STAT331 HW1, Due Friday, March/14

- Show that the generalized gamma distribution family is closed under the power transformation, i.e, if T follows a generalized gamma distribution, then $T^s, s > 0$ also follows a generalized gamma distribution. Show furthermore that any generalized gamma distribution can be written as a power transformation of a Gamma distribution.

- Assume that T and C are possibly correlated failure and censoring times, respectively. Furthermore, assume that

$$\text{pr}(T > t, C > s) = S(t, s), t, s > 0$$

is the continuously differentiable survival function for the bivariate random variable (T, C). Let $U = \min(T, C), \delta = I(T \leq C)$.

1. Show that

$$\text{pr}(U > u, \delta = 1) = \int_u^\infty \left\{ -\frac{\partial S(t,s)}{\partial t} \bigg|_{t=s=x} \right\} dx$$

and

$$\text{pr}(U > u, \delta = 0) = \int_u^\infty \left\{ -\frac{\partial S(t,s)}{\partial s} \bigg|_{t=s=x} \right\} dx$$

2. Show that

$$S(t, s) = e^{-t^2 - s^2 - 2ts} \quad \text{and} \quad S(t, s) = e^{-2(t^2+s^2)}$$

induce the same joint distribution of (U, δ). (This is a toy example on the relationship between identifiability and informative censoring.)

- Suppose (T, C) denote a survival time and potential censoring time, respectively, and that Z is a covariate (such as a binary indicator of treatment group or gender). Suppose that $T \perp C \mid Z$. Does it follow that $T \perp C$? Suppose on the other hand that $T \perp C$. Does it follow that $T \perp C \mid Z$? Prove the statements that are true and give specific examples if not.