PAPR REDUCTION FOR BEAMFORMING OFDM VIA CONSTELLATION-BEAM MODIFICATION

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ABSTRACT

The high peak-to-average power ratio (PAPR) of orthogonal frequency multiplexing (OFDM) greatly reduces the transmitter power efficiency. We propose a method that yields significant and consistent PAPR reductions for all numbers of users (receivers) in a beamforming MISO-OFDM system, such as WiMAX, by simultaneously exploiting the degrees of freedom in the constellation domain and in the null-space of the channel matrix. We also provide an efficient algorithm that is compatible with various constellation configurations in practical systems. With no sacrifice in data rate or channel efficiency, the proposed method achieves a maximum of 6.59 dB and a minimum of 2.11 dB PAPR reduction in an eight-antenna QPSK beamforming MISO-OFDM scenario.

I. INTRODUCTION

The combination of beamforming multiple-input-single-output (MISO) and orthogonal frequency multiplexing (OFDM) provides a robust communications link in fading environments by maximizing received signal strength via beam-steering, and has been incorporated in standards such as 802.16 and WiMAX. However, like SISO-OFDM, MISO-OFDM suffers from a potentially high peak-to-average power ratio (PAPR), caused by the superposition of multiple sub-channel signals in the time domain. A high PAPR requires the power amplifier at the transmitter to be built with a larger dynamic range, resulting in a greater power consumption and energy inefficiency.

A number of PAPR reduction methods have been proposed for SISO-OFDM [1]. The tone-reservation technique inserts signals into unused sub-channels in order to cancel the time-domain peaks in the data-bearing channels, while sacrificing data rate due to the reduction in data-bearing channels [2][3]. Some methods utilize the pre-known phase knowledge to improve PAPR via encoding, such as SLM and PTS, but they require significant side information and are not compatible with many standards [1][4][5]. The active-constellation-extension (ACE) method exploits the modifications in the constellation domain [6]. Several regions in the frequency domain are identified as allowable regions for extension, in which the outer-most constellations can be moved freely while still being correctly detected by a standard maximum likelihood receiver. ACE is able to reduce PAPR without sacrificing data rate and is compatible with most existing standards [6].

While many SISO-OFDM PAPR reduction schemes can be readily extended to MISO-OFDM [1][7], the transmit weight optimization (TWO) exploits the spatial characteristics that is unique to MISO systems [8]. When the number of users (B) is less than the number of transmit antennas (P), the B by P channel matrix has a $(P-B)$-dimensional null-space, in which the transmit power will be completely eliminated by the channel and not received by the user. Intuitively, TWO injects transmit power into the null-space in exchange for a lower PAPR. Analogous to the tone-reservation method in SIFO-OFDM, TWO can be perceived as a form of *beam reservation*, where modifications are strictly contained in the null-space of the channel. However, although TWO offers substantial reductions when B is small compared to P, its performance decreases as the degrees of freedom in the transmit matrix diminishes, and finally fails to yield any reduction when $B = P$ (Figure 1).

Interestingly, while TWO's PAPR reduction gain diminishes as the number of user grows, the extension of ACE in MISO-OFMD systems exhibits a complementary user-dependency. ACE yields little reduction when the number of users is small, but the performance improves significantly as the number of the users increase, due to a greater number of constellation points that are now available to be modified (Figure 1). Since the number of users may vary over time in a real MISO-OFDM system, it is very desirable for a single PAPR reduction scheme to have a consistent performance across all possible numbers of users, while being computationally efficient and compatible with existing standards. We present in this paper a method, the constellation-beam modification (CBM), that combines the complementary user-dependency of ACE and TWO to obtain significant improvement in PAPR reduction for all numbers of users.

In a MISO-OFDM system with eight transmit antennas and one to six users, CBM outperforms both TWO and ACE for all number of users and achieves a maximum improvement of 1.41 dB over using either TWO or ACE alone.

II. METHOD FOR CONSTELLATION-BEAM MODIFICATION

ACE and TWO algorithms exhibit interesting complementary behaviors: TWO yields significant PAPR reductions when the number of users (B) is small, but offers less improvement when the degrees of freedom in the transmit weight matrix drop as B increases. ACE, on the other hand, suffers from limited constellation symbols at low B, while benefiting from the increase in constellation freedom as B grows. This complementary difference in performance motivates a combined algorithm that optimizes using both ACE and TWO to provide a consistently high PAPR reduction.

It is important to note that the ACE and TWO constraint sets are independent, so a joint modification is guaranteed to preserve an undistorted signal reception at the receivers. This orthogonal characteristic of the ACE and TWO constraints suggests that a combined algorithm can be implemented very efficiently, since any combined adjustment can be perceived as *one* additive change to the original signal that simultaneously encapsulates the effects of ACE and TWO.

Mathematically, the CBM method can be formulated as an optimization of PAPR over the intersection of two convex sets:

Minimize:

$$PAPR = \frac{\max\limits_{0\leq m\leq PNL-1} |x_m|^2}{E\left[|x_i|^2\right]} \tag{1}$$

where P represents the number of transmit antennas, N represents the total number of OFDM channels, and L represents the up-sampling factor, subject to:

(1) (ACE) The set $S_A \subseteq \mathbb{C}^{(PNL)}$, consisting of all time-domain signals $x \in \mathbb{C}^{(PNL)}$, of which the corresponding constellations fall under the allowable regions of extension on the constellation domain [6].

(2) (TOW) The set $S_T \subseteq \mathbb{C}^{(PNL)}$, consisting of all time-domain signals resulting in the same constellations at a receiver, specified by an element in S_A [8]. Consequently, for every pair of distinct $x_i, x_j \in S_T$, the difference $(x_i - x_j)$ lies in the null-space of the channel matrix H for all frequencies, i.e. $H(x_i - x_j) = 0$.

The CBM algorithm essentially optimizes over a higher-dimensional convex set with looser constraints. ACE and TWO, however, only make modifications within one of the above sets, while holding the properties specified by the other constraint constant.

III. ALGORITHM FOR CONSTELLATION-BEAM MODIFICATION

Since constellation-beam modification (CBM) can be formulated as an optimization problem over the intersection of two convex sets, a number of convex optimization techniques can be used to find the solution. An efficient project-onto-convex-sets (POCS) algorithm is explored here, which shows significant reduction after a small number of iterations. It can be implemented either with a fixed predetermined peak level or as a gradient-project method that converges towards an optimal value [9]. Also, the discrete time-domain signal is up-sampled by a factor of 4, in order to approximate the analog waveform at the transmit antennas.

Let W_k be the P by B antenna weight matrix for the kth OFDM frequency. Assume W_k is chosen such that $I = H_k W_k$, where H is the channel matrix.

1) Assign frequency-user domain constellation points, $X_k^i \in \mathbb{C}^B$, according to the input data, where i is the iteration number. Construct the initial P by 1 frequency-antenna domain data by $B_k^0 = \left(W_k X_k^0\right)$, Set $i = 0$.

2) Reconstruct the time-antenna domain data by applying a zero-padded inverse FFT:
$$x^i = interp\left[IFFT\left(B^i\right)\right], \tag{2}$$
where x^i is a (PNL) by 1 vector representing the time-domain signal across all antennas, and $interp(*)$ is an interpolation that up-samples the time-domain signal by a factor of L by inserting zeros into the frequency domain.

3) Clip the magnitudes of samples in x^i that exceed a predetermined limit, G_{max}, to get $\overline{x}^i$.

4) Reconstruct the frequency-user domain data $\overline{X_k^i}$ by inverting the beamforming transformation:
$$\overline{X_k^i} = W_k^{pinv}\overline{B_k^i}, \tag{3}$$
where $\overline{B^i}$ is the frequency-antenna domain data after clipping: $\overline{B^i} = FFT(\overline{x}^i)$.

5) Apply active constellation constraints by projecting the constellation points of $\overline{X_k^i}$ onto the allowable regions of extension to obtain X_k^{i+1}. Denote the frequency-user domain descent vector by $\Delta X_k^i = X_k^{i+1} - X_k^i$.

6) Project the frequency-antenna domain clipping $(\overline{B_k^i} - B_k^i)$ onto the the null-space of the channel matrix. Combine the result with the constellation modification obtained in Step 5 to form the frequency-antenna domain descent vector:
$$\Delta B_k^i = [I - W_k(W_k^{pinv})](\overline{B_k^i} - B_k^i) + W_k \Delta X_k^i. \tag{4}$$

7) Update the frequency-antenna domain data:
$$B^{i+1} = B^i + \Delta B^i. \tag{5}$$

8) Let $i = i + 1$, return to Step 2 and repeat until a minimum PAPR is achieved or the number of iterations is exhausted.

It is important to note that only one pair of FFT/IFFT is performed in each iteration, which dominates the computational costs due to a complexity of $O(NlogN)$. The cost of the rest of the operation is $O(N)$. This means the computational overhead by optimizing under a second constraint is very limited compared to performing ACE or TWO alone, which makes the CBM algorithm efficient for practical systems.

Also, the CBM algorithm can be adapted to various constellation configurations, such as QPSK, 16-QAM, or 64-QAM, simply by modifying the constellation constraints used in Step 5 [6].

IV. SIMULATIONS AND RESULTS

Simulations for QPSK, 16-QAM, and 64-QAM have been conducted using over 1000 OFDM blocks with 128 subchannels, 8 antennas, and 1 through 8 users with no unused channels. The algorithm was performed for a maximum of 12 iterations. The channels were modeled using independent Gaussian random complex channel matrices. The analog waveform at the transmit antennas was approximated using an up-sampling factor $L = 4$. In the QPSK case, the simulations yield a maximum PAPR reduction of 6.59 dB with one user, and a minimum reduction of 2.11 dB with eight users.

Figure 1 compares the performance for CBM, TWO and ACE using QPSK. All values are measured at a 1% clipping probability. The CBM method shows significant improvements at five to seven users. Particularly, CBM in the 6-user case yields a 1.41 dB reduction improvement compared to using any of the previous methods alone. When the number of users is close to one or eight, the combined method offers little benefits, due to the diminishing degrees of freedom in either ACE or TWO, respectively.

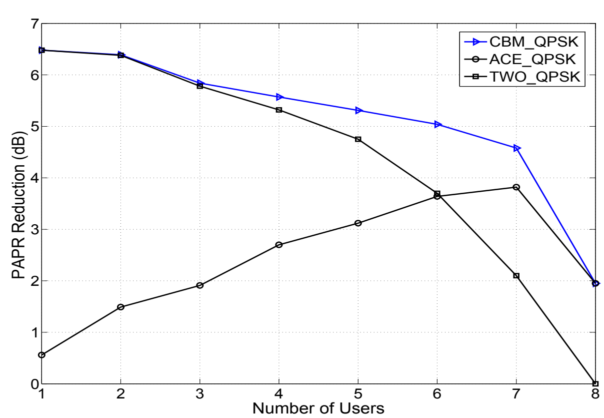

Fig. 1. PAPR reduction comparison of CBM, ACE and TWO using QPSK with eight transmit antennas.

The CBM can also be applied to larger constellations. While TWO's performance is hardly affected when the constellation size increases, ACE contributes less to the total reduction, since the outer-most points now constitute a smaller portion of the whole constellations. Simulations of CBM using 16-QAM shows similar performance to QPSK, while in the case of 64-QAM the reduction with five to eight users decreases by an average of 1 dB (Figure 2).

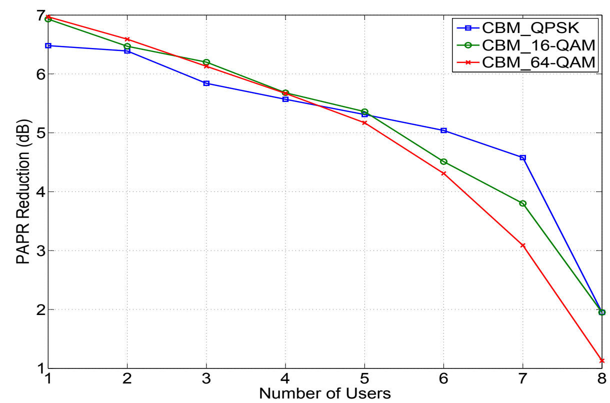

Fig. 2. Performance comparison of CBM using QPSK, 16-QAM and 64-QAM.

The average power does increase, mainly due to the extension of constellations. For the simulated QPSK system with an up-sampling factor $L = 4$, the OFDM average power increases for one to eight users by .43 dB, .45 dB, .49 dB, .57 dB, .75 dB, 1.07 dB, 1.50 dB, and 1.25 dB, respectively (Figure 3). However, since the power consumption will be dominated by the maximum power capacity in systems with linear amplifiers, the effects of an increased average power will be offset by the benefits of a significantly reduced PAPR.

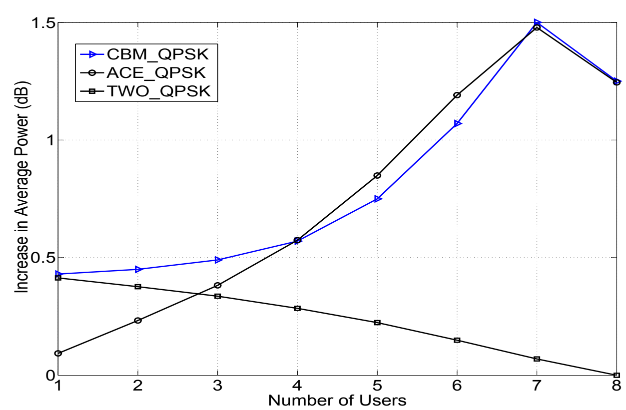

Fig. 3. Average power increase comparison of CBM, ACE and TWO with eight transmit antennas.

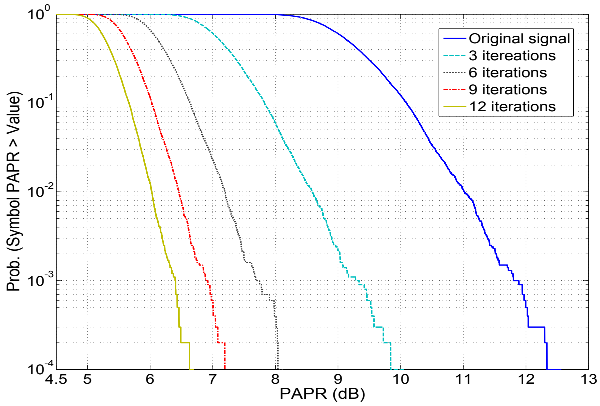

Fig. 4. PAPR Complementary CDFs for six users using CBM.

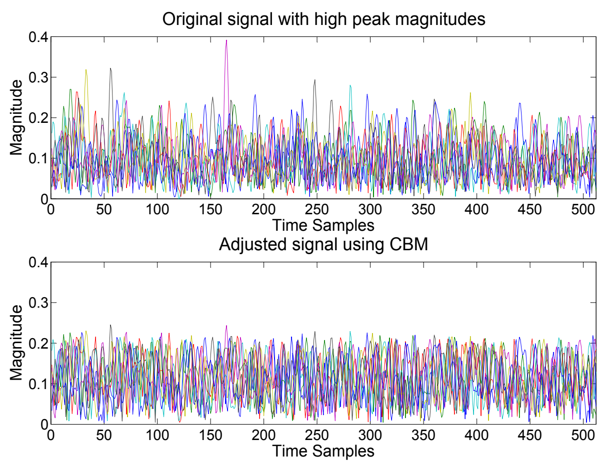

Fig. 5. Superposition of time-domain signals with eight antennas transmitting to six users.

V. DISCUSSION

Simulations using constellation-beam modification have shown significant PAPR reduction in a beamforming MISO-OFDM system. The method is capable of effectively reducing PAPR with any number of users, due to the complementary freedom in constellations and the channel matrix null-space. Thus, a system utilizing 8 antennas and allowing from 1 to 6 users can achieve a very substantial 1.4 dB PAPR improvement (5.0 dB reduction) over the previous TWO approach using CBM. This is essential in many practical systems, because the number of users will vary with system load, and the hardware must be designed for the worst-case performance. The CBM method can also be adapted to other constellation configurations, although the performance may be reduced for large constellation sizes because only the outer-most constellation points can be moved.

Compared to ACE or TWO, the extra cost associated with CBM optimizing in an additional constraint set is $O(N)$, incurred by the linear projection done in Step 5 and 6. This is smaller compared to the $O(NlogN)$ cost for FFT and IFFT, and therefore makes the CBM algorithm attractive for commercial applications. An even more efficient smart gradient projection (SGP) algorithm [6] can reduce the required number of iterations to 3, by adding an extra $O(N)$ step size search along the descent direction obtained in the POCS algorithm [11].

VI. REFERENCES

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