Homework 2 (Due, Friday 04/21)

1. Suppose the observed data are

$$\{(U, \delta) = (1,1), (1,0), (3,0), (4,1), (5,1), (5,0), (9,0), (12,0)\}.$$

Calculate the KM estimator. If the last observation is $(12, 1)$ instead of $(12, 0)$, what is the corresponding KM estimator?

2. The variance of KM estimator $\hat{S}(t)$ can be estimated by the Greenwood formulae:

$$\hat{S}^2(t) \sum_{i=1}^{j} \frac{d_i}{Y(\tau_i)(Y(\tau_i) - d_i)}$$

for $t \in [v_j, v_{j+1})$, where $0 < v_1 < v_2 < \cdots < v_p$ are all the observed failure times.

(a) Give a heuristic construction of an estimator for the covariance

$$cov(\hat{S}(t), \hat{S}(s)),$$

where $t \in [v_j, v_{j+1})$ and $s \in [v_k, v_{k+1})$ with $1 \leq j < k \leq p - 1$.

(b) Construct a 95% confidence region for $(S(t), S(s))$. (Note that the construction is not unique and you only need to present one; Hint: consider the joint distribution of $\{\hat{S}(t) - S(t), \hat{S}(s) - S(s)\}$.)