Package Bidding: Vickrey vs Ascending Auctions

Lawrence M. Ausubel
Department of Economics
University of Maryland
College Park, Maryland 20742 USA
ausubel@econ.umd.edu

Paul R. Milgrom
Department of Economics
Stanford University
Stanford, CA 94305 USA
paul@milgrom.net

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Abstract. The Vickrey and ascending package auctions are found to have identical equilibrium performance in the case where goods are substitutes. In the remaining cases, the Vickrey auction retains its incentive advantages at the cost of setting prices that are so low that the outcome is not in the core of the corresponding exchange economy. The Vickrey auction also introduces biases that distort investments in new technology. By contrast, the ascending package auction has its equilibrium outcomes in the core, provides neutral investment incentives, and is easier for bidders to understand and manage.

1. Introduction

Sellers of large or complex assets need to consider how to divide and package the assets. Packaging decisions are potentially important whenever a bidder's value for a package is different from the sum of the values of the separate parts. For example, a wireless telephone company purchasing telecommunications spectrum licenses prefers to acquire rights to pairs of bands that allow proper physical separation between the in-bound and out-bound signals. A flower wholesaler in Holland may incur fixed costs for shipping and handling that make single lot transactions unprofitable. Since flowers are highly perishable, it may also want to limit its purchases to what it can quickly resell. In real estate sales, some potential buyers may be interested in a whole complex of properties while others simply want space for individual homes or businesses.

In practice, sellers accommodate packaging preferences such as these in a variety of ways. Government-run spectrum auctions are invariably preceded by political processes in which potential buyers press their cases about such matters as the allowed uses of the spectrum and the scope of the licenses in terms of bandwidth, band composition, and geographic coverage. In private auctions of relatively homogeneous goods, winning bidders may be allowed to purchase as many similar lots as they like at the winning price before bids are taken for the remaining lots. In the real estate example, bids might be taken both for a whole complex and for its individual properties, and the two constellations of prices compared.

A complete approach to resource allocation problems of the kind described above must treat the packaging and auctioning problems as a single unit. Processes like that described for the real estate example, which determine the packaging, pricing and allocation decisions, are called "package auctions" or "combinatorial auctions." Typically, bidders in these auctions describe the packages that they wish to acquire and place bids for the named packages.

The package auction design that is best known among economists is the (generalized) Vickrey auction. In the generalized Vickrey model, the items for sale are taken to be M exogenously given "goods" and each bidder submits bids on every one of the 2^M-1 packages.[1] With distinct goods, such an auction may become impractically complicated for the bidders when M is still a single-digit number. Although there are special cases in which the Vickrey auction works well with larger numbers of goods, the sheer complexity of the general problem with many distinct kinds of goods has led auction designers to investigate alternative, dynamic auctions, which are often easier for bidders to comprehend and manage.[2]

There is a second practical issue that recommends package auctions. It is that the current alternatives to package bidding adopted by spectrum and electricity regulators have significant drawbacks of their own that package auctions can avoid. When the items for sale are substitutes, large bidders in multi-unit auctions find it in their interest to withhold some of their demand, in order to avoid driving up prices or to divide the spoils with other bidders. Such "demand reduction" leads to inefficiency of the final allocation (Ausubel and Cramton, 1996). In the two package auctions studied in this paper, there is never a strict incentive for bidders to reduce demand in this way.[3]

Auctions operate in a variety of environments, each involving different costs and constraints, and a credible general auction design must perform well across a range of different environments. We have identified several goals that arise repeatedly in the design of package auctions. The first is computational: there must be some sense in which, if bidders bid straightforwardly and bid evaluation costs are trivial, the auction outcomes will be good ones according to revenue and efficiency criteria. Second, because package bidding is often very complex, simplicity is an important objective of auction design. Dynamic designs are sometimes favored over similar one-shot designs for their relative comprehensibility and because they eliminate the need for bidders to evaluate closely every possible package. Third, the incentives for individuals or coalitions to deviate from straightforward bidding should be small or zero. Finally, the incentives for individuals and coalitions to deviate from efficient pre-auction investment decisions should also be small or zero.

Vickrey's package auction has well-known and unique advantages. According to theorems by Green and Laffont (1979) and by Holmstrom (1979), it is essentially the

[1] There are also versions of the auction with M types of goods and n_m goods of type m. Vickrey's (1961) original model treated the case of one type of good. The generalization to multiple types of goods is due to Clarke (1971) and Groves (1973).

[2] In a static auction, bidders need to decide in advance which packages to evaluate. As we show in Ausubel and Milgrom (2001), a dynamic auction can sometimes provide valuable information to bidders about which packages are worth evaluating.

[3] While the model considered here has only private values, it is reasonable to expect that ascending package auctions will offer further advantages over the sealed-bid Vickrey auction when values are interdependent. See Milgrom and Weber (1982).

only design to provide dominant strategy incentives and yield efficient auction outcomes. Upon closer examination, however, these advantages come at a high cost. The Vickrey auction enjoys incentive advantages over the ascending package auction *only* in those circumstances in which it may charge "low" prices for the assets being sold—prices so low that the allocation lies outside the core of the buyer-seller game. Moreover, unlike ascending package auctions,[4] the Vickrey auction distorts pre-auction technology choices.[5] The ascending package auction may sometimes be preferred to the Vickrey design because its does not share these properties.

The results supporting these claims are only announced here. The formal details are developed in Ausubel and Milgrom (2001).

2. The Model

The environment is one with private values. There is a set $\mathbf{M}=\{1,\ldots,M\}$ of items to be sold and a set N of players. The players consist of a single seller, identified by $l=0$, and bidders $l=1,\ldots,|N|-1$. Bidder l's preferences are described by its valuation function v_l, according to which any package A_l has value $v_l(A_l)$. If bidder l acquires the package for a price b, then l's payoff from the game is $v_l(A_l)-b$. We normalize by assuming that the null package has zero value.

In the Vickrey auction, a bid is a vector $(b_l(A_l):\varnothing \neq A_l \subset \mathbf{M})$ that specifies a price for each package. The auctioneer selects a partition $\{A_l^*\}_{l=1}^{|N|-1} \in \arg\max \sum_{l=1}^{|N|-1} b_l(A_l)$, that is, an allocation of packages to the bidders that maximizes the total bid. The price that each bidder pays is the opportunity cost of the goods it acquires, as given by the Vickrey-Clarke-Groves formula:

$$p_n = \max_A \sum_{l\neq n} b_l(A_l) - \sum_{l\neq n} b_l(A_l^*) .$$

In our model of the ascending package auction,[6] bidders iteratively submit bids, each comprising a package and a proposed payment. After each round, the auctioneer considers the entire list of bids that have been submitted in the current or prior rounds, and determines the collection of compatible bids—at most one from each bidder—that maximizes revenues. These are denoted the "provisionally-winning bids," i.e., the bids that would win if the auction ended right then. However, bidders are given additional opportunities to submit bids in subsequent rounds. The auction concludes after no further bids are submitted and each winning bidder then pays the amount of its own best bid for the package it wins. Thus, the auction can be regarded as a generalization of the simple English auction for one good to the case of packages.

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[4] Rassenti, Smith and Bulfin (1982) are often credited with the first experimental study of package auctions. Bernheim and Whinston (1986) have an early and important theoretical analysis of the sealed-bid, first-price package auction. Banks, Ledyard and Porter (1989) have an early study of ascending package auctions which helped to pioneer the modern applications. For a recent survey of some related work, see DeVries and Vohra (2001).

[5] Additional problems with the Vickrey auction are discussed in Ausubel and Milgrom (2001).

[6] Some facets of the ascending package auction and ascending proxy auction are described in greater detail in Ausubel and Milgrom (2001), Ausubel (1999), and various patent applications of the authors.

More precisely, for each bidder l and package A_l, at the beginning of round t there is a best bid $b_l^{t-1}(A_l)$ carried over from the previous round and a minimum bid $\underline{b}_l^t(A_l) = b_l^{t-1}(A_l) + \varepsilon(l, A_l)$. In round t, a bidder may raise its bid on one or more packages A_l to the minimum $\underline{b}_l^t(A_l)$, or it may submit no new bids. At the end of the round, the auctioneer determines a profile of packages $A^* = (A_1^*, ..., A_{|N|-1}^*)$ to maximize the total round-winning bid:

$$A^* \in \arg\max\left\{\sum_{l=1}^{|N|-1} b_l^t(A_l) : A_l \subset \mathbf{M}, A_l \cap A_k = \varnothing \text{ for } k \neq l\right\}.$$

To focus on the most important economic issues, we assume that the bid increments $\varepsilon(l, A_l)$ are negligibly small and specified so that there is a unique package allocation in each round that maximizes the total bid.

In our suggested implementation, the bidding process is accelerated by having a "proxy bidder" place the bids on each bidder's behalf. Instructions to the proxy bidder are analogous to those given to proxy bidders at Internet auction sites and the rule that the proxy bidder uses is a generalization of those rules. The instructions to l's proxy bidder take the form of a valuation vector $\hat{v}_l$. We suppose that these proxy instructions can be periodically revised, but that there is at some time a final opportunity for all bidders to revise their instructions.

At round t, the proxy bidder decides whether and how to bid as follows. If bidder l is a provisional winner at $t-1$, then l's proxy bidder sets $\hat{\pi}_l^t = \hat{v}_l(A_l^*) - b_l^{t-1}(A_l^*)$; otherwise, it sets $\hat{\pi}_l^t = 0$. It then identifies a package A_l that maximizes the "potential earnings" $\hat{v}_l(A_l) - \underline{b}_l^t(A_l)$. If the potential earnings from A_l exceed $\hat{\pi}_l^t$, then the proxy submits the minimum new bid on that package. Otherwise, it submits no new bid.

The auction ends when no proxy bidder makes a new bid. At that time, the provisional winners become final winners and the winners pay amounts equal to their own winning bids.

3. The Ascending Proxy Auction

The early rounds of the ascending package auction are most important when there is uncertainty about which packages are potentially interesting. The initial stages can then be regarded as a kind of communication phase in which bidders try to form bidding coalitions. We do not study that process in this paper. Instead, we focus on the last stage of the auction, when final bids are made.

The main insight that guides our analysis is that the package auction with proxy bidders is a new kind of "deferred acceptance algorithm," closely related to the ones found in matching theory.[7] Generally, deferred acceptance algorithms are multi-round algorithms in which one side of a market (the "buyers") makes offers while the other side

[7] The most closely related matching process is the one described by Kelso and Crawford (1982). See Roth and Sotomayor (1990) for a detailed survey of matching theory.

(the "sellers") remains passive. The dynamics all occur inside the algorithm in a sequence of rounds. The inputs to the algorithm are lists of preferences for the buyers and the sellers. In the present application, the seller ranks offers by the total revenue they generate, while buyers specify preferences in terms of a valuation function. The algorithm begins with each buyer making a bid directed to a single seller[8] that corresponds to its most preferred outcome, relative to whatever reserve price applies. For example, if the reserve prices are low, each buyer's first bid will likely be for the package of all lots at its reserve price. After each round in which a particular bidder is not a provisional winner, the algorithm moves on to make the bid corresponding to the buyer's next most preferred outcome. On behalf of each seller, the algorithm holds on to the best bids received to date, but defers accepting these bids until the process is complete. The process ends when there is a round with no new bids. At that time, the provisional winners from the last round are declared final winners and their pending bids are accepted.

In line with the results of matching theory, our results are focused around the concept of the core of a cooperative game. To describe our results, we let 0 denote the seller and we generate a cooperative game (N,w) in which, for each coalition $S \ni 0$:

$$w(S) = \max\left\{\sum_{l \in S \setminus 0} v_l(A_l) : A_l \subset \mathbf{M}, A_l \cap A_k = \varnothing \text{ for } k \neq l\right\}.$$

Coalitions excluding the seller have value 0.

In this paper, we interpret core payoffs as the "competitive" prices for the buyers' and seller's services. To justify this interpretation, let us regard the situation as one in which each party has services or assets that it can sell as a package. Let π_l denote the price for the services of player l. Suppose there are competing brokers who, by assembling coalition S, can create value $w(S)$. The zero profit constraint then implies that for each coalition S, $w(S) \leq \sum_{l \in S} \pi_l$. Under our assumptions, value is maximized for the economy when the coalition of the whole forms, so the brokers lose money unless that maximum value is at least what they pay for inputs: $w(N) \geq \sum_{l \in N} \pi_l$. These inequalities define competitive equilibrium prices for the services of the seller and buyers in the model economy.

Notice that the first set of inequalities above coincides exactly with the constraints that no coalition S can block the *value allocation* π. The additional inequality coincides with the *feasibility* constraint that the total value allocation is limited to $w(N)$. Therefore, π is a competitive equilibrium price vector for the buyers' and seller's services if and only if $\pi \in Core(N,w)$.

The coalitional game notation (N,w) also provides a convenient way to characterize the payoffs of the Vickrey auction. The payoff of a bidder l in the Vickrey auction is the value the player adds to the coalition of the whole: $\pi_l^V(N) = w(N) - w(N \setminus l)$. For the

[8] Matching markets get their special structure from the condition that the participants on one side of the market can trade with at most one participant of the other side. In this description, each buyer can be matched to just one seller.

seller, the Vickrey revenue is the residual: $\pi_0^V(N) = w(N) - \sum_{l \in N \setminus 0} \pi_l^V(N)$. A similar characterization defines the Vickrey payoff vector $\pi_l^V(S)$ when participation is restricted to members of a coalition *S*.

By construction, each bidder *l*'s Vickrey payoff is its highest payoff at any point in the core. Any larger payoff to player *l* would leave the total feasible payoff to the coalition $N \setminus l$ at less than $w(N \setminus l)$, so such a payoff vector would be blocked. The payoff vector that pays *l* its Vickrey payoff and pays the seller the balance (i.e., $w(N) - w(N \setminus l)$) is unblocked by inspection. Thus, the Vickrey payoff vector is Pareto-preferred by bidders to all core allocations. We return in Theorem 2, below, to the problem of characterizing when the Vickrey payoff vector is itself contained in the core.

Our theorems about the "ascending proxy auction" refer to the one-shot revelation-game version of the ascending package auction in which instructions to proxy bidders cannot be revised. This analysis is also the basis for any analysis of the multi-stage version, using a backwards induction argument. By an argument similar to ones used to analyze deferred acceptance algorithms in matching theory, we have shown the following:

Theorem 1. Suppose that each bidder reports its actual valuation function to its proxy bidder. Then, the outcome of the ascending proxy auction is an element of $Core(N, w)$, the core of the cooperative game. There is no other point in $Core(N, w)$ at which each winning bidder earns a strictly higher profit.[9]

4. The Core, Substitutes and Nash Equilibrium

The geometry of the core depends on certain properties of the coalitional value function *w*, which in turn depends on properties of the individual valuation functions. One relevant property of *w* is submodularity, which means roughly that any individual bidder has a lower marginal value to more inclusive coalitions.

Definition. The function *w* is *submodular for bidders* if for every two coalitions *S* and *T* that include the seller, $w(S) + w(T) \geq w(S \cap T) + w(S \cup T)$.

The following theorem establishes precise conditions under which the ascending proxy auction leads to an outcome of $\pi^V(S)$, the Vickrey payoff allocation.

Theorem 2. The following statements are equivalent:

(i) The coalitional value function, *w*, is submodular for bidders.

(ii) For every coalition $S \subset N$ that includes the seller, $\pi^V(S) \in Core(S, w)$, i.e., the Vickrey payoff vector is in the core of the game with participation restricted to *S*.

[9] The conclusion of this theorem holds even when there are binding budget constraints—a fact that also provides the basis for extensions of Theorem 5 below. The ability of the ascending proxy auction to lead to core allocations even in the face of budget constraints is another potentially important improvement on the performance of the Vickrey auction.

(iii) For every coalition $S \subset N$ that includes the seller, the payoff vector resulting from truthful reporting in the ascending proxy auction is the Vickrey payoff vector, $\pi^V(S)$.

(iv) For every coalition $S \subset N$ that includes the seller, truthful reporting is a Nash equilibrium of the ascending proxy auction restricting participation to the bidders in *S*.

Condition (i) means that the marginal contribution of a bidder to a coalition is diminishing in the size (inclusiveness) of the coalition. Condition (ii) is a statement about the seller's revenues. Since every bidder's Vickrey payoff is the best it can get at any point in the core, the failure of the Vickrey outcome to lie in the core reflects auction revenues for the seller that are "uncompetitively" low. Condition (iii) characterizes the outcome of the ascending proxy auction when condition (i) holds: it coincides with the outcome of the Vickrey auction. Condition (iv) asserts that truthful reporting is a Nash equilibrium of the ascending proxy auction.

To understand the connection between the third and fourth result, recall that the package auction mechanism always picks points in the core. Consequently, it gives at least $w(N \setminus l)$ to coalition $N \setminus l$, regardless of l's report. Similarly, regardless of l's report, the actual total payoff cannot exceed $w(N)$. So, regardless of l's report, l's actual payoff cannot exceed $w(N) - w(N \setminus l)$: if truthful reporting yields Vickrey payoffs, then truthful reporting is optimal.

Theorem 2 directly formalizes two of our main claims. One was that, for the submodular coalitional value functions, the ascending package auction has incentive properties that are nearly as good as those of the Vickrey auction: truthful reporting is a Nash equilibrium; and, with incomplete information about opponents' valuations, truthful reporting is an *ex post* Nash equilibrium.[10] The other was that the Vickrey auction's incentive properties are more strictly favorable than those of the ascending package auction game *only* when the seller's Vickrey payoff is so low that the payoff vector lies outside the core.

In view of these conclusions, comparing the performance of the two auctions in various real environments requires two additional developments. First, taking valuation information as primitive, when is it likely that the coalitional value functions will satisfy the submodularity condition? Second, how does the performance of the two auctions compare in the alternative cases, when the coalitional value functions are not submodular?

Some new notation and a definition are useful for stating the results. Let *V* be the set of functions from which bidder valuations are drawn. A valuation function $v \in V$ is

[10] Both the Vickrey auction and the ascending package auction have other Nash equilibria as well, and the Vickrey auction also has a dominant strategy property that the ascending auction lacks.

additive if for all $A \cap B = \varnothing$, $v(A \cup B) = v(A) + v(B)$. Let V_{sub} denote the set of valuations for which goods are substitutes.[11]

Theorem 3. Suppose that the set of possible valuations V includes all of the additive valuation functions and that there are at least four bidders (i.e., $|N| \geq 5$). Then, the following are equivalent:

(i) $V \subset V_{sub}$.

(ii) For every profile of valuations $(v_1,...,v_{|N|-1}) \in V^{|N|-1}$, the corresponding coalitional value function is submodular for bidders.

(iii) For every profile of valuations $(v_1,...,v_{|N|-1}) \in V^{|N|-1}$, $\pi^V(N) \in Core(N,w)$.

(iv) For every profile of valuations $(v_1,...,v_{|N|-1}) \in V^{|N|-1}$, there exists a competitive equilibrium of the corresponding exchange economy.[12]

5. When Goods May Not be Substitutes

Theorems 2 and 3 suggest that, setting aside issues about the costs of bidding, the performances of the Vickrey and ascending auctions are most likely to be similar when the goods for sale are substitutes. This leads us to the part of the analysis that motivates much of the research in package auctions: the case of goods that are not substitutes, which is also the case in which the Vickrey outcome is not in the core.

Both the Vickrey auction and the ascending proxy auction have some implausible Nash equilibria that rest on excessive bidder pessimism. Here is an example to illustrate the problem. In the example, there are three bidders and two goods. The bidders' valuations are tabulated below.

Table 1

Goods:	1	2	Package
Bidder 1	6	0	6
Bidder 2	0	6	6
Bidder 3	4	4	8

In the Vickrey auction with the tabulated valuations, there is a Nash equilibrium in which bidder 3 bids 10 for the package while bidders 1 and 2 each bid zero for everything. The bidders' valuations satisfy the substitutes condition, so there is a corresponding equilibrium in the ascending proxy auction. The usual analysis of the Vickrey auction rules out this strategy profile on the basis that the strategies are weakly dominated, but we shall need a more refined criterion for the ascending proxy auction. The equilibrium is characterized by the idea that bidders 1 and 2 are discouraged. Each

[11] Goods are mutual substitutes if the demand function, restricted to the domain of prices for which demand is single-valued, has the property that an increase in the price of one good never reduces the demand for any other good.

[12] Milgrom (2000) first proved the equivalence of (i) and (iv).

bids low because it expects the other to do so and forecasts that raising its bid cannot raise its profits.

We are continuing to explore a variety of refinements to clarify the likely outcome. The equilibrium refinement we apply in this paper rules out discouraged bidders quite directly, as follows. For any strategy profile $v=(v_1,...,v_{|N|-1})$, let $\omega(v)$ denote the outcome induced by *v*. Consider a Nash equilibrium strategy profile $\hat{v}=(\hat{v}_1,...,\hat{v}_{|N|-1})$ and, for any *l*, define $\Sigma_{-l}(\hat{v})=\{v_{-l}:\omega(\hat{v}_l,v_{-l})=\omega(\hat{v}_l,\hat{v}_{-l})\}$. Bidder *l* is said to be *discouraged* at $\hat{v}$ if there exists an alternative strategy v_l' for bidder *l* such that $\pi_l(v_l',v_{-l})\geq\pi_l(\hat{v}_l,v_{-l})$ for all $v_{-l}\in\Sigma_{-l}(\hat{v})$, with strict inequality holding for some $v_{-l}\in\Sigma_{-l}(\hat{v})$. Verbally, bidder *l* is *discouraged* at $\hat{v}$ if $\hat{v}_l$ is weakly dominated by another bid in the game where bidder *l*'s opponents are restricted to strategy profiles which yield the same outcome against $\hat{v}_l$ as does $\hat{v}_{-l}$. Intuitively, the criterion is that the bidder firmly expects the equilibrium outcome to result unless it increases some bid, but is unsure about what will happen if it does increase its bids. An *undiscouraged bidder equilibrium* is a Nash equilibrium profile in strategies satisfying $\hat{v}_l\leq v_l$ at which no bidder is discouraged. This concept is a first attempt to capture the idea that a bidder with multiple best replies does not make low bids merely because the bidder believes it is hopeless to bid higher.

In the tabulated example considered above, an "undiscouraged" bidder 1 will not bid zero. Given multiple best replies, the undiscouraged bidder chooses among them based on the possibility that there *might* be a way to earn *more* than the equilibrium payoff. That could happen, for example, if bidder 2 bid more aggressively than the equilibrium profile specifies or if bidder 3 bid less aggressively.

Theorem 4. In an undiscouraged bidder equilibrium v^* of the ascending proxy auction, the following inequality must hold for all bidders *l*=1,…,|*N*|−1 and all packages A_l: $v_l^*(A_l)\geq v_l(A_l)-\pi_l(v^*)$. The associated payoff vector satisfies $\pi(v^*)\in Core(N,w)$. No other point in $Core(N,w)$ is Pareto-preferred by the bidders. Conversely, for every such bidder-Pareto-optimal payoff allocation $\hat{\pi}\in Core(N,w)$, there is an undiscouraged bidder equilibrium with payoff vector $\hat{\pi}$.

The inequality, $v_l^*(A_l)\geq v_l(A_l)-\pi_l(v^*)$, in Theorem 4 means that bidder *l* bids at least his true valuation minus his equilibrium payoff for every package. In contrast to the dominant strategy equilibrium of the Vickrey auction, the equilibria of the ascending proxy auction identified in Theorem 4 always lead to payoffs that are in the core and therefore are weakly lower for every bidder than the Vickrey payoffs. It follows that the seller's total revenue, which is equal to the total value minus the bidders' profits, is greater in the ascending proxy auction than in the Vickrey auction.

Theorem 5. The seller's revenue at an undiscouraged bidder equilibrium of the ascending proxy auction is equal to its revenue at the dominant strategy equilibrium of the Vickrey auction when $\pi^V(N)\in Core(N,w)$ and is strictly greater when $\pi^V(N)\notin Core(N,w)$.

6. Choice of Organization and Technology

Another consideration in selecting among auctions is whether they create a "level playing field" among competing technologies, encouraging choices that are based on the merits of the technology in creating value rather than on the discounts provided to some technologies by the auction rules. We are particularly interested in package auctions in which different technologies imply incompatible divisions of the assets—a case that we call *incompatible technologies*.

More precisely, we study a situation in which there are two groups of buyers, and it is known in advance that efficiency demands that all of the winners be members of the same group. The question concerns what coordination, if any, takes place among members of the group before the auction. As an example, suppose that the values of each of two regional wireless phone services are $x/2$, while the combined value if the providers coordinate their service before the auction is $x+y$, where y may be positive or negative. Suppose that the value of a competing national data service is z. Then, the total profits of the wireless phone providers in a Vickrey auction if they coordinate their services and bid as a single entity will be $\max(0, x+y-z)$. If they do not coordinate, their bids will still win if $z < x$. In that case their prices will each be $z - x/2$ and their individual profits will therefore be $\max(0, x-z)$; their total prices and profits will be twice that amount. Consequently, their total profit from coordinating services is larger precisely when both $x+y-z>0$ and $y+z-x>0$. The first inequality holds if the coordinated bidders win and the second holds when, as winners, they earn more as a coordinated unit. These conditions are to be compared to the ones that identify when coordinating strictly increases social value: $x+y-z>0$ and $y>0$. The difference establishes that the organization/coordination incentives created by a Vickrey auction do not coincide with the first-best incentives. This happens because the winning bidders in a Vickrey auction pay different total prices depending on how they are organized.

By way of contrast, the total price that the wireless phone bidders must pay to win in the ascending proxy auction is z, regardless of whether the bidders coordinate their services. For competitions among incompatible technologies, the ascending proxy auction is effectively a coalitional second-price auction: the "winning technology" pays a total price equal to the value of the asset to the second-best technology. This provides better *ex ante* incentives for coordination (and associated investment) than those provided by the Vickrey auction.

<u>Theorem 6</u>. Consider package auctions among incompatible technologies. In the dominant-strategy equilibrium of the Vickrey auction, the total price paid by the winning coalition sometimes depends on its organization, and incentives for coalition formation are sometimes inefficient. In an undiscouraged bidder equilibrium of the ascending proxy auction, the total payoff to *ex ante* coordination among bidders is exactly equal to the associated increase in total value and coordination incentives are efficient.

7. Conclusion

We have investigated the performance of two kinds of package auctions. The ascending proxy auction, which we introduce here, is motivated partly by the ease that bidders may experience in using it. In important ways, the new design compares favorably with the better-known Vickrey package auction. When all goods are substitutes, the two auctions theoretically achieve identical equilibrium performance. In the cases where their equilibrium performances differ, the Vickrey auction always leads to lower total revenues and, indeed, to revenues that are uncompetitively low in the sense that the Vickrey payoff allocation is “blocked” by a coalition including the seller and some of the bidders. In addition, the Vickrey auction introduces biases in pre-auction investments that are absent when the ascending proxy auction is used. In our related paper, we show that the ascending auction also has better resistance to collusion and better handling of budget constraints than the Vickrey auction. We offer the ascending proxy auction as a realistic and theoretically sound auction method for actual applications where package bidding may be useful.

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