in the household. They all implemented a minimum income per capita to R$70, but for different groups of households in turn. The reform of June 2012 implemented this minimum for households with children up to 6 years of age, while the one of November of 2012 implemented it for households with children up to 15 years of age. The reform in February of 2013 changed this minimum to all the remaining households. These reforms were the only to affect the marginal benefits faced by the participants, which is the variation necessary to capture intensive margin responses.

The fourth reform in June of 2014 was discussed in section **??**. Finally, the reform in June of 2016 increased the minimum per capita income, the basic benefit and the first threshold to 85, the second threhold to 170, the benefit per child to 39 and the benefit per teenager to 46 *reais*.

Figure 20 plots these schedules for households with 3 members, during the period of analysis.

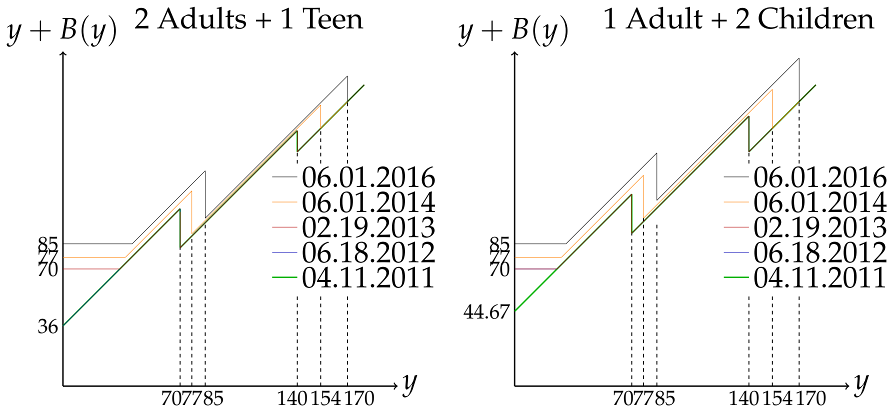


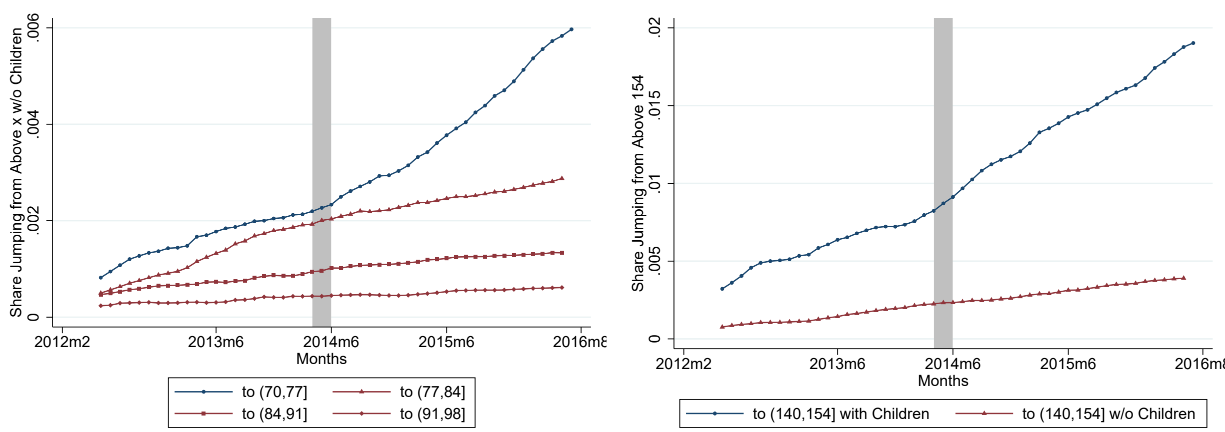
Figure 20: Bolsa Família Schedule Reforms

A.3 Alternative Shares

The evidence in Section 3.3 uses the number of households that were above the relevant threshold and updated as a measure of noneligible applicants that could potentially jump. Here I present evidence of jumps using the number of households above the relevant threshold as an alternative measure of these potential jumpers. Figures 21a and 21b replicate Figures 8a and 9 using the implied alternative share definition:

$$share^{w/oKids}_{x,m} \equiv \frac{\text{N. of hhlds. w/o Kids updating from above } x \text{ to } (x-7,x] \text{ up to month } m}{\text{N. of hhlds. w/o Kids above } x \text{ up to month } m},$$

$$share^{withKids}_{x,m} \equiv \frac{\text{N. of hhlds. with Kids updating from above } x \text{ to } (x-7,x] \text{ up to month } m}{\text{N. of hhlds. with Kids above } x \text{ up to month } m}.$$



(a) Without Children From Above x (b) From Above 154 to (140, 154]

Figure 21: Share of Households Jumping

There is still a break on the share of households jumping to the relevant intervals around the months of the reforms. However, there trends of these alternative shares are not parallel across treatment and control group before the reform. In Figure 21a, the trends of households jumping to $(77, 84]$ seem parallel to the households jumping to $(70, 77]$, but this is because R$84 corresponds the minimum wage in 2013 (a pattern that could also be seen in Figure 8a. For this reason, I prefer to report my results with the share definition in Section 3.3.

A.4 Effect on the Timing of Updates of Comparison Groups

For jumpers to the first threshold, households without children jumping to neighboring intervals ($(77, 84]$, $(84, 91]$ and $(91, 98]$) are used as a control group for those updating to $(70, 77]$. In this case, the sets of potential jumpers in the control groups are households without children above 84, 91, and 98 *reais*, which correspond to the end of each neighboring interval. Since these effects of the reform are conditional on households' updating their information, it is important to investigate whether the reform affected the timing of the reports differently across treatment and control groups.

Figure 22a plots the distribution of the months of updates for households above R$77 (solid line) and R$91 (dashed line). These distributions are almost indistinguishable because only 2.5% of the households without children updating from above 77 were below 91. Even though the reform affected the timing of updates, it did not differently affect these groups of households.

(a) Households without Children

(b) Households Above 154

Figure 22: Date of Updates Distributions Compared to Control Groups

Figure 22b plots the months of updates distributions for households with children and without children above R\$154. These distributions differ significantly. However, it is not clear that households with children update disproportionally more after the reform than households without children. Once again, the reform does not seem to have impacted the timing of updates differently across these two groups. Even though the reform affected the timing of the updates, this effect does not seem to differ among the treatment and control groups used in Section 3.3.

A.5 Inference for Main Results

The estimates of *share* in Figures 8a come in principle from a differences in difference specification. Note, however, that any household with income above R\$ 91 could potentially jumped to both the interval that became attractive after the reform $(70, 77]$ and the placebo interval $(84, 91]$. In other words, treatment and control groups overlap in the data.

For this reason, it is useful to think about the statistical test analogous to the calculation in the Section 3.3. Consider the following system of seemingly unrelated regressions (SUR):

$$\begin{cases} Jumped91_{it} = \sum_{q=-8}^{+9} Quarter\_q_{it}\gamma_q^{91} + u_{it}^{91}, \\ Jumped77_{it} = \sum_{q=-8}^{+9} Quarter\_q_{it}\gamma_q^{77} + u_{it}^{77}. \end{cases}$$

Where $Jumpedx$ is an indicator that household i jumped from above x to $(x-7, x]$ up to month t, and $Quarter\_q_{it}$ is an indicator that t belongs to quarter q.[27] This test can be thought as a comparison on the difference between the quarter fixed effects $\gamma_q^{77} - \gamma_q^{91}$ with the month of the reform being omitted. The difference between these two parameters after the reform captures the effect of interests under the assumption this fixed effects would remain parallel in the absence of the reform. Although this assumption is not testable, the parallel pre-trend assumption can be tested by assessing whether the same difference is zero before the reform.

It is important to run these regressions together to capture the errors correlation structure. In practice, the data set is expanded so that each household i in month t appears twice. The first observations ($d = 0$) considers only jumps to the placebo interval $(84, 91]$ and is analogous to a control group. The second ($d = 1$) considers only the possibility of jumps to the affected interval $(70, 77]$ and is analogous to a treatment group. In this expanded

[27] I use quarterly, instead of monthly, level variation for power issues.

dataset, I run a standard differences-in-differences.

$$Jumped_{itd} = \alpha dup_{itd} + \sum_{q=-8}^{+9} \beta_q Quarter\_q_{itd} + \gamma_t + u_{it}$$

Where $Jumped_{itd}$ is an indicator if household i had jumped to either to $(84, 91]$ in the original observation ($d = 0$) or to $(70, 77]$ in the duplicated observations ($d = 1$) up to month t; dup_{itd} is an indicator if the observation is duplicated ($d = 1$); $Quarter\_q_{itd}$ is the interaction of dup_{itd} an indicator for t to belong in quarter q away from the month of the announcement (omitted interaction); and γ_t are the months fixed effects.

Figure 23 presents the coefficients β_qs. Under the identifying assumption that the share of households jumping to $(70, 77]$ would remain parallel to the share of households jumping to $(84, 91]$ in the absence of the reform, each coefficient for $q > 0$ indicates the effect of the reform up to quarter q away from the reform on the probability of jumping to the treated interval $(70, 77]$ relative to the control interval $(84, 91]$. One cannot reject the hypothesis that the trends of these probabilities were the same before the reform, but the effects of the reform become significant at a 5% level three quarters after the announcement. The coefficient of interest is the effect of the reform on the share of jumpers up to the end of the period of analysis $\beta_9 = 0.006$ as in the reduced form analysis with t-statistic around 12.

The analysis for the second threshold is straightforward because the control (households without children) and the treatment (households with children) groups are well defined. The differences in differences specification is:

$$Jumped_{it} = \alpha Kids_{it} + \sum_{q=-8}^{+9} \beta_q Quarter\_q_{it} + \gamma_t + u_{it}$$

Where $Jumped_{it}$ is an indicator if household i had jumped to $(140, 154]$ up to month t; $Kids_{it}$ is an indicator if the household i had children in month t; $Quarter\_q_{it}$ is the interaction of $Kids_{it}$ and an indicator for t to belong in quarter q away from the month of the announcement (omitted interaction); and γ_t are the months fixed effects.

Figure 24 presents the coefficients β_qs. Under the identifying assumption that the share of households with and without children jumping to $(140, 154]$ would remain parallel in the absence of the reform, each coefficient for $q > 0$ indicates the effect of the reform up to quarter q away from the reform on the probability of jumping. I cannot reject the hypothesis that the trends of these probabilities were the same before the reform, but the effects

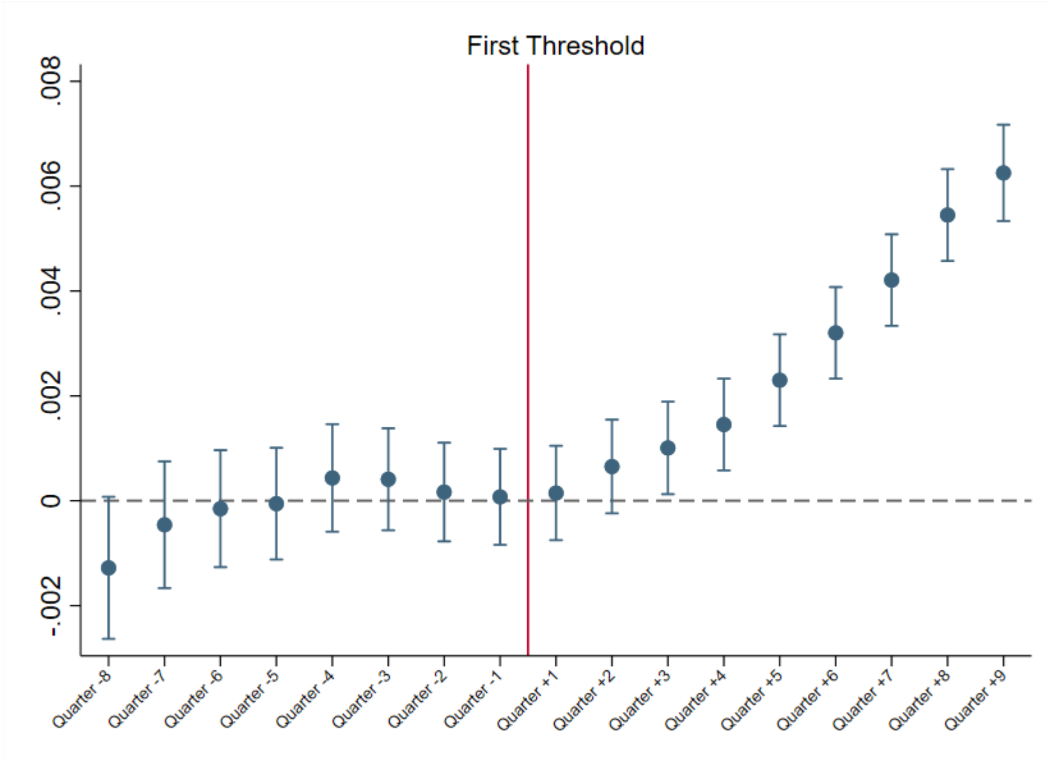


Figure 23: Difference and Differences Results for *share*[77]

of the reform become significant at a 5% level three quarters after the announcement. The coefficient of interest is the effect of the reform on the share of jumpers up to the end of the period of analysis $\beta_9 = 0.014$ as in the reduced form analysis with t-statistic around 12.

A.6 Additional Evidence of Jumps

This appendix replicates the evidence in figures 8a and 9 but in terms of number of jumpers in each month.

Even those these trends are noisier the effect of the reform is also visual in the absolute number of jumpers.

I now present the figures that replicate Figures 8a and 9 but only among households that remained with the constant household composition during their last udpate, i.e. households that did not change their number of members during their last update.

The effects of the reform are similar, but perhaps a little smaller for households with children.

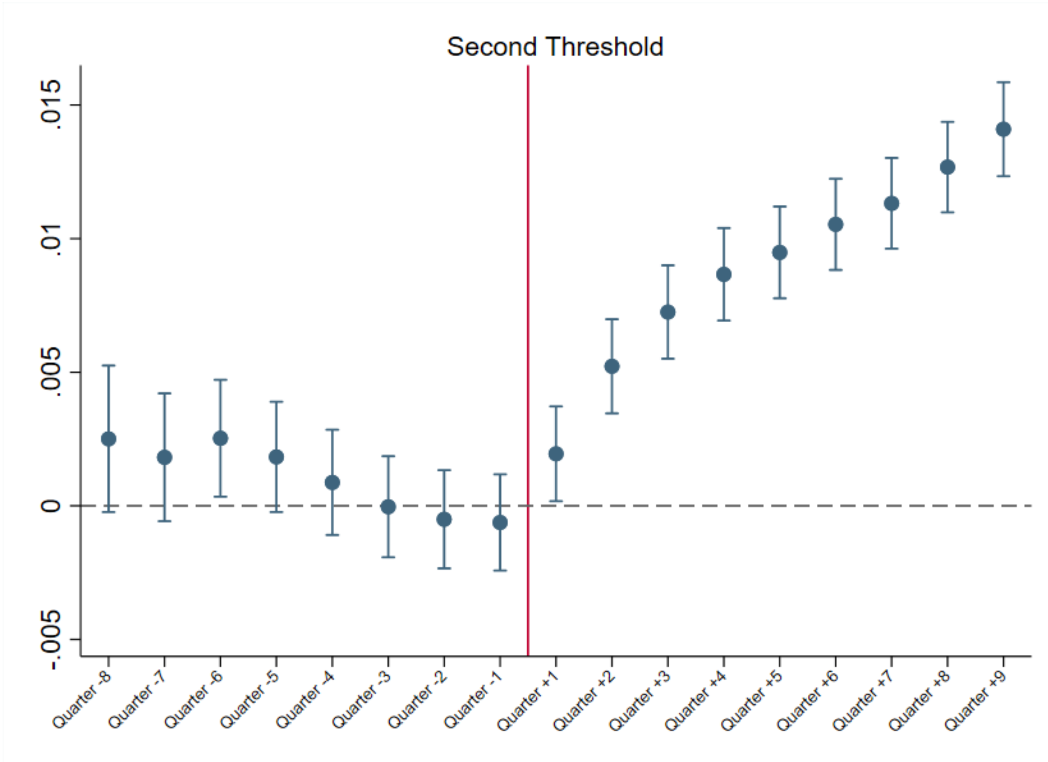


Figure 24: Difference and Differences Results for $share^{154}$

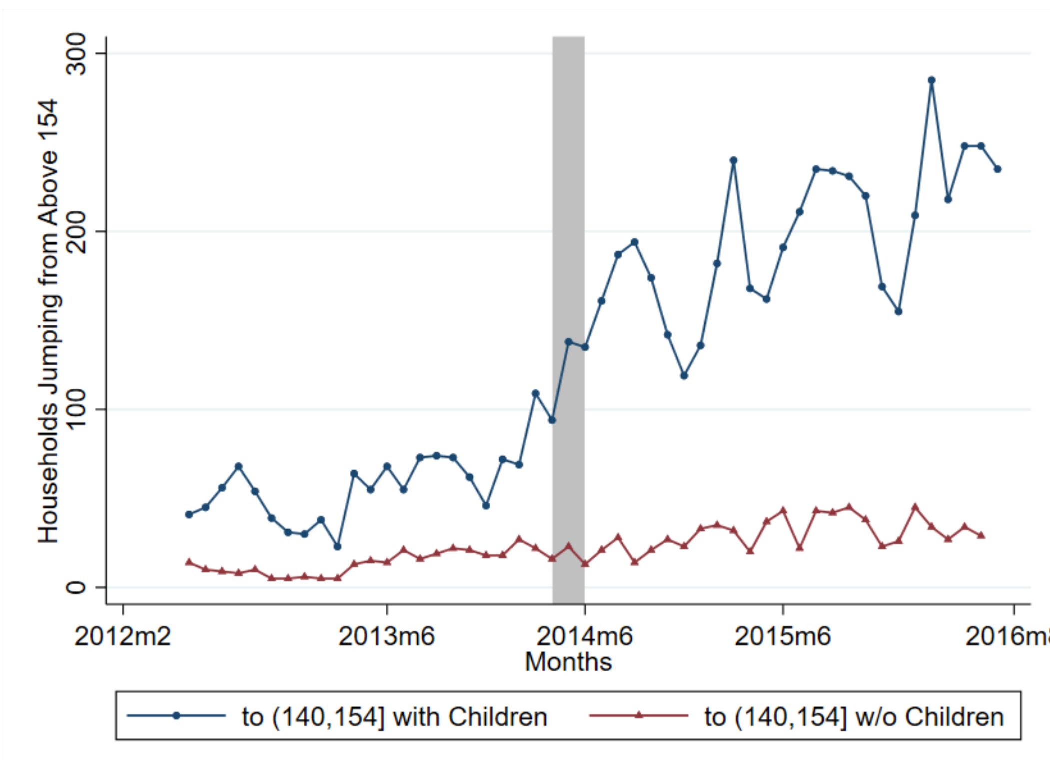


Figure 25: Number of Households Jumping from Above 154 to $(140, 154]$

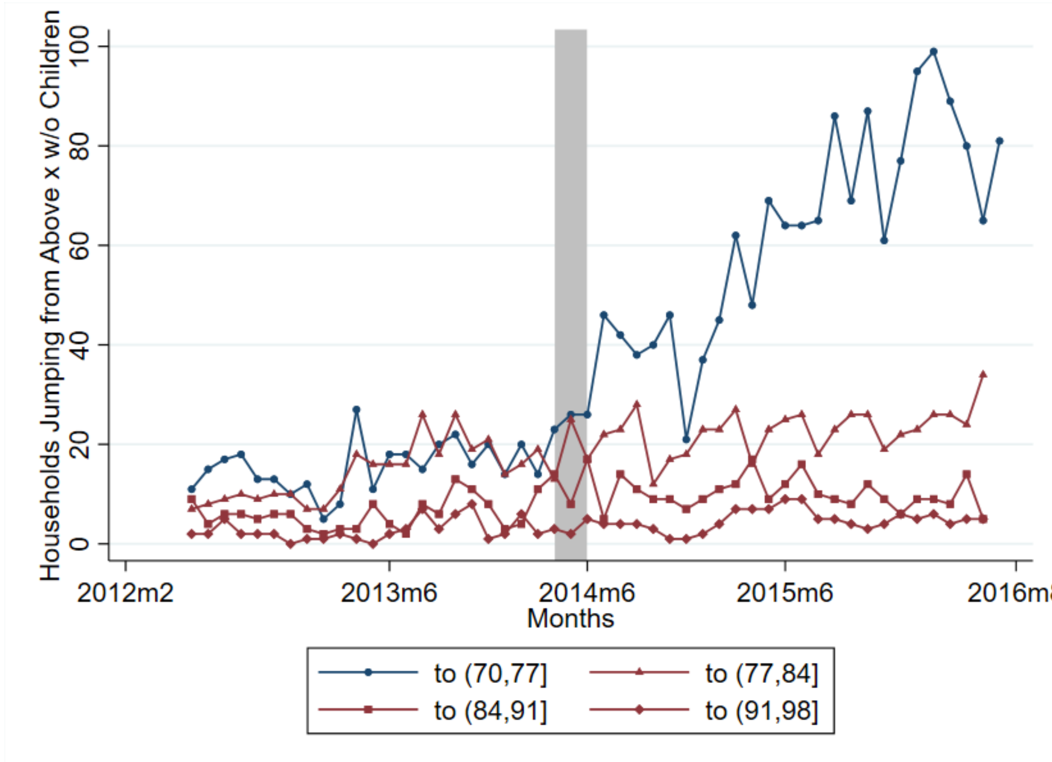


Figure 26: Number of Households Jumping from Above 77 to $(70, 77]$

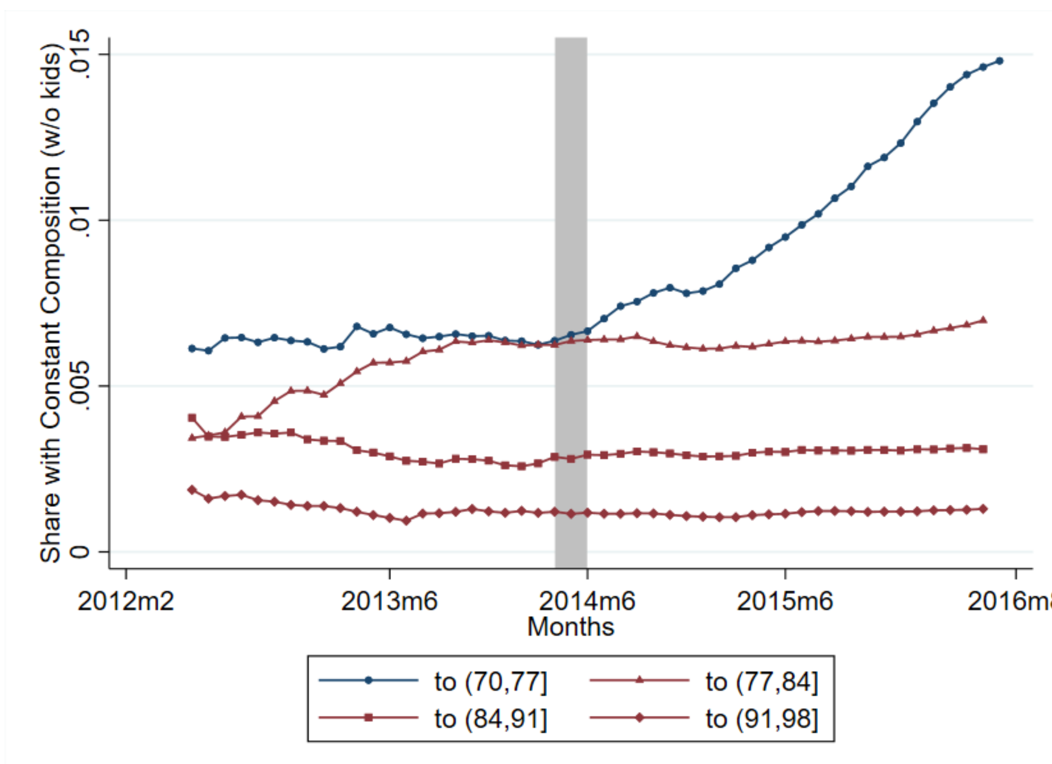


Figure 27: Share of Households with Constant Composition Jumping from Above 77

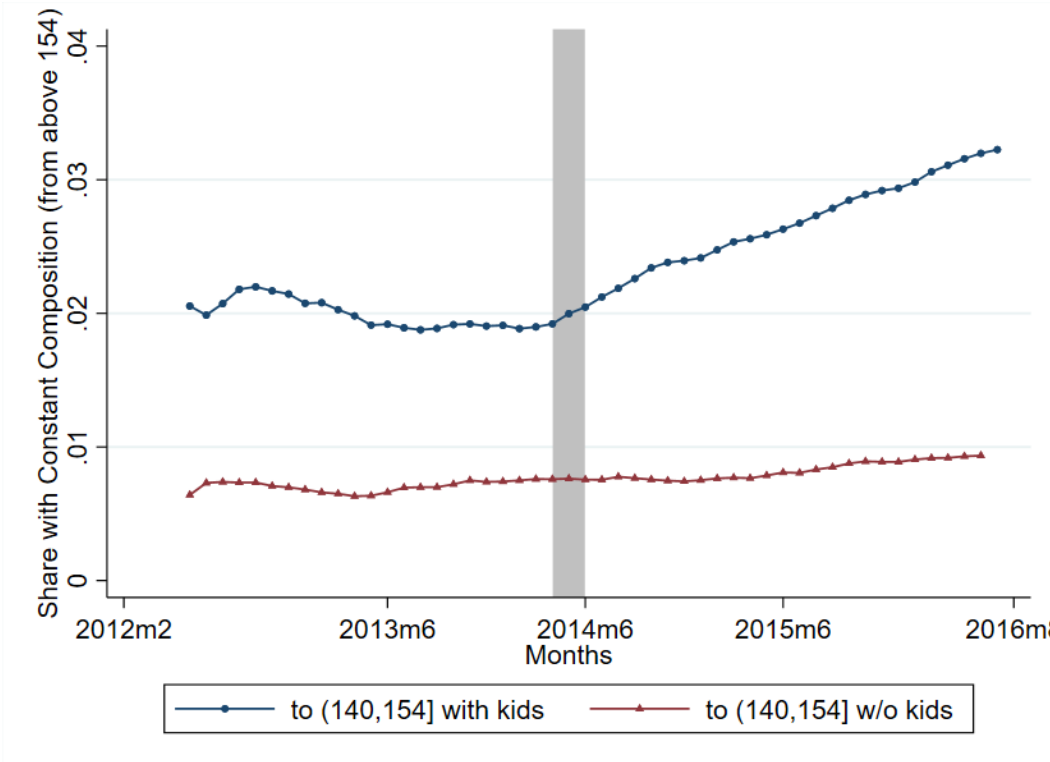


Figure 28: Share of Households with Constant Composition Jumping from Above 154

Finally, I present as alternative placebo series for the share of jumpers without children to $(70, 77]$, the share of jumpers to $(56, 63]$ and $(63, 70]$ and the share of jumpers. There are two sets of alternative of placebo series to the share of households with children jumping to $(140, 154]$: the ones jumping to intervals above 154 ($(154, 168]$;$(168, 182]$ and $(182, 196]$) and the ones jumping below ($(112, 126]$ and $(126, 140]$).

All the placebo test are either smooth around the months of the reform. Some of these series are less stable because they include some fraction of the minimum wage in certain years.

A.7 Mean Reversion Analysis

Figure 32 plots the shares of household in each quartile of the income distribution of households above the eligibility threshold that persist in the same quartile and that moved to other quartiles or to below the threshold. Figure 32a plots the shares of households without children with income above 77 *reais* before January 2012. The quartiles are kept constant at the pre 2012 levels throughout the period. The first Panel plots the share of households initially in the first quartile that moved to below the threhold (solid blue line),

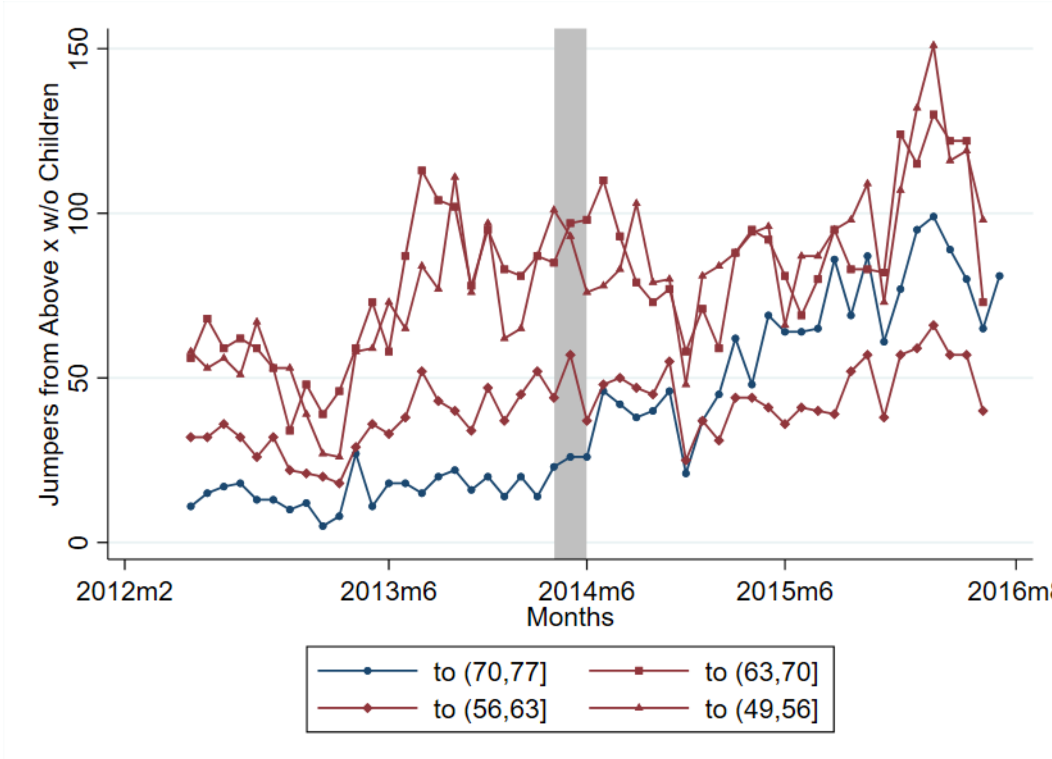


Figure 29: Share of Households without Children Jumping from Above x

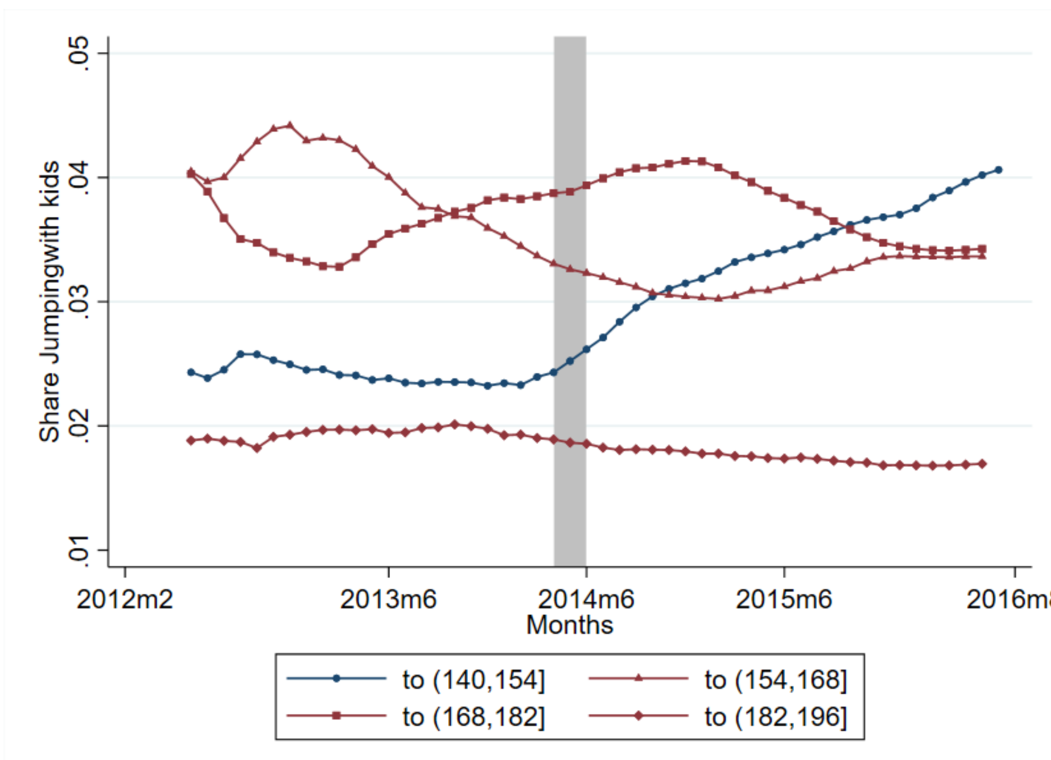


Figure 30: Share of Households with Children Jumping from Above x

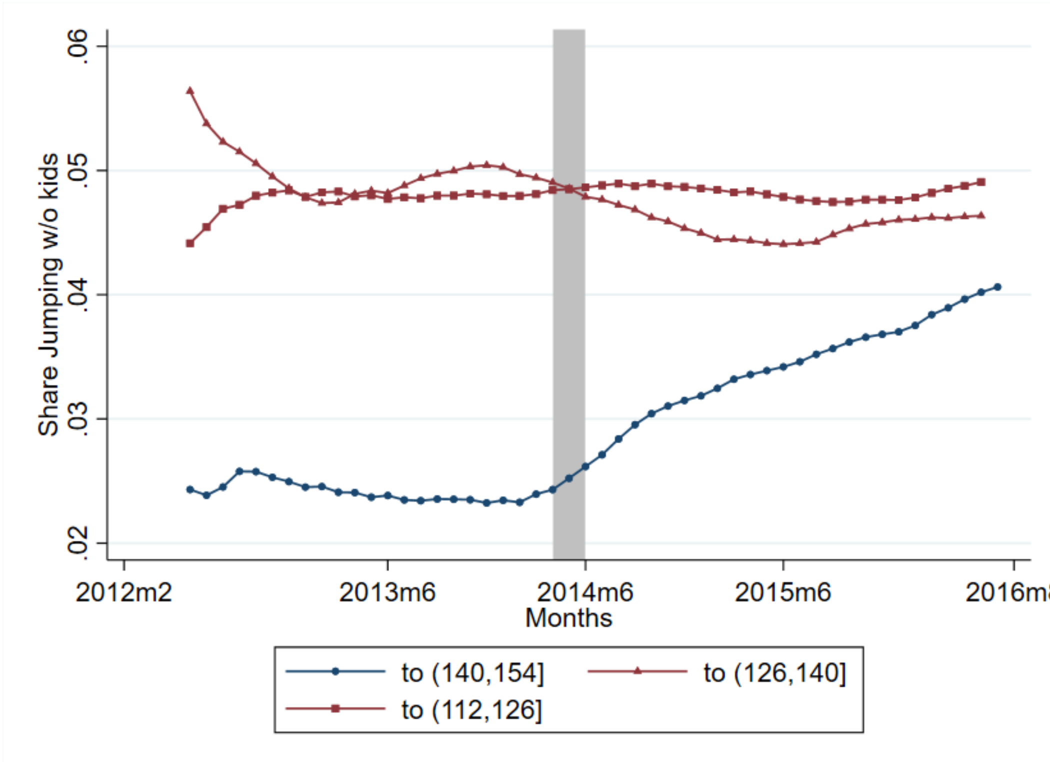


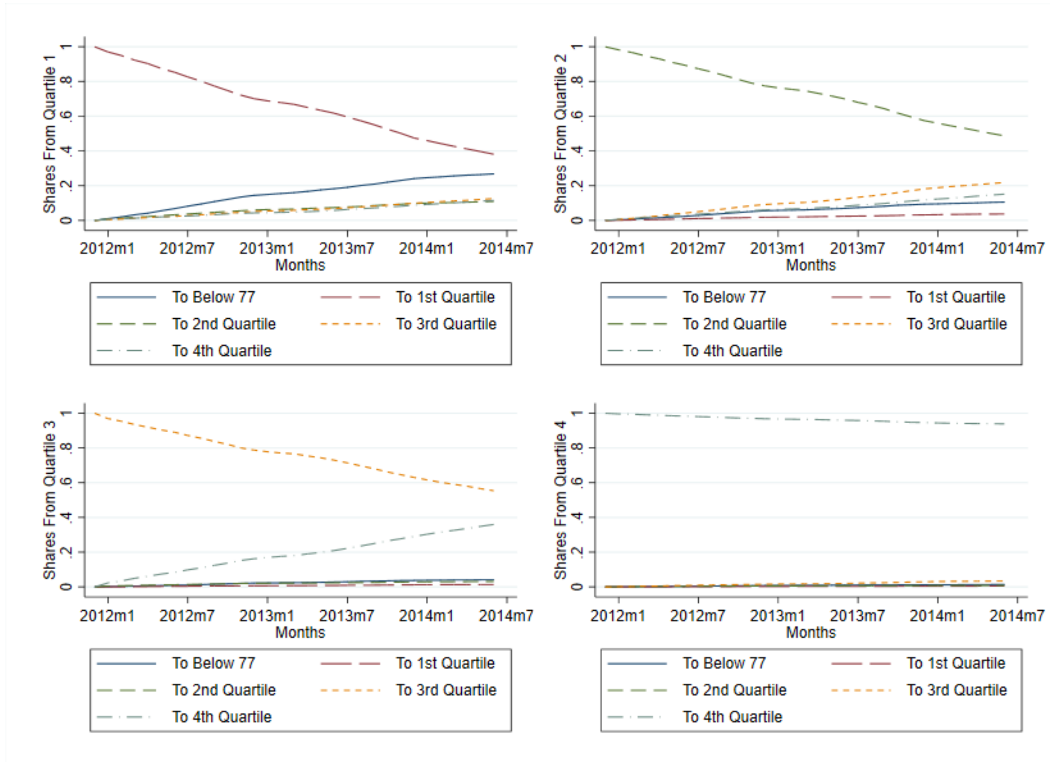
Figure 31: Share of Households with Children Jumping from Above x

persisted in the first quartile (long dashed red line), move to the second (green dashed line), third (yellow short dashed line) and fourth quartile (light blue dash-dotted line).

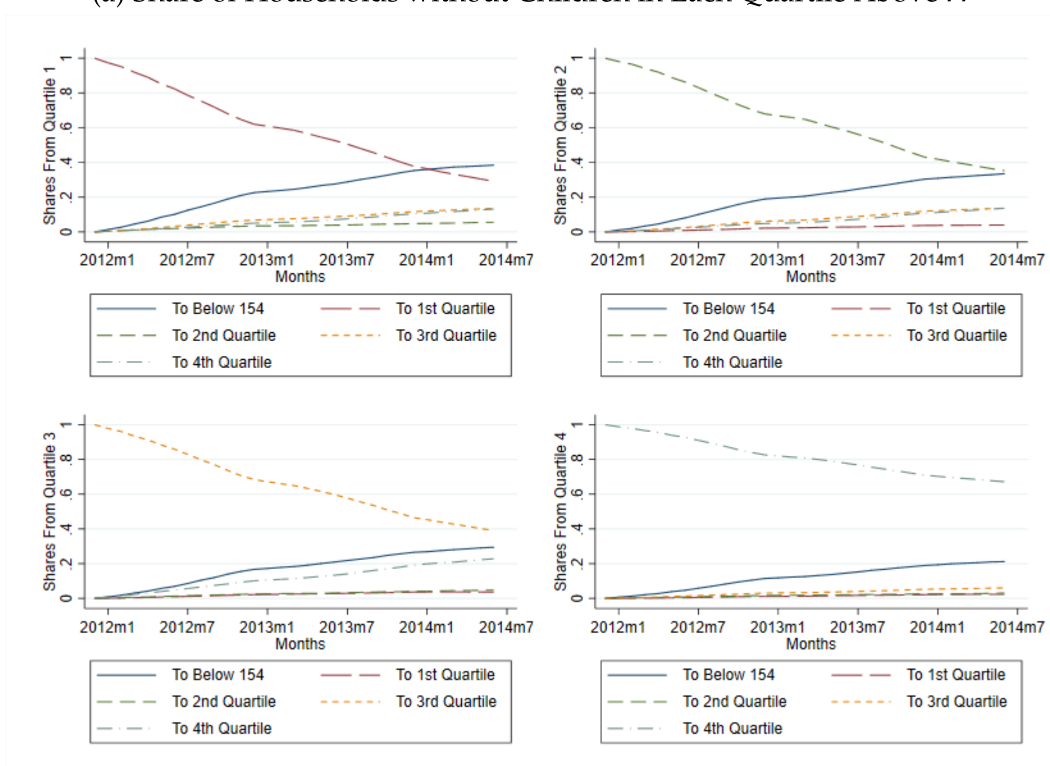
It is clear that the households in this first quartile moved to many other parts of the income distribution even before the reform. Most of the movement out of the first quartile is directed to the interval below the new eligibility threshold $[0, 77]$. This movement could be either reversion to the mean or a late response to the notch which was placed in 2009 at the 70 *reais* level. The movements to other parts of the scheduled are likely unrelated to be unrelated to the schedule and are capturing the changes in income distribution. These movements are less prominent.

The other three Panels of Figure 32a plot the movements out of the second (upper right), third (lower left) and fourth quartiles (lower right). There is substitantial movement from the third to the second quartile and to a lesser extent from the second to the third quartile. Most of the households in the fourth quartile persist in it.

Figure 32b plots the same shares for households with children above their new eligibility threshold (154 *reais*). There is substantion movement from all quartiles to the interval below 154. This could be again interpreted as a late response to the notch (the second



(a) Share of Households without Children in Each Quartile Above 77



(b) Share of Households Entering with Income in $(140, 154]$

Figure 32: Share of Households without Children in Each Quartile Above 77

notch was also placed at 140 in September of 2009). There is also some movent from households in the third to the second quartile.

These patterns indicate some movement from the third to the second quartile for all households and from the second to the third quartile among households without children above 77.

A.8 Proof of Lemma 1

Under Assumption 1, $MRS(n_L, m, c, y) > MRS(n_H, m, c, y)$ for any $n_L < n_H$, m, c and y. Also, under Assumption 2 $MRS_y(n, m, c, y) > 0$ for any (n, m). So it has to be the case that $MRS(n_L, m, c, y_L) = MRS(n_H, m, c, y_H)$ for some $y_H > y_L$.

A.9 Isoelastic Utility and Assumption 3

This appendix shows that the isoelastic utility satisfies assumption 3 if preferences are heterogenous in ability type n and elasticities m.

Proof. The utility of an agent (n, m) choosing consumption c and income y is:

$$u(c, y; n, m) = c - \frac{1}{1 + \frac{1}{m}} \left(\frac{y}{n}\right)^{1+\frac{1}{m}}$$

To evaluate the shape of the $Convex(m; B(\cdot), y)$ function, note that $u_c(c, y; n, m) = 1$, $u_{cy}(\cdot) = 0$, $u_y(\cdot) = \frac{1}{n}\left(\frac{y}{n}\right)^{\frac{1}{m}}$, $u_{cc}(\cdot) = 0$ and $u_{yy}(\cdot) = \frac{1}{n^2 m}\left(\frac{y}{n}\right)^{\frac{1-m}{m}}$.

The definition of MRS implies that that $n(m; y, MRS) = \left(\frac{y}{MRS^m}\right)^{\frac{1}{1+m}}$. Hence the convexity function is given by:

$$\begin{aligned} Convex(m; y, c, MRS) &= -u_{yy}\left(c, y; \left(\frac{y}{MRS^m}\right)^{\frac{1}{1+m}}, m\right) \\ &= \frac{MRS}{ym}, \end{aligned}$$

which goes from ∞ to 0 when m moves from 0 to ∞ for any $MRS \geq 0$ and allocation $(y, c) \in \mathbb{R}_+^2$. □

A.10 Proof of Propositions 1

Consider the welfare function (7) in terms of the primitive utilities $u(c,y;n,m,r)$:

$$W\Big(B(\cdot)\Big) = \int\int\int \Bigg\{ G\Bigg(u\Big(y(n,m,r) + B\big(y(n,m,r)\big), y(n,m,r); n,m,r\Big)\Bigg) \\ -\lambda B\big(y(n,m,r)\big)\Bigg\} dF(n,m,r).$$

Note that because of assumption 1, $y(n,m,r)$[28] is increasing in n for any (m,r).[29] Hence one can define $n(m,r,y)$ as the inverse of $y(n,m,r)$ with respect to the first argument. It is useful to write the first term of the welfare explicitly in terms of the heterogeneity parameters (m,r) and income level y. By transformation of variables $f_{NMR}(n,m,r) = f_{YM}(y(n,m,r),m,r)\frac{dy}{dn}$ and therefore the welfare function can be rewritten as

$$= \int_0^\infty \int_0^\infty \int_0^{\bar{m}_{z,r}} \left[G\Bigg(u\Big(z + B(z), z; n(m,r,z), m, r\Big)\Bigg) - \lambda B(z)\right] f_{YMR}(z,m,r) dm dr dz \\ = \int_0^\infty \left(\int_0^\infty \int_0^{\bar{m}_{z,r}} G(\cdot) f_{M|RY}(m|r,z) dm f_{R|Y}(r|z) dr - \lambda B(z)\right) h(z) dz.$$

where $m_{z,r}$ is the elasticity type such that $(n(\bar{m}_z,r,z),\bar{m}_z,r)$ is indifferent between that z and t_0. Assumptions 3 and 4 ensure that such type exists for all $z > t_0$.

Consider a perturbation I on the transfer to the poor with income up to t_0 as in Figure 13a. Let $m(I,r)$ be defined such that $(n(m(I,r),r,z),m(I,r),r)$ is the indifferent type located at z with attention r under the perturbed schedule. Note that $m(I,r) < \bar{m}_{z,r}$ because of assumption 3 and the fact that $y(n(m(I,r),r,z),m(I,r),r) = z$ under the schedule before the perturbation. It is useful to rewrite welfare as:

$$= \int_0^\infty \Bigg(\int_0^{m(I,r)} G(\cdot) - \lambda B\Big(y(n(m,r,z),r,m)\Big) f_{M|RY}(m|r,z) dm f_{R|Y}(r|z) dr \\ + \int_{m(I,r)}^{\bar{m}_{z,r}} G(\cdot) - \lambda B\Big(y(n(m,r,z),m,r)\Big) f_{M|RY}(m|r,z) dm f_{R|Y}(r|z) dr \Bigg) dH(z)$$

[28] All the terms are also a function of the transfer schedule, but I suppress $B(\cdot)$ to save notation.

[29] The one exception is at the threshold, at which point income is constant with respect to type. This only matters for the effect discussed in footnote 20 which is not relevant empirically.

In what follows, I take the partial derivative of this welfare function with respect to I and apply the Leibnitz rule:

$$\frac{\partial W}{\partial I} = \lambda \int_0^{t_0} \left(g(z) - 1 - \frac{\lambda B'(z)}{1 + B'(z)} \eta(z) \right) dH(z)$$
$$+\lambda I \int_{t_0}^{\infty} \int_0^{\infty} \underset{I \to 0}{lim} f_{M|RY}(m(I,r)|r,z) \frac{dm(I,r)}{dI} dF(r|z) dH(z).$$

Where $\eta(z) = \int_0^{\infty} \int_0^{\bar{m}_z} \frac{\partial y}{\partial I}\Big|_z dF(m,r|z)[1 + B'(z)]$ is the income effect parameter at income level z.[30] The partial derivative of the first term include the usual welfare, mechanical and income effects. Note that because $B'(z) = 0$ for all $z < t$, the income effect will vanish. The partial derivative of the integrand in the second term also vanishes as the interval of integration goes to zero as $I \to 0$. The derivative of the limit of the integrand with respect to I does not disappear for households at income levels $z > t_0$ because $y(n(m,r,z),m,r) = z$ for $m \in [0, m(I,r))$ and $y(n(m,r,z),m,r) = t_0$ for $m \in (m(I,r), \bar{m}_{z,r})$. There is a small set of jumpers and the effect of this jump on the utility of the jumpers is also small, since they were initially indifferent. By the envelope theorem, the effect on welfare is second order. Hence, this second term captures the jumping effects from each income level above the eligibility threshold $z > t_0$ on the budget of the government valued at rate λ.

This last jumping term can be rewritten as follows:

$$\int_0^{\infty} \underset{b \to 0}{lim} f_{M|RY}(m(I,r)|r,z) \frac{dm(I,r)}{dI} dF(r|z) = \int_0^{\infty} \underset{I \to 0}{lim} \frac{dF_{M|RY}(m(I,r)|r,z)}{dm} \frac{dm(I,r)}{dI} dF(r|z) =$$
$$-\frac{\partial P(y_a = t_0 | y_b = y)}{\partial I}$$

So the partial derivative becomes:

$$\frac{\partial W}{\partial I} = \lambda \int_0^{t} \Big(g(z) - 1 \Big) dH(z) - \lambda I \int_t^{\infty} \frac{\partial P(y_a = t_0 | y_b = y)}{\partial I} dH(z). \tag{12}$$

In order to take the partial derivative of the welfare function with respect to the threshold

[30] Also remember that $g(z) \equiv \frac{1}{\lambda} \int \int G'(u) \left(u_c \left(1 + \frac{\partial c}{\partial I} \right) + u_y \frac{\partial y}{\partial I} \right) dF(m,r|z)$ is the average social marginal value of consumption for tax payers with income z expressed in terms of value of public funds.

perturbation t, it is useful to rewrite equation (7) as:

$$W(B(\cdot)) = \int_0^{t_0+t} \left(\int_0^\infty \int_0^{\bar{m}_z} G(\cdot) dF(m,r|z) - \lambda I_0 \right) dH(z)$$
$$+ \int_{t_0+t}^\infty \int_0^\infty \int_0^{\bar{m}_z} \left(G(\cdot) - \lambda B\Big(y(n(m,r,z),m,r)\Big)\right) dF(m,r|z) dH(z),$$

since $B(z) = I_0$ for any $z \leq t$. Noting that only inattentive households $r = 0$ will be at $t_0 + dt$ and applying the Leibiniz rule the derivative becomes:[31]

$$\frac{\partial W}{\partial t} = \lambda I_0(\bar{g}(t_0) - 1)h(t_0) - \lambda \int_{t_0}^\infty \frac{\partial}{\partial t} \int_0^\infty \int_0^{\bar{m}_{z,r}} B\Big(y\big(n(z,m,r),m,r\big)\Big) dF(m,r|z) dH(z),$$

where $\bar{g}(t_0) = \int_{t_0}^{t_0+I_0} \int_0^\infty \frac{G'\Big(u\big(c,t_0;n(t_0,m,0),m,0\big)\Big)u_c(\cdot)}{\lambda I_0} dF_{M|RY}(m|0,t_0)dc$.[32]

Note that even though $B(z) = 0$ for all $z > t$, the last derivative does not vanish because of jumping effects. Using the same argument as above, I get that:

$$\frac{\partial W}{\partial t} = \lambda I_0(\bar{g}(t_0) - 1)h(t) - \lambda I \int_t^\infty \frac{\partial P(y_a = t_0 | y_b = y)}{\partial t} dH(z). \tag{13}$$

Substituting equations (12) and (18) in the total derivative of the welfare function $dW = \frac{\partial W}{\partial I} dI + \frac{\partial W}{\partial t} dt$:

$$dW = \lambda \Bigg\{ (\bar{g} - 1)H(t_0)dI + I_0\big(\bar{g}(t_0) - 1\big)h(t_0)dt$$
$$- \int_{t_0}^\infty \left(\frac{\partial P(y_a = t_0|y_b = z)}{\partial I} dI + \frac{\partial P(y_a = t_0|y_b = z)}{\partial t} dt \right) dH(z) \Bigg\}.$$

Finally, remembering that $share^J(z) = \frac{\partial P(y_a = t_0|y_b = z)}{\partial I} dI + \frac{\partial P(y_a = t_0|y_b = z)}{\partial t} dt$, the expression in Proposition 1 follows.

[31] As mentioned above, there should be an extra term accounting for the effect of the perturbation on the utility of households that moved from t_0 to $t_0 + t$. Since the number of such households in the data is small (3% of the households bunching), I refrain from including this term here here.

[32] Note that for inattentive households $r = 0$, $\frac{\partial y}{\partial t} = 0$.

A.11 Incorporating Misreporting

In this section, I allow households to differ also in their propensity to evade l so that $(n, m, r, l) \sim F(\cdot)$. They have preferences over consumption c, real income y and misreported income $\tilde{y}$, so that total income y equals the sum of mis-reported $\tilde{y}$ and reported income $\bar{y}$, i.e. $y = \tilde{y} + \bar{y}$. These preferences have a utility representation $u(c, y, \tilde{y}; n, m, l)$. Households face a utility cost of misreporting analogous to a labor supply cost. Higher propensity to evade types are able to hide income at a lower cost.

The benefit schedule $B(\cdot)$ is set on the households' reported income $\bar{y}$, so that the household problem is:

$$\max_{c,y,\bar{y}} \{u(c, \bar{y} + \tilde{y}, \tilde{y}; n, m, l)\} \; s.t. \tag{14}$$

$$c \leq \bar{y} + \tilde{y} + rB(\bar{y}).$$

The demand for misreported income of type (n, m, r, l) under the transfer schedule $B(z) = (I_0 + I) * 1(z \leq t_0 + t)$ and at reported income $\bar{y}$ is:

$$\tilde{y}\Big(\bar{y}; n, m, r, l, t, I\Big) = \arg\max_{\tilde{y}} u\Big(\bar{y} + \tilde{y} + r(I_0 + I)1(\bar{y} \leq t_0 + t), \bar{y} + \tilde{y}, \tilde{y}; n, m, l\Big).$$

Note that the first order condition that defines $\tilde{y}(\bar{y})$[33] is $u_c + u_y + u_{\tilde{y}}$, which is not differentiable with respect to $\bar{y}$ at the threshold t_0, because of income effects and the fact that $B(\cdot)$ is discontinuous at that point. For tractability, I assume away income effects

Assumption 6. *There are no income effects so that that utility is quasi-linear in consumption:*

$$u(c, y, \tilde{y}; n, m, l) = c - v(y, \tilde{y}, n, m, l),$$

Where $v(\cdot)$ is a function that captures labor-supply and misreporting utility costs.

Without income effects, the hidden income chosen in equilibrium does not depend on the schedule, as I show next.

Lemma 2. *For any type (n, m, r, l) and reported income $\bar{y}$:*

$$\frac{\partial \tilde{y}(\bar{y}; t, I)}{\partial t} = \frac{\partial \tilde{y}(\bar{y}; t, I)}{\partial I} = 0.$$

[33] In order to save notation, we will denote $\tilde{y}\Big(\bar{y}; n, m, B(\cdot)\Big)$ as $\tilde{y}(\bar{y})$ from now on, unless it is not clear to which types or schedules this supply function is referring to.

Proof. The household first order condition with respect to $\tilde{y}$ is $1 + u_y + u_{\tilde{y}} = 0$. Applying the implicit function theorem to this equation and noting that $u_{yc} = u_{\tilde{y}c} = 0$ we get the result. Note that this is true even at $\bar{y} = t_0$ because the jump in consumption at this point does not affect the misreporting choices. □

This definition allows us to rewrite the household problem as

$$\max_{c,\bar{y}} \tilde{u}(c, \bar{y}; n, m, l) \ s.t. \tag{15}$$
$$c \leq \tilde{y}(\bar{y}) + \bar{y} + rB(\bar{y}).$$

Where $\tilde{u}(c, \bar{y}; n, m, l) = c - v\Big(\tilde{y}(\bar{y}) + \bar{y}, \tilde{y}(\bar{y}); n, m, l\Big)$ is the indirect utility as function of consumption and reported income.

The following assumption guarantees that for any (m, r, l), the reported income will be increasing in ability type n. This corresponds to a single-crossing condition in of indifference curves in the $(\bar{y}, c)$ plane.[34]

Assumption 7. *For any (m, l):*

$$\frac{(u_y + u_{\tilde{y}})u_{yy}}{u_{yy} + u_{\tilde{y}\tilde{y}} + 2u_{\tilde{y}y}} - u_y$$

Is increasing in n.

Lemma 3. *Under assumption 7, $\bar{y}(n, m, r, l)$ is increasing in n for any (m, r, l).*

Proof. Note that $\tilde{u}_{\bar{y}} = u_y + (u_y + u_{\tilde{y}})\tilde{y}'(\bar{y})$. Applying the implicit function theorem on the first order condition that defines $\tilde{y}(\bar{y})$, $1 + u_y + u_{\tilde{y}} = 0$, yields

$$\tilde{y}'(\bar{y}) = -\frac{u_{yy}}{u_{yy} + u_{\tilde{y}\tilde{y}} + 2u_{\tilde{y}y}},$$

So that:

$$\tilde{u}_{\bar{y}} = u_y - \frac{(u_y + u_{\tilde{y}})u_{yy}}{u_{yy} + u_{\tilde{y}\tilde{y}} + 2u_{\tilde{y}y}},$$

Which is increasing in n for any (m, l) under assumption 7. Note that this is independent of r.

[34]Generally, this condition would depend on the transfer/tax schedule. I am using the fact that $B'(\cdot) = 0$ for every z, and that under Assumption 6, changes in virtual income will not affect choices.

Towards a contradiction assume that $n_L > n_H$ but $\bar{y}_H > \bar{y}_L$ where $\bar{y}_x = \bar{y}(n_x, m, r, l)$. By revealed preference:

$$\tilde{u}\Big(c_L(\bar{y}_L), \bar{y}_L; n_L, m, l\Big) \geq \tilde{u}\Big(c_L(\bar{y}_H), \bar{y}_H; n_L, m, l\Big) \text{ and}$$
$$\tilde{u}\Big(c_L(\bar{y}_H), \bar{y}_H; n_H, m, l\Big) \geq \tilde{u}\Big(c_H(\bar{y}_L), \bar{y}_L; n_H, m, l\Big),$$

where $c_x(\bar{y}) = \tilde{y}(\bar{y}, n_x, m, l) + \bar{y} + B(\bar{y})$. By continuity of $u(\cdot)$ in n and c and the continuity of $\tilde{y}$ (and therefore of $c(\cdot)$) in n, there exists a n_i such that:

$$\tilde{u}\Big(c_i(\bar{y}_L), \bar{y}_L; n_i, m, l\Big) = \tilde{u}\Big(c_i(\bar{y}_H), \bar{y}_H; n_i, m, l\Big).$$

Type (n_i, m, r, l) defines the path $(c(s), \bar{y}(s))$ from $s = 0$, $(c_L, \bar{y}_L)$, to $s = 1$, $(c_H, \bar{y}_H)$.

Without loss of generality assume that $n_i > n_H$ (otherwise the argument is symmetric for $n_i < n_L$). Then:

$$\frac{d\tilde{u}(c(s), \bar{y}(s))}{ds} = 0 = c'(s) + \tilde{u}_{\bar{y}}(c(s), \bar{y}(s); n_i, m, l)\bar{y}'(s) > c'(s) + \tilde{u}_{\bar{y}}(c(s), \bar{y}(s); n_H, m, l)\bar{y}'(s)$$

Where the inequality follows from the fact that $\tilde{u}_{\bar{y}}$ is decreasing in n and that $\bar{y}'(s) > 0$. Integrating both sides:

$$0 > \tilde{u}(c_H, \bar{y}_H; n_H, m, l) - \tilde{u}(c_L, \bar{y}_L; n_H, m, l).$$

Which yields a contradiction. □

The following assumption ensures that $\tilde{u}_{\bar{y}}$ is increasing in $\bar{y}$, i.e. the marginal cost of reporting an extra unit of income optimally is decreasing with the reported income.

Assumption 8. *For any (m, l), $u_y - \frac{(u_y + u_{\tilde{y}})u_{yy}}{u_{yy} + u_{\tilde{y}\tilde{y}} + 2u_{\tilde{y}y}}$ is increasing in $\bar{y}$ or:*

$$u_{yy} - \frac{[(u_{yy} + u_{\tilde{y}y})u_{yy} + (u_y + u_{\tilde{y}})u_{yy}](u_{yy} + u_{\tilde{y}\tilde{y}} + u_{y\tilde{y}}) - (u_{yyy} + u_{\tilde{y}\tilde{y}y} + u_{y\tilde{y}y})(u_y + u_{\tilde{y}})u_{yy}}{(u_{yy} + u_{\tilde{y}\tilde{y}} + u_{y\tilde{y}})^2} > 0.$$

Next I show that for every elasticity, attention and propensity to evade type (m, r, l) and marginal rate of substitution between consumption and reported income $\tilde{MRS}$, the reported income chosen in equilibrium is increasing in ability.

Lemma 4. *Under Assumptions 6, 7 and 8, $\bar{y}(n, m, r, l; c, \tilde{MRS})$ is increasing n.*

Proof. Under Assumptions 6 and 7, $\tilde{u}_{\bar{y}}(c, \bar{y}; n_L, m, l) < \tilde{u}_{\bar{y}}(c, \bar{y}; n_H, m, l)$ for any $n_L < n_H$,

m, l, c and $\bar{y}$. Suppose that $\tilde{u}_{\bar{y}}(c, \bar{y}_L; n_L, m, l) = \tilde{MRS}$. under Assumption 8, it has to be the case that $\tilde{MRS} == \tilde{u}_{\bar{y}}(c, \bar{y}_H; n_H, m, l)$ for some $\bar{y}_H > \bar{y}_L$. □

Let $n(\bar{y}, m, l, \tilde{MRS})$ be the inverse of $\bar{y}(n, m, l; c, \tilde{MRS})$ with respect to the first argument.

Definition 2. *The convexity of the indifference curves in the $(\bar{y}, c)$ plane at a particular allocation and marginal rate of substitution for each propensity to evade and elasticity type (m, l) is:*

$$Convex(m, l; c, \bar{y}, \tilde{MRS}) \equiv -\tilde{u}_{yy}\Big(c, \bar{y}; n(\bar{y}, c; m, l, \tilde{MRS}), m, l\Big).$$

The next assumption guarantees that this convexity is decreasing with the elasticity type.

Assumption 9. *For any allocation $(\bar{y}, c) \in \mathbb{R}^2_+$, $\tilde{MRS}$ and l, $Convex(m, l; c, \bar{y}, \tilde{MRS})$ is decreasing and continuous in m and:*

$$\lim_{m \to 0} Convex(m, l; c, \bar{y}, \tilde{MRS}) = \infty \text{ while } \lim_{m \to \infty} Convex(m, l; c, \bar{y}, \tilde{MRS}) = 0.$$

Let $\bar{H}(\cdot)$ be the reported income cdf of applicants under the observed schedule. Welfare under the schedule $(t_0 + t, I_0 + I)$ is a function of the perturbations t and I:

$$W(t_0 + t, I_0 + I) = \int_0^\infty \int\int\int G\Big(\tilde{u}\big(c(z; n, m, l), z; n(z, m, r, l), m, l\big)\Big) dF_{MRL|\bar{Y}}(m, r, l|z) d\bar{H}(z) \\ -\lambda \int_0^{t_0+t} (I_0 + I) d\bar{H}(z), \tag{16}$$

where $c(z; n, m, l) = z + \tilde{y}(z; n, m, l) + (I_0 + I) * 1(z \leq t_0 + t)$[35] is the consumption chosen by households with reported income z, $G(\cdot)$ is an increasing and concave function that captures the redistributive motives of the planner, $F_{MRL|\bar{Y}}(\cdot)$ is the joint distribution of elasticity, attention and propensity to evade types conditional on reported income, and λ is the marginal cost of public funds.

Let $g(z) \equiv \frac{1}{\lambda} \int\int\int G'(\tilde{u}) dF(m, r, l|z)$[36] be the average social marginal value of consumption for applicants with reported income z expressed in terms of the marginal value of public funds.

Let $\bar{g} = \frac{\int_0^{t_0} g(z) d\bar{H}(z)}{\bar{H}(t_0)}$ be the average social marginal value of consumption among the el-

[35]Note that although $\tilde{y}(z; n, m, l)$ does not depend on r because of Assumption 6, $n(z; n, m, l, r)$ does. For a given income levels above the threshold $z > t_0 + t$, only households with lower attention to the schedule will locate their instead of jumping to the threshold.

[36]Note that because of Assumption 6, $\frac{\partial \tilde{y}}{\partial I} = 0$, so that there is no behavioral wedge.

igible poor; $\bar{g}(t_0) = \int\int\int_{t_0}^{t_0+I_0} \frac{G'(\tilde{u}(\bar{y}(t_0;n,m,l)+\bar{c},t_0;n(t_0,m,0,l),m,l))}{\lambda I_0} d\bar{c}dF_{M,L|RY}(m,l|0,t_0)$ be the average social marginal value of consumption for households at (t_0, t_0+dt) between reported consumption levels t_0 and $t_0 + I_0$; and $\overline{share}^J(dt, dI) = \frac{\int_{t_0}^{\infty} \overline{share}^J(dt,dI;z)dH(z)}{1-H(t_0)}$ be the average share of jumpers across all income levels beyond the threshold. The following proposition characterizes the welfare effect for a reform that perturbs the threshold and the transfer by infinitesimal amounts (dt, dI).

Proposition 2. *Under assumptions 7, 8, 9, and 4, an infinitesimal reform that changes the transfer given to the poor by dI and the eligibility threshold by dt impacts welfare by:*

$$dW = \lambda\Big((\bar{g}-1)\bar{H}(t_0)dI + \big(\bar{g}(t_0)-1\big)I_0\bar{h}(t_0)dt - I_0\big(1-\bar{H}(t_0)\big)\overline{share}^J(dt, dI)\Big).$$

Proof. Consider the welfare function (7) in terms of the reduced-form utilities $\tilde{u}(c, \bar{y}; n, m, r, l)$:

$$W\Big(B(\cdot)\Big) = \int\int\int\int \Bigg\{G\Bigg(\tilde{u}\Big(c(n,m,r,l), \bar{y}(n,m,r,l); n,m,r,l\Big)\Bigg) \\ -\lambda B\big(\bar{y}(n,m,r,l)\big)\Bigg\}dF(n,m,r,l),$$

where $c(n,m,r,l)$ and $\bar{y}(n,m,r,l)$ are the consumption and reported income chosen in equilibrium by type (n,m,r,l) under the perturbed schedule $B(z) = 1(z < t_0 + t) * (I_0 + I)$. Note that from Lemma 3, $\bar{y}(n,m,r,l)$[37] is increasing in n for any (m,r,l). Hence one can define $n(\bar{y}; m,r,l)$ as the inverse of $\bar{y}(n,m,r,l)$ with respect to the first argument. It is useful to write the first term of the welfare explicitly in terms of the heterogeneity parameters (m,r,l) and reported income level $\bar{y}$. By transformation of variables $f_{NMR}(n,m,r,l) = f_{YM}(\bar{y}(n,m,r,l), m,r,l)\frac{d\bar{y}}{dn}$ and therefore the welfare function can be rewritten as

$$\int\int\int\int_0^{\bar{m}_{z,r,l}} \Bigg[G\Bigg(\tilde{u}\Big(c(z;n,m,r,l), z; n(z;m,r,l), m,r,l\Big)\Bigg) - \lambda B(z)\Bigg] f_{YMRL}(z,m,r,l)dmdrdld\bar{H}(z) \\ = \int\Bigg(\int\int\int_0^{\bar{m}_{z,r,l}} G(\cdot)f_{M|RLY}(m|r,l,z)dmf_{RL|Y}(r,l|z)drdl - \lambda B(z)\Bigg)d\bar{H}(z).$$

where $m_{z,r,l}$ is the elasticity type such that $(n(\bar{m}_z, r, l, z), \bar{m}_z, r, l)$ is indifferent between reported income $\bar{y} = z$ and $\bar{y} = t_0$. Assumptions 9 and 4 ensure that such type exists for all $z > t_0$.

[37] All the terms are also a function of the transfer schedule, but I suppress $B(\cdot)$ to save notation.

Consider a perturbation I on the transfer to the poor with income up to t_0 as in Figure 13a. Let $m(I,r,l)$ be defined such that $(n(m(I,r,l),r,l,z),m(I,r,l),r,l)$ is the household located at $\bar{y}=z$ with type (r,l) under the perturbed schedule. Note that $m(I,r,l)<\bar{m}_{z,r,l}$ because of Assumption 9 and Lemma 2. It is useful to rewrite welfare as:

$$=\int_0^\infty\Bigg(\int\int\int_0^{m(I,r,l)} G(\cdot)-\lambda B\Big(\bar{y}(n(m,r,z),m,r,l)\Big)f_{M|RLY}(m|r,l,z)dmf_{RL|Y}(r,l|z)drdl$$

$$+\int\int\int_{m(I,r,l)}^{\bar{m}_{z,r,l}} G(\cdot)-\lambda B\Big(\bar{y}(n(m,r,z),m,r,l)\Big)f_{M|RLY}(m|r,l,z)dmf_{RL|Y}(r,l|z)drdl\Bigg)dH(z)$$

In what follows, I take the partial derivative of this welfare function with respect to I and apply the Leibnitz rule:

$$\frac{\partial W}{\partial I}=\lambda\int_0^{t_0}(g(z)-1)\,d\bar{H}(z)+\lambda I\int_{t_0}^\infty\int\int\lim_{I\to 0}f_{M|RY}(m(I,r,l)|r,l,z)\frac{dm(I,r,l)}{dI}dF(r,l|z)d\bar{H}(z).$$

The partial derivative of the first term include the usual welfare and mechanical effects. The partial derivative of the integrand in the second term also vanishes as the interval of integration goes to zero as $I\to 0$. The derivative of the limit of the integrand with respect to I does not disappear for households at income levels $z>t_0$ because $\bar{y}(n(m,r,l,z),m,r,l)=z$ for $m\in[0,m(I,r,l))$ and $\bar{y}(n(m,r,l,z),m,r,l)=t_0$ for $m\in(m(I,r,l),\bar{m}_{z,r,l})$. There is a small set of jumpers and the effect of this jump on the utility of the jumpers is also small, since they were initially indifferent. By the envelope theorem, the effect on welfare is second order. Hence, this second term captures the jumping effects from each reported income level above the eligibility threshold $z>t_0$ on the budget of the government valued at rate λ.

This last jumping term can be rewritten as follows:

$$\int\int\lim_{b\to 0}f_{M|RLY}(m(I,r,l)|r,l,z)\frac{dm(I,r,l)}{dI}dF(r,l|z)=$$
$$\int\int\lim_{I\to 0}\frac{dF_{M|RLY}(m(I,r,l)|r,l,z)}{dm}\frac{dm(I,r,l)}{dI}dF(r,l|z)=-\frac{\partial P(\bar{y}_a=t_0|\bar{y}_b=z)}{\partial I}$$

So the partial derivative becomes:

$$\frac{\partial W}{\partial I}=\lambda\int_0^t\Big(g(z)-1\Big)d\bar{H}(z)-\lambda I\int_t^\infty\frac{\partial P(\bar{y}_a=t_0|\bar{y}_b=z)}{\partial I}d\bar{H}(z).\tag{17}$$

In order to take the partial derivative of the welfare function with respect to the threshold t_0, it is useful to rewrite equation (16) as:

$$W(B(\cdot)) = \int_0^{t_0+t} \left(\int\int\int_0^{\bar{m}_z} G(\cdot) dF(m,r,l|z) - \lambda I_0 \right) d\bar{H}(z)$$
$$+ \int_{t_0+t}^{\infty} \int\int\int_0^{\bar{m}_z} \left(G(\cdot) - \lambda B\Big(\bar{y}(n(m,r,l,z),m,r,l)\Big) \right) dF(m,r,l|z) d\bar{H}(z),$$

since $B(z) = I_0$ for any $z \leq t_0 + t$. Noting that only inattentive households $r = 0$ will be at $t_0 + dt$ and applying the Leibiniz rule the derivative becomes:

$$\frac{\partial W}{\partial t} = \lambda I_0 (\bar{g}(t_0) - 1)\bar{h}(t_0) - \lambda \int_{t_0}^{\infty} \frac{\partial}{\partial t} \int\int\int_0^{\bar{m}_{z,r}} B\Big(\bar{y}\big(n(z,m,r,l),m,r,l\big)\Big) dF(m,r,l|z) d\bar{H}(z),$$

where $\bar{g}(t_0) = \int\int\int_{t_0}^{t_0+I_0} \frac{G'\Big(\tilde{u}\big(\bar{y}(t_0;n,m,l)+\bar{c},t_0;n(t_0,m,0,l),m,0,l\big)\Big)}{\lambda I_0} d\bar{c} dF_{ML|RY}(ml|0,t_0)$.

Note that even though $B(z) = 0$ for all $z > t$, the last derivative does not vanish because of jumping effects. Using the same argument as above, I get that:

$$\frac{\partial W}{\partial t} = \lambda I_0 (\bar{g}(t_0) - 1)\bar{h}(t) - \lambda I \int_t^{\infty} \frac{\partial P(\bar{y}_a = t_0 | \bar{y}_b = z)}{\partial t} d\bar{H}(z). \tag{18}$$

Substituting equations (12) and (18) in the total derivative of the welfare function $dW = \frac{\partial W}{\partial I} dI + \frac{\partial W}{\partial t} dt$:

$$dW = \lambda \Bigg\{ (\bar{g} - 1)\bar{H}(t_0) dI + I_0 \big(\bar{g}(t_0) - 1\big)\bar{h}(t_0) dt$$
$$- \int_{t_0}^{\infty} \left(\frac{\partial P(\bar{y}_a = t_0 | \bar{y}_b = z)}{\partial I} dI + \frac{\partial P(\bar{y}_a = t_0 | \bar{y}_b = z)}{\partial t} dt \right) d\bar{H}(z) \Bigg\}.$$

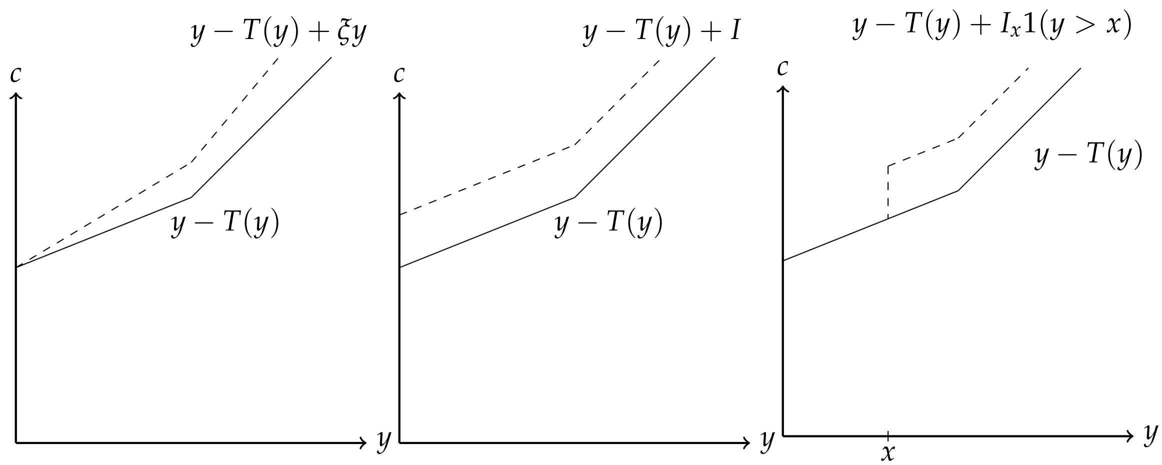
Finally, remembering that $\overline{share}^J(z) = \frac{\partial P(\bar{y}_a = t_0 | \bar{y}_b = z)}{\partial I} dI + \frac{\partial P(\bar{y}_a = t_0 | \bar{y}_b = z)}{\partial t} dt$, the expression in Proposition 2 follows. $\square$

A.12 Optimal Taxation with Jumps in Labor Supply

I show here that the optimal tax formulas in Dodds (2017) can be rewritten in term of recoverable elasticities. First, I define the elasticity concepts and then characterize the optimum. The derivations are in Appendix A.13.

Let $T(\cdot)$ denote the tax as a function of income. The solution of the household problem is $y(n,m) \equiv arg\max_y u(y - T(y), y; n, m)$. Let ξ, I and I_x define perturbations in the transfer schedule's slope, intercept, and levels before x, respectively. Figure 33 illustrate these perturbation concepts. The solution to this perturbed schedule is:

$$y(\xi, I, I_x; n, m) \equiv arg\max_y u\Big(y - T(y) + \xi y + I + I_x 1(y < x), y; n, m\Big).$$



(a) Slope Perturbation (b) Intercept Perturbation (c) x-Level Perturbation

Figure 33: Pertubation Definitions

Let y_b be defined as the income chosen by a random type (n, m) before a given perturbation and Y_a the income chosen by this same type after the same perturbation:

$$y_b \in \arg\max_y U\Big(y - T(y), y; n, m\Big)$$
$$y_a \in \arg\max_y U\Big(y - T(y) + \xi y + I + I_x 1(y > x), y; n, m\Big)$$

The uncompensated income elasticity and the income effect parameter are defined as the expected elasticity for individuals located at each income level [38] y i.e.

$$\varepsilon^u(y) = E\left(\frac{\partial y_a}{\partial \xi}\bigg|_{\xi=I=I_x=0} y_b = y\right)\frac{1-T'(y)}{y} \; and$$
$$\eta(y) = E\left(\frac{\partial y_a}{\partial I}\bigg|_{\xi=I=I_x=0} y_b = y\right)\Big(1-T'(y)\Big),$$

respectively. The compensated elasticity is defined by the Slutsky equation $\varepsilon^c(y) = \varepsilon^u(y) + \eta(y)$.

The jumping semi-elasticities in for agent at income level y with respect to a change in the transfer at x is defined as:

$$\varepsilon^j(y,x) = \frac{\partial P(y_a \neq y | y_b = y)}{\partial I_x}.$$

The planner problem is

$$\max_{T(\cdot)} \int_{n \in N} G\Big(u(c,y;n,m)\Big) dF(n,m) \; s.t.$$
$$\int_0^\infty T(y) dH(y) \geq 0$$
$$y(n,m) = \underset{y}{argmax} \; u(y - T(y), y; n, m) \; \forall n, m,$$

where $H(\cdot)$ is the income distribution and R is the revenue allocated to the transfer program.

The following proposition characterizes the optimal policy.

Proposition 3. *Under assumptions 1, 2, 3 and 4, the optimal transfer schedule is characterized by:*

$$\frac{T'_{opt}(y)}{1-T'_{opt}(y)}\varepsilon^c(y) y h(y) = \int_0^y \left(1 - \beta_z + \frac{T'_{opt}(z)}{1+T'_{opt}(z)}\eta(z)\right) dH(z)$$
$$+ \int_y^\infty [T(z_+) - T(z)]\varepsilon_J(z,y) dH(z), \; and$$
$$\int_0^\infty T_{opt}(y) dH(y) = 0.$$

[38] Although these elasticities concepts differ from the Saez (2001)'s formulation, they are standard in the more recent optimal tax models (see for instance, Scheuer and Werning (2016))

Where $\beta_z = \frac{G' u_c}{\lambda}$ is the social marginal value of consumption at income level z.

A.13 Optimal Tax Formula Derivation

The Lagrangian of the planner's problem is:

$$\mathcal{L} = \int \int \left\{ G\Big(u\big(y(n,m) - T(y(n,m)), y(n,m); n, m\big)\Big) + \lambda T\big(y(n,m)\big) \right\} dF(n,m)$$

Where λ is the Lagrangian multiplier on the planner's budget constraint ($\int T(z) dH(z) \geq 0$).

Note that because of assumption 1, $y(n,m)$ is increasing in n for any m and tax schedule $T(\cdot)$. Hence, I can define $n(m,y)$ as the inverse of $y(n,m)$ with respect to the first argument. It is useful to write the Lagrangian explicitly in terms of the heterogeneity parameters m and income level z. By transformation of variables $f_{NM}(n,m) = f_{YM}(y(n,m),m)\frac{dy}{dn}$ and therefore:

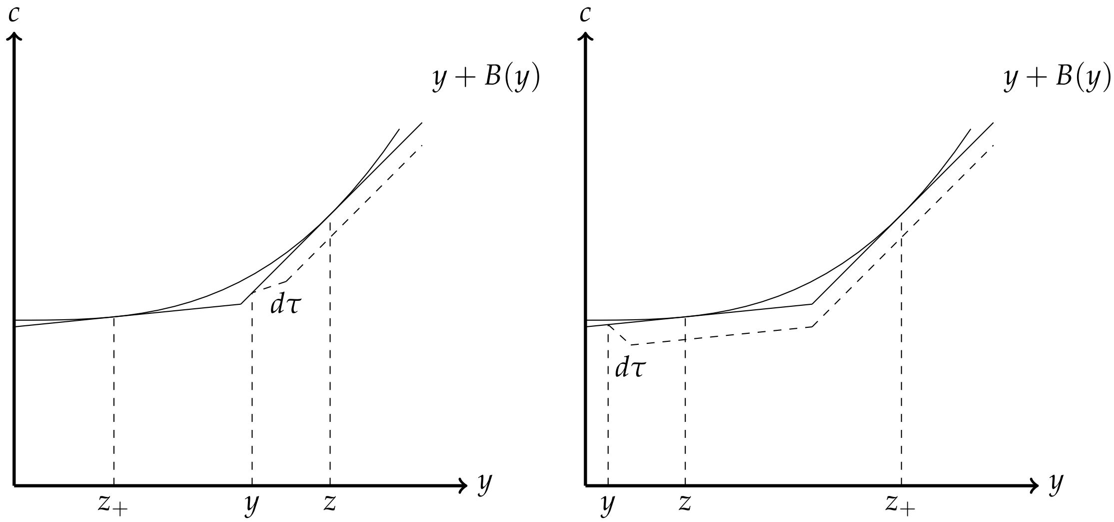
$$= \int_0^\infty \int_{-\infty}^{\bar{m}_z} \left\{ G\Big(u\big(z - T(z), z; n(m,z), m\big)\Big) + \lambda T(z) \right\} f_{YM}(z,m) dz dm$$
$$= \int_0^\infty \int_{-\infty}^{\bar{m}_z} \left\{ G\Big(u\big(z - T(z), z; n(m,z), m\big)\Big) + \lambda T(z) \right\} f_{M|Y}(m|z) dm h(z) dz.$$

Assumptions 3 and 4 ensure that for all z in a convex region of schedule there will be a type $(n(\bar{m}_z, z), \bar{m}_z)$ indifferent between that z and another income level z_+. For everyone located at an income level z outside of the convex schedule, $\bar{m}_z = \infty$, so that all those agents have a unique maximum at z.

Consider a perturbation τ on the slope of the schedule from y until $y + dy$ as in Figure 34. Let $m(\tau)$ be defined such that $(n(m(\tau), z), m(\tau))$ is the indifferent type located at z under the perturbed schedule. Note that $m(\tau) < \bar{m}_z$ because of assumption 3 and the fact that $\bar{m}_z$ prefers z_+ to z under the perturbed schedule. It is useful to rewrite the Lagrangian as:

$$= \int_0^\infty \int_{-\infty}^{m(\tau)} \left\{ G\Big(u\big(z - T(z), z; n(m,z), m\big)\Big) + \lambda T(z) \right\} f_{M|Y}(m|z) dm h(z) dz$$
$$+ \int_0^\infty \int_{m(\tau)}^{\bar{m}_z} \left\{ G\Big(u\big(z - T(z), z; n(m,z), m\big)\Big) + \lambda T(z) \right\} f_{M|Y}(m|z) dm h(z) dz.$$

In the first order condition of this Lagrangian with respect to τ, I apply the Leibnitz rule to the first two terms. The derivative of the integrand in the first term generates the usual



(a) Perturbation Above z_+

(b) Perturbation Below z_+

Figure 34: Perturbations under the Optimum

terms as in Saez (2001).

The derivative of the integrand in the second term vanishes as the interval of integration goes to zero as $d\tau \to 0$. The derivative of the limit of the integrand with respect to τ, however, does not disappear because $y(n(m,z),m) = z$ for all $m \in (-\infty, m(\tau))$ and $y(n(m,z),m) = z_+$ for all $m \in (m(\tau), \bar{m}_z)$. Since the envelope theorem guarantees that the effect of this jump on the utility of the agents (and therefore on welfare) is second order, one can write the first order condition as:[39]

$$\int_0^y \left(1 - \beta_z + \frac{T'_{opt}(z)}{1 - T'_{opt}(z)}\eta(z)\right) dH(z) - \frac{T'_{opt}(y)}{1 - T'_{opt}(y)}\varepsilon^c(y) y h(y)$$
$$+ \int_0^\infty [T(z) - T(z_+)] \lim_{\tau \to 0} f_{M|Y}(m(\tau)|z) \frac{dm(\tau)}{dI_y} h(z) dz = 0.$$

The first two terms include the traditional mechanical, income and elasticity effects. The second term is the effect on the constraint of positive transfers. The last term captures the jumping effects at each income level $z \in (\underline{y}, \infty)$ from z to z_+.

[39]I am dividing the first derivative of the Lagrangian with respect to τ by the product of the interval of the perturbation dy and the multiplier on the planner's constraint λ.

This last jumping term can be rewritten as follows:

$$\lim_{\tau \to 0} f_{M|Y}(m(\tau)|z) \frac{dm(\tau)}{dI_y} = \lim_{\tau \to 0} \frac{dF_{M|Y}(m(\tau)|z)}{dm} \frac{dm(\tau)}{dI_y} = -\frac{\partial P(y_a \neq y | y_b = y)}{\partial I_y} = -\varepsilon^j(z, y)$$

Note that the nature response of $P(y_a \neq z | y_b = z)$ to dI_y depends on the position of z_+ relative to the perturbation level y. If $z_+ < y$, this response $\varepsilon^j(z, y) = \frac{\partial P(y_a \neq z | y_b = z)}{\partial I_y}$ corresponds to the substitution jumping effect (see Figure 34a). If $z_+ > y$, the same response is equivalent to the jumping income effect (see Figure 34b). In either case, the jumping effect at each income level z above the perturbation is $[T(z_+) - T(z)]\varepsilon^j(z, y)$. So the first order condition becomes:

$$\int_0^y \left(1 - \beta_z + \frac{T'_{opt}(z)}{1 - T'_{opt}(z)} \eta(z)\right) dH(z) - \frac{T'_{opt}(y)}{1 - T'_{opt}(y)} \varepsilon^c(y) y h(y) + \int_y^\infty [T(z_+) - T(z)] \varepsilon^j(z, y) dH(z) = 0,$$

which is the first expression in Proposition 3. The last expression in the proposition guarantees that the planner will balance its budget.

A.14 Inference for $share^E$ estimates

This appendix presents the results in Section 5.3 in a regression framework analogous to the one at the end of Appendix A.6.

I focus on households that enter the BF program since January of 2012. To estimate the share of entrants at the new threshold because of the reform, once again the dataset is expanded so that each household i in month t appears twice. The first observations ($d = 0$) considers only entrants to the placebo interval $(77, 84]$ and is analogous to a control group. The second ($d = 1$) considers only the possibility of entrants to the affected interval $(70, 77]$ and is analogous to a treatment group. In this expanded dataset, I run a standard differences-in-differences.

$$Entrant_{itd} = \alpha dup_{itd} + \sum_{q=-8}^{+9} \beta_q Quarter\_q_{itd} + \gamma_t + u_{it}$$

Where $Entrant_{itd}$ is an indicator if household i had entered to either to $(77, 84]$ in the original observation ($d = 0$) or to $(70, 77]$ in the duplicated observations ($d = 1$) from January 2012 up to month t; dup_{itd} is an indicator if the observation is duplicated ($d = 1$); $Quarter\_q_{itd}$ is the interaction of dup_{itd} an indicator for t to belong in quarter q away from the month of the announcement (omitted interaction); and γ_t are the months fixed effects.

Figure 35 presents the coefficients β_qs. Under the identifying assumption that the share of households entering at $(70, 77]$ would remain parallel to the share of households entering to $(77, 84]$ in the absence of the reform, each coefficient for $q > 0$ indicates the effect of the reform up to quarter q away from the reform on the probability of entering at the affected interval $(70, 77]$. One cannot reject the hypothesis that the trends of these shares were the parallel before the reform, but the effects of the reform become significant at a 5% level three quarters after the announcement. The coefficient of interest is the effect of the reform on the share of jumpers up to the end of the period of analysis $\beta_9 = 0.003$ as in the reduced form analysis with t-statistic around 8.

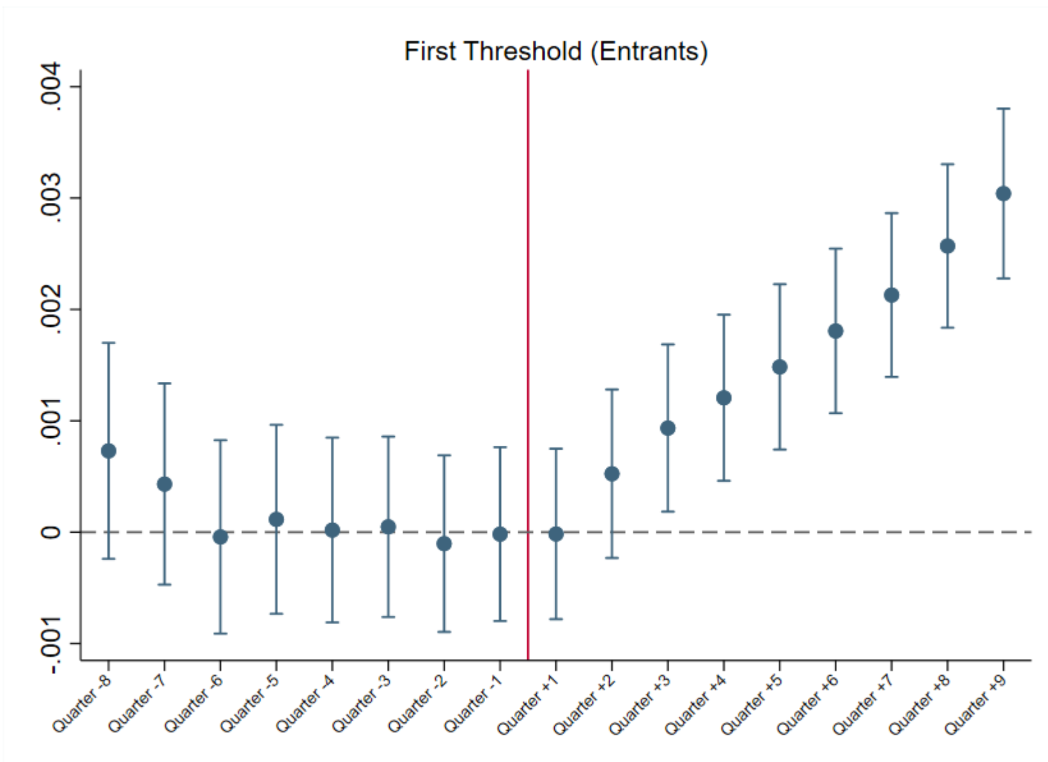


Figure 35: Difference and Differences Results for $share^{77}$

The analysis for the second threshold is straightforward because the control (households without children) and the treatment (households with children) groups are well defined.

The differences in differences specification is:

$$Entrant_{it} = \alpha Kids_{it} + \sum_{q=-8}^{+9} \beta_q Quarter\_q_{it} + \gamma_t + u_{it}$$

Where $Entrant_{it}$ is an indicator if household i had entered at $(140, 154]$ from January 2012 up to month t; $Kids_{it}$ is an indicator if the household i had children in month t; $Quarter\_q_{it}$ is the interaction of $Kids_{it}$ and an indicator for t to belong in quarter q away from the month of the announcement (omitted interaction); and γ_t are the months fixed effects.

Figure 36 presents the coefficients β_qs. Under the identifying assumption that the share of households with and without children entering at $(140, 154]$ would remain parallel in the absence of the reform, each coefficient for $q > 0$ indicates the effect of the reform up to quarter q away from the reform on the probability of a household entering at the new threshold. I cannot reject the hypothesis that the trends of these shares were parallel before the reform after the first quarter in the data,[40] but the effects of the reform become significant at a 5% level two quarters after the announcement. The coefficient of interest is the effect of the reform on the share of jumpers up to the end of the period of analysis $\beta_9 = 0.006$ as in the reduced form analysis with t-statistic around 10.

A.15 Counterfactual Analysis

As pointed in Section 3, the reduced-form effect of the reform on the share of households jumping to the notch is the sufficient statistic for the welfare analysis of the reform. However, in order to calculate the welfare impact of counterfactual reforms, it becomes necessary to disentangle this reduced form effect into the share of jumpers because of a percentage change in the threshold $share^t(y)$ and in the transfer $share^I(y)$.

This Section describes how to estimate these parameters. Section A.15.1 discusses how to decompose the jumping effects into responses to changes in the threshold and in the transfer. This decomposition highlights that the share of jumpers to a pure transfer and to a pure threshold reforms are necessary to conduct counterfactual analysis. I estimate these shares in Section A.15.2 by exploiting variation in the change in the transfer across households. The results are presented in Section A.15.2, and the counterfactual analysis in A.15.3.

[40] The coefficient on the first quarter of the dataset is positive and significant.

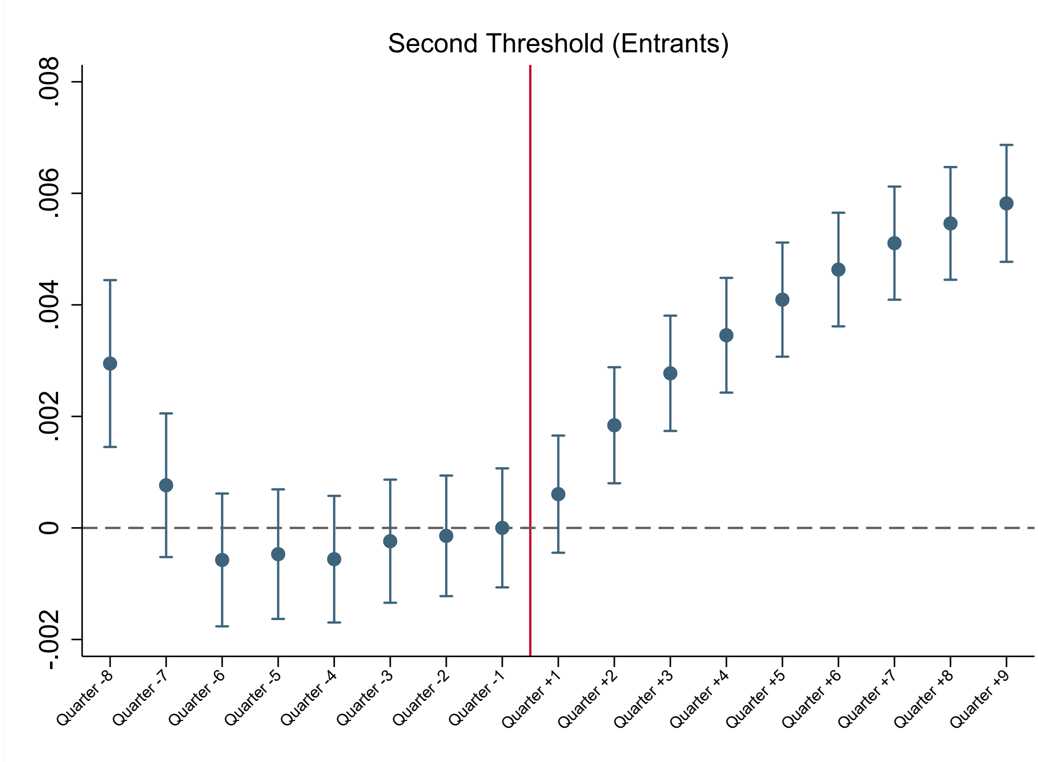


Figure 36: Difference and Differences Results for *share*[154]

A.15.1 Decomposing the Jumping Effect

To decompose the jumping effect, I rewrite the last two terms of equation in Proposition 1, using the following approximation:

$$
\begin{aligned}
share^J(dt, dI) &= \int \left(\frac{\partial P(y_a = t_0 | y_b = z)}{\partial I} dI + \frac{\partial P(y_a = t_0 | y_b = z)}{\partial t} dt \right) dH(z) \qquad (19)\\
&= \int \left(share^I(z) \frac{dI}{t_0 + I_0} + share^t(z) \frac{dt}{t_0} \right) dH(z) = share^I \frac{dI}{t_0 + I_0} + share^t \frac{dt}{t_0},
\end{aligned}
$$

where $share^I(z)$ and $share^I(z)$ were defined in equation 5, and $share^I = \int_{t_0}^{\infty} share^I(z) dH(z)$ and $share^t = \int_{t_0}^{\infty} share^t(z) dH(z)$ are the average shares of jumpers to pure transfer and threshold reforms across all income levels above the initial threshold. Therefore, one can approximate the welfare change of a (ϵ^t, ϵ^I) reform as:

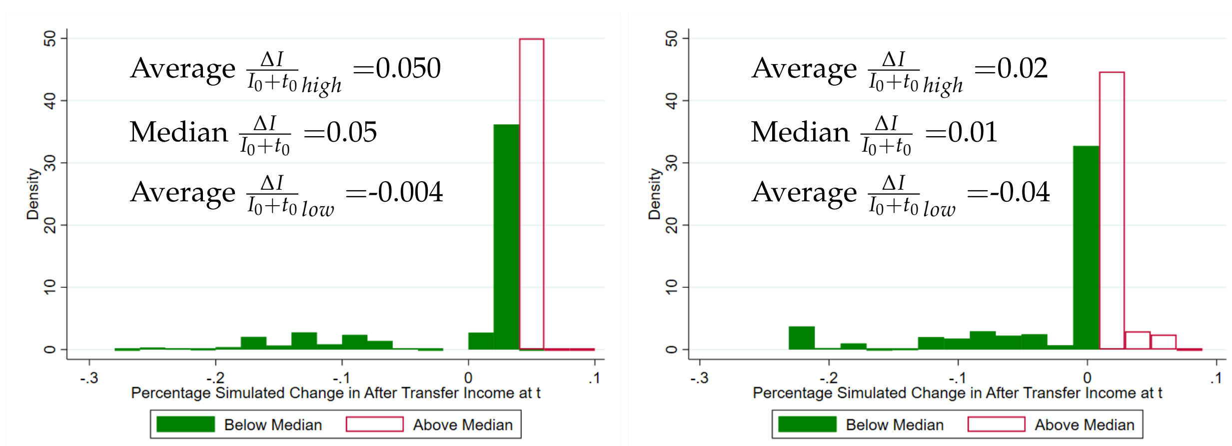
$$
\frac{dW}{\lambda} \approx \underbrace{\epsilon_I(\bar{g} - 1) H(t_0)}_{TransferEffect} + \underbrace{\epsilon_t(\bar{g}(t_0) - 1) I_0 h(t_0)}_{ThresholdEffect} \underbrace{- I_0 \left(share^I \frac{\epsilon^I}{t_0 + I_0} + share^t \frac{\epsilon^t}{t_0} \right) \Big(1 - H(t_0)\Big)}_{Jumping\ Effect}. \qquad (20)
$$

The income distribution $H(z)$ is directly observed from the data, Δt and ΔI are given by the schedule reform.[41] In what follows, I estimate $share^t$ and $share^I$ which are the behavioral parameters in the above equation. This allow me to compute the welfare effect of counterfactual reforms that change the threshold and the transfer by arbitrary amounts (ϵ^t, ϵ^I).

A.15.2 Estimating the Partial Shares *share*^{I} and *share*^{t}

The BF reform affected the after transfer income differently according to the the household composition. For instance, households with one child and three adults had an increase in their after transfer per capita income at the second notch from 57.33 to 58.33 *reais* (1.7% increase), while a household with one adult and two children had their after transfer income at the notch increased from 68 to 70 *reais* (2.9% increase).

Figure 37a and 37b plot the distributions of $\frac{\Delta^{sim} I}{I_0+t_0}$, i.e. the simulated percentage transfer given at the notch, for households without children above the first threshold and for households with Children above the second threshold before the reform. I construct the simulated change in transfer, as the simulated change the household would go through with the reform if their household composition would remained constant. Since households may have respond to the reform by changing their composition, this ensures all the variation is exogenous to the behavioral responses to the reform.



(a) First Threshold without Children

(b) Second Threshold with Children

Figure 37: $\frac{\Delta I}{I_0+t_0}$ Distribution among Households Above New Threshold

The green solid bars represent the distributions for households with changes below the

[41] I use the average ΔI to conduct this analysis.

median and the red empty bars represent the same distribution for households with changes above the median. Unfortunately, there is not a lot of variation in these simulated changes. The median change is about 5% for households with children and 1% for households without. The average above the median is close to it for both group of households. Some outliers ensure that the average for the group below the median is negative further away from it. In what follows, I investigate whether households that experienced larger changes in the after transfer income at the threshold jumped more.

In Figure 38, the blue line marked with circles plots the share of households with children that jumped from above 154 to the $(140, 154]$ interval among households that updated, while the red line marked with triangles plots the same shares for households without children. Panels A and B display the analysis for households that faced changes in the after transfer income at the threshold below and above the median, respectively.

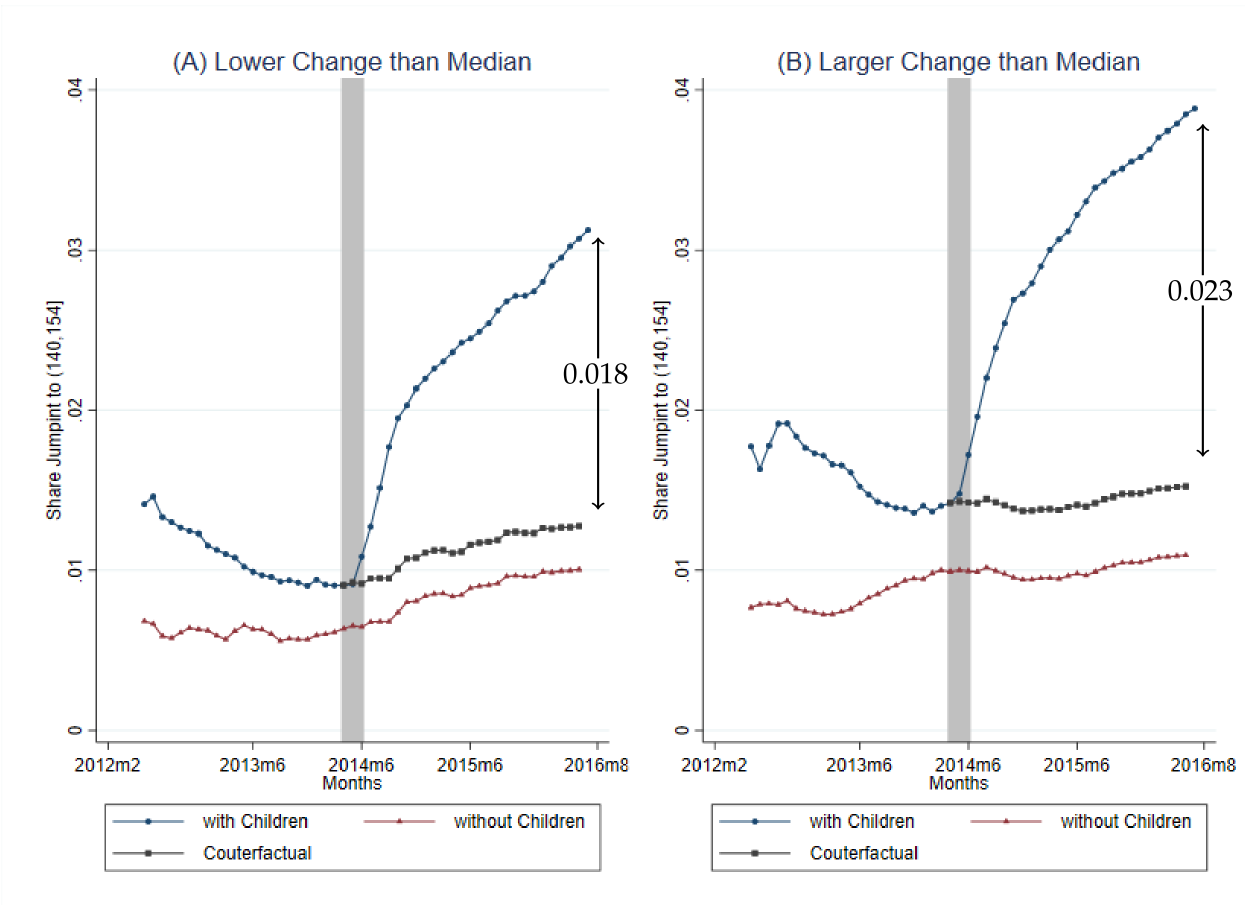


Figure 38: Share of Jumpers Below and above $\frac{\Delta I}{t_0+I_0}$ Median (2nd Threshold)

Under the identifying assumption that these trends would remain parallel after the reform,[42] 1.8% (2.3%) of the households facing lower (larger) changes than the median jumped because of the reform. Using the approximation in equation (19), for households below and above the median on can back out $share^t$ and $share^I$. In order to adjusting for the entrant jumpers, I re-normalize the share of partial jumpers multiplying by ratio between the share of jumpers and the share of jumpers among pre-reform applicants $\frac{\hat{share}^J}{\hat{share}}$ estimated in Sections 4.2 and 3.3 respectively. I find that $\hat{share}^I = 0.103$ and $\hat{share}^t = 0.216$.

The first share is interpreted as follows: a 1% increase in the after transfer income at the old threshold 140 would increase the probability of households jumping from above the new threshold 154 by 0.103 percentage points. The second share is interpreted as: a 1% increase of the threshold of eligibility would increase the probability of households jumping from above 154 by 0.216 percentage points.

The same analysis for the households without children jumping to the first threshold indicates that households that faced larger changes in the after transfer income do not respond more to the reform, as depicted in Figure 39.

The above result would imply a negative jumping elasiticty with respect to the after transfer income $share^I < 0$. This might be a result of the small variation in simulated changes in transfer across households. For this reason, I refrain from doing this counterfactual analysis for this group of households. Appendix A.16 shows how the assumption over preferences can be used to provide bounds for the jumping elasticities.

A.15.3 Counterfactual Results

The welfare effect of a small reform can be approximated by rewriting equation (20) for a discrete reform.

$$\frac{\Delta W}{\lambda} \approx \underbrace{\epsilon_I(\bar{g}-1)H(t_0)}_{TransferEffect} + \underbrace{\epsilon_t(\bar{g}(t_0)-1)I_0 h(t_0)}_{ThresholdEffect} \underbrace{- I_0\left(share^I \frac{\epsilon^I}{t_0+I_0} + share^t \frac{\epsilon^t}{t_0}\right)\left(1-H(t_0)\right)}_{Jumping\ Effect}. \tag{21}$$

Table 7 present the inputs for the welfare calculation of these alternative reforms.

Because households are more responsive to changes in the threshold, reforms that increase

[42] The analysis here is analogous to the one in Section 3.3.

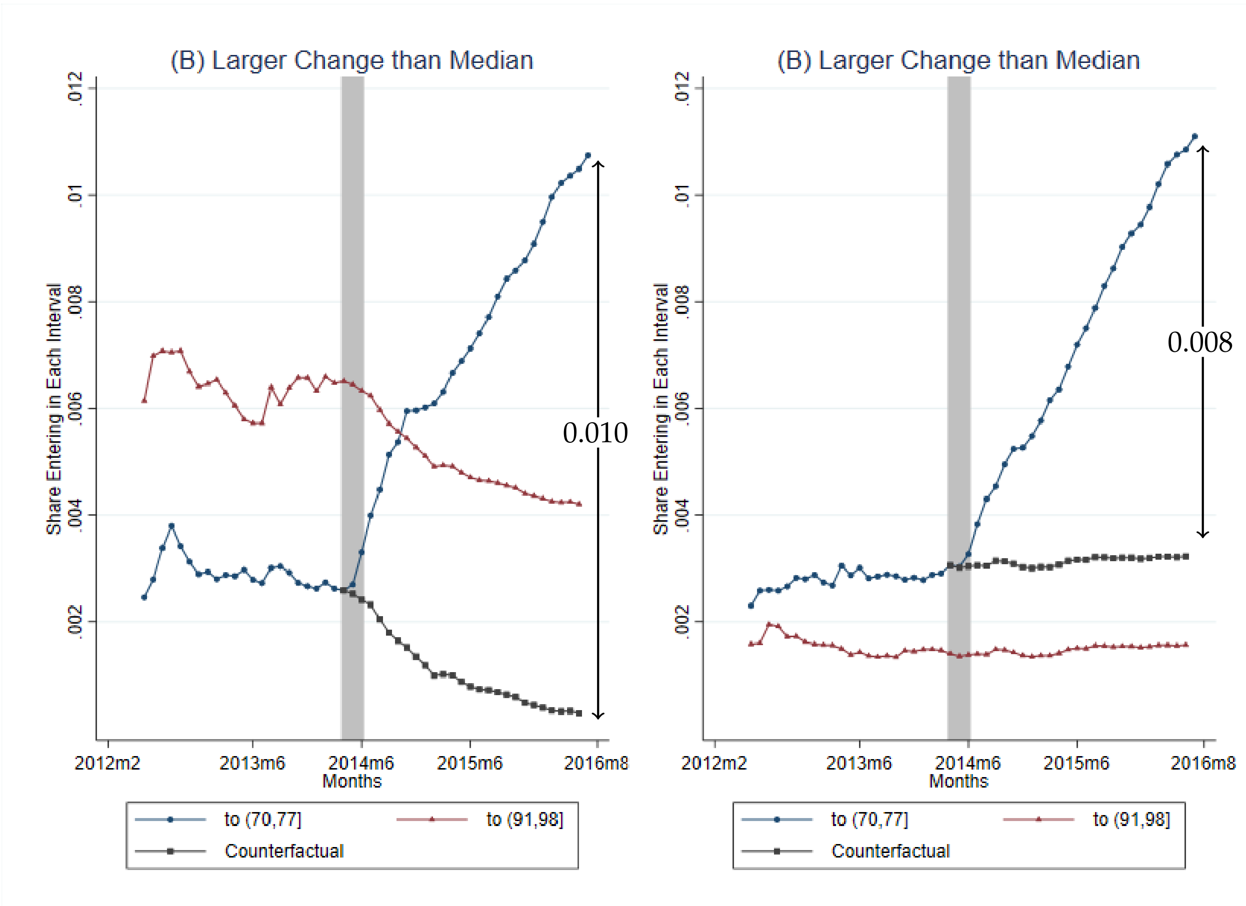


Figure 39: Share of Jumpers Below and above $\frac{\Delta I}{t_0+I_0}$ Median (1st Threshold)

Table 7: Counterfactual Analysis

Group	$H(t_0)\epsilon_I$	$I_0 h(t_0)\epsilon_t$	Jumping Effect
t=140	$0.661\epsilon_I$	$0.018\epsilon_t$	$0.013\epsilon_t + 0.005\epsilon_I$

the eligibility cut-off more than proportionally than the transfer tend to be less efficient than the ones that increase only the transfer. In general, increasing the transfer but not the cutoff would generate the largest welfare gains, if the welfare weights on the poor are not too low.

A.16 Structural Approach

Consider households solving problem (4) and choosing income y as depicted in Figure 40a. At the initial schedule (black solid line), the government transfers I_0 to households with income up to t_0. Household m_H[43] (represented by the red solid indifference curve) is the highest elasticity type (with the lowest convexity of the indifference curve) that will locate at y initially. This household's indifference curve is tangent to the notch in the original schedule.

Now consider a reform that increases the transfer by dI and the eligibility threshold by dt as the one depicted in the dashed schedule in the same figure. Under this reformed schedule $(t_0 + dt, I_0 + dI)$, m_H would clearly prefer to locate at the new notch and so would households with elasticity types just below m_H. Household m_L (with the blue dashed indifference curve) has the lowest elasticity among households that would jump to the new notch. The following lemma guarantees that

Lemma 5. *The proportion of types between m_L and m_H along the line $(n(y,m),m)$ is proportional to the distance between the consumption at the indifference curve going through (y,y) at any income level $z < y$.*

Proof. See Appendix A.17. □

Hence, the set of jumpers after the (dt, dI) reform correspond to types between m_L and m_H.

Even though the reform changed the transfer and threshold, the counterfactual analysis requires the elasticities or the probability of jumping with respect to a pure transfer $(0, dI)$ and pure threshold $(dt, 0)$ reforms. Figure 40b illustrates the $(0, dI)$ reform. It is useful to consider what happens around the notches under the initial schedule (t_0, I_0) and after both the pure transfer reform $(t_0, I_0 + dI)$ and after the the actual reform $(t_0 + dt, I_0 + dI)$ as in Figures 40c and 40d. I denote $m(dI)$ the household that is indifferent between y

[43]Note that because I focus on households initially at y, this corresponds to type $(n(y, m_H), m_H)$. Throughout this section, I refer to types only mentioning the degree of convexity to simplify the notation. Household m here denotes type $(n(y,m),m)$.

and the t_0 after the $(0, dI)$ reform. Therefore the set of jumpers corresponds to all types between m_H and $m(dI)$.

Under the assumption that the distribution of types is smooth, $F(m|z)$ can be approximated by a uniform distribution for $m \in (m_L, m_H)$. Hence the proportion of households between any two types$m_1 \neq m_2$ is proportional to the distance between their two indifference curves tangent to the schedule at y evaluated at any income level $z < y$. In Figure 40d, dx is the distance between the $m(dI)$ and m_L indifference curves at income t_0. From the argument above, one can see that the proportion of jumpers to the reform (dt, dI) is proportional to the segment $dI + dx$, while the proportion of jumpers to the counterfactual reform $(0, dI)$ is proportional to dI. This implies the following relation:

$$\frac{\frac{\partial P(y_a=t_0|y_b=y)}{\partial I}dI}{\frac{\partial P(y_a=t_0|y_b=y)}{\partial t}dt + \frac{\partial P(y_a=t_0|y_b=y)}{\partial I}dI} = \frac{P(m(dI) < m < m_H|y_b = y)}{P(m_L < m < m_H|y_b = y)} \approx \frac{dI}{dt + dx}.$$

Let $share(dt, dI; y) = \frac{\partial P(y_a=t_0|y_b=y)}{\partial t}dt + \frac{\partial P(y_a=t_0|y_b=y)}{\partial I}dI$ denote the share of households at y that would jump after the (dt, dI) reform. Rearranging the above relation yields:

$$\frac{\partial P(y_a = t_0|y_b = y)}{\partial I} \approx \frac{share(dt, dI)}{dt + dx}.$$

Even though I do not observe dx, one can bound it. Note that m_L's indifference curve is tangent to the new observed notch $(t_0 + dt, I_0 + dI)$ and that has slope $-\frac{u_y(z+I_0+dI,z)}{u_c(z+I_0+dI,z)} \in -\left(0, \frac{u_y(t_0+dt+I_0+dI,t_0+dt)}{u_c(t_0+dt+I_0+dI,t_0+dt)}\right)$ for all $z \in (t_0, t_0 + dt)$. Hence the interception between segments starting at $(t_0 + dt, I_0 + dI)$ with these slopes and the vertical line at t are natural bounds for dx. The upper bound dx^I_{UP} is equal to the dt, since $B'(z) = 0$ for $z < t_0$.[44]

The lower bound dx^I_{LB} is determined by the line that connects the new notch $(t_0 + dt + I_0 + dI, t_0 + dt)$ to the original position of the household (y, y) depicted as the gray solid line in Figures 40c and 40d. It is given by $dx^I_{LB} = \frac{I_0+dI}{y-t_0-dt}dt$. Finally, since $\varepsilon^I(y) = \frac{\partial P(y_a=t_0|y_b=y)}{\partial I}(t_0 + I_0)$:

$$\varepsilon^I(y) \in \left(\frac{share(dt, dI; y)}{dt + dI}(t_0 + I_0), \frac{share(dt, dI; y)}{dx^I_{LB} + dI}(t_0 + I_0)\right). \tag{22}$$

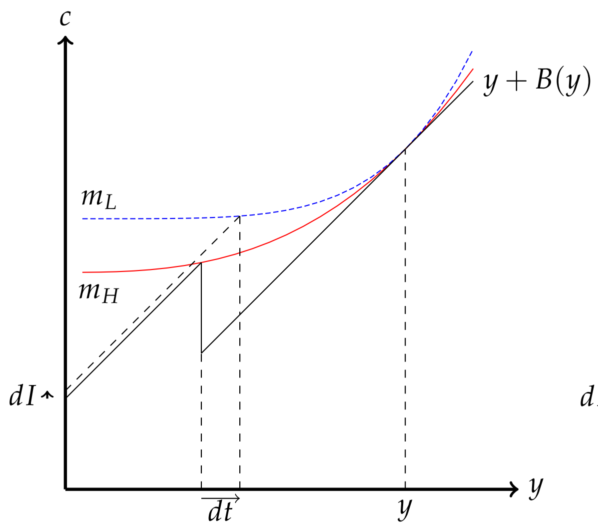
Using a symmetric argument for the case a of a reform that changes only the threshold

[44]More generally, $dx_{UB} = (1 + B'_-(t_0))dt$.

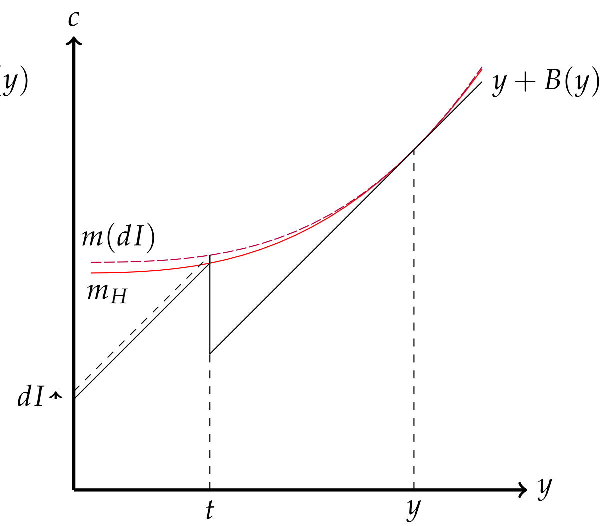
$(dt, 0)$, I get that:

$$\varepsilon^t(y) \in \left(\frac{share(dt, dI; y)}{dx^t_{LB} + dI} \frac{dx^t_{LB}}{dt} t_0, \frac{share(dt, dI; y)}{dt + dI} t_0 \right). \tag{23}$$

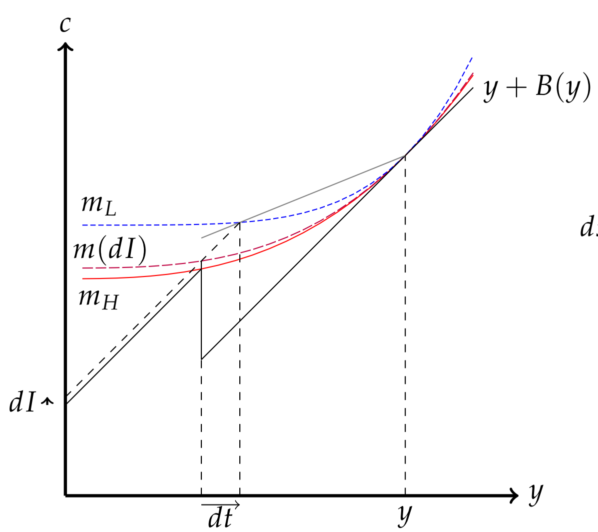
where $dx^t_{LB} = \frac{I_0}{y - t_0} dt$.



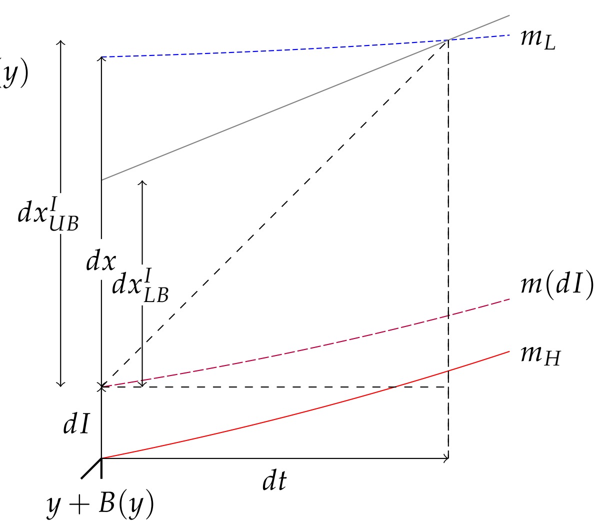
(a) (dt, dI) Reform



(b) $(0, dI)$ Reform



(c) $(0, dI)$ and (dt, dI) Reforms



(d) Zoom at the Notch

Figure 40: Share of Jumpers After Different Reforms

Let $\varepsilon^I_{LB}(y)$, $\varepsilon^I_{UB}(y)$, $\varepsilon^t_{LB}(y)$ and $\varepsilon^t_{UB}(y)$ the lower and upper bounds for $\varepsilon^I(y)$ and $\varepsilon^t(y)$ respectively. I define $share(\Delta t, \Delta I)$ as the share of jumpers after a non-infinitesimal reform that changes the threshold by Δt and the transfer by ΔI such as the one in the *Bolsa Família* program. Equation (22) provides a natural way of estimating the bounds for transfer elasticity as

$$\hat{\varepsilon}^{I}_{LB}(y) = \frac{share(\Delta t, \Delta I; y)}{\Delta t + \Delta I}(t_0 + I_0) \text{ and } \hat{\varepsilon}^{I}_{UB}(y) = \frac{share(\Delta t, \Delta I; y)}{\Delta x^{I}_{LB} + \Delta I}(t_0 + I_0), \tag{24}$$

where $\Delta x^{I}_{LB} = \frac{I_0 + \Delta I}{y - t_0 - \Delta t}\Delta t$. Analogously, equation (23) motivates the following estimators for $\varepsilon^{t}(y)$'s bounds.

$$\hat{\varepsilon}^{t}_{LB}(y) = \frac{share(\Delta t, \Delta I; y)}{\Delta x^{t}_{LB} + \Delta I}\frac{\Delta x^{t}_{LB}}{\Delta t} t_0 \text{ and } \hat{\varepsilon}^{t}_{UB}(y) = \frac{share(\Delta t, \Delta I; y)}{\Delta t + \Delta I} t_0, \tag{25}$$

where $\Delta x^{t}_{LB} = \frac{I_0}{y - t_0}\Delta t$.

A.16.1 Identification

It is useful to explicitly state the identifying assumptions which ensure the increase in the share of jumpers after a reform $(\Delta t, \Delta I)$ recovers the elasticities of interest. There are two key assumptions. (1) The counterfactual number of jumpers in the absence of the reform would follow similar trends as the one observed before the reform; (2) the distribution of types $F(n, m)$ is smooth. While the first assumption ensures that the share of households that jumped because of the reform is recoverable, the second assumption allows the use of the bounds presented in section 4.2.

Section 3 presented some evidence that the first assumption holds in the application. The share of households jumping after the reform among groups unaffected by the reform (households without children in Figure 9 and households jumping to intervals unaffected by the reform in Figure 8a) do not present any structural break. This suggests that the trends before the reform in the treated group is a useful to predict this counterfactual share of jumpers in the absence of the reform. The second identifying assumption is not testable.

A.17 Proof of Lemma 5

Let $c(z; m)$ be the consumption of the household of type $(n(y, m), m)$ that makes it indifferent to the bundle (y, y) when producing income z. The second order Taylor approximation of $c(z,m)$ around y yields:

$$c(z,m) \approx c(y,m) + c'(y,m)(z-y) + \frac{c''(y,m)}{2}(z-y)^2.$$

Since I am considering households with the same MRS at (y,y), $c'(y,m_1) = c'(y,m_2)$. Hence

$$c(z,m_1) - c(z,m_2) \approx \frac{c''(y,m_1) - c''(y,m_2)}{2}(z-y)^2. \tag{26}$$

Using the first order Taylor approximation of $c''(y,m)$ around m_0:

$$c''(y,m) = c''(y,m_0) + c''_m(y,m_0)(m-m_0).$$

Substituting this on (26), I get:

$$c(z,m_1) - c(z,m_2) \approx \frac{c''_m(y,m_0)}{2}(z-y)^2(m_1-m_2).$$

This implies that the distance between m_1 and m_2's indifference curves at any income level $z < y$, $c(z,m_1) - c(z,m_2)$, is proportional to the difference in types $m_2 - m_1$. If the distribution is smooth, it can be approximated by a uniform distribution. In this case $c(z,m_1) - c(z,m_2)$ is proportional to $F(m_1|z) - F(m_2|z)$ too.

A.17.1 Structural Approach Results

The counterfactual analysis depends on aggregate share of jumpers the jumping elasticities at each income level. In order to bound the elasticities with respect to t and I, I use equations (25) and (24). In the case of households without children jumping to the first threshold $t_0 = 70$, $I_0 = 34.16$, $\Delta t = R\$7$ and $\Delta I =$ R\$3.42. As indicated in Figure **??**, $share(7, 3.62) = 1.62 \times 10^{-2}$ so that the implied lower bound on the transfer elasticity and upper bound on the threshold elasiticity are $\hat{\varepsilon}^I_{LB} = 0.16$ and $\hat{\varepsilon}^t_{UB} = 0.11$ respectively.

In order to recover the other bounds, one needs the initial income level y. Among households in the first quartile of the income distribution above 77 *reais* before the reform, the average income was R\$111.63. This implies $\Delta x^t_{LB} = 5.74$ and $\Delta x^I_{LB} = 7.60$. Therefore the upper bound for the transfer elasticity and the lower bound for the threshold elasticity are respectively $\hat{\varepsilon}^I_{UB} = 0.15$ and $\hat{\varepsilon}^t_{LB} = 0.10$.

Table 8: Bounds on Jumping Elasticities

Quartile	Avg. Inc.	$\hat{\varepsilon}^{I}_{LB}$	$\hat{\varepsilon}^{I}_{UB}$	$\hat{\varepsilon}^{t}_{LB}$	$\hat{\varepsilon}^{t}_{UB}$
First Threshold $t = 70$					
1st	111.63	0.16	0.15	0.10	0.11
2nd	212.29	0.04	0.07	0.01	0.02
3rd	367.46	0.04	0.09	0.01	0.03
4th	840.17	0.02	0.05	0.00	0.01
Second Threshold $t = 140$					
1st	167.23	0.22	0.08	0.18	0.17
2nd	200.19	0.13	0.15	0.09	0.10
3rd	249.09	0.15	0.27	0.08	0.12
4th	503.73	0.16	0.53	0.04	0.12