Economic vs. computational efficiency in markets
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How can we share a scarce good in a fair way?

Philosophers have thought about this question for centuries. In recent years, it has been studied from a new computational perspective.

In this project, we want to formally quantify economic efficiency in markets that perform minimal computation.

Given impractically large (exponentially) amounts of computation, optimal social welfare is achievable. Instead, we want to quantify economic efficiency in practice, when computation is minimal. Thus we look at the simplest possible markets.

Such markets are important in practice for sharing bandwidth between users on a large network...

Our understanding is limited to one-sided markets

The most studied one-sided market is a mechanism by Kelly [2] for sharing bandwidth over a single network link.

How will users behave? For that, we need additional concepts:

Definition. A providersubmit a bid b, and pays \( b \) per unit.

The happiness of user \( q \) is measured using a utility function \( U(q) \). We'll assume that once all the bids are in, we have a Nash equilibrium:

We propose the first efficient two-sided market:

A user's utility function:

\[
U(q, b_r) = \sum_{(s,t) \in \text{path}} c_{st} - b_r
\]

A provider's utility function:

\[
U(r, b_q) = \sum_{(s,t) \in \text{path}} c_{st} - b_q
\]

1. Each user submits a bid \( b \), and pays \( b \) per unit.
2. The market computes a single market-clearing price \( p \).
3. Provider \( r \) is told to offer a units of bandwidth. User \( q \) gets a fraction of total production and is paid \( p \).

At equilibrium, total supply must equal total demand.

\[
\sum_{q} b_q = \sum_{r} a_r = \sum_{(s,t) \in \text{path}} c_{st} \]

Outline of main proofs

Proof of theorem 3. First, from our differentiability and convexity assumptions, we obtain necessary and sufficient first order Nash equilibrium conditions.

We then express the price of anarchy as the solution of an optimization problem. We minimize over all possible allocations and utilities under the constraint that they form a Nash equilibrium:

\[
\min \left\{ \frac{\sum_{(s,t) \in \text{path}} c_{st}}{\sum_{(s,t) \in \text{path}} c_{st}} : \text{subject to convexity and utility constraints} \right\}
\]

Further, we find that the best price of anarchy occurs when the elasticity of \( f(p) \) equals one. This elasticity is uniquely achieved by our mechanism.

This market is optimal within its class

Kelly's mechanism is optimal within a large class of mechanisms. These mechanisms have a natural extension to two-sided markets.

Definition. A smooth two-sided market-clearing mechanism is a tuple of functions \( (f, g, h) \) such that for all \( q, r \), \( f(q) > 0 \) and \( h(q, r) = f(q) \) such that for all \( q, r \), \( g(q) > 0 \) and \( g(q) + h(q, r) = f(q) \). Then for all \( q, r \), \( \frac{h(q, r)}{g(q)} \leq \frac{\|f\|_{\infty}}{\|g\|_{\infty}} < 1 \), where \( \|f\|_{\infty} \) is the maximum of \( f \) over all \( q \). These mechanisms have a natural extension to two-sided markets.

We find that the best price of anarchy occurs when the elasticity of \( f(p) \) equals one. This elasticity is uniquely achieved by our mechanism.

Within our computationally efficient market system, there is a 40% loss in economic efficiency. Designing this market was non-trivial, and most similar mechanisms are inefficient. This raises questions about the economic efficiency of real markets.

It is not clear how prices are formed in real markets, but a version of our mechanism has been studied in economics as a model of price formation [4]. Thus our results are also relevant within an existing body of economic literature on prices.

References