## Vector Semantics \& Embeddings

Word Meaning

## What do words mean?

N-gram or text classification methods we've seen so far

- Words are just strings (or indices $\mathrm{w}_{\mathrm{i}}$ in a vocabulary list)
- That's not very satisfactory!

Introductory logic classes:

- The meaning of "dog" is DOG; cat is CAT $\forall x \operatorname{DOG}(x) \rightarrow$ MAMMAL(x)
Old linguistics joke by Barbara Partee in 1967:
- Q: What's the meaning of life?
- A: LIFE

That seems hardly better!

## Desiderata

What should a theory of word meaning do for us?
Let's look at some desiderata
From lexical semantics, the linguistic study of word meaning

Lemmas and senses
lemma
mouse (N)
sense
2. a hand-operated device that controls a cursor...

Modified from the online thesaurus WordNet

A sense or "concept" is the meaning component of a word Lemmas can be polysemous (have multiple senses)

## Relations between senses: Synonymy

Synonyms have the same meaning in some or all contexts.

- filbert / hazelnut
- couch / sofa
- big / large
- automobile / car
- vomit / throw up
- water / $\mathrm{H}_{2} \mathrm{O}$


## Relations between senses: Synonymy

Note that there are probably no examples of perfect synonymy.

- Even if many aspects of meaning are identical
- Still may differ based on politeness, slang, register, genre, etc.


## Relation: Synonymy?

water $/ \mathrm{H}_{2} \mathrm{O}$
" $\mathrm{H}_{2} \mathrm{O}$ " in a surfing guide?
big/large
my big sister != my large sister

## The Linguistic Principle of Contrast

Difference in form $\rightarrow$ difference in meaning

## Abbé Gabriel Girard 1718

## LA' JUSTESSE <br> DE LA

LANGUE FRANÇOISE,
$\circ v$
Les differentes significations
DESMOTS QUIPASSENT POUR

SYNONIMES.
ParM.l'Abbé GIRARD C.D.M.D.D.B.


Re: "exact" synonyms
"je ne crois pas qu'il y air demor fynonime dans aucune Langue.

## [I do not believe that there is a synonymous word in any language]

Chez I aurent d'Houry, Imprimeur-
L-braire, au bas de ta ruc dela Harpe, vis. da vis la rue $S$. Scverini, au Saint Efprir.

Avie Approbution in Erivilegs dis Roy.

## Relation: Similarity

Words with similar meanings. Not synonyms, but sharing some element of meaning
car, bicycle
cow, horse

## Ask humans how similar 2 words are

| word1 | word2 | similarity |
| :--- | :--- | :--- |
| vanish | disappear | 9.8 |
| behave | obey | 7.3 |
| belief | impression | 5.95 |
| muscle | bone | 3.65 |
| modest | flexible | 0.98 |
| hole | agreement | 0.3 |

## Relation: Word relatedness

Also called "word association"
Words can be related in any way, perhaps via a semantic frame or field

- coffee, tea: similar
- coffee, cup: related, not similar


## Semantic field

## Words that

- cover a particular semantic domain
- bear structured relations with each other.


## hospitals

surgeon, scalpel, nurse, anaesthetic, hospital restaurants
waiter, menu, plate, food, menu, chef
houses
door, roof, kitchen, family, bed

## Relation: Antonymy

Senses that are opposites with respect to only one feature of meaning
Otherwise, they are very similar!

```
dark/light short/long fast/slow rise/fall
hot/cold up/down in/out
```

More formally: antonyms can

- define a binary opposition or be at opposite ends of a scale
- long/short, fast/slow
- Be reversives:
- rise/fall, up/down


## Connotation (sentiment)

- Words have affective meanings
- Positive connotations (happy)
- Negative connotations (sad)
- Connotations can be subtle:
- Positive connotation: copy, replica, reproduction
- Negative connotation: fake, knockoff, forgery
- Evaluation (sentiment!)
- Positive evaluation (great, love)
- Negative evaluation (terrible, hate)


## Connotation

## Words seem to vary along 3 affective dimensions:

- valence: the pleasantness of the stimulus
- arousal: the intensity of emotion provoked by the stimulus
- dominance: the degree of control exerted by the stimulus

|  | Word | Score |  | Word | Score |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Valence | love | 1.000 | toxic | 0.008 |  |
|  | happy | 1.000 | nightmare | 0.005 |  |
| Arousal | elated | 0.960 |  | mellow | 0.069 |
| Dominance | frenzy | 0.965 | napping | 0.046 |  |
|  | powerful | 0.991 | weak | 0.045 |  |
|  | leadership | 0.983 | empty | 0.081 |  |

## So far

## Concepts or word senses

- Have a complex many-to-many association with words (homonymy, multiple senses)

Have relations with each other

- Synonymy
- Antonymy
- Similarity
- Relatedness
- Connotation


## Vector Semantics \& Embeddings

Word Meaning

## Vector Semantics

## Vector Semantics \& Embeddings

## Computational models of word meaning

Can we build a theory of how to represent word meaning, that accounts for at least some of the desiderata?

We'll introduce vector semantics
The standard model in language processing!
Handles many of our goals!

## Ludwig Wittgenstein

PI \#43:
"The meaning of a word is its use in the language"

## Let's define words by their usages

One way to define "usage":
words are defined by their environments (the words around them)

Zellig Harris (1954):
If $A$ and $B$ have almost identical environments we say that they are synonyms.

## What does recent English borrowing ongchoi mean?

## Suppose you see these sentences:

- Ong choi is delicious sautéed with garlic.
- Ong choi is superb over rice
- Ong choi leaves with salty sauces

And you've also seen these:

- ...spinach sautéed with garlic over rice
- Chard stems and leaves are delicious
- Collard greens and other salty leafy greens

Conclusion:

- Ongchoi is a leafy green like spinach, chard, or collard greens
- We could conclude this based on words like "leaves" and "delicious" and "sauteed"


## Ongchoi: Ipomoea aquatica "Water Spinach"


rau muống ...


Yamaguchi, Wikimedia Commons, public domain

## Idea 1: Defining meaning by linguistic distribution

Let's define the meaning of a word by its distribution in language use, meaning its neighboring words or grammatical environments.

## Idea 2: Meaning as a point in space (Osgood et al. 1957)

## 3 affective dimensions for a word

- valence: pleasantness
- arousal: intensity of emotion
- dominance: the degree of control exerted

|  | Word | Score |  | Word | Score |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Valence | love | 1.000 | toxic | 0.008 |  |  |
|  | happy | 1.000 | nightmare | 0.005 |  |  |
| Arousal | elated | 0.960 | mellow | 0.069 | NRC VAD Lexicon |  |
|  | frenzy | 0.965 | napping | 0.046 | (Mohammad 2018) |  |
| Dominance | powerful | 0.991 | weak | 0.045 |  |  |
|  | leadership | 0.983 | empty | 0.081 |  |  |
|  |  |  |  |  | 0.0 |  |

Hence the connotation of a word is a vector in 3-space

# Idea 1: Defining meaning by linguistic distribution 

Idea 2: Meaning as a point in multidimensional space

Defining meaning as a point in space based on distribution
Each word = a vector (not just "good" or " $\mathrm{w}_{45}$ ")
Similar words are "nearby in semantic space"
We build this space automatically by seeing which words are nearby in text


## We define meaning of a word as a vector

Called an "embedding" because it's embedded into a space (see textbook)
The standard way to represent meaning in NLP
Every modern NLP algorithm uses embeddings as the representation of word meaning
Fine-grained model of meaning for similarity

## Intuition: why vectors?

Consider sentiment analysis:

- With words, a feature is a word identity
- Feature 5: 'The previous word was "terrible"'
- requires exact same word to be in training and test
- With embeddings:
- Feature is a word vector
- 'The previous word was vector [35,22,17...]
- Now in the test set we might see a similar vector $[34,21,14]$
- We can generalize to similar but unseen words!!!


## We'll discuss 2 kinds of embeddings

tf-idf

- Information Retrieval workhorse!
- A common baseline model
- Sparse vectors
- Words are represented by (a simple function of) the counts of nearby words

Word2vec

- Dense vectors
- Representation is created by training a classifier to predict whether a word is likely to appear nearby
- Later we'll discuss extensions called contextual embeddings


## From now on： <br> Computing with meaning representations instead of string representations

荃者所以在鱼，得鱼而忘荃 Nets are for fish；
Once you get the fish，you can forget the net．
言者所以在意，得意而忘言 Words are for meaning；
Once you get the meaning，you can forget the words
庄子（Zhuangzi），Chapter 26

## Vector Semantics

## Vector Semantics \& Embeddings

## Words and Vectors

## Vector Semantics \& Embeddings

## Term-document matrix

Each document is represented by a vector of words

|  | As You Like It | Twelfth Night | Julius Caesar | Henry V |
| :---: | :---: | :---: | :---: | :---: |
| battle | 1 | 0 | 7 | 13 |
| good | 14 | 80 | 62 | 89 |
| fool | 36 | 58 | 1 | 4 |
| wit | 20 | 15 | 2 | 3 |

## Visualizing document vectors



## Vectors are the basis of information retrieval

|  | As You Like It | Twelfth Night | Julius Caesar | Henry V |
| :---: | :---: | :---: | :---: | :---: |
| battle <br> good <br> fool <br> wit | $\left[\begin{array}{l}1 \\ 14 \\ 36 \\ 20\end{array}\right.$ | 0 <br> 00 | 7 <br> 58 | $\left(\begin{array}{c}7 \\ 62 \\ 1 \\ 2\end{array}\right.$ |

Vectors are similar for the two comedies
But comedies are different than the other two
Comedies have more fools and wit and fewer battles.

## Idea for word meaning: Words can be vectors too!!!

|  | As You Like It | Twelfth Night | Julius Caesar | Henry V |
| :---: | :---: | :---: | :---: | :---: |
| battle <br> good <br> fool | 1 | 0 | 7 | 13 |
| wit | 114 | 80 | 62 | 89 |

battle is "the kind of word that occurs in Julius Caesar and Henry V"
fool is "the kind of word that occurs in comedies, especially Twelfth Night"

## More common: word-word matrix (or "term-context matrix")

Two words are similar in meaning if their context vectors are similar

is traditionally followed by cherry pie, a traditional dessert often mixed, such as strawberry rhubarb pie. Apple pie computer peripherals and personal digital assistants. These devices usually a computer. This includes information available on the internet

|  | aardvark | $\ldots$ | computer | data | result | pie | sugar | $\ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| cherry | 0 | $\ldots$ | 2 | 8 | 9 | 442 | 25 | $\ldots$ |
| strawberry | 0 | $\ldots$ | 0 | 0 | 1 | 60 | 19 | $\ldots$ |
| digital | 0 | $\ldots$ | 1670 | 1683 | 85 | 5 | 4 | $\ldots$ |
| information | 0 | $\ldots$ | 3325 | 3982 | 378 | 5 | 13 | $\ldots$ |



## Words and Vectors

## Vector Semantics \& Embeddings

## Cosine for computing word similarity

## Vector Semantics \& Embeddings

## Computing word similarity: Dot product and cosine

The dot product between two vectors is a scalar:

$$
\operatorname{dot} \operatorname{product}(\mathbf{v}, \mathbf{w})=\mathbf{v} \cdot \mathbf{w}=\sum_{i=1}^{N} v_{i} w_{i}=v_{1} w_{1}+v_{2} w_{2}+\ldots+v_{N} w_{N}
$$

The dot product tends to be high when the two vectors have large values in the same dimensions Dot product can thus be a useful similarity metric between vectors

## Problem with raw dot-product

Dot product favors long vectors
Dot product is higher if a vector is longer (has higher values in many dimension)
Vector length:

$$
|\mathbf{v}|=\sqrt{\sum_{i=1}^{N} v_{i}^{2}}
$$

Frequent words (of, the, you) have long vectors (since they occur many times with other words).
So dot product overly favors frequent words

## Alternative: cosine for computing word similarity

$$
\operatorname{cosine}(\vec{v}, \vec{w})=\frac{\vec{v} \cdot \vec{w}}{|\vec{v}||\vec{w}|}=\frac{\sum_{i=1}^{N} v_{i} w_{i}}{\sqrt{\sum_{i=1}^{N} v_{i}^{2}} \sqrt{\sum_{i=1}^{N} w_{i}^{2}}}
$$

Based on the definition of the dot product between two vectors $a$ and $b$

$$
\begin{aligned}
\mathbf{a} \cdot \mathbf{b} & =|\mathbf{a}||\mathbf{b}| \cos \theta \\
\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|} & =\cos \theta
\end{aligned}
$$

## Cosine as a similarity metric

-1: vectors point in opposite directions
+1 : vectors point in same directions
0 : vectors are orthogonal


But since raw frequency values are non-negative, the cosine for term-term matrix vectors ranges from 0-1

## Cosine examples

$\cos ($ cherry, information $)=$

$$
\frac{442 * 5+8 * 3982+2 * 3325}{\sqrt{442^{2}+8^{2}+2^{2}} \sqrt{5^{2}+3982^{2}+3325^{2}}}=.017
$$

$\cos ($ digital, information $)=$

$$
\frac{5 * 5+1683 * 3982+1670 * 3325}{\sqrt{5^{2}+1683^{2}+1670^{2}} \sqrt{5^{2}+3982^{2}+3325^{2}}}=.996
$$

Visualizing cosines (well, angles)


## Vector Semantics \& Embeddings

## Cosine for computing word similarity

## Vector Semantics \& Embeddings

## TF-IDF

## But raw frequency is a bad representation

- The co-occurrence matrices we have seen represent each cell by word frequencies.
- Frequency is clearly useful; if sugar appears a lot near apricot, that's useful information.
- But overly frequent words like the, it, or they are not very informative about the context
- It's a paradox! How can we balance these two conflicting constraints?


## Two common solutions for word weighting

tf-idf: tf-idf value for word $t$ in document $d$ :

$$
w_{t, d}=\mathrm{tf}_{t, d} \times \mathrm{idf}_{t}
$$

Words like "the" or "it" have very low idf
PMI: (Pointwise mutual information)

- $\operatorname{PMI}\left(w_{1}, w_{2}\right)=\log \frac{p\left(w_{1}, w_{2}\right)}{p\left(w_{1}\right) p\left(w_{2}\right)}$

See if words like "good" appear more often with "great" than we would expect by chance

## Term frequency (tf) in the tf-idf algorithm

We could imagine using raw count:

$$
\mathrm{tf}_{t, d}=\operatorname{count}(t, d)
$$

But instead of using raw count, we usually squash a bit:

$$
\mathrm{tf}_{t, d}= \begin{cases}1+\log _{10} \operatorname{count}(t, d) & \text { if } \operatorname{count}(t, d)>0 \\ 0 & \text { otherwise }\end{cases}
$$

## Document frequency (df)

$\mathrm{df}_{t}$ is the number of documents $t$ occurs in.
(note this is not collection frequency: total count across all documents)
"Romeo" is very distinctive for one Shakespeare play:
Collection Frequency Document Frequency
Romeo 1131
action $113 \quad 31$

## Inverse document frequency (idf)

## $\operatorname{idf}_{t}=\log _{10}\left(\frac{N}{\mathrm{df}_{t}}\right)$

$N$ is the total number of documents in the collection

| Word | df | idf |
| :--- | :--- | :--- |
| Romeo | 1 | 1.57 |
| salad | 2 | 1.27 |
| Falstaff | 4 | 0.967 |
| forest | 12 | 0.489 |
| battle | 21 | 0.246 |
| wit | 34 | 0.037 |
| fool | 36 | 0.012 |
| good | 37 | 0 |
| sweet | 37 | 0 |

## What is a document?

Could be a play or a Wikipedia article
But for the purposes of tf-idf, documents can be anything; we often call each paragraph a document!

## Final tf-idf weighted value for a word

## Raw counts:

$$
w_{t, d}=\mathrm{tf}_{t, d} \times \mathrm{idf}_{t}
$$

|  | As You Like It | Twelfth Night | Julius Caesar | Henry V |
| :--- | :---: | :---: | :---: | :---: |
| battle | 1 | 0 | 7 | 13 |
| good | 114 | 80 | 62 | 89 |
| fool | 36 | 58 | 1 | 4 |
| wit | 20 | 15 | 2 | 3 |

tf-idf:

|  | As You Like It | Twelfth Night | Julius Caesar | Henry V |
| :--- | :--- | :--- | :--- | :--- |
| battle | 0.246 | 0 | 0.454 | 0.520 |
| good | 0 | 0 | 0 | 0 |
| fool | 0.030 | 0.033 | 0.0012 | 0.0019 |
| wit | 0.085 | 0.081 | 0.048 | 0.054 |

## Vector Semantics \& Embeddings

## TF-IDF

## Vector Semantics \& Embeddings

## Word2vec

## Sparse versus dense vectors

tf-idf (or PMI) vectors are

- long (length $|\mathrm{V}|=20,000$ to 50,000)
- sparse (most elements are zero)

Alternative: learn vectors which are

- short (length 50-1000)
- dense (most elements are non-zero)


## Sparse versus dense vectors

## Why dense vectors?

- Short vectors may be easier to use as features in machine learning (fewer weights to tune)
- Dense vectors may generalize better than explicit counts
- Dense vectors may do better at capturing synonymy:
- car and automobile are synonyms; but are distinct dimensions
- a word with car as a neighbor and a word with automobile as a neighbor should be similar, but aren't
- In practice, they work better

Common methods for getting short dense vectors
"Neural Language Model"-inspired models

- Word2vec (skipgram, CBOW), GloVe

Singular Value Decomposition (SVD)

- A special case of this is called LSA - Latent Semantic Analysis
Alternative to these "static embeddings":
- Contextual Embeddings (ELMo, BERT)
- Compute distinct embeddings for a word in its context
- Separate embeddings for each token of a word


## Simple static embeddings you can download!

Word2vec (Mikolov et al)
https://code.google.com/archive/p/word2vec/

GloVe (Pennington, Socher, Manning)
http://nlp.stanford.edu/projects/glove/

## Word2vec

Popular embedding method
Very fast to train
Code available on the web
Idea: predict rather than count
Word2vec provides various options. We'll do:
skip-gram with negative sampling (SGNS)

## Word2vec

Instead of counting how often each word w occurs near "apricot"

- Train a classifier on a binary prediction task:
- Is w likely to show up near "apricot"?

We don't actually care about this task

- But we'll take the learned classifier weights as the word embeddings

Big idea: self-supervision:

- A word c that occurs near apricot in the corpus cats as the gold "correct answer" for supervised learning
- No need for human labels
- Bengio et al. (2003); Collobert et al. (2011)


## Approach: predict if candidate word c is a "neighbor"

1. Treat the target word $t$ and a neighboring context word $c$ as positive examples.
2. Randomly sample other words in the lexicon to get negative examples
3. Use logistic regression to train a classifier to distinguish those two cases
4. Use the learned weights as the embeddings

## Skip-Gram Training Data

Assume a +/- 2 word window, given training sentence:
...lemon, a [tablespoon of apricot jam, a] pinch...

$$
\begin{array}{llll}
\mathrm{c} 1 & \mathrm{c} 2 & {[\text { target }]} & \mathrm{c} 3 \\
\mathrm{c} 4
\end{array}
$$

## Skip-Gram Classifier

(assuming a +/- 2 word window)
...lemon, a [tablespoon of apricot jam, a] pinch... c1 c2 [target] c3 c4

Goal: train a classifier that is given a candidate (word, context) pair (apricot, jam) (apricot, aardvark)

And assigns each pair a probability:

$$
\begin{aligned}
& P(+\mid w, c) \\
& P(-\mid w, c)=1-P(+\mid w, c)
\end{aligned}
$$

## Similarity is computed from dot product

Remember: two vectors are similar if they have a high dot product

- Cosine is just a normalized dot product

So:

- Similarity(w,c) $\propto$ w•c

We'll need to normalize to get a probability

- (cosine isn't a probability either)


## Turning dot products into probabilities

$\operatorname{Sim}(\mathrm{w}, \mathrm{c}) \approx \mathrm{w} \cdot \mathrm{c}$
To turn this into a probability
We'll use the sigmoid from logistic regression:

$$
\begin{aligned}
P(+\mid w, c) & =\sigma(c \cdot w)=\frac{1}{1+\exp (-c \cdot w)} \\
P(-\mid w, c) & =1-P(+\mid w, c) \\
& =\sigma(-c \cdot w)=\frac{1}{1+\exp (c \cdot w)}
\end{aligned}
$$

## How Skip-Gram Classifier computes $P(+\mid w, c)$

$$
P(+\mid w, c)=\sigma(c \cdot w)=\frac{1}{1+\exp (-c \cdot w)}
$$

This is for one context word, but we have lots of context words. We'll assume independence and just multiply them:

$$
\begin{aligned}
P\left(+\mid w, c_{1: L}\right) & =\prod_{i=1}^{L} \sigma\left(c_{i} \cdot w\right) \\
\log P\left(+\mid w, c_{1: L}\right) & =\sum_{i=1}^{L} \log \sigma\left(c_{i} \cdot w\right)
\end{aligned}
$$

## Skip-gram classifier: summary

A probabilistic classifier, given

- a test target word w
- its context window of $L$ words $c_{1: L}$

Estimates probability that w occurs in this window based on similarity of $w$ (embeddings) to $c_{1: L}$ (embeddings).

To compute this, we just need embeddings for all the words.

## These embeddings we'll need: a set for w, a set for c



## Vector Semantics \& Embeddings

## Word2vec

## Vector Semantics \& Embeddings

## Word2vec: Learning the embeddings

## Skip-Gram Training data

...lemon, a [tablespoon of apricot jam, a] pinch... c1 c2 [target] c3 c4
positive examples +
t c
apricot tablespoon
apricot of
apricot jam
apricot a

## Skip-Gram Training data

...lemon, a [tablespoon of apricot jam, a] pinch...
c1
c2 [target] c3 c4
positive examples +
t
apricot tablespoon
apricot of
apricot jam
apricot a
For each positive example we'll grab k negative examples, sampling by frequency

## Skip-Gram Training data

...lemon, a [tablespoon of apricot jam, a] pinch... c1 c2 [target] c3 c4
positive examples +
$\frac{\mathrm{t}}{\mathrm{c}}$
apricot of
apricot jam
apricot a
negative examples -

| t | c | t | c |
| :--- | :--- | :--- | :--- |
| apricot | aardvark | apricot | seven |

apricot forever
apricot where apricot dear apricot coaxial apricot if

## Word2vec: how to learn vectors

Given the set of positive and negative training instances, and an initial set of embedding vectors
The goal of learning is to adjust those word vectors such that we:

- Maximize the similarity of the target word, context word pairs ( $w, c_{\text {pos }}$ ) drawn from the positive data
- Minimize the similarity of the ( $w, c_{\text {neg }}$ ) pairs drawn from the negative data.


## Loss function for one $w$ with $c_{\text {pos }}, C_{\text {neg1 }} \ldots C_{\text {negk }}$

Maximize the similarity of the target with the actual context words, and minimize the similarity of the target with the $k$ negative sampled non-neighbor words.

$$
\begin{aligned}
L_{\mathrm{C} E} & =-\log \left[P\left(+\mid w, c_{p o s}\right) \prod_{i=1}^{k} P\left(-\mid w, c_{\text {neg }_{i}}\right)\right] \\
& =-\left[\log P\left(+\mid w, c_{p o s}\right)+\sum_{i=1}^{k} \log P\left(-\mid w, c_{n e g_{i}}\right)\right] \\
& =-\left[\log P\left(+\mid w, c_{p o s}\right)+\sum_{i=1}^{k} \log \left(1-P\left(+\mid w, c_{n e g_{i}}\right)\right)\right] \\
& =-\left[\log \sigma\left(c_{p o s} \cdot w\right)+\sum_{i=1}^{k} \log \sigma\left(-c_{n e g_{i}} \cdot w\right)\right]
\end{aligned}
$$

## Learning the classifier

How to learn?

- Stochastic gradient descent!

We'll adjust the word weights to

- make the positive pairs more likely
- and the negative pairs less likely,
- over the entire training set.


## Intuition of one step of gradient descent



## Reminder: gradient descent

- At each step
- Direction: We move in the reverse direction from the gradient of the loss function
- Magnitude: we move the value of this gradient $\frac{d}{d w} L(f(x ; w), y)$ weighted by a learning rate $\eta$
- Higher learning rate means move $w$ faster

$$
w^{t+1}=w^{t}-\eta \frac{d}{d w} L(f(x ; w), y)
$$

## The derivatives of the loss function

$$
\left.\left.\begin{array}{rl}
L_{C E} & =-\left[\log \sigma\left(c_{\text {pos }} \cdot w\right)+\sum_{i=1}^{k} \log \sigma\left(-c_{\text {neg }}^{i}\right.\right.
\end{array} \cdot w\right)\right] .
$$

## Update equation in SGD

Start with randomly initialized C and W matrices, then incrementally do updates

$$
\begin{aligned}
& c_{\text {pos }}^{t+1}=c_{\text {pos }}^{t}-\eta\left[\sigma\left(c_{\text {pos }}^{t} \cdot w^{t}\right)-1\right] w^{t} \\
& c_{\text {neg }}^{t+1}=c_{\text {neg }}^{t}-\eta\left[\sigma\left(c_{\text {neg }}^{t} \cdot w^{t}\right)\right] w^{t} \\
& w^{t+1}=w^{t}-\eta\left[\left[\sigma\left(c_{p o s} \cdot w^{t}\right)-1\right] c_{p o s}+\sum_{i=1}^{k}\left[\sigma\left(c_{\mathrm{neg}_{i}} \cdot w^{t}\right)\right] c_{n e g_{i}}\right]
\end{aligned}
$$

## Two sets of embeddings

SGNS learns two sets of embeddings
Target embeddings matrix W
Context embedding matrix C
It's common to just add them together, representing word $i$ as the vector $\mathrm{w}_{\mathrm{i}}+\mathrm{c}_{\mathrm{i}}$

## Summary: How to learn word2vec (skip-gram) embeddings

## Start with V random d-dimensional vectors as initial embeddings

Train a classifier based on embedding similarity

- Take a corpus and take pairs of words that co-occur as positive examples
- Take pairs of words that don't co-occur as negative examples
- Train the classifier to distinguish these by slowly adjusting all the embeddings to improve the classifier performance
-Throw away the classifier code and keep the embeddings.


## Vector Semantics \& Embeddings

## Word2vec: Learning the embeddings

## Vector Semantics \& Embeddings

## Properties of Embeddings

The kinds of neighbors depend on window size
Small windows (C= +/-2) : nearest words are syntactically similar words in same taxonomy

- Hogwarts nearest neighbors are other fictional schools
- Sunnydale, Evernight, Blandings

Large windows ( $\mathrm{C}=+/-5$ ) : nearest words are related words in same semantic field

- Hogwarts nearest neighbors are Harry Potter world:
- Dumbledore, half-blood, Malfoy


## Analogical relations

The classic parallelogram model of analogical reasoning (Rumelhart and Abrahamson 1973)
To solve: "apple is to tree as grape is to
Add $\overrightarrow{\text { tree }}$ - $\overrightarrow{a p p l e}$ to $\overrightarrow{g r a p e}$ to get $\overrightarrow{\text { vine }}$ tree


## Analogical relations via parallelogram

The parallelogram method can solve analogies with both sparse and dense embeddings (Turney and Littman 2005, Mikolov et al. 2013b)

$$
\overrightarrow{\text { king }}-\overrightarrow{\text { man }}+\overrightarrow{\text { woman }} \text { is close to } \overrightarrow{\text { queen }}
$$

$$
\overrightarrow{\text { Paris }}-\overrightarrow{\text { France }}+\overrightarrow{\text { Italy }} \text { is close to } \overrightarrow{\text { Rome }}
$$

For a problem a:a*::b:b*, the parallelogram method is:

$$
\hat{b}^{*}=\operatorname{argmax} \operatorname{distance}\left(x, a^{*}-a+b\right)
$$

## Structure in GloVE Embedding space



## Caveats with the parallelogram method

It only seems to work for frequent words, small distances and certain relations (relating countries to capitals, or parts of speech), but not others. (Linzen 2016, Gladkova et al. 2016, Ethayarajh et al. 2019a)

Understanding analogy is an open area of research (Peterson et al. 2020)

## Embeddings as a window onto historical semantics

Train embeddings on different decades of historical text to see meanings shift
~30 million books, 1850-1990, Google Books data


William L. Hamilton, Jure Leskovec, and Dan Jurafsky. 2016. Diachronic Word Embeddings Reveal Statistical Laws of Semantic Change. Proceedings of ACL.

## Embeddings reflect cultural bias!

Bolukbasi, Tolga, Kai-Wei Chang, James Y. Zou, Venkatesh Saligrama, and Adam T. Kalai. "Man is to computer programmer as woman is to homemaker? debiasing word embeddings." In NeurIPS, pp. 4349-4357. 2016.

## Ask "Paris : France :: Tokyo : x" <br> - x = Japan

Ask "father : doctor :: mother : x"

- x = nurse

Ask "man : computer programmer :: woman : x"

- x=homemaker

Algorithms that use embeddings as part of e.g., hiring searches for programmers, might lead to bias in hiring

## Historical embedding as a tool to study cultural biases

Garg, N., Schiebinger, L., Jurafsky, D., and Zou, J. (2018). Word embeddings quantify 100 years of gender and ethnic stereotypes. Proceedings of the National Academy of Sciences 115(16), E3635-E3644.

- Compute a gender or ethnic bias for each adjective: e.g., how much closer the adjective is to "woman" synonyms than "man" synonyms, or names of particular ethnicities
- Embeddings for competence adjective (smart, wise, brilliant, resourceful, thoughtful, logical) are biased toward men, a bias slowly decreasing 1960-1990
- Embeddings for dehumanizing adjectives (barbaric, monstrous, bizarre) were biased toward Asians in the 1930 s , bias decreasing over the $20^{\text {th }}$ century.
- These match the results of old surveys done in the 1930s


## Vector Semantics \& Embeddings

## Properties of Embeddings

