Due Diligence and Security Design*

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Abstract

This paper studies equilibrium security design in a setting in which firms possess private information about the quality of their projects but investors have access to an information production technology. In the model, firms and investors enter into contracts directly and seek counterparties in a competitive environment. Once a deal is reached, investors have an opportunity to perform due diligence on the target firm before deciding whether to accept or reject the offer. I show that when information acquisition is endogenous, firms with higher quality projects may actually be more information-averse, choosing to deter screening by issuing more valuable and more information-insensitive securities while firms with lower quality projects do the opposite. Separation of this kind is more likely when investors can adapt their signals to the contracts offered and their beliefs, and such flexibility gives rise to commonly observed security designs, such as debt and simple combinations of debt and equity.

Keywords: Asymmetric information, competition, security design, private issuance

JEL codes: D82, D41, D86, G24, G32

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1 Introduction

Many financial transactions are organized in the following way. Once a proposed deal is finalized, the investor is given a chance to conduct due diligence on the target firm and possibly pull out of the deal. Discovery occurs at the discretion of the investor, who decides how much scrutiny to apply to the firm, what kind of information to collect, etc. In this paper, I model financing transactions that are arranged this way and ask what this implies for the design of securities and the degree and nature of due diligence conducted.

In the model, firms hold risky projects but require external financing to undertake them. Though firms initially possess superior information about the quality of their projects, investors have access to an information production technology. Trade is decentralized but competitive; there are large sets of both investors and firms, and rather than bargain over securities and prices, agents simply choose their preferred contracts and seek counterparties offering the same terms. The timing is as follows. Any party interested in a particular contract enters the corresponding “market” in search of a trading partner. With some probability, firms and investors meet and (because their offers match) a deal is reached. However, before the agreement is concluded, investors have the option to purchase a costly private signal of their own design before deciding whether to accept or reject the offer.

In principle, there is a market for (virtually) every possible security design. However, commitment is limited. Because firms cannot observe what information is acquired about them, signal-contingent claims cannot be written into contracts, and whatever due diligence is conducted (mathematically, the design of the signal) must be ex-post rational from the perspective of investors.

In equilibrium, pooling does not occur in any market. That is, different types of firms offer different contracts. What’s more, different types offer different security designs which induce different levels of due diligence. Interestingly, firms with higher quality projects signal their type by offering more valuable and more information insensitive securities, trading larger payments to investors on average for less intense screening. Firms with lower quality projects do the opposite, the implication being that such “low types” are somehow more amenable to investor scrutiny. Section 5.1 provides sufficient conditions for a two-type economy to have a unique equilibrium in which high types forego due diligence altogether by issuing higher interest debt while low types issue a combination of lower interest debt and equity in a market subject to screening.

This is surprising, because higher quality projects are always more likely to be approved by investors. The key assumption is that investor signals are not endowed but the result of an endogenous information production decision, and the mechanisms underlying the result constitute the key insights of this paper. There are three forces. First, an investor has less incentive to screen a higher quality firm which has already revealed its type. Second, because investor signals are endogenously imperfect, entailing some probability of rejection even when the true state (the return) is good, firms with higher quality projects may have more to lose from due diligence even though they are
always approved at higher rates. Finally, better firms are inherently more disposed to security designs which deter screening; more information-insensitive securities promise flatter (more constant) payments to investors, implying residual payments to firms which are more steeply increasing in the cash flow - a payoff structure more attractive to firms with higher quality projects.

In addition to highlighting these mechanisms, the unique equilibrium described above provides a unified theory of the security designs most commonly observed in practice. The model applies most directly to firms which (due to youth, size or general opaqueness) don’t have access to public capital markets. Rather than sell securities at auction or through an investment bank, such firms enter into contracts directly with investors in less organized markets. Debt is of course ubiquitous in this setting and throughout the financial system, and many ‘private issues’ in this market resemble a combination of senior debt and equity. For instance, Kaplan and Strömberg (2003) report that in 48% of the deals in their sample, investors essentially receive a portfolio both preferred equity and common stock, while in an additional 31.8%, they receive preferred equity with the option to convert to common stock.

The predicted relationship between security design and (the extent/degree of) due diligence also broadly matches the data. In practice, private buyers of debt, such as banks, typically screen borrowers in a perfunctory manner, while private buyers of both debt and equity (or similar composite securities), such as venture capital (VC) firms, perform extensive due diligence before entering into contracts. For example, Cosh et al. (2009) find that VC firms reject applicants at a rate of 48% compared to 17% for banks. Whereas eligibility for bank loans is often decided on the basis of preset criteria (also Cosh et al. (2009)), selection into VC financing - a variable associated with higher future TFP growth independent of any monitoring effect - is not reducible to observable characteristics (see Chemmanur et al. (2011)).

Taken together, the results suggest that firms with higher quality projects avoid screening by trading with banks, while firms with lower quality projects instead obtain financing from VC firms, angel investors, and the like. Though the idea that quality correlates with information-aversion is perhaps counterintuitive, this prediction is consistent with a common perception: that it is the firms with the greatest failure rates and least certain cashflows which avoid banks and seek funding from alternative sources. Of course, this is anecdotal, and given the premise that firms are privately informed, difficult to test empirically. However, the evidence is at least consistent with separation (i.e. the sorting of firms into different submarkets); Cosh et al. (2009) notes large cross sectional variation in the type and number of financing sources approached across firms, and Robb and Robinson (2014) and Gomes and Phillips (2012) find no systematic preference for one form of financing over another, no hierarchy or “pecking order.”

Beyond shedding light on firms’ financing choices, the analysis highlights the fragility of private issuance markets generally. In the comparative statics of the model, as the marginal cost of information approaches infinity, the market unravels and trade shuts down in equilibrium. For reasons
discussed in Section 6, the signaling devices and pooling securities identified by other researchers fail
to operate in this setting, and information acquisition is critical to sustaining trade. Due diligence
is thus a vital ingredient of the model.

**Related Literature.** The two most closely related papers are Nachman and Noe (1994) and Yang
and Zeng (2015). Omitting information acquisition, the economy studied in this paper resembles
that of Nachman and Noe (1994). The main difference is the way in which competition is modeled;
their setting assumes actuarially fair pricing, whereas here price (the amount invested) is embedded
into the contract. Nachman and Noe (1994) thus provides a useful benchmark for understanding
the implications of directed search. In Yang and Zeng (2015), firms and investors also enter into
contracts to finance a project, and investors are endowed with the same information production
technology studied in this paper. However, the authors do not consider asymmetric information or
competition. In their setting, the issuing firm makes a single take-it-or-leave-it offer to an investor,
and the two parties are symmetrically informed ex ante.

A number of papers consider the security design problem of a firm with private information
about its assets or investment projects. As in the present work, the sensitivity of given security’s
value to private information is critical, and debt - the least sensitive - often arises. For example,
in Myers and Majluf (1984) and Nachman and Noe (1994), debt optimally minimizes mispricing
in pooling equilibrium. This primacy of debt is fairly robust to the environment. For instance,
in Leland and Pyle (1977), Demarzo and Duffie (1999), and DeMarzo (2005), equilibrium is fully
separating and there is no mispricing, but debt minimizes signaling costs. On the other hand,
several papers obtain the *most* sensitive security - the residual complement of debt (a leveraged
equity/call option payoff, implemented as a warrant for all equity outstanding) - under alternative
assumptions.¹ Common to all these theories, however, is a unique prediction of the optimal security
given the primitives of the model. Here, in contrast, firms signal their type with their choice of
security design.

Like this paper, Boot and Thakor (1993), Fulghieri and Lukin (2001), and Daley et al. (2016)
consider an economy with both privately informed firms and investor signals. In Boot and Thakor
(1993) and Fulghieri and Lukin (2001), individual investor signals determine demand (and hence
market-maker valuations) in a noise trading environment, while in Daley et al. (2016) public rat-
ings influence the prices offered by competitive bidders. In these settings, information production
always benefits firms holding the highest quality assets; whatever the security and whatever in-
vestors’ ex ante beliefs, more information implies more revenue from the sale. Thus, high types may
wish to encourage information production or amplify the impact of ratings by issuing maximally
information-sensitive securities. In this model, however, the contract is settled before any due dili-

¹In obtaining warrants as an optimal security, Fulghieri et al. (2014) consider non-standard distributional assump-
tions, DeMarzo et al. (2005), Axelson (2007), and Garmaise (2007) consider investor private information, Boot and
Thakor (1993) and Fulghieri and Lukin (2001) consider investor information acquisition, and Daley et al. (2016)
consider public ratings.
gence is conducted. Signals can only influence investors’ decisions to accept or reject a match and not the terms of trade, so this force is not present.

Investor decisions in this paper more closely match those of Dang et al. (2015a), Dang et al. (2015b), Yang (2015), and Yang and Zeng (2015), which consider binary choices (accept, reject) together with endogenous information acquisition. Similar security designs arise in these models; debt optimally mitigates endogenous adverse selection, but if information is sufficiently valuable to the entrepreneur, a combination of debt and equity may dominate. However, like Yang and Zeng (2015), these papers assume single-offer bargaining and do not consider ex-ante information asymmetry.

The notion of competitive equilibrium used in this paper builds on a framework first developed in Gale (1996) and applied and adapted in Dubey and Geanakoplos (2002), Bisin and Gottardi (2006), Guerrieri et al. (2010), Bisin et al. (2011), Chang (2012), Guerrieri and Shimer (2014), Kurlat (2016), and Williams (2016). Because the terms of a contract can affect what is actually being traded when information is incomplete, these papers treat different contracts as distinct commodities with separate markets. Since prices cannot adjust to clear markets, alternative clearing mechanisms must be assumed, the simplest of which is random rationing. The degree of rationing is a measure of illiquidity, and Guerrieri et al. (2010), Chang (2012), and Guerrieri and Shimer (2014) demonstrate that static models of of rationing across markets and models of directed search are essentially equivalent, an interpretation I exploit. Relative to this literature, the addition of information acquisition and the analysis of security design are novel.

2 The Model

2.1 The Economy

Firms

A measure $L(t)$ of risk neutral firms holds projects of type $t \in \{1, ..., T\}$. A fixed investment of $K$ in a type $t$ project yields a random return $X \sim F_t(\cdot)$, where $\{F_t\}$ is a set of cumulative distribution functions with common support in $[0, \bar{x}] \equiv \mathbb{X}$ (where $\bar{x}$ is possibly equal to $\infty$) satisfying $E_t X < \infty \forall t$. Each firm possesses a single project but no liquid wealth with which to undertake it. Firms know their type, but investors do not.

Investors

There is an endogenous measure $M$ of identical risk-neutral households/investors satisfying $M > \sum_t L(t) \equiv L$. There are more investors than firms, so investors can be rationed on net (more on this below). Each investor has liquid wealth $W > \max_t \{E_t X\}$ from which to invest, and hence can finance any project.
Investors are also endowed with an information production technology. For any given project, investors can purchase a signal $\sigma$ correlated with its return $X$. The cost of $\sigma$ depends on its joint distribution with $X$, and in principle any measurable $\sigma$ is feasible. In particular, the cost is proportional to the expected reduction in Shannon’s entropy - a measure of the spread of a distribution - achieved by conditioning on $\sigma$ (i.e. the entropy difference between the prior and posterior distributions of $X$).

Letting $f$ denote the density of $X$, entropy is defined as:

$$e = -\mathbb{E} \ln (f (X))$$

Thus, the cost of signal $\sigma$ is $\bar{c} \cdot \Delta e$ ($\bar{c}$ a positive constant), where

$$\Delta e = -\mathbb{E} \ln (f (X)) + \mathbb{E} [\ln (f (X | \sigma)) | \sigma]$$

(1)

Note that because there is no restriction on the signal, nothing prevents investors from ultimately becoming more informed than firms. Importantly, investors may have an incentive to purchase a signal even in the event firms’ private information is fully revealed.

**Contracts**

Firms must enter into contracts with investors to finance their projects; if a firm fails to obtain external financing, its project is not initiated. In return for an investment $I \geq K$, investors receive a claim on the project as a function of $X$, as specified by a security $S : \mathbb{X} \to \mathbb{R}^+$. Although firms only need to raise $K$ to proceed, they can in principle raise more. Such excess payments might represent a partial sale of the project or wages for an entrepreneur.

Not all securities are feasible. Let $\mathcal{F}$ be the set of real-valued functions on $\mathbb{X}$ measurable with respect to the Borel algebra. Securities $S (\cdot) \in \mathcal{F}$ must satisfy the following conditions:

1. Limited liability: $0 \leq S (x) \leq x \ \forall x$.
2. Monotonicity: $S (\cdot)$ is nondecreasing.
3. Continuity
4. Differentiability almost everywhere

Limited liability means the firm cannot promise to pay investors more than the project return. This restriction can be justified by the assumption that firms or entrepreneurs have no other assets or by simply recasting $X$ as total wealth ex-post. In what follows, it will not be important to rule out losses for project owners (in fact, the next section introduces entry costs for firms).

Monotonicity is standard; decreasing securities allow firms to reduce their obligations by increasing their cash flows, creating a potential arbitrage opportunity in further financings.\(^2\)

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\(^2\)See Innes (1990) for a discussion.
Continuity and differentiability are technical conditions which are not at all restrictive in practice (when the sample space is, in fact, discrete). Nor are they very restrictive for continuous state spaces; note that the ‘almost everywhere’ exception permits infinitely many kinks. Moreover, if the so-called ‘dual monotonicity’ assumption common in the literature is imposed - requiring nondecreasing-ness of both \( S(x) \) and its residual \( x - S(x) \) - the condition is immediately satisfied: \( S(\cdot) \) is necessarily Lipschitz continuous and hence differentiable almost everywhere.

Denote the set of securities which satisfy all four conditions by \( S \). Then the contract space \( A \) is defined by

\[
A = \{(I, S(\cdot)): S \in S, I \geq K\} \cup \emptyset
\]

where the empty set represents the option not to trade.

**Markets**

Agents do not bargain over the terms of their contracts. Instead, trade is competitive. Investors and firms simply choose their preferred contracts and then seek counterparties offering the same terms. There is a unique search market for every contract in \( A \), and in principle agents can trade in any market they wish (the selection of a contract and subsequence search for a trading partner effectively ‘opens’ the corresponding market).

Upon entering a market, investors and firms are randomly matched (or left unmatched). In the event of a match, an investor has the option to produce information about the firm’s project before deciding whether to accept or reject the offer. Figure 1 illustrates the timing (note the traffic light color coding).

As long as markets are in some sense rival, we may assume that investors and firms commit to a single market without loss of generality; it makes no difference whether firms of type \( t \) elect a mixed strategy in which they enter some market \( \alpha \in A \) with probability \( q \) or whether \( q \)-share of type \( t \) firms enter \( \alpha \) exclusively. Likewise, whether \( M \) and \( L(t) \) are interpreted as populations or abstract measures of the resources investors and firms can devote to search (and freely distribute across markets) is irrelevant.

**Payoffs**

Conditional on a match in market \( \alpha = (I, S) \in A \), the payoff of each party depends on the signal purchased by the investor. Though it is possible to express utilities in terms of an arbitrary signal \( \sigma \), it is easier to characterize the solution to the investor’s problem and state the payoffs in reduced form.

Since the decision is binary, investors will optimally purchase a Bernoulli signal. Any more refined information structure will only cost more without improving the investor’s decision rule (see Woodford (2008)). Thus, without loss of generality, we may assume \( \sigma \) takes on a value of

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3Throughout, the terms ‘market’ and ‘contract’ will be used interchangeably.
either $0$ or $1$, where $\sigma = 1$ if and only if the project is approved. It follows that the investor’s information production decision can be modeled as a choice of the function $p : X \to [0, 1]$, where

$$p(x) = \Pr\{\sigma = 1 \mid X = x\}.$$ Note that $p(\cdot)$ fully specifies the joint distribution of $X$ and $\sigma$.

Given the simplicity of the signal, it is advantageous to invoke a symmetry property of entropy: for any two jointly distributed random variables, the expected reduction in entropy associated with conditioning on one variable is the same as the reduction in entropy associated with conditioning on the other. That is, it does not matter which variable is taken to be the signal and which the state, and the entropy difference $\Delta e$ can be rewritten by swapping $\sigma$ and $X$ in (1).\(^4\)

Since Bernoulli distributions are fully characterized by the probability of success, the mapping from Bernoulli distributions to entropies reduces to a function of a single variable, $-h : [0, 1] \to \mathbb{R}_+$, where $h(p) = p \ln p + (1 - p) \ln (1 - p)$.\(^5\) Thus, by symmetry, the expected difference in entropy associated with signal choice $p(\cdot)$ is

$$\Delta e(p(\cdot)) = \mathbb{E}h(p(X)) - h(\mathbb{E}p(X))$$

Given the convexity of $h$, it is clear that $\Delta e$ is always non-negative.

In choosing $p(\cdot)$, the investor considers the tradeoff between the cost of the signal $\bar{c} \Delta e(p(\cdot))$ and the value added to her ex ante contract payoff $\mathbb{E}[p(X)(S(X) - I)]$. The expectation terms in both expressions are calculated with respect to the investor’s belief prior. Let $\mu(\alpha, t)$ denote the match-conditional probability of meeting a firm of type $t$ in market $\alpha$ according to investor beliefs.

\(^4\)This has the added benefit of allowing us to dispense with the assumption that $X$ has a density.

\(^5\)By convention, $0 \ln (0) = \lim_{p \to 0} p \ln p = 0$
Then the cdf of the investor’s prior is given by the mixture

\[ \sum_{t} \mu(\alpha, t) F_t \]

Write \( \mathbb{E}_t \) for the expectation conditional on \( X \sim F_t \), \( \mathbb{E}_{\mu(\cdot, \cdot)} \) for the expectation conditional on \( X \sim \sum_{t} \mu(\alpha, t) F_t \), and \( \Delta e_{\mu(\cdot, \cdot)}(p(\cdot)) \) for \( \mathbb{E}_{\mu(\cdot, \cdot)} h(p(X)) - h(\mathbb{E}_{\mu(\cdot, \cdot)} p(X)) \). Letting \( \mathcal{P} = \{ p \in \mathcal{F} : 0 \leq p(x) \leq 1 \forall x \} \), the investor’s problem, given contract \( \alpha = (S, I) \) and prior \( \mu(\alpha, \cdot) \), is

\[
\max_{p(\cdot) \in \mathcal{P}} \left\{ \mathbb{E}_{\mu(\cdot, \cdot)} [p(X)(S(X) - I)] - \bar{c} \cdot \Delta e_{\mu(\cdot, \cdot)} (p(\cdot)) \right\}
\]

This problem has a unique solution, as established in the following lemma:

**Lemma 1.** Given \( \mu(\alpha, \cdot) \) and \( \alpha = (S, I) \), there is a unique optimal signal. The solution \( p^*(\cdot \mid \alpha, \mu(\cdot, \cdot)) \) takes the following form:

\[
\Pr \{ p^*(X \mid \alpha, \mu(\cdot, \cdot)) = 1 \} = 1 \text{ (the investor accepts without screening) if }
\mathbb{E}_{\mu(\cdot, \cdot)} \left[ \exp \left( -\bar{c}^{-1} (S(X) - I) \right) \right] \leq 1 \Rightarrow \mathbb{E}_{\mu(\cdot, \cdot)} \left[ \exp \left( \bar{c}^{-1} (S(X) - I) \right) \right] > 1
\]

\[
\Pr \{ p^*(X \mid \alpha, \mu(\cdot, \cdot)) = 0 \} = 1 \text{ (the investor rejects without screening) if }
\mathbb{E}_{\mu(\cdot, \cdot)} \left[ \exp \left( \bar{c}^{-1} (S(X) - I) \right) \right] \leq 1 \Rightarrow \mathbb{E}_{\mu(\cdot, \cdot)} \left[ \exp \left( \bar{c}^{-1} (S(X) - I) \right) \right] > 1
\]

\[
\Pr \{ 0 < p^*(X \mid \alpha, \mu(\cdot, \cdot)) < 1 \} = 1 \text{ (the investor screens, with a positive conditional probability of rejection in every state) if neither of the above conditions holds (both of the expectations above are greater than 1). Letting } \pi \text{ denote } \mathbb{E}_{p^*}(X \mid \alpha, \mu(\cdot, \cdot)), p^*(x \mid \alpha, \mu(\cdot, \cdot)) \text{ is implicitly defined by}
\]

\[
S(x) - I = \bar{c} \left( h'(p^*(x \mid \alpha, \mu(\cdot, \cdot))) - h'(\pi) \right)
\]

**Proof.** See Yang (2015), Proposition 2. \( \square \)

Thus, assuming optimal information acquisition \( p^*(\cdot \mid \alpha, \mu(\cdot, \cdot)) \), the payoff of a matched investor in market \( \alpha = (S, I) \), given beliefs \( \mu(\alpha, \cdot) \) is

\[
v(\alpha \mid \mu) := \mathbb{E}_{\mu(\cdot, \cdot)} [p^*(X \mid \alpha, \mu(\cdot, \cdot))(S(X) - I)] - \bar{c} \cdot \Delta e_{\mu(\cdot, \cdot)} (p^*(\cdot \mid \alpha, \mu(\cdot, \cdot)))
\]

and the payoff of a matched firm of type \( t \) is

\[
u(\alpha, t \mid \mu) := \mathbb{E}_t [p^*(X \mid \alpha, \mu(\cdot, \cdot)) (X - S(X) + I - K)]
\]

Note that the expectation in the latter case is taken with respect to the firm’s private information.
The overall utility from choosing contract $\alpha$ reflects not only the payoff conditional on a match but the cost of entry and the probability of matching in the corresponding market. Investors incur cost $\kappa$, satisfying $0 < \kappa < \max_t \{E_tX\}$, and firms incur cost $\iota$, satisfying $0 < \iota < K$, in entering a market with a positive probability of trade. If investors optimally reject a contract $\alpha$ outright and/or matching in the corresponding market is ruled out, offering $\alpha$ is equivalent to choosing not to trade and hence not subject to entry costs. For the purposes of calculating deviation payoffs (a point relevant for refining off-equilibrium beliefs; see below), entry costs are sunk to any firm or investor already in an active market.

### 2.2 Equilibrium

A candidate equilibrium consists of a quintuple of functions $\{g, f, \lambda, \gamma, \mu\}$ specifying the liquidity/tightness and firm population profiles associated with each market as well as investors’ and firms’ aggregate entry decisions. The definitions are as follows:

1. Functions $g : A \to \mathbb{R}_+$ and $\{f(\cdot, t) : A \to \mathbb{R}_+\}_{t=1}^T$, of finite support, map markets to the measures of investors and firms of type $t$, respectively, that enter those markets. Aggregates must respect population totals, so that $\sum_\alpha f(\alpha, t) = L(t)$ and $\sum_\alpha g(\alpha) = M$.

2. Functions $\lambda : A \to \mathbb{R}_+$ and $\gamma : A \to \mathbb{R}_+$, map markets to (common) beliefs as to firms’ and investors’ probability of matching, respectively, in those markets. As probabilities, $\lambda$ and $\gamma$ must satisfy $0 \leq \lambda(\alpha) \leq 1$ and $0 \leq \gamma(\alpha) \leq 1$ for all $\alpha \in A$.

3. Functions $\{\mu(\cdot, t) : A \to \mathbb{R}_+\}_{t=1}^T$ map markets to investors’ match-conditional probability of meeting a firm of type $t$ in those markets. Since $\mu(\alpha, \cdot)$ is a probability mass function, $\mu$ must satisfy $\sum_\alpha \mu(\alpha, t) = 1$ for all $\alpha \in A$.

The equilibrium objects $\lambda$, $\gamma$, and $\mu$ are taken as given by investors and firms and play the role of prices in traditional competitive equilibrium, while functions $g$ and $f$ are analogous to consumption bundles or allocations.

Functions $\lambda$, $\gamma$, and $\mu$ jointly determine the probability of trade in each market. Since entry costs are only incurred in markets with a positive probability of trade, they depend on $\lambda$, $\gamma$, and $\mu$. Firm entry costs are given by $\iota(\alpha) := \iota \cdot 1_{\{\lambda(\alpha) > 0, \epsilon^*P_e^*X|\alpha, \mu(\alpha, \cdot)) > 0\}}$. Clearly, $\iota(\alpha) = \iota$ if firms trade $\alpha$ with positive probability and $\iota(\alpha) = 0$ otherwise. Likewise, investor entry costs are given by $\kappa(\alpha) = \kappa \cdot 1_{\{\gamma(\alpha) > 0, \epsilon^*P_e^*X|\alpha, \mu(\alpha, \cdot)) > 0\}}$.

A set $\{g, f, \lambda, \gamma, \mu\}$ is an equilibrium if it satisfies four basic conditions:

1. Firm optimality; if $f(\tilde{\alpha}, t) > 0$ (some firms of type $t$ choose $\tilde{\alpha} \in A$), then

$$u(\tilde{\alpha}, t \mid \mu) \lambda(\tilde{\alpha}) - \iota(\tilde{\alpha}) = \max_{\alpha \in A} \{u(\alpha, t \mid \mu) \lambda(\alpha) - \iota(\alpha)\}$$
2. Investor optimality; if \( g(\tilde{\alpha}) > 0 \) (some investors choose \( \tilde{\alpha} \in A \)), then

\[
v(\tilde{\alpha} | \mu) \gamma(\tilde{\alpha}) - \kappa(\tilde{\alpha}) = \max_{\alpha \in A} \{ v(\alpha | \mu) \gamma(\alpha) - \kappa(\alpha) \}
\]

3. Market clearing: \( f(\alpha, t) \lambda(\alpha) = g(\alpha) \gamma(\alpha) \mu(\alpha, t) \) for all \( \alpha \) and \( t \).

4. Investor free entry: if \( g(\tilde{\alpha}) > 0 \), then

\[
v(\tilde{\alpha} | \mu) \gamma(\tilde{\alpha}) - \kappa(\tilde{\alpha}) = 0
\]

The optimality and free entry conditions are straightforward. Market clearing requires that the number of matches firms of type \( t \) can expect in market \( \alpha \) (as calculated by multiplying type \( t \)'s presence in the market \( f(\alpha, t) \) by its probability of a match \( \lambda(\alpha) \)) is equal to the number of matches investors can expect with firms of type \( t \) (as calculated by multiplying investors’ presence in the market \( g(\alpha) \) by its probability of a matching with \( t, \gamma(\alpha) \mu(\alpha, t) \)).

Though not explicitly included as a separate condition, rational expectations are incorporated in the definition of equilibrium above, in two respects. First, the assumption that firms maximize \( u(\alpha, t | \mu) \), a utility function which embeds optimal investor signals, implies that firms have rational expectations about due diligence in each market. Second, the substitution of beliefs for actual probabilities of trade in the definition of market clearing implies that these beliefs are accurate. In fact, manipulating the market clearing condition yields the Bayesian formula for \( \mu \). Summing across \( t \), we have

\[
\lambda(\alpha) \sum_t f(\alpha, t) = g(\alpha) \gamma(\alpha).
\]

Dividing the equation by the sum, and rearranging, we have

\[
\mu(\alpha, t) = \frac{f(\alpha, t)}{\sum_s f(\alpha, s)}
\]

The probability of meeting type \( t \), conditional on a match, is equal to type \( t \)'s presence in the market divided by the total number of firms in that market.

### 2.3 Refined Equilibrium

The definition of equilibrium in section 2.2 does not restrict the values of \( \lambda, \gamma, \) or \( \mu \) in markets which are not ‘open’ in equilibrium; that is, markets which neither firms nor investors enter according to \( g \) and \( f \). This leads to the existence of many equilibria. In fact, virtually no contract is ruled out; any equilibrium can be supported by setting \( \lambda \) and \( \gamma \) equal to zero in alternative markets, effectively shutting them down. Investors expect no deviation by firms, firms expect no deviation by investors, and as a result on one deviates.

Refinement 1, which Gale (1996) called “orderly beliefs”, resolves this issue directly by requiring that one side of the market always matches with certainty:

**Refinement 1** (Simple Matching Function). \( \lambda(\alpha) < 1 \) implies \( \gamma(\alpha) = 1 \) and vice versa. \( \triangle \)
In open markets, imposing Refinement 1 is equivalent to introducing a simple matching function whereby if firm presence exceeds investor presence in a given market, investors match with certainty while firms match with probability equal to the investor-firm ratio (and vice versa). To see this, consider the market clearing condition. Summing the equation yields $\lambda(\alpha) \sum_t f(\alpha, t) = g(\alpha) \gamma(\alpha)$. Rearranging, we have $\frac{\lambda(\alpha)}{\gamma(\alpha)} = \frac{g(\alpha)}{\sum_t f(\alpha, t)}$. Thus, if $g(\alpha) < \sum_t f(\alpha, t)$, Refinement 1 implies $\lambda(\alpha) = g(\alpha)$ and $\gamma(\alpha) = 1$, with the analogous result for the opposite case. Of course, such a matching function implies that one side always matches with probability 1, so the conditions are equivalent in open markets.

Although Refinement 1 only fully determines $\lambda$ and $\gamma$ in open markets, it essentially requires beliefs everywhere to be consistent with the matching function just described. In effect, when forming mutual expectations about off-equilibrium markets, firms and investors first make some prediction as to the investor-firm ratio and then derive $\lambda$ and $\gamma$ by the rule above.\(^6\)

Though essential, Refinement 1 is only minimally restrictive. In virtually all models of this kind, a great many equilibria survive its imposition. Moreover, it is especially weak in this setting, because investor ‘reject’ decisions can also effectively shut down markets. For instance, if, say, type $t$ returns are so poor that investors always optimally reject the project, then setting $\mu(\alpha, t) = 1$ and $\lambda(\alpha) = 0$ in an off-equilibrium market $\alpha$ has the same effect as sending both $\gamma(\alpha)$ and $\lambda(\alpha)$ to 0. An additional refinement which also restricts $\mu$ is therefore necessary to rule out unreasonable equilibria and make meaningful predictions.

Following Gale (1996), Chang (2012), Guerrieri and Shimer (2014), Williams (2016), and other papers which model competition in the same way, I apply an adaptation of the D1 criterion of Cho and Kreps (1987), which defines admissible beliefs in relation to equilibrium payoffs.\(^7\) In general, the spirit of D1 is as follows: if, say, type $t'$ prefers to deviate whenever type $t$ prefers to deviate but not vice versa, the uninformed party places no weight on $t$ in the event said deviation is observed. The meaning of ‘whenever’ depends on the context. D1 was originally defined for games in which the payoff of deviation depends on the actions taken in response, so the set of actions consistent with profitable deviation for each type determines admissible beliefs. In the adaptation of Gale (1996) and the papers just listed, however, only market conditions are relevant. Uninformed ‘buyers’ are constrained to believe that only ‘sellers’ willing - given equilibrium payoffs - to accept the lowest probability of trade are available in any off-equilibrium market. This model is unique in that firms’ utilities depend on both market conditions - the probability of a match, $\lambda$ - and investor actions - the signal purchased, which if rationalizable corresponds to some $\mu$. Thus, a new adaptation of D1 is required.

\(^6\)Several papers in the directed search literature, such as Chang (2012), Guerrieri and Shimer (2014), and Williams (2016), implicitly incorporate this refinement by simply defining a matching function and considering beliefs not over matching probabilities but buyer-seller ratios.

\(^7\)The D1-criterion is so-called because it is also the first level of the inductive “universal divinity” requirement of Banks and Sobel (1987).
One possible adaptation would have investors consider the set of \{\mu, \lambda\} pairs consistent with each type's deviation and rule out any type whose set is a strict subset of another's. While this captures the essence of D1, in practice it eliminates almost no equilibria. The dimensionality of the signal space is simply too large for investors to rule out deviations in this way. Another adaptation would admit \mu only if, fixing \mu in firm preferences (so that \lambda is the only free parameter), \mu survives the Gale (1996) refinement discussed above. However, this approach is too restrictive in practice, because such a fixed point may not exist. The following refinement represents a middle ground which yields intuitive restrictions on the set of equilibria and selects a unique equilibrium under certain conditions:

**Refinement 2** (D1 Adapted). Suppose \(\alpha\) is a contract which is not traded in equilibrium. Let \(u^*_t\) be the equilibrium payoff of a firm of type \(t\) and define

\[
k(t) = \inf \{k > 0 : ku(\alpha, t|\mu(\alpha, t) = 1) \geq u^*_t\}
\]

If \(\min_t k(t) < 1\) and \(k(t') > \min_t k(t)\) for any \(t'\), then \(\mu(\alpha, t') = 0\). \(\triangle\)

In this adaptation, firms' propensity to deviate is measured by the lowest matching probability they're willing to accept, assuming investors correctly infer their type (and purchase information accordingly). Essentially, investors form their beliefs by first making a simple conjecture: firms internalize the fact that investors have rational expectations in equilibrium. In that case, they will never offer an off-equilibrium contract in the hope of fooling their counterparties, and the only ambiguity involved in opening a new market is the the probability of a match.

### 2.4 The Type Set

Section 2.1 states that each type's return distribution is private information without characterizing the set \(\{F_t\}\). I make the following four assumptions:

#### Assumption 1 (SFOSD).

\(F_{t'}(x) < F_t(x)\) for \(x \in (0, \bar{x})\) and \(t' > t\).

#### Assumption 2 (Mixtures).

For all \(t \in \{1, \ldots, T\}\), \(F_t = \eta F_T + (1 - \eta) F_1\) for some \(\eta \in [0, 1]\).

#### Assumption 3 (Different Hazard Rates).

For any \(v \in \mathbb{R}\), the set

\[
\left\{x \in X : \frac{1 - F_T(x)}{1 - F_1(x)} = v\right\}
\]

has Lebesgue measure zero. In addition, \(\lim_{x \to \bar{x}} \frac{1 - F_T(x)}{1 - F_1(x)} = \inf_x \frac{1 - F_T(x)}{1 - F_1(x)}\).

#### Assumption 4 (A Very Bad Type).

\(\mathbb{E}_1 X < \nu < K\)

Assumption 1 establishes the most important characteristic of the type set; higher type projects are better in the sense of strict first order stochastic dominance (SFOSD). This implies that \(\mathbb{E}_t g(X)\)
is increasing in $t$ for any nondecreasing, nonconstant function $g$. In particular, given the monotonicity assumption, the value of any security is increasing in the associated project’s type.

By Assumption 2, every type’s return distribution can be expressed as a mixture of the return distributions of the worst and best types. This assumption ensures (and is necessary to ensure) that the set of possible investor belief priors $\left\{ \sum_t \mu_t F_t : \mu \in \Delta^T \right\}$ is fully ordered by first order stochastic dominance. It also implies that the set of functions $\left\{ \frac{1-F_{t'}}{1-F_t} : t' > t \right\}$ share a common order relation in $x$-space. Under these conditions, it is straightforward to extend the results to a setting with more than two active types (where a type is ‘active’ if it trades with positive probability in equilibrium). However, strictly speaking, Assumption 2 is not necessary for any of the results. An alternative, contract-specific ordering of belief priors (which does not require any assumptions on $\{F_t\}$) may be found in investors’ best response function $E_p^*(X | \alpha, \mu(\alpha, \cdot))$, which maps all mixtures $\mu$ to a single axis. However, Assumption 2 simplifies the proofs, and though it may be relaxed, ordinal equivalence of $\left\{ \frac{1-F_{t'}}{1-F_t} : t' > t \right\}$ (the implication above) must be assumed in any case.

The first part of Assumption 3 guarantees that there is no region or neighborhood over which the function $\frac{1-F_{t'}}{1-F_t}$ is constant, i.e. the upper-tail likelihood ratio is always either increasing or decreasing. This is equivalent to assuming that $F_1$ and $F_T$ have different hazard rates everywhere. Note, however, that Assumption 3 does not imply that $F_T$’s hazard rates are always lower (a condition which would imply Assumption 1).

While SFOSD is a global condition, different hazard rates is a local condition. The significance is that project type is payoff-relevant at every point in the state space, a property which can be exploited to derive contract perturbations which separate types.

The second part of Assumption 3 complements the first part, ensuring such separability even for contracts on the border of $A$ (for which the set of possible perturbations is limited). Essentially, it requires that the relative superiority of $F_T$ diminishes in the limit, so that the ratio $\frac{1-F_{t'}}{1-F_t}$ eventually falls to its lowest point. That $F_1$ ‘catches up’ in this way can justify an interpretation of lower type projects as more speculative even if always less valuable ex ante. In fact, if there exists some $\phi \in [0, \bar{x})$ such that $\frac{1-F_{t'}}{1-F_t}$ is decreasing after $\phi$ (i.e. over $[\phi, \bar{x})$), then $F_1$ has smaller hazard rates in this region and actually dominates $F_T$ in the upper tail. Formally, letting $\tilde{F}_t(x | v) = Pr_t \{ X \leq v + x | X > v \}$ denote the cdf of project $t$ conditional on the return $X$ exceeding some $v \in \mathbb{R}_+$, if $\frac{1-F_{t'}}{1-F_t}$ is decreasing over $[\phi, \bar{x}]$, then $\tilde{F}_t(x | v)$ SFOSD $\tilde{F}_{t'}(x | v)$ for all $v \geq \phi$ and all $t' > t$. This is despite that fact that $F_{t'}$ dominates $F_1$ overall; the FOSD property is thus, in a sense, ‘split’ (see Lemma 3 in the Online Appendix). This is natural in the context of private issuance markets, where new/young businesses may promise immense profits conditional on success even if they have high failure rates and low expected returns ex ante.

Finally, Assumption 4 introduces a ‘very bad’ type with expected project returns too low to justify not only the capital requirement $K$ but the firm’s cost of entry as well. This low type can only possibly profit ex ante by wholly or partially selling its project to unsuspecting investors via
contracts which offer $I > K + \iota$. The role of Assumption 4 is to sustain no-trade as an equilibrium.

2.5 Remarks on the Interpretation of the Model

Taking the statement of the model literally, in the event of a match, investors either accept or reject an offer, and then the world ends. Given the lack of further trade opportunities and the potential for mutual gains, it might seem curious why a failed offer does not simply give rise to a process of renegotiation. In a single-offer bargaining context, such as those of Dang et al. (2015a), Dang et al. (2015b), Yang (2015), and Yang and Zeng (2015), this is a reasonable critique. However, this is a model of competition, not strategic interaction. Swapping rationing rates (e.g. $\lambda$ and $\gamma$ above) for Poisson arrival rates, static models of rationing across markets and models of directed search are essentially equivalent, as illustrated by Guerrieri et al. (2010), Chang (2012), and Guerrieri and Shimer (2014). Thus, trade probabilities less than 1 should be interpreted as discounts which reflect the time it takes to execute a successful transaction. Matching probabilities reflect the tightness of the market, while rejection probabilities represent an additional layer of delay caused by investor screening.

The model may be viewed as an abstraction of the following environment: firms solicit financing by first submitting applications specifying business plans, the securities offered, etc. Given investor preferences (their selection of ‘submarkets’ of interest), a certain proportion of these lead to requests for an interview/presentation (a match). Given investor screening decisions, a certain proportion of matches then lead to approval (a positive signal), and eventually a transaction takes place. This is not unlike labor markets, housing markets, or other search markets. For instance, a worker seeking employment may restrict attention to vacancies satisfying certain wage and other criteria. The frequency of interviews would then depend on the availability of acceptable jobs, and the frequency of offers the rigorousness of employers’ due diligence. Of course, in all these settings, there would be some bargaining at the margin, but the idea that the terms are mostly set is not far fetched.

Flexibility in investor signals is important because the relevance of a given information structure will generally depend on the security design and the investor’s belief prior. For instance, an investor offered a debt security will only be concerned with the likelihood of default. Similarly, a signal which only discriminates between high cash flow states will add little value to a prior with a thin upper tail.

As for the particular information production function specified, Shannon’s entropy provides a natural measure of the cost of uncertainty because it can be interpreted at the number of bits of information needed to record a stream of data generated by a given distribution.\(^8\) However, the results are likely to hold for any specification which satisfies the symmetry property noted in Section 2.1 (interchangeability of the state and the signal) and under which the ‘uncertainty’ of a Bernoulli distribution with probability of success $p$ shares the key axiomatic features of $e(p)$:

---

\(^8\)See Shannon (1948).
arg \max e(p) = 0.5, e(p) = e(1 - p), and a concave, parabolic shape (see Figure 2).

![Figure 2: The entropy of a Bernoulli distribution](image)

### 3 Characterizing the Set of Equilibria

This section presents necessary conditions for any \( \{g, f, \lambda, \gamma, \mu\} \) to be an equilibrium. The main result establishes full separation; different types enter different markets. This and several other propositions are derived in the same way: an allocation (violating the hypotheses of the proposition) is proposed, deviations to small perturbations of traded contracts are considered, and a contradiction is found.

To definitively rule out equilibrium, such perturbations must benefit both parties. Otherwise, a sufficiently low matching probability can deter whichever side favors the (slightly) off-equilibrium contract from entering the corresponding market. However, given that at least one of \( \gamma \) and \( \lambda \) equals 1 by Refinement 1, if both parties stand to profit from a match in an alternative market, it follows that either investors or firms (or both) prefer deviation. Of course, these payoffs depend on investor beliefs, so it’s necessary to establish that investors expect projects of equal or greater quality in off-equilibrium markets. In each case, the argument invokes Refinement 2.

In this and subsequent sections, sketches of the proofs follow formal statements of the results. Full proofs of all the propositions may be found in the Online Appendix. Throughout, the phrase “market \( \alpha \) is open” is synonymous with “contract \( \alpha \) is traded with positive probability in equilibrium.”

**Proposition 1** (No pooling). *Pooling does not occur in any open market. That is, if \( t \) chooses \( \alpha \), \( t' \neq t \) chooses \( \alpha' \), and either \( \alpha \) or \( \alpha' \) is traded with positive probability, then \( \alpha \neq \alpha' \).*
Proposition 1 hinges critically on Assumption 3. To understand the importance of the upper tail likelihood ratio $\frac{1-F_{t'}}{1-F_t}$ (where $t' > t$), note that by monotonicity each increment in the value of $S$ carries over to higher cash-flow states. Thus, indicator functions which equal 1 if and only if $X$ exceeds some threshold, i.e. securities defined by

$$1_{X \geq \tilde{x}}(x) = \begin{cases} 
1 & x \geq \tilde{x} \\
0 & \text{otherwise}
\end{cases}$$

for some $\tilde{x} \in \mathbb{X}$, provide a basis for $S$, and any $S \in \mathbb{S}$ can be constructed as the linear combination $S = \int_0^{\tilde{x}} S' (\tilde{x}) 1_{X \geq \tilde{x}} d\tilde{x}$. The weight on $1_{X \geq \tilde{x}}$ is the derivative precisely at the threshold $\tilde{x}$.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{basis-security.png}
\caption{Basis Security $1_{X \geq \tilde{x}}$}
\end{figure}

Obviously, by the formula above, $E_t S = \int_0^{\tilde{x}} S' (\tilde{x}) [E_t 1_{X \geq \tilde{x}}] d\tilde{x}$. Since $E_t 1_{X \geq \tilde{x}} = 1 - F_t (\tilde{x})$, it follows that the cost of the ‘increment’ $S' (\tilde{x})$ to type $t$ is given by $1 - F_t (\tilde{x})$. Thus, the ratio $\frac{1-F_{t'}}{1-F_t} (\tilde{x})$ measures the benefit (cost) to type $t'$ of a reduction (increase) in the slope of $S$ at $\tilde{x}$ relative to the benefit (cost) to type $t$.

The proof proceeds as follows. Suppose, for instance, that firm types $t$ and $t' > t$ pool in some market $\alpha = (S, I)$. By Assumption 3, hazard rates across types are never equal, so that $\frac{1-F_{t'}}{1-F_t}$ is always varying. This implies the existence of regions $H, L \subset \mathbb{X}$ satisfying the following conditions:

1. None of the constraints defining $\mathbb{S}$ (monotonicity, etc.) bind on $H$ and $L$

2. If $x_h \in H$ and $x_l \in L$, then $\frac{1-F_{t'}}{1-F_t} (x_h) > \frac{1-F_{t'}}{1-F_t} (x_l)$

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Given this, it is possible to construct a contract perturbation $\tilde{\alpha} = (\tilde{S}, I)$, where $\tilde{S} = S + \delta \epsilon$, $\delta \in \mathbb{R}_+$, and $\epsilon : \mathbb{X} \rightarrow \mathbb{R}$, which is more favorable to $t'$. In fact, by setting $\epsilon' < 0$ on $H$ and $\epsilon' > 0$ on $L$, so that $\epsilon$ benefits firms precisely where the relative impact to $t'$ is high and hurts firms precisely where the relative impact to $t'$ is low, the ratio of $t'$’s deviation payoff to $t$’s deviation payoff can be made arbitrarily large. Thus, whatever firms’ payoffs from $\alpha$, there exists a perturbation which attracts type $t'$ at a lower matching probability.

This argument ignores the effect of investor screening, and implicitly assumes that both $\alpha$ and $\tilde{\alpha}$ are accepted with probability 1 conditional on a match. However, because an investor optimally responds to a first order stochastic dominant shift in her prior (e.g. a higher type) with a higher conditional probability of acceptance in every state, the advantage to $t'$ is only amplified by information acquisition.

By Refinement 2, it follows that $\mu(\tilde{\alpha}, t') = 1$. For sufficiently small $\delta$, we then have both $v(\tilde{\alpha} | \mu) > v(\alpha | \mu)$ and $u(\tilde{\alpha}, t' | \mu) > u(\alpha, t' | \mu)$. This is a contradiction.

The sketch above assume two types, but the argument for a general pool of firms is similar. There exists a slightly off-equilibrium contract which benefits high types (those whose projects are undervalued in the original market) and which investors expect to be offered by a more favorable profile of firms. If the perturbation is small enough, this alone makes deviation profitable for investors as well.

**Proposition 2** (No-trade Equilibrium). There is an equilibrium in which all firms choose not to trade (i.e. choose $\emptyset$).

This result is a consequence of Assumption 4 (i.e. the existence of a ‘very bad’ type). If all firms choose not to trade, accepting a common (normalized) payoff of 0, they all have the same propensity to deviate. The worst type is just as likely as any other to offer any particular contract. As a result, beliefs $\mu(\alpha, 1) = 1$, $\gamma(\alpha) = 1$, and $\lambda(\alpha) = 0$ for all $\alpha$ sustain no-trade as an optimal choice. Pessimistic expectations of meeting the ‘very bad’ type deter investors, while firms, unable to match in any market, are indifferent between entry and their outside option.

**4 Due Diligence and Security Design**

This section explores the interaction between investor information acquisition and security design. The astute reader will have noticed that due diligence plays almost no role in the results of Section 3 - Proposition 1 is a consequence of the size of the contract space and the fact that types are locally differentiated (Assumption 3), while Proposition 2 stems from the existence of a ‘very bad’ type. Once the question turns to what contracts are actually traded, however, investor screening becomes crucial.

As before, perturbation arguments establish the results, but rather than construct profitable deviations, the proofs demonstrate that traded securities solve optimization problems. It is as if the
securities which emerge in equilibrium are designed by social planners or the firms themselves.

**Definition.** $S$ is a debt security if $S(x) = \min(x, D)$ for some face value $D \in \mathbb{R}_+$. A debt contract is a pair $(I, S)$ for which $S$ is a debt security. The gross interest rate of a debt contract is $\frac{D}{I}$.

**Proposition 3** (Unscreened Debt). *If market $\alpha$ is open in equilibrium, investors accept $\alpha$ with probability 1 ($\alpha$ is not subject to screening), and $\gamma(\alpha) < 1$ (investors are rationed), then $\alpha$ is a debt contract with $I = K$.*

![Figure 4: Debt](image)

To prove Proposition 3, it is first shown that under the conditions above, $\alpha$ solves an optimization problem. Once this problem is properly reduced and characterized, the emergence of debt is intuitive.

Suppose $\alpha$ satisfies the hypotheses of the proposition, and $t$ is the unique type which enters the corresponding market (given Proposition 1, this is without loss of generality). Since investors are rationed at $\alpha$, any sufficiently small perturbation which yields a higher payoff to $t$ (conditional on a match) without adversely affecting beliefs $\mu$ will also make investors better off. This implies that $\alpha$ is a local maximum of $u(\tilde{\alpha}, t | \mu)$, subject to the constraint that the investors’ prior $\sum_t \mu(\tilde{\alpha}, t) F_t$ weakly (first order stochastically) dominates $F_t$. That is, $\alpha$ solves

$$\max_{\tilde{\alpha} \in N(\alpha)} u(\tilde{\alpha}, t | \mu)$$

subject to

$$\sum_t \mu(\tilde{\alpha}, t) F_t \text{ FOSD } F_t$$

(3)
where $N(\alpha)$ denotes some neighborhood around $\alpha$.

Proposition 3 also postulates that $\alpha$ is accepted with probability 1. In other words, $\alpha \in A_t$, where $A_t$ is the set of markets in which investors endogenously choose not to screen firms of type $t$. By Lemma 1, $A_t = \{ \alpha = (S, I) \in A : \mathbb{E}_t [\exp(-\bar{c}^{-1}(S(X) - I))] \leq 1 \}$. Given the characterization (3), contracts in the interior of $A_t$ are immediately ruled out; a perturbation may be constructed as in proof of Proposition 1 which both maintains or improves investors’ beliefs and increases firms’ payoffs.

For contracts $\alpha$ on the border of $A_t$, i.e. for which $\mathbb{E}_t [\exp(-\bar{c}^{-1}(S(X) - I))]$ exactly equals 1, the constraint in (3) does not bind. SFOSD implies that if $\mathbb{E}_t [\exp(-\bar{c}^{-1}(S(X) - I))] = 1$, then $\mathbb{E}_{\tilde{t}} [\exp(-\bar{c}^{-1}(S(X) - I))] > 1$ for any $\tilde{t} < t$. Thus given Lemma 1, lower types are locally screened. It follows that lower types experience a discontinuous drop in utility around $\alpha$, and investors rule out types $\tilde{\alpha} < \alpha$ in any sufficiently small neighborhood by Refinement 2. Thus, \[ \sum_{\tilde{t}} \mu(\tilde{\alpha}, t) \text{ FOSD } F_t \text{ for all } \tilde{\alpha} \in N(\alpha), \text{ and any } \alpha \text{ which solves } (3) \text{ also solves } \text{max}_{\tilde{\alpha} \in N(\alpha)} u(\tilde{\alpha}, t \mid \mu(\tilde{\alpha}, t) = 1) \]

This drops the constraint from (3) and fixes investor priors at $F_t$ (since the true priors weakly dominate $F_t$, this is a weaker condition). In words, $\alpha$ is a full information local optimum of firm $t$’s utility.

As it happens, there is a unique local optimum of $u(\tilde{\alpha}, t \mid \mu(\tilde{\alpha}, t) = 1)$ in the set $A_t$: the global maximum in $A_t$. This is necessarily a debt contract with $I = K$, as stated in Proposition 3. The intuition is simple. Among securities which yield the same expected return to investors, debt - the constant payoff security subject to the limited liability constraint - minimizes the incentive to acquire information. By a sort of duality, this implies that debt maximizes firms’ utilities subject to the constraint that firms’ optimal response is not to perform due diligence.

**Proposition 4** (Screened Senior Debt and Equity). If market $\alpha$ is open in equilibrium and $\alpha$ is accepted with probability less than 1 (investors choose to screen $\alpha$), then $\alpha = (S, I)$ has the following form:

1. $I = K$

2. There exists $\hat{x}$ and $\hat{S}$ such that

$$S(x) = \begin{cases} x & 0 \leq x \leq \hat{x} \\ \hat{S}(x) & x > \hat{x} \end{cases}$$

where $\hat{S}$ satisfies $\hat{x} < \hat{S}(x) < x$ and $0 < \hat{S}'(x) < 1$ for all $x$.

The significance of Proposition 4 is that if a traded contract is subject to screening, then the
security resembles a combination of senior debt and equity.\textsuperscript{9} The identity is not exact; as shown in 5, the ‘equity’ portion is concave. However, this curvature can be easily obtained if, say, the entrepreneur’s shares vest at a rate which is increasing in the success of the project, a practice which is quite common.\textsuperscript{10}

The derivation of Proposition 4 is straightforward. Suppose $t$ enters market $\alpha$ in equilibrium. As argued above, it must be that no contract perturbation is mutually profitable (conditional on a match) for both investors and firms. This implies that $\alpha$ solves

$$
\max_{\tilde{\alpha} \in N(\alpha)} u(\tilde{\alpha}, t \mid \mu)
$$

subject to

$$
v(\tilde{\alpha} \mid \mu) \geq v(\alpha \mid \mu(\alpha, t) = 1)
$$

for some $N(\alpha)$.

However, Proposition 4 also stipulates that $t$’s contract $\alpha$ is rejected with positive probability. This fact can be used to simplify (4). If $t$ is subject to due diligence, so is any type $\tilde{t} < t$ in a sufficiently small neighborhood of $\alpha$. Since lower types face less favorable screening in any off-equilibrium market and (by revealed preference) weakly prefer their equilibrium payoff to mimicking $t$ (i.e. trading $\alpha$, subject to an investor signal adapted for type $t$), they require a higher probability of trade in any market $\tilde{\alpha}$ sufficiently close to $\alpha$. This rules out types $\tilde{t} < t$ by Refinement 2. In other

\textsuperscript{9}Or, alternatively, participating convertible preferred equity

\textsuperscript{10}See Kaplan and Strömberg (2003).
words, there exists a neighborhood $N(\alpha)$ such that $\mu(\tilde{\alpha}, \tilde{t}) = 0$ for all $\tilde{t} < t$ and all $\tilde{\alpha} \in N(\alpha)$, meaning investor priors weakly dominate $F_t$ in $N(\alpha)$. Given that both investors and firms benefit from more optimistic beliefs, it follows that any contract which solves (4) also solves

$$\max_{\tilde{\alpha} \in N(\alpha)} u(\tilde{\alpha}, t \mid \mu(\tilde{\alpha}, t) = 1)$$

subject to $v(\tilde{\alpha} \mid \mu(\tilde{\alpha}, t) = 1) \geq v$

(5)

where $v = v(\alpha \mid \mu(\alpha, t) = 1)$.

In other words, $\alpha$ must be (locally) pairwise Pareto efficient. In the absence of information acquisition, the ex ante aggregate payoff is constant (fixing some project $t$), so all contracts are pairwise efficient. That is, (5) has no implications for contracts not subject to screening in equilibrium. However, among contracts $\alpha$ locally rejected with positive probability (given $\mu(\alpha, t) = 1$), (5) has a unique local optimum: the global solution. Proposition 4 characterizes this solution.

The shape of the resulting security is intuitive. Increasing-ness in $x$ encourages efficient screening. Obviously, it is socially desirable for a positive signal to be more likely the higher the true return. Examining the formula $p^*$ in Lemma 1, it is clear that a greater payout $S(x)$ induces a larger $p^*(x \mid \alpha, \mu(\alpha, \cdot))$. Thus, for the probability of acceptance to be increasing in the state, $S$ must also be everywhere increasing.

As for the cutoff $\hat{x}$, consider the firm’s problem in (5). In setting $S(x)$, the firm faces a key tradeoff. A larger $S(x)$ has the direct effect of reducing the firm’s residual payoff in state $x$. However, it also relaxes the constraint, permitting lower values of $S$ elsewhere, and increases the conditional probability of acceptance both locally and in all states of the world. Since the value of project approval in high return states pushes $S$ up everywhere, for low enough $x$ the second effect dominates and the limited liability constraint binds.

5 Due Diligence and Market Signaling

Given the relationship between due diligence and security design just established, firms signal their type with their choice of contracts. Section 5.1 presents and explicates the main result, and Section 5.2 develops a a simple, two-state example which illustrates the underlying mechanisms.

5.1 A Unique Trade Equilibrium in Two Active Types

**Definition 1.** Let $\bar{\alpha}_t = \arg\max_{\alpha} u(\alpha, t \mid \mu(t) = 1)$ be the contract which delivers type $t$’s highest possible payoff in separating equilibrium (this contract is unique).

**Definition 2.** Let $A_t = \{(S, I) \in A : E_t[\exp(-c^{-1}(S(X) - I))] \leq 1\}$ be the set of contracts which are not screened under $\mu(t) = 1$.
Proposition 5. Suppose that there are three types, i.e. $T = 3$, and let $t = 2$. Suppose further that type $t$ projects are poor enough that they are subject to screening in all markets - that is, $A_t = \emptyset$ - and the contract which firms of type $t$ most prefer is worth less to investors than their cost of entry, i.e. $v(\alpha_t | \mu(t) = 1) < \kappa$. If, in addition, type $T$ projects are good enough that $\bar{\alpha}_T \in A_T$ and $v(\alpha | \mu(t) = 1) > \kappa$, there exists a unique trade equilibrium, in which type $T$ issues debt in a market without investor screening, type $t$ issues a combination of debt and equity (of the form described in Proposition 4) in a market subject to screening, and type 1 does not trade.

In separating equilibrium, high types pay more on average but forego due diligence. Their selection guarantees funding at a cost. Low types accept the risk of screening and give up an equity stake but compensate by issuing less expensive securities, including lower interest rate debt. Figure 6 depicts each type’s contract.

![Figure 6: Separating Securities](image)

Low types choose a combination of lower interest debt and equity (purple, dotted line) while high types choose higher interest debt (blue, solid line).

This result is striking for several reasons. First, full separation typically cannot occur in models where trade is motivated by an investment opportunity rather than relative impatience, or risk sharing, etc, so the first step in explaining Proposition 5 is to identify what’s different about this setting. Two factors often rule out separation:

1. Given a project (fixing type), any contract yields the same aggregate payoff

2. As an equilibrium condition, investors extract the same value from all securities
Suppose both factors hold, and firms of different types issue different securities, choosing prices to satisfy investor indifference. Obviously, the high type’s contract is worth less to investors when the project is actually of lower quality. It follows that investors prefer buying the low type’s security to buying the high type’s security (at the same price) from a low type. Since all contracts yield the same aggregate payoff (fixing the project), this implies that low types wish to imitate high types. Intuitively, low types profit from mispricing, ruling out separation.

In this setting, however, neither condition applies. As touched on in Section 4, the (pairwise) social surplus depends on the signal purchased by the investor and hence is variable. At the same time, rationing in different markets may equalize investor payoffs even if the contracts themselves promise different returns. As a result, separation is not ruled out by the assumptions of the model.

Second, and more surprising, is how separation occurs. Given that better projects are always more likely to be approved by investors, one might expect more favorable securities from the perspective of firms to be associated with more extensive due diligence in equilibrium, and high types to signal their quality by accepting more intense screening. Instead, firms reveal their type in the opposite way.

This is a counterintuitive result, and it is interesting to examine the factors that give rise to it. The key assumption is that investor signals are not arbitrary - they reflect an information production decision, and as such, satisfy incentive compatibility. This constrains investor signals in a way that makes the more intuitive equilibrium structure described above unlikely. In particular, although high types may wish to sell less costly securities in a market with enough screening to deter imitation, given the impossibility of commitment, investors will generally not be disposed to cooperate. For one, high types are most attracted to securities which allow them to retain significant residual risk. Since these securities entail flatter, more constant payments to investors, they are more information insensitive, undercutting separation. Moreover, even if investors optimally respond by performing due diligence, such securities tend to induce noisy signals which may actually be costlier for high types, who have more to lose from error-prone decisions in high return states (this is despite the fact that better projects are always more likely to be approved). Finally, supposing separation is achieved anyway, once a high type firm has revealed its quality, the incentive to acquire information diminishes.

Section 5.2 illustrates these forces in the context of a simple, two state example.

5.2 Separation: A Simple Example

This section analyzes a version of the model developed in Section 2.1 in which there are only two possible project return realizations (i.e. two states). The exposition abstracts from the details of competition, equilibrium, etc. and simply presents graphical depictions of firms’ preferences, the solution to the investor’s problem, and the effects of investor screening on security payoffs.

This example is not, strictly speaking, a special case of the general model. A continuous state
space plays a role in pinning down equilibrium security designs, and risky debt is not well defined when there are only two states. However, separation occurs in the same way, so the exercise is instructive.

Suppose there are two states, \( b \) (bad) and \( g \) (good), and two types of firms, \( l \) (low) and \( h \) (high). Projects cost \( k \), and return \( x_i \) in state \( i \), where

\[
x_b < k < x_g
\]

In this sense, the project is risky and state \( g \) is good. The good state occurs with probability \( q_l \) for type \( l \), where \( q_l < q_h \). In this sense, the high type is better.

In return for an investment \( k \), firms sell securities. A generic security \( s \) pays investors \( s_i \) in state \( i \), while firms receive the residual payoff \( r_i = x_i - s_i \).

Security payoffs can be equivalently represented with a simple change of variables. Given a security \( s \), let \( \Delta_s = s_g - s_b \) and \( \Delta_r = r_g - r_b \) be the payoff spread across states for the investor and the firm, respectively. Firm \( i \)'s objective can then be written

\[
r_b + q_i \Delta_r
\]

Stating preferences in this way, firms’ utility functions have linear indifference curves in \((\Delta_r, r_b)\)-space. Type \( i \) indifference curves have slope \(-q_i\). Thus, there is single crossing. The low type requires less compensation in base pay \( r_b \) for any reduction in the payoff spread \( \Delta_r \).

Figure 7 depicts firms’s preferences. Every security is represented by a particular point in the coordinate plane. Firms value both more base pay \( r_b \) and more payoff spread \( \Delta_r \), so utilities are increasing in the northeast direction. High types value \( \Delta_r \) more, so their indifference curves are steeper. Given any security (as a possible option/endowment for all firms) and plotting each type’s points of indifference, low types potentially separate by choosing a security in the upper left quadrant, while high types potentially separate by choosing a point in the lower right quadrant.

Now suppose some securities are rejected with probability \( p_i \) in state \( i \), based on a private signal. Furthermore, suppose \( p \) is everywhere endogenous, and investors have rational expectations. As before, signal costs are given by the entropy difference (multiplied by a constant). The investor solves the same problem (2) adapted to two states; the selection of the function \( p(\cdot) \) reduces to a choice or two variables, \( p_h \) and \( p_l \):

\[
\max_{p_l, p_h} \{ qp_h (s_h - k) + (1 - q) p_l (s_l - k) - c \Delta e (p_h, p_l) \}
\]

This problem has an analytical solution. Effective firm payoffs, denoted with a tilde, are then given by multiplying firms’ residual returns by investors’ optimal probability of acceptance in the
corresponding state: $\tilde{r}_i = p_i^* r_i$. Changing variables again, we have

$$\tilde{r}_b = r_b p_b^*$$
$$\tilde{\Delta}_r = \tilde{r}_g - \tilde{r}_b$$

The solution to the investor’s problem can be represented graphically in two steps. First, the set of screened securities is marked out in the plot. Second, gross security payoffs are mapped to effective payoffs (accounting for screening) in the same coordinate plane. The following adding-up identities aid in understanding the figures:

$$r_b + s_b = x_b$$
$$\Delta_r + \Delta_s = \Delta_x$$

In other words, any gain to the firm translates 1 for 1 into a loss to the investor, along either dimension.

Figure 8 depicts the set of screened securities, by type. An investor who believes she’s trading with the low (high) type will purchases a signal if and only if the security lies in the blue (red) region.
Presented with a low (high) type project, investors endogenously screen any security in the blue (red) region, reject with certainty above this region, and accept with certainty below this region.

Note that since firm utilities are increasing as you go Northeast, investor utilities are increasing as you go Southwest (by adding up). It’s thus immediately clear that, given a low (high) type project, securities below the blue (red) region are accepted with probability 1 and securities above the region are rejected outright. Investors cut high types more slack, so the red region is thinner and shifted up.

Notice how each set fans out as $\Delta_r$ falls. A smaller $\Delta_r$ translates into a larger $\Delta_s$, i.e. a more steeply increasing and hence more information sensitive security. As expected, more information sensitive securities induce more information production.

The next step is to plot the effect of screening on security values. Figure 9 charts a path in the set of screened securities (e.g. the blue or red regions above) and traces gross and effective payoffs as $\Delta_r$ increases and $r_b$ falls.

Though effective payoffs are always below gross payoffs, the difference expands dramatically as $\Delta_r$ increases. As the security becomes less and less information sensitive, the signal purchased by the investor becomes noisier, leading to inefficient screening. Firm payoffs are loaded into the high state just as the conditional probability the project is mistakenly rejected in the high state increases.

Putting the plots together, the intuition for why high types signal their quality by avoiding due diligence becomes clear. Figures 10 and 11 explore two possible equilibrium constructions. In
Figure 9: Screened Securities’ Gross and Effective Payoffs
As securities become more information insensitive, the cost of screening increases.

Figure 10, the high type selects an unscreened security, choosing a point below the red region. Can the low type separate by choosing a screened security? Traveling up the high type’s indifference curve, the security lies in the blue region (the low type is screened), and the effective payoff ends up in the correct quadrant. Thus, the answer is yes.

Figure 11 explores the opposite construction. The low type selects an unscreened security, choosing a point below the blue region. Can the high type separate by choosing a screened security? Traveling down the low type’s indifference curve, the red region is only reached at the very end, by which point the signal is so noisy, and the effective payoff so poor, that neither type is willing to switch. Thus, the answer is no.

Again, this is despite the fact the cost of any signal is decreasing in project quality; the purple arrows are always shorter for high types. Of course, with arbitrary signals, it’s easier to construct an equilibrium in which high types signal their quality by encouraging screening. But imposing incentive compatibility in information acquisition, the opposite is true, and with the right refinement, the unique equilibrium is one in which high types separate by deterring information production.

The argument can also be made algebraically. Suppose in equilibrium investors accept security
Figure 10: Feasible Separation

The high type avoids information acquisition, while the low type chooses a screened security.

$s$ without screening and reject $\tilde{s}$ with probability $p_i$ in state $i$. A firm prefers $\tilde{s}$ if and only if

$$q \begin{cases} \leq \frac{\tilde{r}_b - r_b}{\Delta_r - \Delta_r} & \text{if } \tilde{\Delta}_r < \Delta_r \\ \geq \frac{\tilde{r}_b - r_b}{\Delta_r - \Delta_r} & \text{if } \tilde{\Delta}_r > \Delta_r \end{cases}$$

Thus, if $\tilde{\Delta}_r < \Delta_r$ and $0 < \frac{\tilde{r}_b - r_b}{\Delta_r - \Delta_r} < 1$ (implying $\tilde{r}_b > r_b$), there exists $q_b$ and $q_g$ such that type $l$ prefers the screened security while type $h$ prefers the opposite.

Unpacking $\tilde{\Delta}_r < \Delta_r$ uncovers the securities and signals for which type $l$ prefers screening while type $h$ does not:

$$p_b [x_g - x_b - (\tilde{s}_g - \tilde{s}_b)] + (p_g - p_b) [x_g - \tilde{s}_g] < x_g - x_b - (s_g - s_b)$$

This is more likely to hold as $s_g - s_b$ decreases, $\tilde{s}_g - \tilde{s}_b$ increases, $p_b$ decreases, and $p_g - p_b$ decreases.

But these are precisely the same conditions on $S$ associated with incentive compatibility for investors when the screening decision is endogenous. A smaller differential $s_g - s_b$ reduces the incentive to screen $s$ (in the extreme case of risk-less debt, $s_g = s_b$, the signal is worthless), and a larger differential $\tilde{s}_g - \tilde{s}_b$ increases the incentive to screen $\tilde{s}$. At the same time, the investor is less likely to purchase information as $q$ increases.

The coincidence is even stronger if the signal design itself is endogenous, extending to the
conditions on \( p \). Examining (6), the first order condition for an interior solution (investor does not accept or reject the contract outright) is

\[
p_i = \frac{\pi}{(1 - \pi) e^{\frac{k-s_i}{s_i}} + \pi}
\]

where \( \pi = p_g q + p_b (1 - q) \) is the unconditional probability of acceptance.

From the expression for \( p_i \), it is clear that \( p_i \) is increasing in \( s_i \), and hence \( p_g - p_b \) (signal power) is increasing in \( s_g - s_b \). Solving the system yields

\[
\pi = \frac{q (\eta - \lambda) + \eta (\lambda - 1)}{(\eta - 1)(\lambda - 1)}
\]

where \( \lambda = e^{\frac{k-s_b}{s_b}} > 1 \) and \( \eta = e^{\frac{k-s_g}{s_g}} < 1 \).\(^{11}\) Differentiating, we have \( \frac{\partial p_i}{\partial q} > 0 \), \( i \in \{b, g\} \), and \( \frac{\partial [p_g - p_b]}{\partial q} > 0 \) for \( q \) sufficiently small.

Thus, both \( p_b \) and (eventually) \( p_g - p_b \) falls with \( q_i \), as required. The signal choice complements separation.

\(^{11}\)If \( s_b \geq k \), the investor is guaranteed to profit and hence accepts without acquiring information. Likewise, if \( s_g \leq k \) the investment is manifestly unprofitable and hence the project is rejected without information acquisition. The fact that \( s \) is screened thus implies \( \lambda > 1 \) and \( \eta < 1 \).
6 Comparative Statics

Proposition 6. As the marginal cost of information $\bar{c}$ goes to infinity, the only equilibrium which survives is no-trade.

If due diligence is ruled out, the market unravels. Given the presence of other potential signaling devices, this result is surprising. However, note that since un-financed projects are worthless, rationing alone cannot separate types, as in Guerrieri and Shimer (2014) and other directed search models. Moreover, since there is no trade motive beyond financing investment, high types cannot signal their quality by retaining cash flows, as in Demarzo and Duffie (1999) and DeMarzo et al. (2005).

The result is also surprising because, as previously stated, if $\bar{c} = \infty$ (information acquisition is not possible), the economy resembles that of Nachman and Noe (1994), who derive pooling equilibria. The difference lies in the different ways competition is modeled. In Nachman and Noe (1994), the uncertainty associated with issuing an off-equilibrium security concerns the sale price. Hence, their refinement is based on the lowest price firms are willing to accept. However, since prices are bounded below by $K$, if a security sells for exactly $K$ in equilibrium, the refinement fails to rule out any beliefs about off-equilibrium securities which increase firm payoffs. This sustains pooling in their model. Here, in contrast, contracts fully specify the size of the investment, and firms face uncertainty not over prices but rather the thickness of off-equilibrium markets. Since beliefs about trade probabilities are unconstrained, the analogous refinement does not admit the equilibria of Nachman and Noe (1994).

Such equilibria are arguably unstable, however. The knife-edge case $p = K$ no longer exists if, say, firms randomly draw $K$ from some continuous distribution (so that the investment required is variable but unrelated to project quality). In that case, no matter how narrow the support of the distribution, the securities they characterize will support neither a general pooling equilibrium nor a separate pool for each realization of $K$. In contrast, the assumption of a common $K$ is innocuous in this setting.

7 Conclusion

The model presented in this paper captures and sheds light on some of the most salient features of private issuance markets, including the use of debt and simple combinations of debt and equity, the relationship between due diligence and security design, and the self-sorting of firms into different submarkets (bank finance, VC finance, etc.). The paper provides a unified theory of debt and non-debt securities and the first example of a model in which firms signal their type through their selection of distinct security designs, identifying new mechanisms whereby, counterintuitively, firms with higher quality projects may wish to discourage information production.
While not explicitly considered, the topics and phenomena addressed here have important macro-financial implications. Private information acquisition by investors of course entails a pass-through of private information to secondary markets, where asymmetric information can trigger fire sales, contagion, and flights to quality if sufficiently severe (as in Guerrieri and Shimer (2014) and Kurlat (2016)). And it is through debt liabilities that the balance sheet and real effects of crisis are greatly amplified. An extension could thus explore the connection between due diligence, security design, and financial fragility.
References


