Game theory and the spectrum auctions

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Abstract

We describe two game theoretic models that have been used to assess past and proposed future rules for the US spectrum auctions. © 1998 Elsevier Science B.V. All rights reserved.

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1. Introduction

The year 1994 witnessed the introduction of the '*simultaneous ascending auction*' – a new kind of auction designed to improve the allocation of high-value resources compared to more traditional auctions. The new auction was used first in the United States, where it was applied to the sale of licenses to use the radio spectrum for paging and wireless telephone services. The initial 'test' of the auction rules was a US$617 million sale of ten paging licenses in July 1994. Auctions with higher stakes and more licenses soon followed. In spectrum auction #4, which began in December 1994, 99 licenses were sold and the sale proceeds totaled approximately US$7.7 billion. The perceived successes of these high-profile American auctions have led imitators to adapt the auction for a variety of applications around the world.

Much of the excitement about the spectrum auctions came from the nature, size and politics of the application. Before these auctions, the United States

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government had used comparative hearings and lotteries to assign radio spectrum licenses. The adoption of auctions was an important step in developing a rational plan to privatize the spectrum. The auction sales proceeds also focused political attention on the value of spectrum that had earlier been given away, as well as on how best to use the windfall revenues that the spectrum sales represented in the federal budget process.

For economists and auction specialists, the auction form itself was remarkable for several reasons. First, it relied extensively on modern information processing technologies. The simultaneous ascending auction is an interactive process in which bidders, often located far from the auction site, are kept informed about bidding on tens or hundreds or thousands of licenses. They respond to the information by making higher bids on selected licenses. None of this would have been possible before the development of interactive network technology.

Second, the auction design made detailed use of the ideas of economic theory and the recommendations of economic theorists. Indeed, the US communications regulator adopted all its important rules from the detailed proposals for a simultaneous ascending auction that were made by Preston McAfee, Robert Wilson and me.[1]

Third, the evidence demonstrates convincingly that the simultaneous ascending auction realized the main economic advantages that its proponents had claimed for it. Most important among these was that, unlike older auction forms that it displaced, the simultaneous ascending auction in the US promoted price-guided substitution among similar licenses. The clearest evidence for this is the near equality of prices in the new auctions among licenses that are close substitutes. For reference, one may take the spectrum auctions in New Zealand, which were completed just before the US spectrum auctions began. These had involved price ratios of as much as four-to-one among spectrum licenses that were nearly perfect substitutes. By contrast, in the initial US application (officially known as "auction # 1"), shifting just $1 million of the $617 million total of winning bids from the relatively high-priced licenses to lower-priced ones would have exactly equalized the prices among the nearly perfect substitutes sold in that auction.[2]

Several parts of economic theory have proved helpful in planning the simultaneous ascending auction and in thinking about how the design might be improved. The complete version of this paper presented at the European Economic Association meeting explained how tatonnement theory as well as game theory played a role in assessing various proposed rules. Due to space

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[1] McAfee and McMillan (1996) and Milgrom (1997) give accounts of the roles played by the theorists.
[2] Cramton (1995).

limitations, this paper omits the *tatonnement* models and presents just two of the uses of game theory in evaluating the auction rules.

We begin by reviewing a simplified version of the auction rules in Section 2. Section 3 reports two applications of game theory to the simultaneous ascending auction design. Game theoretic analysis revealed a kind of free-rider problem in some proposed bidding mechanisms, leading to a rejection at least for the initial auctions of plans for 'combinatorial bidding', in which bidders specify prices for collections of licenses rather than for individual licenses. Game theory is also being used to reevaluate the closing rule for the US spectrum auctions to determine whether alternative rules can preserve the flexibility of the current rules while reducing the susceptibility of the auction to informal collusive agreements.

2. Simultaneous ascending auction rules in brief

A simultaneous ascending auction is an auction for a multiple items in which bidding occurs in rounds. At each round, bidders simultaneously make sealed bids for any licenses in which they are interested. After the bidding, round results are posted. For each item, these results consist of the identities of the new bids and bidders and the '*standing high bid*' and the corresponding bidder. The initial standing high bid for each item is zero and the '*corresponding bidder*' is the auctioneer. As the auction progresses, the new standing high bid at the end of a round for an item is the larger of the previous standing high bid or the highest new bid and the corresponding bidder is the one who made that bid. In addition to the round results, the minimum bids for the next round are also posted. These are computed from the 'standing high bid' by adding a pre-determined bid increment, such as 5% or 10%.

Bidder activity during the auction is controlled by what the regulator called the '*Milgrom–Wilson activity rule*'. It works as follows. First, a quantity measure for spectrum is established, which provides a rough index of the value of the license. Typically, the quantity measure for a spectrum license is based on the bandwidth of the licensed spectrum and the population of the geographic area covered by the license. At the outset of the auction, each bidder establishes its *initial eligibility* for bidding by making deposits covering a certain quantity of spectrum. During the auction, a bidder is considered *active* for a license at a round if it makes an eligible new bid for the license or if it owns the standing high bid from the previous round. At each round, a bidder's *activity* is constrained not to exceed its eligibility. If a bid is submitted that exceeds the bidder's eligibility, the bid is simply rejected.

The auction is conducted in three stages. In the first stage, a bidder who wishes to maintain its eligibility must be active on licenses covering some fraction f_1 of its eligibility. If a bidder with eligibility x is active on $y < f_1 x$ during

this stage, then its eligibility is reduced at the next round to y/f_1. In the second and third stages, a similar rule applies but using fractions f_2 and f_3. In recent auctions in the US, the fractions used have been $(f_1, f_2, f_3) = (0.6, 0.8, 0.95)$. Thus, in stage 3, bidders know that the auction is nearing its close in the sense that the remaining demand for licenses is just $1/f_3$ times the current activity level.

There are several different options for rules to close the bidding and the spectrum regulator is presently reevaluating these. One proposal, made by Preston McAfee, specified that when a license had received no new bids for a fixed number of rounds, bidding on that license would close. That proposal was coupled with a suggestion that the bid increments for licenses should reflect the bidding activity on a license. A second proposal, made by Robert Wilson and me, specified that bidding on all licenses should close simultaneously when there is no new bidding on any license. To date, the latter rule is the only one that has been used in the spectrum auctions.

When the auction closes, the licenses are sold at prices equal to the standing high bids to the corresponding bidders. The rules that govern deposits, payment terms, and so on are quite important to the success of the auction,[3] but they are mostly separable from the other auction rule issues and receive no further comment here.

3. Auctions and game theory

The part of economic theory most noted for its contribution to the auction design is game theory. Here we consider two uses of game theory. The first formalizes the free rider problem that was the reason for rejecting combinatorial bidding.[4] The second is a study of how the auction closing rules affect the likelihood of collusive outcomes. To focus on the primary issues, we assume for these analyses that 'jump bids' in which bidders bid more than the posted minimum or raise their own bids are excluded by the rules and that ties are broken at random.

3.1. Combinatorial bidding and free riding

One of the main issues in the early debates about the spectrum auction was whether all bidding should be for individual licenses or whether, instead, bids for

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[3] Failure to establish these rules properly led to billions of dollars of bidder defaults in the United States 'C-block auction'. Similar problems on a smaller scale occurred in some Australian spectrum auctions.

[4] Remarkably, politics had little influence on the US spectrum auction design. There were no entrenched bureaucratic interests with pre-conceived notions of how auctions should be run, and few others willing to challenge the recommendations of the economists and take responsibility for suggesting their own sets of rules.

Table 1

	A	B	AB	Budget
1	4	–	–	3
2	–	4	–	3
3	$1 + \varepsilon$	$1 + \varepsilon$	$2 + \varepsilon$	2

combinations of licenses should be allowed. According to one combinatorial bidding proposal, bids would be accepted for licenses or packages of licenses. After all bidding had ceased, the collection of bids that maximize total revenues would be the winning bids, and licenses would be assigned accordingly. Our model of this auction below assumes that in the event of ties, package bids are selected in preference to bids on individual licenses and that bids must be entered as whole numbers.

The spectrum regulator's initial notice proposed the use of a kind of combinatorial bidding, and several participants to the proceedings advocated them vigorously. There are cogent arguments favoring the use of some kinds of combinatorial bidding in some circumstances, but they are omitted here for brevity. Instead, we focus here on the game theoretical disadvantages of combinatorial bids that motivated the regulator's decision to abandon them for its initial spectrum auctions.

The primary argument against allowing combination bids is that such bids can give rise to a free rider problem among bidders on the individual licenses, leading to avoidable inefficiencies. The table below provides a simplified version of the example I presented in the actual deliberations to show how that can happen. In this example, there are three bidders – labeled 1, 2 and 3 – and two licenses – A and B. Bidders 1 and 2 are willing to pay up to 4 for licenses A and B, respectively, and neither is eligible to acquire the other license.[5] With ε small and positive, bidder 3 has the lowest values for the licenses but is distinguished by its desired to acquire both. To keep the strategy spaces small and ease the analysis, we impose economically insignificant budget constraints on the bidders, as shown in Table 1.

With the specified values, the sole efficient license assignment has bidders 1 and 2 acquiring licenses A and B, respectively. With bids restricted to be whole numbers, that corresponds to a subgame perfect equilibrium of the simultaneous ascending auction in which bidders 1 and 2 make minimum bids at each round as necessary to acquire their respective licenses of interest, while bidder 3 bids 1 for each license and then gives up.

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[5] In the actual auctions, bidders were ineligible to acquire additional wireless telephone licenses for areas they already served. This restriction was motivated by competition policy.

Table 2

	Raise bid	Do not raise
Raise bid	2,2	2,3
Do not raise	3,2	0,0

If a combinatorial auction is used, bidder 3 can refrain from bidding for licenses A and B directly, bidding instead for the pair AB. This strategy creates a free rider problem for bidders 1 and 2. A high bid by bidder 1 on license A helps bidder 2 to acquire license B. A symmetric observation applies to bidder 2. Each would prefer that the other raise the total of the individual bids sufficiently to beat 3's bid.

Even in the complete information case shown here, this free rider problem can lead to the possibility of an inefficient mixed strategy equilibrium. The corresponding equilibrium strategies are as follows. Bidder 3 bids 2 for the license combination AB whenever that price is less than 2, and then never bids again. Bidder 1 raises the price of license A by 1 whenever it does not own the standing high bid for that license. Otherwise, if at any time during the auction the license prices are 1 for A, 1 for B, and 2 for AB, then bidder 1 raises its high bid on license A with probability $\frac{2}{3}$. Bidder 2's strategy is symmetrical to bidder 1's but focused on license B instead of license A.

The key to understanding this equilibrium is to recognize the payoffs in the subgame after the prices are 1 for A, 1 for B, and 2 for package AB. The payoff matrix for bidders 1 and 2 in that subgame is shown in (Table 2).

This subgame has a symmetric equilibrium in which each bidder raises the bid with probability $\frac{2}{3}$, and backward induction from that supports the equilibrium strategies described above. At the equilibrium, there is a $\frac{4}{9}$ probability that 3 acquires both licenses even though its value for those licenses is just $\frac{1}{4}$ of the total of the competitors' values. This example is representative of a robust set of examples, including especially ones with asymmetric information that makes the free rider problem even harder to resolve.

To summarize:

Proposition 1. Introducing combinatorial bidding into a simultaneous ascending auction can introduce inefficient equilibrium outcomes that would be avoided in the auction without combinatorial bidding.[6]

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[6] Not all kinds of combinatorial bidding create such problems. The generalized Vickrey auction (also known as the Groves-Clarke pivot mechanism) uses combinatorial bidding without suffering this problem.

3.2. Collusion and closing rules

The rules of the spectrum auctions have specified that bidding remains open for all licenses until there were no new bids for any license. Thus, if a license that changes hands at some round is a substitute or complement for another license, the losing bidder can react by bidding for the substitute and the winning bidder can react by bidding for the complement. From the perspective of *tatonnement* theory, such flexibility is essential if the price adjustment mechanism is to locate efficient license assignments.

From a strategic perspective, however, simultaneous closings create avoidable opportunities for collusion. To illustrate this in a simple model, suppose that there are two bidders, 1 and 2, and two licenses, A and B. Each bidder has a value for each license of 10 and a value of 20 for the pair. The auction rules are the same as in the preceding section, with a simultaneous close of bidding on all licenses when there is no bidding on any license. The next two theorems, the proofs of which are straightforward, show that both 'competitive' and 'collusive' outcomes are consistent with equilibrium in this game.

Proposition 2. The following strategies constitute a subgame perfect of the game with simultaneous closing of bidding on all licenses. For each bidder, if the price of either license is below 10 and the bidder is not the standing high bidder, then bid on that license.

These strategies lead to the competitive equilibrium outcome with prices of 10 for both licenses and zero profits for the bidders.

Proposition 3. The following strategies constitute a subgame perfect equilibrium of the game with simultaneous closing of bidding on all licenses. For bidder A:

If B has never bid on license 1, then if license 1 has received no bids, A bids 1 on license 1; otherwise, A does not bid again.

If B has ever bid on license 1, then bid according to the strategy described in Proposition 2.

Bidder B bids symmetrically, exchanging the roles of licenses 1 and 2 and bidders A and B.

This equilibrium outcome results in prices of just 1 for each license and total profits of 18 for the two bidders, which is the maximum profit possible at any outcome of the game. This maximally collusive outcome is supported by the threat to revert to 'purely competitive' behavior if the other party to the arrangement ever bids on the 'wrong' license.

One simple alternative closing rule that eliminates this collusive outcome specifies that bids will be accepted on a license only if there were new bids on that *same* license in the immediately preceding round. This rule excludes the

possibility that a bidder can retaliate if the other cheats on the arrangement. For example, suppose that the auction is supposed to end after round n with a price of $n \leq 8$ on license 1, won by bidder A. Then, bidder 2 has nothing to lose and, in the trembling hand logic of equilibrium, something to gain by raising the price at round $n + 1$. Consequently, we have the following result.

Proposition 4. In the game in which bidding on each license closes when there is a round with no bids on that license, the price of each license is at least 9 at every sequential equilibrium.

The preceding model excludes the main disadvantage of license-by-license closings, which is that it makes it impossible for bidders to respond to changing relative prices during the auction. This substitution possibility was the reason that simultaneous closing of all licenses was included in the Milgrom–Wilson design. It is possible to create hybrid rules that delay the closing of all licenses until late in the auction and adopt sequential closings then to permit substitution while excluding one kind of collusion. Work on such rules is an ongoing exercise.

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