THE WORLD INCOME DISTRIBUTION*

DARON ACEMOGLU AND JAUME VENTURA

We show that even in the absence of diminishing returns in production and technological spillovers, international trade leads to a stable world income distribution. This is because specialization and trade introduce de facto diminishing returns: countries that accumulate capital faster than average experience declining export prices, depressing the rate of return to capital and discouraging further accumulation. Because of constant returns to capital accumulation from a global perspective, the world growth rate is determined by policies, savings, and technologies, as in endogenous growth models. Because of diminishing returns to capital accumulation at the country level, the cross-sectional behavior of the world economy is similar to that of existing exogenous growth models: cross-country variation in economic policies, savings, and technology translate into cross-country variation in incomes. The dispersion of the world income distribution is determined by the forces that shape the strength of the terms-of-trade effects—the degree of openness to international trade and the extent of specialization.

I. INTRODUCTION

Figure I plots income per worker relative to the world average in 1990 against its 1960 value, and draws the 45 degree line for comparison. This picture of the world income distribution raises two questions: first, why are there such large differences in income across countries? For example, some countries, such as the United States or Canada, are more than 30 times as rich as others such as Mali or Uganda. Second, why has the world income distribution been relatively stable since 1960? A number of growth miracles and disasters notwithstanding, the dispersion of income has not changed much over this period: most observations are around the 45 degree line and the standard deviation of income is similar in 1990 to what it was in 1960 (1.06 versus 0.96).[1]

* We thank Pol Antràs and Rubén Segura-Cayuela for excellent research assistance. We also thank three anonymous referees, Alberto Alesina, and seminar participants at the University of California at Berkeley, Brown and Duke Universities, the Federal Reserve Bank of Minneapolis, the Federal Reserve Bank of New York, the Massachusetts Institute of Technology, Stanford University, and the University of California, Los Angeles for comments and suggestions.

1. Among subsets of countries with similar institutional structures there is substantial narrowing of differentials. For example, the standard deviation of log income per worker among OECD economies was 0.53 in 1960 and fell to 0.30 in 1990. In contrast, there appears to be significant widening of income differentials during the 100 years before 1960. See, for example, Durlauf [1995] and Quah [1997] on changes in the postwar world income distribution, and Parente and Prescott [1994] and Jones [1997] on its relative stability. Note also that the relative stability of the world income distribution is a postwar phenomenon; see

The Quarterly Journal of Economics, May 2002

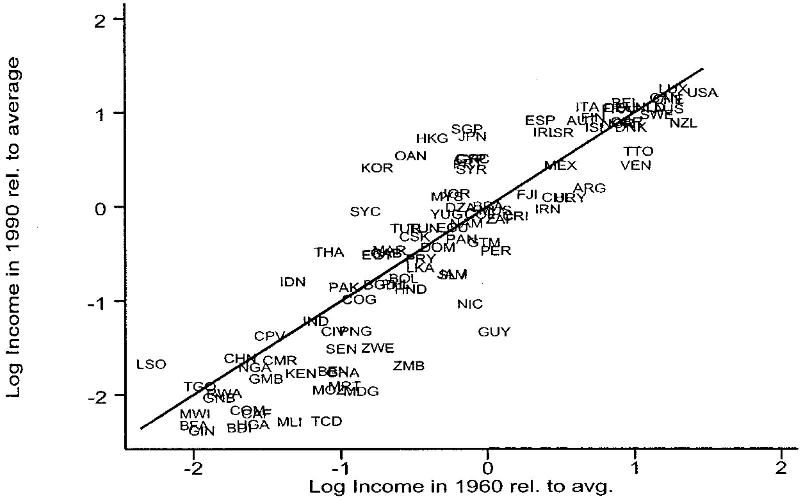

FIGURE I
Log of Income per Worker in 1990 and 1960 Relative to World Average from the Summers and Heston [1991] Data Set
The thick line is the 45 degree line.

Existing frameworks for analyzing these questions are built on two assumptions: (1) "shared technology" or technological spillovers: all countries share advances in world technology, albeit, in certain cases, with some delay; (2) diminishing returns in production: the rate of return to capital or other accumulable factors declines as they become more abundant. The most popular model incorporating these two assumptions is the neoclassical (Solow-Ramsey) growth model. All countries have access to a common technology, which improves exogenously. Diminishing returns to capital in production pull all countries toward the growth rate of the world technology. Differences in economic policies, saving rates, and technology do not lead to differences in long-run growth rates, but in levels of capital per worker and income. The strength of diminishing returns determines how a given set of differences in these

Pritchett [1997] for the widening of the world income distribution since 1870 and Acemoglu, Johnson, and Robinson [2001b] for the reversal in relative economic rankings over the past 500 years, and widening over the past 200 years.

country characteristics translate into differences in capital and income per worker.[2]

This paper offers an alternative framework for analyzing the world income distribution. We show that even in the absence of diminishing returns in production and technological spillovers, international trade—based on specialization—leads to a stable world income distribution. Countries that accumulate capital faster than average experience declining export prices, reducing the value of the marginal product of capital and discouraging further accumulation at home. They also increase the demand for products and the value of the marginal product of capital in the rest of the world, encouraging accumulation there. These terms-of-trade effects introduce de facto diminishing returns at the country level and ensure the stability of the world income distribution. Consequently, cross-country differences in economic policies, saving rates, and technology lead to differences in relative incomes, not in long-run growth rates. How dispersed the world income distribution will be for a given set of country characteristics is determined by the forces that shape the strength of the terms-of-trade effects; namely, the degree of openness to international trade and the extent of specialization.

Some degree of specialization in production is essential for the terms-of-trade effects we emphasize here: if domestic and foreign products were perfect substitutes, countries would be facing flat export demands, and capital accumulation would not affect their terms of trade. That countries specialize in different sets of products appears plausible. Moreover, this assumption has proved to be crucial in explaining some robust features of international trade, such as the substantial two-way trade in products of similar factor intensities and the success of the gravity equation in accounting for bilateral trade flows (see, for example, Helpman [1987] or Hummels and Levihnson [1995]).

We model the world as a collection of economies à la Rebelo [1991], with growth resulting from accumulation of capital. In the absence of international trade, countries grow at different rates

2. A different but related story recognizes technology differences across countries. Despite these differences, backward countries share some of the technological improvements of advanced economies through spillovers. These spillovers ensure the stability of the world income distribution, and also determine how differences in country characteristics translate into income differences. See, for example, Grossman and Helpman [1991], Parente and Prescott [1994], Coe and Helpman [1995], Howitt [2000], Eaton and Kortum [1999], Barro and Sala-i-Martin [1997], and Acemoglu and Zilibotti [2001].

determined by their economic policies, saving rates, and technology. With international trade and specialization, the world as a whole still behaves as the standard AK economy, but now all countries share the same long-run growth rate.

To understand why countries tend to grow at the same rate and what factors determine their relative incomes, consider the familiar steady-state condition equating the rate of return to savings to the effective rate of time preference. In our model, this condition takes the form,

$$\frac{\text{rental rate (domestic capital/world capital, technology)}}{\text{price of investment goods}} = \text{effective rate of time preference.}$$

The rental rate depends negatively on the relative capital of the country because of terms-of-trade effects: countries that produce more face lower export prices and a lower value of the marginal product of capital. This condition also shows how different characteristics affect relative incomes. In the steady state, countries with lower rates of time preference and lower price of investment goods (those with fewer distortions affecting investment) will have lower rental rates, hence higher relative capital and income. Countries with better technologies will be richer, in turn, because they have higher rental rates for a given level of relative capital and income.

Despite rich interactions across countries, cross-country income differences take a simple form, analogous to the basic Solow-Ramsey model. We also show that cross-country income differences and the rate of conditional convergence depend on the strength of the terms-of-trade effects, not on the capital share in output as in the Solow-Ramsey model. For plausible values of the elasticity of export demand and the share of exports in GDP, the terms-of-trade effects are strong enough to generate an elasticity of output to capital sufficient to account for observed differences in incomes.

We also provide evidence of terms-of-trade effects. We look at cross-country growth regressions to isolate differences in growth rates due to accumulation. As emphasized by Barro [1991] and Barro and Sala-i-Martin [1995], countries that are poor relative to their steady-state income level accumulate faster. We show that this faster accumulation is also associated with a worsening in the terms of trade. Our estimates imply that holding technology and other determinants of steady-state income constant, a 1 percentage point faster growth is associated with a 0.6 percentage point deterioration in the terms of trade. With terms-of-trade effects of this magnitude, our model explains a significant fraction of cross-country income differences.

Our main results are derived in Sections II and III in a model with capital as the only factor of production and with exogenous specialization. Section IV extends the model to include labor as an additional factor of production. This extended model generates higher wages and costs of living in richer countries as is the case in the data. Section V generalizes our results to the case where countries choose which goods, and how many goods, to produce. Despite endogenous specialization, the terms-of-trade effects continue to operate and ensure a common long-run growth rate across countries. As a by-product of this analysis, we also obtain a simple theory of cross-country technology differences: countries with lower rates of time preference (higher saving rates) have better technologies, contributing to their higher relative income.

Our study is related to the endogenous growth literature[3] and to papers on cross-country technological spillovers mentioned above. Howitt [2000] is most closely related. He shows that in a model of Schumpeterian endogenous growth, if innovations build on a worldwide "leading-edge technology," all countries grow at the same rate, and policy and saving rate differences affect relative incomes. Howitt's results are therefore parallel to ours, but rely on widespread technological spillovers. We emphasize instead the role of commodity trade and show that even a small amount of commodity trade is sufficient for all countries to share the same long-run growth rate.

Our paper also relates to the literature on international trade and growth. A first strand of the literature emphasizes learning-by-doing, and studies how international trade changes the industrial structure of countries and affects their aggregate rate of productivity growth.[4] A second strand studies how international trade affects the incentives to innovate.[5] A third strand studies how international trade affects the process of capital accumula-

3. See, for example, Romer [1986, 1990], Lucas [1988], Rebelo [1991], Grossman and Helpman [1991], and Aghion and Howitt [1992]. Although we use the formulation of Rebelo with capital accumulation as the engine of growth, our results generalize to a model in which endogenous growth results from technical change as in some of the other papers.

4. Krugman [1987], Stokey [1991], Young [1991], and Brezis, Krugman, and Tsiddon [1993].

5. See, among others, Segerstrom, Anant, and Dinopoulos [1990], Grossman and Helpman [1991], and Rivera, Batiz, and Romer [1991].

tion in the presence of some form of factor price equalization.[6] Our paper is closer to this third line of research, since we also examine the effects of international trade on the incentives to accumulate capital. We depart from earlier papers by focusing on the case without factor price equalization. With factor price equalization, the rental rate of capital is independent of domestic capital and countries can accumulate without experiencing diminishing returns. Without factor price equalization, the rental rate of capital is determined by the domestic capital stock even in the absence of technological diminishing returns.

II. A World of AK Economies

In this section we outline a world of AK economies with trade and specialization. The main purpose of this model is to demonstrate how terms-of-trade effects create a force toward a common growth rate across countries. We establish that any amount of international trade ensures that cross-country differences in technology, saving, and economic policies translate into differences in income levels, *not* growth rates. Countries that accumulate capital faster than average experience declining export prices, reducing the rate of return to capital and discouraging further accumulation. These terms-of-trade effects create diminishing returns to capital at the country level and keep the world distribution stable.

A. Description

The world we consider contains a continuum of countries with mass 1. Capital is the only factor of production. There is a continuum of intermediate products indexed by $z \in [0,M]$, and two final products that are used for consumption and investment. There is free trade in intermediate goods and no trade in final products or assets.

Countries differ in their technology, savings, and economic policies. In particular, each country is defined by a triplet (μ,ρ,ϕ), where μ is an indicator of how advanced the technology of the country is, ρ is its rate of time preference, and ϕ is a measure of the effect of policies and institutions on the incentives to invest.

6. See, for instance, Stiglitz [1970] and Ventura [1997]. See also Cunat and Maffezoli [2001] who analyze growth in a world economy that starts outside the cone of diversification, but eventually reaches factor price equalization.

We denote the joint distribution of these characteristics by $G(\mu,\rho,\phi)$ and assume it is time invariant.

All countries admit a representative consumer with utility function:

$$\int_0^\infty \ln c(t) \cdot e^{-\rho \cdot t} \cdot dt, \tag{1}$$

where $c(t)$ is consumption at date t in the (μ,ρ,ϕ)-country. Throughout the paper we simplify the notation by suppressing time and country indices when this causes no confusion. The budget constraint facing the representative consumer is

$$p_I \cdot \dot{k} + p_C \cdot c = y \equiv r \cdot k, \tag{2}$$

where p_I and p_C are the prices of the investment and consumption goods, k is capital stock, and r is the rental rate. For simplicity, we assume that capital does not depreciate. Since there is no international trade in assets, income y must be equal to consumption, $p_C \cdot c$, plus investment, $p_I \cdot \dot{k}$.

To introduce specialization, we adopt the Armington [1969] assumption that products are differentiated by origin.[7] Let μ be the measure (number) of intermediates produced by the (μ,ρ,ϕ)-country, with $\int \mu \cdot dG = M$. A higher level of μ corresponds to the ability to produce a larger variety of intermediates, so we interpret μ as an indicator of how advanced the technology of the country is. In all countries, intermediates are produced by competitive firms using a technology that requires one unit of capital to produce one intermediate.

Each country also contains many competitive firms in the consumption and investment goods sectors with unit cost functions:

$$B_C(r,p(z)) = r^{1-\tau} \cdot \left(\int_0^M p(z)^{1-\epsilon} \cdot dz \right)^{\tau/(1-\epsilon)}, \tag{3}$$

$$B_I(r,p(z)) = \phi^{-1} \cdot r^{1-\tau} \cdot \left(\int_0^M p(z)^{1-\epsilon} \cdot dz \right)^{\tau/(1-\epsilon)}, \tag{4}$$

7. We make this crude assumption to simplify the analysis and highlight the implications of specialization for growth patterns in the simplest way. In Section V we model how countries choose the set of intermediates that they produce and therefore provide a microfoundation for this assumption.

where $p(z)$ is the price of the intermediate with index z. These equations state that the production of consumption and investment goods requires the services of domestic capital and intermediates. The parameter τ is the share of intermediates in production, and it will also turn out to be the ratio of exports to income. This ratio is usually interpreted as a measure of openness. The parameter ϵ is the elasticity of substitution among the intermediates and also the price elasticity of foreign demand for the country's products. The inverse of this elasticity is often interpreted as a measure of the degree of specialization. We assume that $\epsilon > 1$. This assumption rules out immiserizing growth—the country becoming poorer by accumulating more (see Bhagwati [1958]). Note that the technologies for consumption and investment goods are identical except for the shift factor ϕ. We use this parameter as a crude measure of the effect of policies and institutions on the incentives to invest. Examples of the policies and institutions we have in mind include the degree of enforcement of property rights or the distortions created by the tax code.[8]

B. World Equilibrium

A competitive equilibrium of the world economy consists of a sequence of prices and quantities such that firms and consumers maximize and markets clear. Our assumptions ensure that such an equilibrium exists and is unique. We prove this by construction.

Consumer maximization of (1) subject to (2) yields the following first-order conditions:

$$\frac{r + \dot{p}_I}{p_I} - \frac{\dot{p}_C}{p_C} = \rho + \frac{\dot{c}}{c}, \tag{5}$$

$$\lim_{t \to \infty} \frac{p_I \cdot k}{p_C \cdot c} \cdot e^{-\rho \cdot t} = 0. \tag{6}$$

Equation (5) is the standard Euler equation and states that the rate of return to capital, $(r + \dot{p}_I)/p_I - \dot{p}_C/p_C$, must equal the rate of time preference plus a correction factor that depends on the

8. Jones [1995], Chari, Kehoe, and McGrattan [1997], and Parente, Rogerson, and Wright [2000] have emphasized the importance of such policies in explaining cross-country differences in income levels, and a range of empirical studies have shown the importance of institutional differences in affecting investment and economic performance (e.g., Knack and Keefer [1995], Barro [1997], Hall and Jones [1999], and Acemoglu, Johnson, and Robinson [2001a]).

slope of the consumption path. Equation (6) is the transversality condition. Integrating the budget constraint and using the Euler and transversality conditions, we find that the optimal rule is to consume a fixed fraction of wealth:

$$p_C \cdot c = \rho \cdot p_I \cdot k. \tag{7}$$

Equation (7) implies that countries with more patient consumers—low ρ—will have lower consumption to capital ratios.

Next consider firm maximization. The production functions (3) and (4) ensure that all intermediates are produced in equilibrium. Since firms in the intermediates sector are competitive, they set their price equal to marginal cost, which is the rental rate of capital. So the price of any variety of intermediate produced in the (μ, ρ, ϕ)-country is equal to

$$p = r, \tag{8}$$

where r is the rental rate of capital in the (μ,ρ,ϕ)-country. We use the ideal price index for intermediates as the numeraire; i.e.,

$$\int_0^M p(z)^{1-\epsilon} \cdot dz = \int \mu \cdot p^{1-\epsilon} \cdot dG = 1. \tag{9}$$

Since all countries export practically all of their production of intermediates and import the ideal basket of intermediates, this choice of numeraire implies that p is also the terms of trade of the country, i.e., the price of exports relative to imports.[9]

Firms in the consumption and investment sectors take prices as given and choose factor inputs to maximize profits. The logarithmic preferences in (1) ensure that the demand for consumption goods is always strong enough to induce some production in equilibrium, so price equals cost:

$$p_C = r^{1-\tau}. \tag{10}$$

On the other hand, if the country starts with a large capital stock, consumers may want to dissave, and there may not be any pro-

9. Although each country is small relative to the world, it has market power because of complete specialization in the production of intermediates. So, each country may want to act as a monopolist, imposing an optimal tariff or an export tax. Whether they do so or not does not affect our results, and we ignore this possibility. In any case, a cooperative equilibrium with free trade policies is superior to a noncooperative equilibrium in which all countries actively use trade policy, so we may think that countries have solved this coordination problem and have committed not to use trade policy.

duction of investment goods. We rule this possibility out by assuming that the initial capital stock is not too large. This ensures that price equals cost for the investment good as well:

$$p_I = \phi^{-1} \cdot r^{1-\tau}. \tag{11}$$

Finally, we need to impose market clearing for capital. By Walras' law, this is equivalent to imposing trade balance.[10] Each country spends a fraction τ of its own income on foreign intermediates, while the rest of the world spends a fraction $\tau \cdot \mu \cdot p^{1-\epsilon}$ of their income on this country's intermediates. Therefore, trade balance requires

$$y = \mu \cdot p^{1-\epsilon} \cdot Y, \tag{12}$$

where $Y \equiv \int y \cdot dG$ is world income. Equation (12) implies that when the measure of varieties, μ, is larger, a given level of income y is associated with better terms of trade, p, and higher rental rate of capital, since $r = p$. Intuitively, a greater μ implies that for a given aggregate capital stock, there will be less capital allocated to each variety of intermediate, so each will command a higher price in the world market. Conversely, for a given μ, a greater relative income y/Y translates into lower terms of trade and rental rate.

C. World Dynamics

The state of the world economy is fully described by a distribution of capital stocks. A law of motion for the world economy consists of a rule to determine the trajectory of this distribution from any starting position. This law of motion is given by the following pair of equations for each country:[11]

$$\dot{k}/k = \phi \cdot r^{\tau} - \rho, \tag{13}$$

$$r \cdot k = \mu \cdot r^{1-\epsilon} \cdot \int r \cdot k \cdot dG. \tag{14}$$

10. Market clearing for capital implies that $k = k_n + \mu \cdot k_i$, where k_n is capital used in the nontraded sector, and k_i is capital used in the production of each intermediate. Given the Cobb-Douglas assumption, we have $k_n = (1 - \tau) \cdot y/r$. Also because demand for each intermediate is of the constant elasticity form and a fraction τ of world income Y is spent on intermediates, we have $k_i = \tau \cdot p^{1-\epsilon} \cdot Y/p$. Using $y = r \cdot k$, the market clearing condition for capital is equivalent to (12).

11. To obtain (13), we substitute equations (7) and (11) into the budget constraint (2). To obtain (14), we simply rewrite equation (12) using (2) and (8).

For a given cross section of rental rates, the set of equations in (13) determines the evolution of the distribution of capital stocks. For a given distribution of capital stocks, the set of equations in (14) determine the cross section of rental rates.

The world economy has a unique and stable steady state in which all countries grow at the same rate.[12] To describe this steady state, define the world growth rate as $x \equiv \dot{Y}/Y$, and the relative income of a (μ,ρ,ϕ)-country as $y_R \equiv y/Y$. Then, setting the same growth rate for all countries, i.e., $\dot{k}/k = \dot{y}/y = x$, we obtain the steady-state cross section of rental rates as

$$r^* = \left(\frac{\rho + x^*}{\phi}\right)^{1/\tau}, \tag{15}$$

where an asterisk is used to denote the steady-state value of a variable; for example, x^* is the steady-state world growth rate. Since $p = r$, equation (15) also gives the steady-state terms of trade of the country, p^*. It is important to note that in steady state terms of trade and rental rates are constant. This highlights that the world income distribution is stable not because of continuously changing terms of trade, but because countries that accumulate more face lower terms of trade, reducing the interest rate and the incentives for further accumulation. In the steady state, both the distribution of capital stocks and relative prices are stable.

Using equations (9), (8), (12), and (15), we can provide a complete characterization of the world distribution of income in the steady state:

$$y_R^* = \mu \cdot \left(\frac{\phi}{\rho + x^*}\right)^{(\epsilon-1)/\tau}, \tag{16}$$

$$\int \mu \cdot \left(\frac{\phi}{\rho + x^*}\right)^{(\epsilon-1)/\tau} \cdot dG = 1. \tag{17}$$

Equation (16) describes the steady-state world income distribution and states that rich countries are those that are patient (low ρ), create incentives to invest (high ϕ), and have access to better technologies (high μ). Equation (17) implicitly defines the steady-state world growth rate and shows that it is higher if countries

12. Stability follows immediately since there is a single differential equation describing the behavior of each country given by (13), and this differential equation is stable because, from equation (14), a greater k leads to a lower r.

have "good" characteristics, i.e., low values for ρ and high values for ϕ and μ.

International trade and specialization play an essential role in shaping the world income distribution. To see this, use equations (8), (10), (11), and (12) to write the terms of trade and the rate of return to capital as follows:

$$\text{terms of trade} = p = \left(\frac{\mu}{y_R}\right)^{1/(\epsilon-1)}, \tag{18}$$

$$\text{rate of return} = \frac{r + \dot{p}_I}{p_I} - \frac{\dot{p}_C}{p_C} = \phi \cdot p^{\tau}. \tag{19}$$

These are the two key relative prices in our economy. Equation (18) states that for a given measure of country technology μ, the terms of trade of the country are decreasing in its relative income. Intuitively, a greater level of income translates into greater production for each variety of intermediates in which the country specializes, and this greater supply reduces the relative prices of these intermediates. Equation (19) states that for given economic policies ϕ, the rate of return to capital is increasing in the terms of trade. This is also intuitive: a higher price for the country's exports raises the value of the marginal product of capital and hence the rate of return to capital. Equations (18) and (19) together explain why countries face diminishing returns to capital.

These equations also illustrate the sources of income differences across countries. To provide incentives for accumulation, the steady-state rate of return to capital must equal the effective rate of time preference, $\rho + x^*$. Equation (19) implies that for countries with greater patience and better economic policies, lower terms of trade are sufficient to ensure accumulation (i.e., to ensure that the rate of return is equal to $\rho + x^*$). Equation (18), on the other hand, translates lower terms of trade and better technology into a greater relative income level, y_R. So countries with low values for ρ and high values for ϕ and μ will be richer.

Equations (18) and (19) also give the intuition for the stability of the world income distribution. A country with a relative income level below its steady-state value has terms of trade above its steady state (equation (18)). Terms of trade above steady state in turn translate into a rate of return to capital that exceeds the effective rate of time preference (equation (19)). This induces faster accumulation than the rest of the world, increasing relative income. As this occurs, the terms of trade worsen, the rate of return declines, and the rate of capital accumulation converges toward the world growth rate.

As in most growth models, both the shape of the steady-state world income distribution and the speed of convergence toward this steady state depend on the strength of diminishing returns. While in standard models diminishing returns are postulated as a property of technology, in our model it is derived from changes in relative prices resulting from international trade and specialization. Naturally, the strength of diminishing returns depends on the volume of trade and the extent of specialization. There are stronger diminishing returns when the volume of trade and the extent of specialization are greater (high τ and low ϵ). When τ is low, equation (19) shows that the rate of return to capital is less sensitive to changes in the terms of trade. In the limit, as $\tau \to 0$, we converge to a closed economy, the rate of return is independent of the terms of trade, and there are no diminishing returns. In this case, as in the standard endogenous growth models, very small differences in country characteristics are sufficient to create arbitrarily large differences in incomes. Similarly, when ϵ is high, equation (18) shows that terms of trade are less sensitive to differences in relative incomes. In the limit as $\epsilon \to \infty$, we are back to the endogenous growth world.

III. Empirical Implications and Evidence

World income has experienced secular growth during the past 200 years. And over the postwar era, as suggested by Figure I, most countries have grown at similar rates. Our model provides a unified framework for interpreting these facts. Since there are constant returns to capital accumulation from a global perspective, the rate of growth of the world economy is endogenous. However, since there are diminishing returns to capital accumulation at the country level, the cross-sectional behavior of the world economy is similar to that of existing exogenous growth models: cross-country variation in economic policies, savings, and technologies translate into cross-country variation in incomes. We now discuss how our model can be used to interpret cross-country income differences and patterns of conditional convergence, and provide some evidence of terms-of-trade effects.

A. Quantitative Implications

Does our model imply cross-country income differences that are quantitatively plausible? To answer this question, first consider the Solow [1956] model: countries save a fraction s of their income, and have access to the Cobb-Douglas aggregate production function, $y = (A \cdot e^{x \cdot t})^{1-\alpha} \cdot k^{\alpha}$, where A is a country-specific efficiency parameter, x is the exogenous rate of technological progress common across all countries, and α is the share of capital in national product. Since $\alpha < 1$, this production function exhibits technological diminishing returns. Define income per effective worker as $\hat{y} = y \cdot e^{-x \cdot t}$. Then, steady-state income is

$$\hat{y}^* = A \cdot \left(\frac{s}{x}\right)^{\alpha/(1-\alpha)}. \tag{20}$$

So countries that save more (high s) and are more efficient (high A) have higher per capita incomes, although all countries share the same growth rate x. The responsiveness of income to savings depends on the capital share α. Mankiw, Romer, and Weil [1992] estimated a version of equation (20) and found that it provides a reasonable fit to cross-country differences in income for $\alpha \approx 2/3$. Similarly, Klenow and Rodriguez-Clare [1997] and Hall and Jones [1999] show that given the range of variation in capital-output ratios and education across countries, the Solow model accounts for the observed differences in income per capita without large differences in the productivity term A if $\alpha \approx 2/3$. This implies a qualified success for the Solow model: given the share of capital in national product of approximately 1/3 as in OECD economies, the framework accounts for cross-country income differences only if there are sizable differences in productivity or efficiency (the A term).

To relate these empirical findings to our model, note that our key equation (16) is in effect identical to (20); in our model, the savings rate is $s = p_I \cdot \dot{k}/y$. So the steady-state savings rate is $s = x^*/(\rho + x^*)$, and substituting this into (16), we have[13]

13. In practice, Mankiw, Romer, and Weil [1992] use the investment-to-output ratio i rather than the savings rate. Summers and Heston [1991] construct i with a correction for differences in relative prices of investment goods across countries, so effectively $i = s/p_I$. Using this definition, an alternative way of expressing the empirical predictions of our model is $y_R = \mu \cdot (i/x^*)^{\epsilon-1}$. In this case, the efficiency parameter, A, is simply equal to μ, and the equivalent of α in equation (20) is $(\epsilon - 1)/\epsilon$. The quantitative predictions of our model are affected little by this change.

$$y_R^* = \mu \cdot \phi^{(\epsilon-1)/\tau} \cdot \left(\frac{s}{x^*}\right)^{(\epsilon-1)/\tau}.$$

Our model therefore implies the same cross-country relationship as the Solow model with two exceptions: (i) the efficiency parameter A captures the effects of both the technology term, μ, and the inverse of the relative price of investment goods, ϕ; and (ii) the elasticity of relative income to savings depends not on the capital share, but on the degree of specialization, ϵ, and the volume of trade, τ. In particular, the equivalent of α in equation (20) is $(\epsilon - 1)/(\tau + \epsilon - 1)$ in our model, so the elasticity of output to savings is decoupled from the capital share.[14]

Does this implied elasticity of output to savings generate plausible quantitative predictions? Given the Cobb-Douglas preferences, the share of traded goods, τ, is the share of exports in GDP. Since, except for the United States and Japan, this number is around 30 percent or higher for rich economies (see World Development Report [1997]), here we take it to be $\tau = 0.3$. On the other hand, ϵ corresponds to the elasticity of export demand. Estimates of this elasticity in the literature are for specific industries, and vary between 2 and 10, although there are also estimates outside this range (see, for example, Feenstra [1994] or Lai and Trefler [1999]). For our purposes, we need the elasticity for the whole economy, not for a specific industry. Below we use cross-country data on changes in terms of trade to estimate an elasticity of $\epsilon = 2.6$. So here we use this as our baseline estimate. With $\epsilon = 2.6$, our model's predictions for cross-country income differences are identical to those of the Solow model with $\alpha = 0.85$. Therefore, in contrast to the simplest neoclassical growth model which yields a small elasticity of output to savings, our model implies a reasonably large elasticity, and in fact, generates cross-country income differences even larger than those observed in the data. If there were, in addition, technological diminishing returns, as in the Solow model, or technological catch-up, as emphasized for example by Howitt [2000], the implied elasticity of output to savings would be lower. This suggests that perhaps terms-of-trade effects emphasized here and technological diminishing returns or technological catch-up are jointly important in

14. In this economy the capital share in national product is equal to 1. In the next section, when we introduce labor, the capital share will no longer be 1, but the elasticity of output to savings will remain unchanged.

determining the effect of differences in saving rates and distortions on cross-country income differences.

Does the model also generate plausible implications for growth dynamics? Barro [1991] and Barro and Sala-i-Martin [1995] represent country-level growth dynamics by regressions of the form,

$$g_t = -\beta \cdot \ln y_{t-1} + Z_t' \cdot \theta + u_t, \tag{21}$$

where g_t is the annual growth rate of income of the country between some dates $t-1$ and t, and Z_t is a set of covariates that determine steady-state income. The parameter $\beta = -dg_t/d\ln y_{t-1}$ is interpreted as the speed of (conditional) convergence toward steady state. These regressions typically estimate a value of $\beta \approx 0.02$ corresponding to a rate of conditional convergence of about 2 percent a year. In our model, the growth rate of output can be expressed as[15]

$$g \equiv \frac{\dot{y}}{y} = x + \frac{\epsilon - 1}{\epsilon} \cdot (\rho + x^*) \cdot \left[\left(\frac{y_R}{y_R^*} \right)^{-\tau/(\epsilon - 1)} - \frac{\rho + x}{\rho + x^*} \right]. \tag{22}$$

When a county is at its steady state value, i.e., $y_R = y_R^*$, it grows at the rate $(x + (\epsilon - 1) \cdot x^*)/\epsilon$, which is a weighted average of the steady-state world growth rate, x^*, and the current world growth rate, x. When the world is also in steady state, i.e., $x = x^*$, the country grows at the world growth rate, x^*. If y_R is below its steady-state value, it grows at a rate that depends on the distance away from this steady state, the elasticity of export demand, ϵ, the share of traded goods, τ, and the rate of time preference, ρ.

The implied speed of convergence is therefore $\beta = -dg/d\ln y = \tau \cdot (\rho + x^*) \cdot (y_R/y_R^*)^{-\tau/(\epsilon-1)}/\epsilon$. As in the Solow-Ramsey model, the speed of convergence is not constant; countries away from their steady states grow faster. Near the steady state, $y_R \approx y_R^*$, we have that $\beta = \tau \cdot (\rho + x^*)/\epsilon$. The baseline values of parameters suggested by Barro and Sala-i-Martin [1995] imply that the term in parentheses is about 0.1.[16] With these values, the Solow model with a capital

15. To obtain this equation, we use the trade balance condition (12) and the budget constraint (2) to get an expression for y in terms of world income, Y, and the capital stock of the country, k: $y = (\mu \cdot Y)^{1/\epsilon} \cdot k^{(\epsilon-1)/\epsilon}$. We first time-differentiate this equation, next substitute from (13) for $\dot{k}/k$ and (12) for r to obtain an expression for $\dot{y}/y$, and then substitute for the steady-state relative income level, y_R^*, from (16).

16. The standard formula includes the rate of population growth, n, and the rate of depreciation of capital, δ, which we have set to zero to simplify notation. It is easy to check that if we allow for positive population growth and depreciation, the speed of convergence would be $\beta = \tau \cdot (\rho + n + \delta + x^*)/\epsilon$. Barro and Sala-i-Martin suggest a parameterization with an annual discount factor of about

share of one-third predicts convergence at approximately 6.6 percent a year, considerably faster than the actual speed of convergence. In contrast, with an elasticity of export demand of $\epsilon = 2.6$ and the share of exports in GDP of $\tau = 0.3$, our model implies that $\beta \approx 0.011$—convergence at approximately 1.1 percent a year, which is slower than observed in the data. The predicted speed of convergence would be higher again with additional technological diminishing returns or technological catch-up.

B. Empirical Evidence on Terms-of-Trade Effects

At the center of our approach is the notion that as a country accumulates more capital, its terms of trade deteriorate. Is there any evidence supporting this notion? A natural place to start is to look at the correlation between growth and changes in terms of trade. Consider equation (12) which links the terms of trade of a country to its relative income. Taking logs and time differences, we obtain

$$\pi_t = (g_t - x_t)/(\epsilon - 1) + \Delta \ln \mu_t, \tag{23}$$

where π_t is defined as the rate of change in the terms of trade between date t and some prior date $t - 1$, g_t is the rate of growth of the country's income, x_t is the rate of growth of world income, and $\Delta \ln \mu_t$ is the change in technology. More generally, this last term stands for all changes that affect income and terms of trade positively, including changes in technology and the world's tastes toward the country's products.

In theory, we can estimate an equation of the form (23) using cross-country data. Unfortunately, in practice, we do not have direct measures of the technology term, $\Delta \ln \mu_t$, so the only option is to estimate (23) without this term, or with some proxies. Figure II plots changes in terms of trade between 1965 and 1985 against the growth rate of income during the same period for the entire set of countries we have data on terms of trade, and separately for non-OPEC countries.[17] It shows that there is no relationship between growth and changes in terms of trade.

0.99 (i.e., $\rho = 0.02$), a depreciation rate of 5 percent, a world growth rate of 2 percent, and a population growth rate of 1 percent per annum. This implies that $\rho + n + \delta + x^* \approx 0.1$.

17. The terms-of-trade data are from Barro and Lee [1993], in turn constructed from the World Bank and United Nations sources. Barro and Lee report five-year averages of the changes in the prices of exports minus the prices of imports. The change in terms of trade 1965–1985 is the geometric average of these changes between 1965 and 1985.

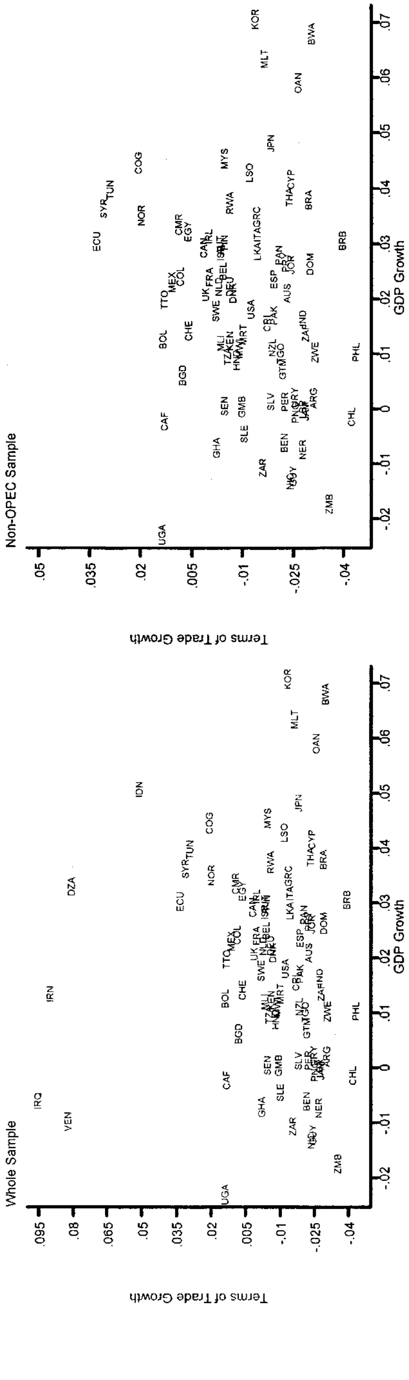

FIGURE II
Changes in Terms of Trade 1965–1985 versus GDP Growth 1965–1985

Does this imply that there are no terms-of-trade effects in the data? Not necessarily. Since changes in technology, as captured by the $\Delta \ln \mu_t$ term, are directly correlated with changes in income, estimates from an equation of the form (23) and the relationship shown in Figure II will be biased. This is the standard identification problem, and to make progress, we need to isolate changes in growth rates that are plausibly orthogonal to the omitted technology term $\Delta \ln \mu_t$. A plausible source of variation would come from countries growing at different rates because they have started in different positions relative to their steady-state income level and are therefore accumulating at different rates to approach their steady state.

How can we isolate changes in income due to accumulation? Here we make a preliminary attempt by using a convergence equation like (21). Recall that these equations relate cross-country differences in growth rates to two sets of factors: (i) a set of covariates, Z_t, which determine the relative steady-state position of the country; and (ii) the initial level of income, which captures how far the country is from its relative steady-state position. Accordingly, differences in growth due to the second set of factors approximate changes in income due to accumulation, and give us an opportunity to investigate whether faster accumulation leads to worse terms of trade.

The estimating equation is

$$\pi_t = \delta \cdot g_t + Z_t' \cdot \omega + \upsilon_t, \tag{24}$$

where, as before, π_t is the rate of change in terms of trade, and g_t is the growth rate of output. We will estimate (24) using Two-Stage Least Squares (2SLS), instrumenting g_t using equation (21). The vector Z_t includes potential determinants of steady-state income, in particular, human capital and institutions variables. The coefficient of interest is δ, which, in our theory, corresponds to $-1/(\epsilon - 1)$ as in (23). The excluded instrument in our 2SLS estimation is the initial level of income, $\ln y_{t-1}$. Intuitively, conditional on income growth and other covariates, the initial level of income should not affect the terms of trade.[18]

18. In the presence of technological convergence, countries below their steady state may also be improving their technologies, and $\ln y_{t-1}$ may be correlated with $\Delta \ln \mu_t$. In this case, our estimate of δ would be biased upwards, stacking the cards against finding a negative δ. More generally, this consideration suggests that we may want to interpret our estimate of the strength of the terms-of-trade effects as a lower bound.

Table I reports cross-sectional regressions of the rate of change of terms of trade between 1965 and 1985 on the growth rate of income over the same period and various sets of covariates as in equation (24) (all data are from Barro and Lee [1993]). The top panel reports the 2SLS estimate of δ, the coefficient on output growth in equation (24). The middle panel gives the first-stage coefficient on $\ln y_{t-1}$, β. Finally, the bottom panel reports the OLS estimate of δ. Naturally, in the first stage and the OLS the same covariates as the 2SLS are included, but the coefficients are not reported to save space. Different columns correspond to different sets of covariates. In the first-stage relationship, the coefficients are very similar to the convergence equations estimated by Barro and Sala-i-Martin [1995], and we do not report them here.

In column (1) we start with a minimal set of covariates that control for human capital differences. These are average years of schooling in the population over age 25 in 1965 and the log of life expectancy at birth in 1965. Both of these variables are typically found to be important determinants of steady-state relative income levels (or country growth rates), so they are natural variables to include in our Z_t vector. We also include a dummy for OPEC countries (in our sample, these are Algeria, Indonesia, Iran, Iraq, and Venezuela). The coefficient on log GDP in 1965, reported in Panel B, shows the standard result of conditional convergence at the speed of approximately 2 percent a year. The estimate of the coefficient of interest, δ, in column (1) is -0.6 with a standard error of 0.27. This estimate implies that a country growing 1 percentage point faster due to accumulation experiences a 0.6 percentage point decline in its terms of trade. This estimate is statistically significant at the 5 percent level. The coefficient on years of schooling is insignificant, while the coefficients on life expectancy and the OPEC dummy are positive and statistically significant. The coefficient on the OPEC dummy implies that, all else equal, the terms of trade of the OPEC countries improved at approximately 0.091 percentage points a year during this period. We return to the interpretation of the other covariates later. Notice also that, as suggested by Figure II, the OLS coefficient reported in Panel C is insignificant and practically equal to 0. The contrast between the OLS and the 2SLS estimates likely reflects the fact that the 2SLS procedure is isolating changes in income that are due to accumulation and hence orthogonal to $\Delta \ln \mu_t$.

TABLE I
IV REGRESSIONS OF GROWTH RATE OF TERMS OF TRADE

	Main regression (1)	Detailing schooling (2)	Adding political indicat (3)	Adding change in Sch (4)	Adding change in Sch (5)	Nonoil sample (6)
Panel A: Two-stage least squares						
GDP Growth 1965–1985	−0.595	−0.578	−0.458	−0.561	−0.455	−0.620
	(0.265)	(0.261)	(0.221)	(0.248)	(0.187)	(0.354)
Years of schooling 1965	−0.001		−0.002	−0.000		−0.001
	(0.002)		(0.002)	(0.002)		(0.002)
Years of primary schooling 1965		−0.002				
		(0.003)				
Years of secondary schooling 1965		−0.002				
		(0.006)				
Years of higher schooling 1965		0.019				
		(0.034)				
Log of life expectancy 1965	0.043	0.045	0.034	0.020		0.046
	(0.024)	(0.024)	(0.021)	(0.027)		(0.030)
OPEC dummy	0.091	0.090	0.092	0.086	0.087	
	(0.009)	(0.009)	(0.009)	(0.010)	(0.009)	
War dummy			−0.013			
			(0.005)			
Political instability			0.007			
			(0.023)			
Log black market premium			−0.005			
			(0.012)			
Change in years of schooling 1965–1985				0.008	0.009	
				(0.004)	(0.003)	
Change in log of life expectancy 1965–1985				−0.000	−0.042	
				(0.078)	(0.045)	
Panel B: First-stage for GDP growth						
Log of GDP 1965	−0.019	−0.020	−0.024	−0.020	−0.020	−0.016
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
R^2	0.35	0.36	0.54	0.47	0.47	0.34
Panel C: Ordinary least squares						
GDP Growth 1965–1985	0.037	0.037	0.038	0.041	−0.005	0.116
	(0.106)	(0.107)	(0.107)	(0.112)	(0.103)	(0.114)
N. of obs	79	79	70	79	79	74

"Growth Rate of Terms of Trade" is measured as the annual growth rate of export prices minus the growth rate of import prices. The OPEC dummy takes value 1 for five countries in our sample (Algeria, Indonesia, Iran, Iraq, and Venezuela). The political instability variable is the average of the number of assassinations per million inhabitants per year and the number of revolutions per year, the war variable is a dummy for countries that fought at least one war over the period 1965–1985, and the log black market premium is the average of the logarithm of the black market premium over the period 1965–1985. All the data are from the Barro-Lee data set.

Excluded instrument is log of output in 1965 in columns (1), (2), (3), and (4) and (6), while in column (5) excluded instruments are log of output in 1965, years of schooling in 1965, and the log of life expectancy in 1965.

In column (2) we enter years of primary, secondary, and tertiary schooling separately, and this has little effect on the estimate of δ. Column (3) adds a number of common controls used by Barro and Sala-i-Martin [1995] to control for differences in institutions and property rights, which are likely to be first-order determinants of productivity and technology and, hence, of steady-state income. These institutional variables are an index of political instability, a dummy for experiencing a war during this period, and the log of the black market premium. The estimate is now -0.46 (standard error = 0.21).[19]

The coefficients on the covariates in columns (1)–(4) are difficult to interpret because they refer to values at the beginning of the sample. For example, a 10 percent higher life expectancy in 1965 is associated with 0.5 percentage point improvement in terms of trade. This may capture the fact that initial level of life expectancy is correlated with subsequent changes in these human capital variables and therefore possibly correlated with $\Delta \ln \mu_t$ as well. In columns (5) and (6) we add the changes in years of schooling and life expectancy between 1965 and 1985 to the basic regression of column (1). In column (5) these changes are entered as additional covariates. In column (6) we instead use the initial levels of years of schooling and life expectancy as excluded instruments in addition to the initial level of income. In both columns the estimate of δ is similar to our baseline estimate, and statistically significant at the 5 percent level. We find that changes in the years of schooling are positive and significant in the second stage, indicating that countries that increased their human capital over this period experienced improvements in their terms of trade. This is reasonable since improvements in human capital are likely to be correlated with $\Delta \ln \mu_t$.[20]

Finally, in column (7) we repeat the basic regression of col-

19. As in typical cross-country growth regression analyses, these institutional variables are treated as exogenous. We also experimented with a specification instrumenting for a measure of institutions among the former colonies using the mortality rates of European colonizers following Acemoglu, Johnson, and Robinson [2001a]. Unfortunately, the restriction to former colonies left us with too small a sample, and the results were insignificant.

20. We experimented with different specifications and various subsets of covariates, with similar results. We also estimated δ using decadal changes, and a random-effect model as in Barro and Sala-i-Martin [1995] and Barro [1997]'s favorite specification. In this case, the results are similar to those reported in Table I. For example, the equivalent of column (1) yields an estimate of δ of -0.88 with a standard error of 0.42, while the equivalent of column (6) which excludes oil producers yields an estimate of -0.85 with a standard error of 0.51.

umn (1) excluding the five OPEC countries from the sample. The estimate of δ is -0.62 (standard error = 0.35), which is no longer significant at the 5 percent level, but significant at the 10 percent level.

Figure III gives a visual representation of the 2SLS estimate reported in columns (3) and (6) of Table I. On the vertical axis we have the component of the changes in the terms of trade that is orthogonal to the covariates included in the regression, and on the horizontal axis, the projection of GDP growth on our instrument, initial income, again orthogonalized with respect to the set of covariates. The OLS regression of the first variable on the second will give precisely the corresponding 2SLS estimate. The figure shows that countries predicted to grow faster because of relatively low initial income (relative to their human capital indicators), such as the Philippines, Thailand, Taiwan, or Korea, typically experienced a worsening in their terms of trade compared with countries with relatively high initial income, such as Mexico, Switzerland, or France.

Overall, the results in Table I provide some preliminary evidence that higher output growth due to accumulation is associated with a worsening in the terms of trade, as implied by our mechanism. Nevertheless, given the relatively low number of observations and the usual difficulties in interpreting cross-country regressions, this result has to be interpreted with caution.

We can also use the magnitudes of the coefficient estimates to compute implied values for the export demand elasticity ϵ. For example, the estimate in column (1), -0.6, implies that $\epsilon \approx 2.6$. This is a reasonable elasticity estimate, within the range of the industry estimates. Returning to the discussion in the previous subsection, recall that with a value of ϵ around 2.6, our model accounts for much of the variability in income levels across countries (in fact, as noted above, it somewhat "overexplains" the observed differences). Therefore, this evidence suggests that terms-of-trade effects may be quantitatively important in understanding the observed patterns of cross-country income differences and growth.

IV. Labor, Wages, and Product Prices

Capital is the only factor of production in the model of Section II. This limits the potential applications of the model. It also makes our approach silent on two important features of the data:

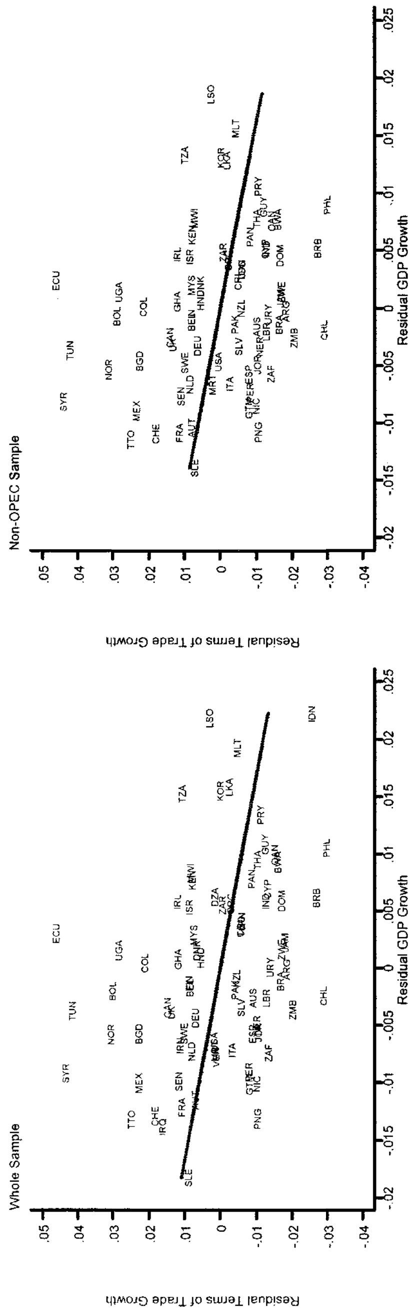

FIGURE III
The Instrumental-Variables Relationship between Changes in Terms of Trade 1965–1985 and GDP Growth 1965–1985
(Instrumented by log GDP 1965)

(1) that wages for comparable workers are higher in richer countries; and (2) that costs of living are higher in rich countries (see Summers and Heston [1991]). Fortunately, it is straightforward to generalize our baseline model of Section II by adding labor as another factor of production: all of the implications we have emphasized so far remain unchanged, and in addition, the model generates higher wages and higher costs of living in richer countries.

A. The Model with Labor

Let us add two assumptions to our basic model. First, the production of the consumption good now requires labor. In particular, we adopt the following unit cost function:

$$B_C(w,r,p(z)) = w^{(1-\gamma)\cdot(1-\tau)} \cdot r^{\gamma\cdot(1-\tau)} \cdot \left[\left(\int_0^M p(z)^{1-\epsilon} \cdot dz\right)^{\tau/(1-\epsilon)}\right], \tag{25}$$

which is identical to equation (10), except for the presence of domestic labor services in production, implying that the consumption goods sector uses labor in addition to capital and traded intermediates.

Second, each consumer supplies one unit of labor inelastically. The budget constraint of the representative consumer then becomes

$$p_I \cdot \dot{k} + p_C \cdot c = y \equiv r \cdot k + w, \tag{26}$$

where w is the wage rate. The rest of the assumptions in subsection A remain the same. The model in this section is therefore the limiting case in which $\gamma \to 1$. In this limit, labor is not used in production, and the wage is zero.

Consumers now maximize the utility function (1) subject to the new budget constraint (26). The solution to this problem still involves the Euler equation (5) and the transversality condition (6). Once again integrating the budget constraint and using the Euler and transversality conditions, we obtain the consumption rule as

$$p_C \cdot c = \rho \cdot \left(p_I \cdot k + \int_0^\infty w \cdot e^{-\int_0^t (r+\dot{p}_I)/p_I \cdot dv} \cdot dt\right). \tag{27}$$

The optimal rule is still to consume a fixed fraction of wealth, which now also includes the net present value of wages.

The existence of labor income has no effect on firms in the intermediate and investment goods sectors. So equations (8) and (11) still apply. But the condition that price equals marginal cost for the firms in the consumption good sector is now given by

$$p_C = w^{(1-\gamma)\cdot(1-\tau)} \cdot r^{\gamma\cdot(1-\tau)}, \tag{28}$$

so prices of consumption goods depend on the wage rate.

Since we now have two factor markets, the trade balance condition, (12), is not sufficient to ensure market clearing, and we need to add a labor market clearing condition to complete the model. Labor demand comes only from the consumption goods sector, and given the Cobb-Douglas assumption, this demand is $(1-\gamma)\cdot(1-\tau)$ times consumption expenditure, $p_C \cdot c$, divided by the wage rate, w. So the market clearing condition for labor is

$$1 = (1-\gamma)\cdot(1-\tau)\cdot((p_C\cdot c)/w). \tag{29}$$

It is useful to note that (29) implies labor income, w, is always proportional to consumption expenditure. Using this fact, we can simplify the optimal consumption rule, (27), to obtain

$$p_C\cdot c = \frac{\rho}{1-(1-\gamma)\cdot(1-\tau)}\cdot p_I\cdot k. \tag{30}$$

The law of motion of the world economy is again described by a distribution of capital stocks, but now this distribution is given by a triplet of equations for each country:[21]

$$\dot{k}/k = \phi\cdot r^{\tau} - \rho. \tag{31}$$

$$r\cdot k + w = \mu\cdot r^{1-\epsilon}\cdot\int (r\cdot k + w)\cdot dG. \tag{32}$$

$$\frac{w}{r\cdot k + w} = \frac{(1-\gamma)\cdot(1-\tau)\cdot\rho}{[\gamma+(1-\gamma)\cdot\tau]\cdot\phi\cdot r^{\tau} + (1-\gamma)\cdot(1-\tau)\cdot\rho}. \tag{33}$$

Equation (31) is the law of motion for capital. It is identical to (13) and gives the evolution of the distribution of capital stocks for

21. To obtain (31), we start with (26), and substitute for w using (29), for $p_C\cdot c$ using (30), and for p_I using (11). Equation (32) is simply the trade balance condition, (12), rearranged with $y = r\cdot k + w$. Finally, we use (11), (29), and (30) to express w as a function of k and r, and then rearrange to obtain (33).

a given distribution of rental rates. Equation (32) determines the cross section of rental rates for a given distribution of capital stocks and wage rates. The third equation, (33), defines the labor share—wage income divided by total income. This equation also shows that the behavior of the labor share simply depends on the rental rate: as the rental rate increases, the labor share falls.

The steady-state world distribution of income follows from equations (31) and (32). In steady state, $\dot{k}/k = x^*$; i.e., all countries will grow at the same rate. This immediately gives the steady-state rental rate as in equation (15) from the previous section. More important for our purposes, the steady-state distribution of income and the world growth rate are still given by equations (16) and (17). Therefore, the intuition regarding the determinants of the cross-sectional distribution of income from subsection C applies exactly. Moreover, the empirical implications, and the fit of the model to existing evidence, discussed in Section III, are also valid.[22] But there are now new implications for the cross section of wages and some key relative prices.

B. Factor and Product Prices

Equations (31), (32), and (33) give the steady-state factor prices. The steady-state rental rate of capital is still given by (14) from Section II. In addition, the steady-state wage rate is

$$w^* = \frac{(1-\gamma)\cdot(1-\tau)\cdot\rho}{[\gamma+(1-\gamma)\cdot\tau]\cdot x^* + \rho}\cdot\mu\cdot\left(\frac{\phi}{\rho+x^*}\right)^{(\epsilon-1)/\tau}\cdot Y. \tag{34}$$

In the cross section the rental rate of capital continues to be lower in richer countries, i.e., countries with low ρ and high ϕ. In contrast, wages tend to be higher in richer countries: countries with better technology (high μ) and with better economic policies (high ϕ) will have higher wages. Both of these follow because richer countries generate a greater demand for consumption, increasing the demand for labor and wages.[23]

22. The equation describing convergence to steady state is also similar. In particular, equation (22) from the previous section still gives the rate of growth of capital income (relative to average capital income), say y_R^k, but now total income also includes labor income. We can write relative income as $y_R \equiv y_R^k/\zeta_R^k$ where ζ_R^k is the share of capital income relative to the average value of this share in the world. So long as factor shares do not change much near the steady state, equation (22) still describes the convergence properties of this more general model.

23. Interestingly, the effect of ρ on wages is ambiguous. Countries with low ρ will accumulate more and tend to be richer, and through the same mechanism,

The contrast between the behavior of the rental rate and the wage rate in the time series is also interesting. While the rental rate of capital remains constant, equation (34) shows that wages in all countries grow at the rate of world income growth. This prediction is consistent with the stylized facts on the long-run behavior of factor prices.

Finally, recall that equation (33) gives the share of labor in national product, so the capital share in national product is no longer equal to 1, while relative income differences are exactly the same as in the model of Sections II and IV. This highlights that, as claimed before, the result that the responsiveness of relative income to savings and economic policies depends on the share of exports in GDP and export demand elasticity was not predicated on a capital share of 1.[24]

What about product prices? While in the model of Section II both consumption and investment goods were cheaper in rich countries, now equation (28) implies that consumption goods tend to be more expensive in richer countries. This is because wages are higher in rich countries. As a result, the cost of living (the geometric average of consumption and investment goods prices) could be higher in rich countries. In fact, since the share of income spent on investment goods is small, differences in consumption good prices are likely to dominate, making the costs of living higher in richer countries.

Next, note that the relative price of investment goods is now $p_I/p_C = \phi^{-1} \cdot (w/r)^{-(1-\gamma)\cdot(1-\tau)}$. This is different from the relative price expression in the previous model because of the second term, which incorporates the fact that consumption goods are more labor-intensive than investment goods. Our model in Section II generated lower relative prices of investment goods in rich countries only because of differences in policies, ϕ. Now we have

---

they will have a greater demand for consumption and higher wages. However, as equation (30) shows, everything else equal, a country with low ρ will consume less, which tends to reduce the demand for consumption and wages. Differentiation gives that

$$\frac{\partial w^*}{\partial \rho} < 0 \Leftrightarrow \quad \epsilon > 1 + \frac{[\gamma + (1-\gamma)\cdot\tau]\cdot x^* \cdot \tau \cdot (\rho + x^*)}{\rho^2 + [\gamma + (1-\gamma)\cdot\tau]\cdot \rho \cdot x^*}.$$

In other words, as long as the elasticity of foreign demand is large enough, countries with low ρ will have higher wages.

24. More generally, although the responsiveness of output to saving rates and policies does not depend on the capital share in national product, it can be shown that it does depend on the capital share in the investment goods sector.

an additional effect reinforcing this: richer countries have higher wages, reducing the relative prices of investment goods.

V. Specialization and Technology Differences

The previous sections have shown how trade and specialization shape the process of world growth and cross-country income differences. At the center stage of our framework are diminishing returns due to terms-of-trade effects: as countries accumulate more capital, they increase the production of the commodities in which they specialize, and their terms of trade worsen. There are two assumptions underlying this mechanism:

1. Each country specializes in a different set of products.
2. The set of products a country produces is fixed (or, at least, it does not grow proportionally with its income).

The importance of these two assumptions is highlighted in equation (18). If countries were not specialized, or if $\epsilon \to \infty$ so that different goods were perfect substitutes, they would face flat export demands. In this case, capital accumulation and greater production of intermediates would not worsen the terms of trade. On the other hand, if the set of products in which a country specializes were proportional to its income, the production of each variety would not change with income. In this case, even with downward-sloping export demands, capital accumulation would not worsen the terms of trade.

In this section we show that these assumptions can be justified as the equilibrium of a model in which countries choose the set of goods they produce. We use a model of specialization due to increasing returns as in Helpman and Krugman [1985] to illustrate our main point. The working paper version [Acemoglu and Ventura 2001] shows that our results also apply if specialization is driven by costly product development, for example, as in Grossman and Helpman [1991].

We introduce two modifications to the model of Section IV. First, we assume that there is an infinite mass of intermediates, and all firms in all countries know how to produce them. Hence, all countries have access to the same technology frontier. The total number of goods produced, M, as well as its distribution among countries, μ, is determined as part of the equilibrium.

Second, we assume that one worker is needed to run the production process for each intermediate. So there is a fixed cost of production equal to the wage w. In addition, one unit of capital is required to produce one unit of each intermediate, so there is also a variable cost in terms of the rental rate of capital, r. The rest of the assumptions from Section IV still apply.

The consumer problem is still to maximize (1) subject to the budget constraint, (26). The solution continues to be given by the Euler equation (5) and the transversality condition (6), and the consumption rule is still represented by (27) from the previous section.

Firms in the consumption and investment goods sectors face the same problem as before, and equations (11) and (25) still determine their prices. But firms in the intermediate goods sector are now subject to economies of scale. Since an infinite number of varieties are available at no cost, no two firms will ever choose to produce the same good. So each producer is a monopolist. With isoelastic demands, all intermediate good monopolists in a country will set the same price, equal to a constant markup over marginal cost (which is equal to the rental rate, r):

$$p = (\epsilon/(\epsilon - 1)) \cdot r. \tag{35}$$

Hence, the terms of trade are no longer equal, but simply proportional, to the rental rate of capital. Because of the markup over marginal cost, each producer makes variable profits equal to ϵ^{-1} times its revenue, $\tau \cdot p^{1-\epsilon} \cdot Y$. As long as these variable profits exceed the cost of entry, there will be entry. So we have a free-entry equation equating variable profits to fixed costs:

$$w = (\tau/\epsilon) \cdot p^{1-\epsilon} \cdot Y, \tag{36}$$

where w, the wage rate, is the fixed cost of producing an intermediate, since one worker is required to run the production process.

The trade balance equation, (12), still applies. The market clearing condition for labor needs to be modified because now μ workers are employed in the intermediate sector:

$$1 - \mu = (1 - \gamma) \cdot (1 - \tau) \cdot (p_C \cdot c)/w. \tag{37}$$

Notice that the consumption-to-capital ratio is no longer constant, so we need to add this ratio, $z \equiv (p_C \cdot c)/(p_I \cdot k)$, as a costate variable, and include the transversality condition to determine the trajectory of the system. In addition, the number of varieties is now endogenous and will be determined from the free-entry condition, equation (36).

The laws of motion of the key variables is given by two blocks of equations for each country:

1. Dynamics. For a given distribution of factor prices, r and w, and varieties, μ, this block determines the evolution of the distribution of capital stocks:[25]

$$\frac{\dot{k}}{k} = \phi \cdot r^{\tau} - \left[1 - \frac{(1-\gamma)\cdot(1-\tau)}{1-\mu}\right] \cdot z. \tag{38}$$

$$\frac{\dot{z}}{z} = \left[1 - \frac{(1-\gamma)\cdot(1-\tau)}{1-\mu}\right] \cdot z - \rho. \tag{39}$$

$$\lim_{t\to\infty} z \cdot e^{-\rho \cdot t} = 0. \tag{40}$$

Equation (38) gives the evolution of the capital stock as a function of the rental rate r, the number of varieties, μ, and the consumption-to-capital ratio, z. It differs from (31) only because the consumption-to-capital ratio now varies over time. Equation (39) gives the evolution of the consumption-to-capital ratio as a function of the number of varieties, μ. Finally, equation (40) is the transversality condition.

2. Factor prices and varieties. Three equations give the cross section of factor prices and the number of varieties of intermediates as functions of the distribution of capital stocks and consumption to capital ratios:[26]

$$r \cdot k + w = \mu \cdot \left(\frac{\epsilon}{\epsilon - 1}\right)^{1-\epsilon} \cdot r^{1-\epsilon} \cdot \int (r \cdot k + w) \cdot dG. \tag{41}$$

$$\frac{w}{r \cdot k + w} = \frac{(1-\gamma)\cdot(1-\tau)\cdot z}{(1-\mu)\cdot\phi\cdot r^{\tau} + (1-\gamma)\cdot(1-\tau)\cdot z}. \tag{42}$$

$$w = \frac{\tau}{\epsilon} \cdot \left(\frac{\epsilon}{\epsilon - 1}\right)^{1-\epsilon} \cdot r^{1-\epsilon} \cdot \int (r \cdot k + w) \cdot dG. \tag{43}$$

Equation (43) is the trade balance equation and differs from (32)

25. To obtain (38), we start with (26), and substitute for w using (37), for $p_C \cdot c$ using the definition $z \equiv (p_C \cdot c)/(p_I \cdot k)$, and for p_I using (11). Equation (39) follows from substituting for w from the market clearing equation (37) into (26) and using the definition $z \equiv (p_C \cdot c)/(p_I \cdot k)$.

26. Equation (41) follows from (12) combined with (35). The wage equation, (42), follows from the market clearing condition for labor, (37), and the definition of z in a manner analogous to the derivation of equation (33) in footnote 21. Finally the free-entry equation, (43), is obtained by substituting for world income, Y.

because, due to monopoly power, the rental rate and the terms of trade are not equal (see equation (35)). Equation (42) gives the labor share. Finally, (43) is the free entry condition.

The dynamics of the world economy are again stable and converge to a unique steady state. This steady state is described by two equations similar to (16) and (17):

$$y_R^* = \mu^* \cdot \left(\frac{\epsilon}{\epsilon - 1}\right)^{1-\epsilon} \cdot \left(\frac{\phi}{\rho + x^*}\right)^{(\epsilon-1)/\tau}, \tag{44}$$

$$\int \mu^* \cdot \left(\frac{\epsilon}{\epsilon - 1}\right)^{1-\epsilon} \cdot \left(\frac{\phi}{\rho + x^*}\right)^{(\epsilon-1)/\tau} \cdot dG = 1. \tag{45}$$

The reason why (44) and (45) differ from (16) and (17) is the presence of monopoly markup. Otherwise, they are identical to (16) and (17), and imply the same cross-sectional relationship between economic policies, saving rates, and technology.

The key modification is that the number of varieties is now endogenous and given by[27]

$$\mu^* = \tau \cdot \frac{\rho + x^* \cdot [1 - (1 - \gamma) \cdot (1 - \tau)]}{\rho \cdot [\tau + \epsilon \cdot (1 - \gamma) \cdot (1 - \tau)] + \tau \cdot x^*}. \tag{46}$$

The only country-specific variable in this equation is ρ. So all countries have similar μ's, but those with lower discount rates (and hence higher saving rates) endogenously specialize in the production of more goods—or loosely speaking, they will "choose better technologies." Intuitively, countries with low ρ accumulate more capital and have a larger capital stock relative to their wage rates. For a given number of goods, they therefore face worse terms of trade. Consequently, they find it profitable to incur the fixed cost of production for more goods.

Now that technology differences, μ's, are endogenous, there are two determinants of cross-country income differences: countries with better economic policies (i.e., high ϕ) will be richer for the same reasons as before. Countries with lower discount rates (i.e., high ρ) will be richer both because of the reasons highlighted in Sections II and IV, and because they will choose to specialize in the production of more intermediates.

27. To obtain this equation, we divide the free-entry condition (43) by the trade balance condition (41) to get $w/(r \cdot k + w) = \tau/(\epsilon \cdot \mu)$. We equate this to the labor share equation, (42), and then substitute the steady-state value of z from equation (39).

Notice that technology differences in this model simply translate into differences in relative incomes, not long-run growth rates. This appears plausible since there is evidence pointing to significant technology differences across countries (e.g., Klenow and Rodriguez-Clare [1997] and Hall and Jones [1999]), and as noted in the Introduction, these differences do not seem to lead to permanent differences in growth rates.

To understand why the steady-state number of goods is independent of the level of capital stock or income (cf. equation (46)), denote the fixed cost of production by f (in equation (36), we had $f = w$). Then using (12), the free-entry condition can be written as $\mu = (\tau/\epsilon) \cdot (y/f)$. This equation states that the number of goods in which a country specializes is proportional to its income divided by the fixed cost of production. The reason why μ is constant is that as y increases f increases also. This is a consequence of the assumption that fixed costs are in terms of the scarce factor. As the country becomes richer, demand for labor increases, causing a proportional increase in the wage rate. So, y/f and μ remain constant.[28]

VI. Concluding Remarks

This paper has presented a model of the world income distribution in which all countries share the same long-run growth rate because of terms-of-trade effects. Countries that accumulate faster supply more of the goods that they specialize in to the world and experience worse terms of trade. This reduces the return to further accumulation and creates a demand pull on other nations. We view this model as an attractive alternative to the existing approaches where common long-run growth rates result only if all countries share the same technology.

Naturally, a theory of diminishing returns due to terms-of-trade effects does not preclude diminishing returns in production or cross-country technological spillovers. It is relatively straight-

28. It is also useful to contemplate what would happen if the fixed cost f depended on the wage rate, but less than proportionately, say $f = w^{\zeta}$. As long as $\zeta > 0$, μ would still increase with income, but less than proportionately. As a result, our key mechanism, that an increase in production translates into worse terms of trade, would continue to hold, since, from equation (18), terms of trade are proportional to μ/y, and are decreasing in y. Nevertheless, in this case, the model would not be well behaved for another reason: as μ increases with income, the world growth rate would increase over time, eventually becoming infinite. This explains our particular choice of $f = w$ to preserve steady growth.

forward—although cumbersome—to write down a model with all of these factors present and complementing each other. The more important question is their relative contribution to explaining the actual world income distribution. Here, we made a preliminary attempt at estimating the extent of terms-of-trade effects. Our results show that a country accumulating faster than others experiences a worsening in its terms of trade, and the estimated strength of the terms-of-trade effect suggests that our mechanism could be important in understanding cross-country differences in income levels.

Naturally, other factors could be driving the negative relationship between faster accumulation and the decline in terms of trade, so future empirical work on this topic is necessary. Nevertheless, to our knowledge, ours is the first investigation of why faster accumulation leads to a lower value of marginal product of capital—it is typically assumed that this is due to technological diminishing returns, despite no direct evidence of this effect. In contrast, we showed, both theoretically and empirically, that faster accumulation may lead to a lower value of marginal product of capital because of its effect on the terms of trade.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

REFERENCES

Acemoglu, Daron, Simon Johnson, and James Robinson, "The Colonial Origins of Comparative Development: An Empirical Investigation," *American Economic Review,* XCI (2001a), 329–401.

Acemoglu, Daron, Simon Johnson, and James A. Robinson, "Reversal of Fortune: Geography and Institutions in the Making of the Modern World Income Distribution," NBER Working Paper No. 8460, 2001b, *Quarterly Journal of Economics,* forthcoming.

Acemoglu, Daron, and Jaume Ventura, "The World Distribution of Income," NBER Working Paper No. 8083, 2001.

Acemoglu, Daron, and Fabrizio Zilibotti, "Productivity Differences," *Quarterly Journal of Economics,* CXVI (2001), 563–606.

Aghion, Philippe, and Peter Howitt, "A Model of Growth through Creative Destruction," *Econometrica,* LX (1992), 323–351.

Armington, Paul S., "A Theory of Demand for Products Distinguished by Place and Production," *International Monetary Fund Stuff Papers,* XVI (1969), 159–178.

Barro, Robert, "Economic Growth in a Cross Section of Countries," *Quarterly Journal of Economics,* CVI (1991), 407–443.

——, *Determinants of Economic Growth: A Cross-Country Empirical Study* (Cambridge, MA: MIT Press, 1997).

Barro, Robert, and Jong-Wha Lee, "International Comparisons of Educational Attainment," *Journal of Monetary Economics,* XXXII (1993), 363–394.

Barro, Robert, and Xavier Sala-i-Martin, *Economic Growth* (New York, NY: McGraw-Hill, 1995).

Barro, Robert, and Xavier Sala-i-Martin, "Technological Diffusion, Convergence, and Growth," *Journal of Economic Growth,* II (1997), 1–26.

Bhagwati, Jagdish, "Immiserizing Growth: A Geometrical Note," *Review of Economic Studies,* XXV (1958), 201–205.

Brezis, Elise, Paul Krugman, and Daniel Tsiddon, "Leapfrogging in International Competition," *American Economic Review,* LXXXIII (1993), 1211–1219.

Chari, V. V., Patrick Kehoe, and Ellen McGrattan, "The Poverty of Nations: A Quantitative Investigation," Working Paper, Federal Reserve Bank of Minneapolis, MN, 1997.

Coe, David, and Elhanan Helpman, "International R&D Spillovers," *European Economic Review,* XXXIX (1995), 859–887.

Cunat, Alejandro, and Marco Maffezoli, "Growth and Interdependence under Complete Specialization," mimeo, Universita Bocconi, 2001.

Durlauf, Steven, and P. Johnson, "Multiple Regimes and Cross-Country Growth Behavior," *Journal of Applied Econometrics,* X (1995), 365–384.

Eaton, Jonathan, and Samuel Kortum, "International Technology Diffusion: Theory and Measurement," *International Economic Review,* XL (1999), 537–570.

Feenstra, Robert, "New Product Varieties and the Measurement of International Prices," *American Economic Review,* LXXXIV (1994), 157–177.

Grossman, Gene, and Elhanan Helpman, *Innovation and Growth in the Global Economy* (Cambridge, MA: MIT Press, 1991).

Hall, Robert, and Charles I. Jones, "Why Do Some Countries Produce So Much More Output per Worker Than Others?" *Quarterly Journal of Economics,* CXIV (1999), 82–116.

Helpman, Elhanan, "Imperfect Competition and International Trade: Evidence from Fourteen Industrial Countries," *Journal of the Japanese and International Economics,* I (1987), 62–81.

Helpman, Elhanan, and Paul Krugman, *Market Structure and Foreign Trade* (Cambridge, MA: MIT Press, 1985).

Howitt, Peter, "Endogenous Growth and Cross-Country Income Differences," *American Economic Review,* XC (2000), 829–846.

Hummels, David, and James Levinsohn, "Monopolistic Competition and International Trade: Reconsidering the Evidence," *Quarterly Journal of Economics,* CX (1995), 799–836.

Jones, Charles I., "Economic Growth and the Relative Price of Capital," *Journal of Monetary Economics,* XXXIV (1995), 359–382.

——, "On the Evolution of the World Income Distribution," *Journal of Economic Perspectives,* XI (1997), 19–36.

Klenow, Peter J., and Andres Rodriguez-Clare, "The Neoclassical Revival in Growth Economics: Has It Gone Too Far?" *NBER Macroeconomics Annual,* B. Bernanke and J. Rotemberg, eds. (Cambridge, MA: MIT Press, 1997).

Knack, Steven, and Philip Keefer, "Institutions and Economic Performance: Cross-Country Tests Using Alternative Institutional Measures," *Economics and Politics,* VII (1995), 207–227.

Krugman, Paul, "The Narrow Moving Band, the Dutch Disease, and the Competitive Consequences of Mrs. Thatcher: Notes on Trade in the Presence of Scale Dynamic Economies," *Journal of Development Economics,* XXIV (1987), 41–55.

Lai, Huiwen, and Daniel Trefler, "The Gains from Trade: Standard Errors with the CES Monopolistic Competition Model," University of Toronto, mimeo, 1999.

Lucas, Robert, "On the Mechanics of Economic Development," *Journal of Monetary Economics,* XXII (1988), 3–42.

Mankiw, N. Gregory, David Romer, and David N. Weil, "A Contribution to the Empirics of Economic Growth," *Quarterly Journal of Economics,* CVII (1992), 407–437.

Parente, Stephen L., and Edward C. Prescott, "Barriers to Technology Adoption and Development," *Journal of Political Economy,* CII (1994), 298–321.

Parente, Stephen, Richard Rogerson, and Randall Wright, "Homework in Development Economics," *Journal of Political Economy,* CVIII (2000), 680–687.

Pritchett, Lant, "Divergence, Big Time," *Journal of Economic Perspectives,* XI (1997), 3–18.

Quah, Danny, "Empirics for Growth and Distribution: Stratification, Polarization and Convergence Clubs," *Journal of Economic Growth,* II (1997), 27–60.

Rebelo, Sergio, “Long-Run Policy Analysis and Long-Run Growth,” *Journal of Political Economy,* CXIX (1991), 500–521.
Rivera-Batiz, Luis A., and Paul M. Romer, “Economic Integration and Endogenous Growth,” *Quarterly Journal of Economics,* CVI (1991), 531–555.
Romer, Paul M., “Increasing Returns and Long-Run Growth,” *Journal of Political Economy,* XCIV (1986), 1002–1037.
——, “Endogenous Technological Change,” *Journal of Political Economy,* XCVIII (1990), S71–S102.
Segerstrom, Paul S., T. Anant, and Elias Dinopoulos, “A Schumpeterian Model of the Product Life Cycle,” *American Economic Review,* LXXX (1990), 1077–1092.
Solow, Robert, “A Contribution to the Theory of Economic Growth,” *Quarterly Journal of Economics,* LXX (1956), 65–94.
Stiglitz, Joseph E., “Factor Price Equalization in a Dynamic Economy,” *Journal of Political Economy,* LXXVIII (1970), 456–488.
Stokey, Nancy, “Human Capital, Product Quality and Growth,” *Quarterly Journal of Economics,* CVI (1991), 587–616.
Summers, Robert, and Alan Heston, “The Penn World Table (Mark 5): An Expanded Set of International Comparisons, 1950–1988,” *Quarterly Journal of Economics,* CVI (1991), 327–368.
Ventura, Jaume, “Growth and Interdependence,” *Quarterly Journal of Economics,* CXII (1997), 57–84.
World Bank, *World Development Report 1997: The State in a Changing World* (Oxford and New York: Oxford University Press for the World Bank, 1997).
Young, Alwyn, “Learning by Doing and the Dynamic Effects of International Trade,” *Quarterly Journal of Economics,* CVI (1991), 369–405.