Development of a Mathematical Problem-Solving App that Supports Transfer of Learning to Symbolic Representations

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Abstract

This paper presents an early-stage concept for a mathematical learning app and its underlying development approach that aims to support transfer of learning in school-age children. The mathematical app is specifically designed to support transfer from game-based, concrete representations of mathematics, to formal, symbolic representations. The approach is guided by a) a theoretical articulation of how the product achieves its intended outcomes (a logic model) and 2) a deep rooting in cognitive research and learning theory. We begin by describing an initial version of the app and research suggesting it showed promised for supporting educationally-relevant outcomes. We then describe how these findings – along with revisiting a relevant theory of learning – motivated modifications to improve the app, resulting in the early-stage concept. We discuss future directions and the importance of educational technology design based on theories of human thinking and learning.

Author Keywords

Game-based math learning; transfer; deconcretization; digital manipulative, mathematical cognition.

CSS Concepts

• **HCI theory concepts and models**

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Introduction

Math serves as a major obstacle for pursuing some college majors and careers, and occurs on a large scale, contributing to our national skills shortage. Compounding the problem is that many K-12 students approach mathematics with a substantially negative attitude — in some cases even developing a phobia (Tobias, 1995) or fixed mindset (Dweck, 2007) that prevents them from excelling in mathematics and pursuing STEM careers.

It has been known since the early 1990s (Nunes et al, 1993) that a significant obstacle to learning math at the K-8 levels is the use of symbolic representations. For example, consider the equation, below.

$$24 - 6y = 12$$

Solving this equation involves both an understanding of the symbolic "semantics" (e.g., y denotes an unknown number), but also the symbolic "grammar" (e.g., 24 can be subtracted from each side, but 6 cannot).

While mastery of that symbolic language is essential for more advanced work in math, Nunes and her colleagues showed that when math problems were presented in natural settings, such as handling money in a market transaction, ordinary people (including K-8 children) rapidly become proficient, even when they could not solve the same problems in symbolic form.

Thus, these findings considered together present a paradox: How are students able to solve mathematical problems in naturalistic contexts, yet, at the same time, struggle with mathematics in formal educational settings? The answer is that the type of representation matters and greatly influences students' problem solving. In the case of symbolic representations, students are faced with an unfamiliar context with its own symbolic grammar – in essence, like learning a foreign language. Devlin (2011) calls this the "Symbol Barrier"; a difficulty that is more one of *language*, rather than mathematics.

Game-based, Concrete Representations of Mathematics that Break the Symbol Barrier

Using smartphone, tablet, and PC technologies, it is possible to present some parts of K-8 mathematics in a manner that breaks the Symbol Barrier, enabling learning to engage with the mathematics, and solve problems, without having to master and use the symbolic notation. (Not unlike using a piano or a guitar to learn and produce music without having to first master musical notation.) This is the approach that underlies the work described in this note, which focuses on the students' transfer of problem-solving skills developed using a digital-based, dynamic, non-symbolic representation to being able to solve the same problems in the more traditional, paper-and-pencil setting required for later progress.

In the case of a technological aid that enables students to master a math technique when presented in a fashion that breaks the Symbol Barrier, its benefits will be limited if the students cannot subsequently transfer their new skills to the symbolic-math world.

Though educational technology holds promise to support students' transfer to symbolic representations, it is not a cure-all for this problem. This was demonstrated with the math game *DragonBox Algebra*. Initial studies of the app found that students using it became proficient in solving rational-coefficients, single-variable linear equations in as little as 90 minutes (Popovic, 2014). Yet, a later study (Long & Aleven, 2014) showed students were unable to transfer

their algebra skills to the traditional paper-and-pencil symbolic representation.

Mitigating, if not always avoiding, such problems requires better incorporation of research and theory on learning to guide interaction design.

BrainQuake

BrainQuake has spent the past six years developing tablet and Web-based math learning apps (presented as puzzle games) to break the Symbol Barrier, with the primary focus being the developing the all-important creative, problem-solving ability (CPSA) that underpins all work in mathematics.

The theory of change behind these apps is that students are provided opportunities to engage in mathematical thinking with familiar, concrete representations. Doing so allows them to compute mathematical solutions much more advanced than would be typically considered for their age. This assertion is supported by cognitive research suggesting that representations greatly influence mathematical problem solving (Landy & Goldstone, 2010) and that students solve problems better with more familiar representations (Koedinger & Nathan, 2014). Once a student has had practice in thinking mathematically and solving problems, the mastery of math's symbolic language are acquired much more easily.

BrainQuake's launch app, *Wuzzit Trouble* (*WT*), presented systems of linear equations in up to four variables as a digital gear mechanism, as seen in Figure 1. Several studies, including two independent, peer-reviewed university studies of *WT* showed that the approach leads to significantly improved performance in both written and interactive digital tests of CPSA (Pope & Mangram, 2015; Kiili et all, 2015).

That strong start led to BrainQuake winning a 30-month, $1.1M award from the US Department of Education to spend two and a half years building out their initial product, described in more detail, below.
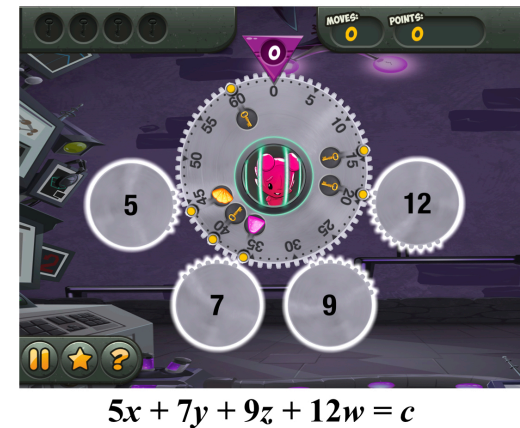

$$5x + 7y + 9z + 12w = c$$

Figure 1. Representation of a mathematical equation in Wuzzit Trouble (screenshot) and the equivalent in formal mathematical notation (below).

Research Partnership to Support Product Development

The development of the BrainQuake's product was supported by a research partnership with WestEd—an educational research and service agency. This collaboration began by working with WestEd to develop a structured logic model (Kao, Matlen, Tiu, & Lin, 2018), shown in Figure 2, that outlined how the product achieved its intended outcomes, creating a shared understanding between our development team and the research partners. This model was co-developed between BrainQuake and WestEd and underwent multiple iterations, taking weeks to create. This time and effort was well-spent, as it served as a

necessary and critical first step in guiding our iterative user research and design work.

Inputs	Activities	Outputs	Impact
Students - low-performing math backgrounds - low math self-confidence - positive perception of games **Middle School Teachers** **Company expertise** - UI/UX - game developmnt - math education theory - assessment - data analytics **Use Context** wifi connection 1:1 comp-student	At least 100 levels of proportional reasoning puzzles (at least 1 hour each week, for at least 3 weeks) Students independently modify wrong answers based on in-game feedback Easy to implement for teachers and students w/o guidance Teachers monitor student gameplay	Students complete all levels at own pace Students stay engaged with game even for more difficult levels Students provide appropriate response to feedback Teachers identify common student areas of confusion Teachers remediate common misconceptions	Increased performance on standardized tests involving proportions Increased attitudes towards mathematics Decreased fear of failure Improved mathematical problem solving Students transfer knowledge of proportions to learn related mathematical concepts

Figure 2. Logic model that guided user research and development.

The point of this paper is not to describe those studies in detail, but instead to summarize the key takeaways. One noteworthy and consistent finding from these studies was that students were highly engaged while playing *WT*, and this enjoyment was maintained across successive puzzle levels (see Figure 3). In fact, enjoyment remained at ceiling even when puzzles were rated highly difficult.

WestEd also conducted a randomized control trial pilot study with 34 grade 5 and 6 teachers and their associated students over the course of a 6 week implementation. This study represented an important but early-stage step in understanding how the BrainQuake puzzle suite is used in classroom contexts. The study revealed that—as in the usability testing—

treatment students were highly engaged while playing the games. This engagement seemed to shield students from developing negative perceptions of mathematics (e.g., 75% of students agreed that they would keep attempting the puzzle until they got it right, 69% were motived to try a different way if they failed at first).
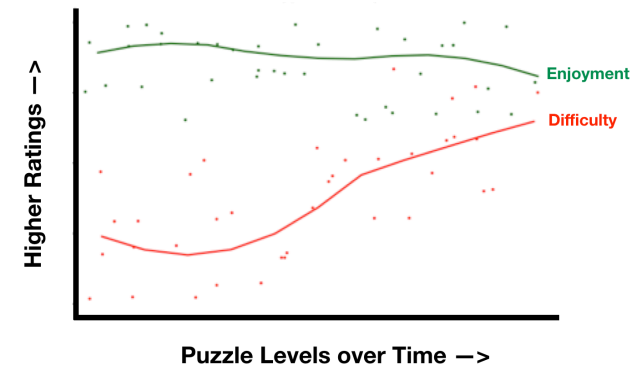

Figure 3. Jittered plot of difficulty and enjoyment ratings across successive puzzle levels. Data are combined across multiple user studies. A consistent finding is that enjoyment stayed at ceiling levels despite increase in difficulty over time.

Due to the relatively short duration of the study, we did not expect to observe large treatment impacts – nevertheless, we explored impact as a check on whether trends were in the expected direction. The analysis confirmed this prediction, the effects were consistently positive favoring the treatment students across multiple models that used different covariates (mean effect sizes = .08 and .22 on a measure of mathematical achievement that involved symbolic representations, and a math attitudes questionnaire, respectively). Students using the app particularly gained in their confidence for searching for novel solutions to problems, which is in alignment with the

BrainQuake logic model –by design, students can arrive at solutions to BrainQuake puzzles in a variety of different ways; thus, supporting strategy search.

Supporting Transfer by Instantiating Cognitive Theory into Game Design

While the prior research on BrainQuake games was promising, there were still opportunities for improvement. Specifically, we found that learning outcomes trended in a positive direction, but effects were relatively small. Why was this the case? This finding presented an opportunity to revisit the logic model and brainstorm new ways to support the link between mathematical thinking with familiar representations and formal mathematics.

Based on past research on cognition, concrete representations (such as representations used in some learning games, including *WT*) and the abstract symbolic representations in mathematics present a tradeoff in learning (Goldstone & Son, 2005). Some studies indicate that students learn more efficiently when initially learning from concrete representations, but that knowledge is less robust relative to when students learn from abstract representations (Koedinger et al. 2008). To balance the benefits of each representational type, gradually fading the concreteness of representations over time by replacing initially concrete representations with more abstract representations can be an effective way to support student learning, a process known as "concreteness fading" (Goldstone & Son, 2005). Fyfe and colleagues (2014) specifically propose a three-stage model for supporting transfer, that involves an initial concrete stage, and end-goal abstract stage, and a linking stage that serves as an intermediate step in connecting the

initial and end stages. In the case of WT, we believed that what was needed was this intermediate step that supports the connection making between the engaging, concrete mathematical representations provided by the app and the more symbolic mathematics we hoped students would transfer to.
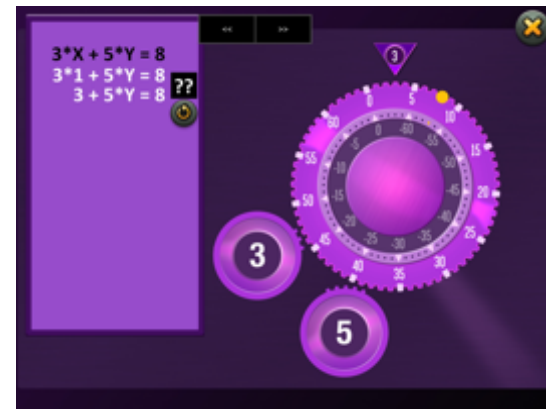

Figure 4. Prototype for intermediate linking representation (digital manipulative) that supports the mapping between concrete and symbolic mathematical representations.

This process motivated a new iteration of the product called *Digital Manipulatives* (DMs; see Figure 4). The DMs seeks to provide the student with interactive, exploratory learning experiences that connect the interactive digital app representation of mathematics with their more traditional, symbolic representations. The specific nature of the way the two representations are connected depends on the nature of the app's interactive representation. The goal is not to *display* the connection in a student-passive way, but rather, to allow students to explore the conceptual space between the interactive digital app and the traditional, symbolic

representation, in order to gain a tangible feeling and understanding of how each is a representation equivalent to the other. In the case of *WT*, this conceptual connection is achieved by a split screen, where manual solution of the Gears puzzle, as in *WT*, is accompanied by the simultaneous development of the corresponding integer equation being solved.

Conclusion

The work described here is one of many initiatives to develop and apply technologies to support teaching and learning in classroom settings. The goal of these efforts is not to replace the teacher, but instead to utilize the power of computing to support educators in scaffolding student learning. Even then, educational technology is not a panacea for solving the difficulties involves in academic learning. In the past two decades, many ideas for designing technology to support educational outcomes have been proposed and implemented. Few, however, have been grounded in a strong cognitive theory of learning. Here we present an early stage concept that is based in, and attempts to implement, underlying cognitive theory that could support students' transfer. The approach we took was also guided by a carefully developed logic model that provided predictions which we could test, and, if those predictions did not fully bear out, could motivate further improvements to the product. We hope this paper will motivate similar incorporation of learning theory into the design of edtech platforms.

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