Statistics and System Performance Metrics for the Two Wave With Diffuse Power Fading Model

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***Abstract*—The Two Wave with Diffuse Power (TWDP) fading model was proposed by Durgin, Rappaport and de Wolf as a generalization of the Rayleigh and Rician fading models. This model can characterize a large range of fading behavior and has a geometric justification in terms of two dominant line of sight components in the presence of a diffuse component. The fact that the pdf of the TWDP model is not in closed-form has hindered the analytical characterization of this otherwise intuitive model. We show that any metric which is a linear function of the envelope statistics of the TWDP fading model can be computed as a finite integral of the corresponding metric for the Rice model. Employing this approach, we obtain simple expressions for some performance metrics for this fading model hitherto not found in the literature, such as the Amount of Fading, the Level Crossing Rate and the Moment Generating Function, by leveraging existing results for Rician fading.**

***Index Terms*—Envelope statistics, fading channels, multipath propagation, Rician fading, small-scale fading, TWDP.**

I. Introduction

Small-scale narrowband fading models such as the common Rayleigh, Rician and Nakagami-m models have been verified to match closely with field trials. The Rayleigh and Rice models are geometrically justified in terms of the Line of Sight (LoS) and Non-Line of Sight (NLoS) components [1] of the fading envelope. The more general Nakagami-m model can model Rayleigh or Rice fading models and is a close fit to land-mobile and indoor multipath propagation but still does not have a physical interpretation for arbitrary values of the fading severity index m [2]. The $\kappa - \mu$ and $\eta - \mu$ fading models proposed in [3] are generalizations of these models and are defined as a sum of multipath clusters with each cluster having a LoS and multiple NLoS components.

A related distribution is the Two Wave with Diffuse Power (TWDP) fading model proposed in [1]. This model consists of two LoS components and multiple diffuse NLoS components, and is supported by field measurements in indoor scenarios [4]. By varying the power of the LoS and NLoS components, the TWDP model encompasses the Rayleigh and Rician models along with the LoS case with no diffuse components. Another fading behaviour which the TWDP can model is when the fading is more severe than Rayleigh fading [5]. This regime, termed *Hyper-Rayleigh* fading, has been observed in Wireless Sensor Networks (WSN) deployed in cavity structures such as an aircraft or a bus [6].

Although the TWDP model can model a variety of fading environments, its complicated statistical characterization has been its main drawback. Since the original pdf of this model has an integral form, an approximate closed-form pdf was also proposed in [1] to facilitate further analytical results for this scenario. This approximate pdf has been widely used to characterize the performance of wireless communication systems in TWDP fading: in [8], the Bit Error Rate (BER) of BPSK modulated signals in TWDP fading was studied. These results were extended to diversity combining reception and over other modulation schemes in [9–11]. Symbol error rates in relay networks over TWDP channels were also studied in [12, 13]. These expressions provide the first analytical characterization for TWDP in a number of scenarios. They are, however, approximations; the exact characterization of most performance metrics in TWDP is an open problem. Recently, an alternative exact expression for the TWDP pdf was given in [7] in terms of an infinite series.

Interestingly, the authors in [1] posited that the TWDP pdf *somewhat resembles* the Rician pdf, but did not further exploit this similarity. We have found that characterizing the envelope statistics of TWDP fading is closely related to a classical problem in communication theory addressed by Rice [14] on the statistical properties of sine waves in Gaussian noise. Esposito and Wilson [15] further developed these ideas and provided expressions for the distribution of two sine waves in the presence of Gaussian noise. Motivated by these results, we show that the envelope statistics of the TWDP fading model conditioned on the phase difference between the LoS components results in the Rician fading model. This allows us to express *any performance metric* which is a linear function of the envelope statistics of the TWDP fading model in terms of a finite integral of the Rician metrics. Using this simple yet powerful approach, we find simple expressions for the pdf, cdf, Amount of Fading and the level crossing rate (LCR). As a key result, we obtain a closed-form expression for the Moment Generating Function, which to the best of our knowledge has not been expressed in the literature so far. This enables us to use the MGF approach to calculate the BER for various modulation schemes.

II. Statistical Characterization of the TWDP Fading Model

A. A brief description of the TWDP fading model

As presented in [16], the complex baseband received signal $s(t)$ in narrowband multipath fading is:

$$s(t) = \Re\left\{u(t)\sum_n \alpha_n e^{j\phi_n}\right\}, \quad (1)$$

where $u(t)$ is the transmitted signal in baseband, α_n and ϕ_n represent the amplitude and phase of the n-th multipath component and $\Re\{.\}$ denotes the real part.

The TWDP fading model [1, eq. 7] consists of two specular components and a diffuse component, as

$$V_r = V_1 \exp(j\phi_1) + V_2 \exp(j\phi_2) + X + jY, \quad (2)$$

where V_r is the received signal, components 1 and 2 are specular components with $\phi_1,\, \phi_2 \sim \mathcal{U}(0, 2\pi)$ and V_1 and V_2 are constant. In the diffuse component $X,\, Y \sim \mathcal{N}(0, \sigma^2)$. The model is conveniently expressed in terms of the parameters K and Δ, defined as

$$K = \frac{V_1^2 + V_2^2}{2\sigma^2}, \quad (3)$$

$$\Delta = \frac{2V_1V_2}{V_1^2 + V_2^2}. \quad (4)$$

Here K, like the Rician parameter, represents the ratio of the power of the specular components to the diffuse power. Δ is defined as the ratio of the peak specular power to the average specular power and serves as the comparison of the power levels of the two specular components. We observe that $\Delta = 1$ only when the two specular components are of equal amplitude, and $\Delta = 0$ when either LoS component has zero power. Special cases of the TWDP model are detailed in [1], encompassing the *One Wave*, *Two Wave*, Rayleigh and Rician fading models. In [5] it is shown that when $K > 0$ and $\Delta \approx 1$ the channel exhibits worse fading than Rayleigh, referred to as Hyper-Rayleigh behavior. As K increases, the fading becomes more severe and with the extreme condition of $K \to \infty$, the most-severe Two Wave fading model emerges

The behavior of the TWDP fading model under these conditions is seen in Fig. II-A. The two-ray model exhibits the worst fading. As the power of the diffuse component increases, the systems becomes more benign with Rayleigh-like behavior. Rice-like conditions, where the power of one LoS component is much larger than the other LoS component and the diffuse components, offer better fading than Rayleigh.

The pdf of the TWDP model was given in [1] as

$$f_R(r) \;=\; r\int_0^\infty e^{-\frac{v^2\sigma^2}{2}} J_0(V_1v)J_0(V_2v)J_0(vr)v dv. \quad (5)$$

where $J_0(\cdot)$ denotes the Bessel function of the first kind with order zero. An alternative expression for this pdf was also given as

$$f_R(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2} - K\right)\times \quad (6)$$
$$\frac{1}{\pi}\int_0^\pi \exp\left(K\Delta\cos\theta\right) I_0\left(\frac{r}{\sigma}\sqrt{2K(1-\Delta\cos\theta)}\right)d\theta,$$

where $I_0(\cdot)$ is the modified Bessel function of the first kind with order zero. Since both (5) and (6) are in integral form,

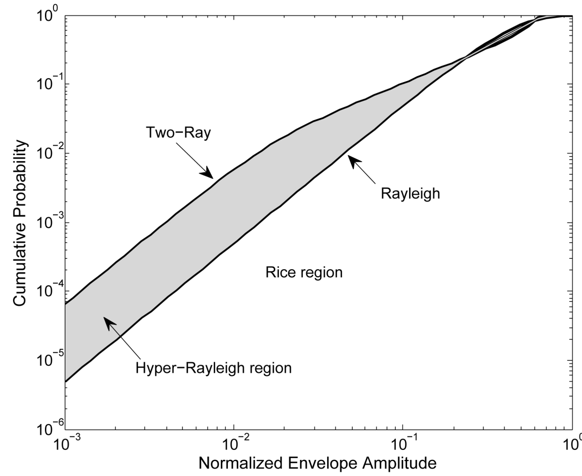

Figure 1. The CDF of the TWDP fading model vs. the normalized envelope amplitude. The worst case arises in the Two-Ray model ($\Delta = 1$, $K \to \infty$).

the authors in [1] presented an approximate representation of the pdf as

$$f_R(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2} - K\right)\sum_{i=1}^{M} a_i D\left(\frac{r}{\sigma^2}; K,\, \Delta\alpha_i\right), \quad (7)$$

where a_i are tabulated constants, the order M should be sufficiently large and

$$D(x; K,\, \alpha_i) = \frac{1}{2}\exp(\alpha_i K) I_0\left(x\sqrt{2K(1-\alpha_i)}\right) + \frac{1}{2}\exp(-\alpha_i K) I_0\left(x\sqrt{2K(1+\alpha_i)}\right), \quad (8)$$

where $\alpha_i = \cos\left(\frac{\pi(i-1)}{2M-1}\right)$.

B. TWDP fading as a generalization of Rician fading

Similar to the procedure followed in [1] to derive (6) from (5), we use an expanded form of the Bessel functions J_0 which results in

$$f_{TWDP}(r) = r\int_0^\infty v\exp\left(\frac{-v^2\sigma^2}{2}\right)J_0(vr)\frac{1}{(2\pi)^2} \times \int_{\theta=0}^{2\pi}\int_{\phi=0}^{2\pi} \exp[jV_1v\cos(\theta) + jV_2v\cos(\phi)]d\theta d\phi dv, \quad (9)$$

where we adopted the notation $f_{TWDP}(r)$ to denote the TWDP pdf for convenience of discussion. We recognize that

$$\begin{aligned} V_1\cos(\theta) + V_2\cos(\phi) &= V_1\cos(\theta) + V_2\cos(\theta - \alpha) \\ &= [V_1 + V_2\cos(\alpha)]\cos(\theta) + V_2\sin(\alpha)\sin(\theta) \\ &= \sqrt{V_1^2 + V_2^2 + 2V_1V_2\cos(\alpha)}\cos(\theta + \theta_0), \end{aligned} \quad (10)$$

where $\alpha = \theta - \phi$ is the phase difference between the two LoS components and $\theta_0 = \arctan(\frac{V_2\sin(\alpha)}{[V_1 + V_2\cos(\alpha)]})$. Using (10) in (9) and noticing that adding a phase term θ_0 in the Bessel function integrand does not modify it as it is integrated over an entire period, we get

$$f_{TWDP}(r) = \frac{1}{2\pi}\int_{\alpha=0}^{2\pi} r \int_{0}^{\infty} v \exp(\frac{-v^2\sigma^2}{2}) J_0(vr) \times J_0\left(\sqrt{V_1^2+V_2^2+2V_1V_2\cos(\alpha)}\right) dv d\alpha. \quad (11)$$

The inner integral of (11) is seen to be a special case of (5) with only one LoS component $\bar{V}_1$ and $\bar{V}_2 = 0$, i.e. it can be seen as an *equivalent* Rician pdf. The equivalent LoS component amplitude $\bar{V}_1$ is given by,

$$\bar{V}_1 = \sqrt{V_1^2+V_2^2+2V_1V_2\cos(\alpha)} \quad (12)$$

$$\bar{K} = K\left(1+\Delta\cos(\alpha)\right). \quad (13)$$

Employing the equivalent Rician pdf in (11), we obtain

$$f_{TWDP}(r) = \frac{1}{2\pi}\int_{0}^{2\pi} f_{Rice}\left(r;\, K[1+\Delta\cos(\alpha)]\right) d\alpha. \quad (14)$$

Thus, we see that the pdf of the TWDP fading model is obtained by finding the Rician pdf with equivalent $\bar{K}$ as given by (3) and taking the mean with respect to α, the phase difference between the LoS components. If we plug the well-known expression for the Rician pdf [2] given by

$$f_{Rice}(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}-K} I_0\left(\frac{r}{\sigma}\sqrt{2K}\right) \quad (15)$$

in (14), we obtain an expression very similar to (6). It is straightforward to show that both are coincident by a simple change of variables.

We have been able to find an insightful connection between the TWDP and the Rician fading models, showing that the former is in fact a natural generalization of Rician fading for two LoS components. This connection can be inferred for an arbitrary number of LoS components; however, as discussed in [1], the applicability of such an $n-$wave model is questionable in practice.

Another intuitive approach to arriving at (14) is as follows: conditioning the received signal amplitude on the phase difference between the LoS components we get

$$V_r = \exp(j\phi_1)\left(V_1 + V_2\exp[j(\alpha)]\right) + V_{diff}. \quad (16)$$

This problem is equivalent to finding the Rician pdf as there is a single LoS component of uniformly distributed phase ϕ_1 and constant amplitude $\bar{V}$ and $\bar{K}$ given in (12) and (13), respectively. Thus, the TWDP fading distribution conditioned on the phase difference α results in the Rician envelope distribution, i.e. $f_{TWDP}(r|\alpha) = f_{Rice}\big(r;\; K[1+\Delta\cos(\alpha)]\big)$.

Given that $\phi_1, \phi_2 \sim \mathcal{U}(0, 2\pi)$, the random variable $\alpha = \phi_2 - \phi_1 \sim \mathcal{U}(0, 2\pi)$. Although $\phi_2 - \phi_1$ is a symmetric triangular distribution from -2π to 2π, we are interested in the phase difference modulo 2π and α results in a uniformly distributed pdf. Employing the uniform distribution in (17), we obtain (14).

Although the consideration of uniformly and independently distributed phase for the LoS components has been verified through field measurements, we can extend the TWDP fading model to a more general case where the phase difference α can follow *any* distribution. The envelope distribution of this model would be the average of f_{Rice} over this distribution of α.

$$f_{TWDP}(r) = \int_{0}^{2\pi} f_{Rice}\left(r;\, K[1+\Delta\cos(\alpha)]\right) f_\alpha(\alpha) d\alpha \quad (17)$$

In the subsequent sections, we describe how the generalization of Rician fading as TWDP fading allows us to express the performance metrics of the latter in terms of existing expressions for the former in a very simple manner.

III. Performance Metrics of the TWDP Distribution

Let H be a metric of a fading model, expressed as a linear function of its envelope pdf in the form

$$H = \int f_R(r) g(r) dr, \quad (18)$$

where $g(.)$ is an arbitrary function defined on $\mathbf{R}$. Then, the metric of the TWDP model $H_{TWDP}(K,\Delta)$ can be expressed in terms of the metric of the Rician fading model $H_{Rice}(K)$ as

$$H_{TWDP}(K,\Delta) = \frac{1}{2\pi}\int_{0}^{2\pi} H_{Rice}\big(K[1+\Delta\cos(\alpha)]\big) d\alpha. \quad (19)$$

This is easily verified by changing the order of integration in (18). This simple approach is new in the literature to the best of our knowledge. We now apply this general technique to find expressions for performance metrics of the TWDP fading model.

A. Probability density function

Using the pdf of the Rician distribution given in (15), the pdf of the TWDP is given by

$$f_{TWDP}(r) = \frac{1}{2\pi}\int_{0}^{2\pi} \frac{r}{\sigma^2}\exp\left(-K[1+\Delta\cos(\alpha)] - \frac{r^2}{2\sigma^2}\right) \times I_0\left(\frac{r\sqrt{2K[1+\Delta\cos(\alpha)]}}{\sigma}\right) d\alpha. \quad (20)$$

The average SNR at the receiver is defined as $\bar{\gamma} = \bar{P}_r/N_0$, where $\bar{P}_r = V_1^2 + V_2^2 + 2\sigma^2$ is the average received power and $N_0/2$ is the Power Spectral Density of the AWGN noise. Since the average SNR is expressed as

$$\bar{\gamma} = (1+K)2\sigma^2/N_0, \quad (21)$$

the pdf of γ is given by

$$f_\gamma(\gamma) = \frac{f_R\left(\sqrt{\bar{P}_r\gamma/\bar{\gamma}}\right)}{2\sqrt{\bar{\gamma}\gamma/\bar{P}_r}}. \quad (22)$$

Using this in the Rice pdf expression, we find the TWDP fading pdf to be

$$f_{TWDP}(\gamma) = \frac{1}{2\pi}\int_{0}^{2\pi} \frac{1+K}{\bar{\gamma}}\exp\left\{-\bar{K}(\alpha) - \frac{\gamma(1+K)}{\bar{\gamma}}\right\} \times I_0\left(2\sqrt{\frac{\gamma\bar{K}(\alpha)[K+1]}{\bar{\gamma}}}\right) d\alpha, \quad (23)$$

where $\bar{K}(\alpha)$ is defined in (13). We observe that $\Delta = 0$ results in the scenario where $\bar{K}(\alpha) = K$ and the resulting pdf is equivalent to the Rician pdf as expected. Furthermore, taking $K = 0$, we get the exponential distribution that characterizes the SNR distribution of Rayleigh fading.

B. Cumulative distribution function

The CDF of the Rice distribution is

$$F_{Rice}(r) = 1 - Q_1(\sqrt{2K}, \frac{r}{\sigma}), \tag{24}$$

where $Q_1(\cdot,\cdot)$ is the Marcum $Q-$function. Hence, the CDF of the TWDP distribution is directly given by

$$F_{TWDP}(r) = 1 - \tfrac{1}{2\pi}\int_0^{2\pi} Q_1\big(\sqrt{2K[1+\Delta\cos(\alpha)]}, \frac{r}{\sigma}\big)\, d\alpha. \tag{25}$$

C. Moment Generating Function

The moment generating function (MGF) of the Rice distribution is given by

$$\mathcal{M}_{Rice}(s) = \frac{1+K}{1+K-s\bar{\gamma}}\exp\left(\frac{Ks\bar{\gamma}}{1+K-s\bar{\gamma}}\right). \tag{26}$$

Given that $\frac{1+K}{\bar{\gamma}}$ is constant both for Rician and TWDP distributions[1], and represents the ratio of noise introduced by the receiver to the power of the diffuse component according to (21), we have that the MGF of the TWDP distribution is

$$\begin{aligned}\mathcal{M}_{TWDP}(s) &= \frac{1}{2\pi}\int_0^{2\pi}\frac{1+K}{1+K-s\bar{\gamma}}\exp\left(\frac{\bar{K}(\alpha)s\bar{\gamma}}{1+K-s\bar{\gamma}}\right)d\alpha\\ &= \frac{1+K}{1+K-s\bar{\gamma}}\exp\left(\frac{Ks\bar{\gamma}}{1+K-s\bar{\gamma}}\right)I_0\left(\frac{Ks\bar{\gamma}\Delta}{1+K-s\bar{\gamma}}\right).\end{aligned} \tag{27}$$

Hence, we have found a closed-form expression for the MGF of the TWDP fading model. Even though the TWDP pdf or cdf cannot be expressed in closed-form, we have shown that the MGF is characterized by a very simple expression. This has two direct implications: first, the moments for the TWDP distribution can also be expressed in closed-form, using Leibniz's rule for the derivative of products. Secondly, the MGF is extensively used to characterize the error rate performance of digital communication systems [2]. Hence, expression (27) is useful to analyze some of the scenarios considered in the literature [8–11] without the need for using the approximate pdf in (7).

D. Moments

The moments for the TWDP distribution can be directly obtained from the MGF. However, it is also possible to calculate these moments from the moments of the Rice distribution, given by

$$\mathbf{E}(\gamma^k) = \frac{k!}{(1+K)^k}\,{}_1F_1(-k,\,1;\,-K)\bar{\gamma}^k, \tag{28}$$

[1] In general, when using a certain performance metric derived for Rician fading to obtain the equivalent metric for TWDP fading, we propose the following rule of thumb: $\bar{K}(\alpha)$ should not be substituted in place of K where a term $\frac{1+K}{\bar{\gamma}}$ appears in the equivalent expression for the Rician metric before integration.

where ${}_1F_1(\cdot,\cdot;\cdot)$ is the Kummer confluent hypergeometric function. Using (19), we have

$$\mathbf{E}_{TWDP}(\gamma^k) = \frac{k!\bar{\gamma}^k}{(1+K)^k 2\pi}\int_0^{2\pi}{}_1F_1\big(-k,\,1;\,-\bar{K}(\alpha)\big)d\alpha. \tag{29}$$

In particular, the first two moments are given by

$$\mathbf{E}_{TWDP}(\gamma) = \bar{\gamma}, \tag{30}$$

$$\mathbf{E}_{TWDP}(\gamma^2) = \frac{\bar{\gamma}^2}{(1+K)^2}\left\{2+4K+K^2\left(1+\frac{\Delta^2}{2}\right)\right\}. \tag{31}$$

E. Amount of Fading

The Amount of Fading (AF) metric, as described in [2], is a simple performance criterion to assess the fading model. It is very useful in the analysis of diversity systems, since it allows us to evaluate the severity of fading by using higher moments of the SNR. The AF is defined as follows

$$AF = \frac{\mathbf{E}[(\gamma-\bar{\gamma})^2]}{\mathbf{E}[\gamma]^2}. \tag{32}$$

The Amount of Fading in the TWDP channel for which a closed-form expression has hitherto not been found is thus easily seen to be

$$AF_{TWDP} = \frac{2+4K+K^2\Delta^2}{2(1+K)^2}. \tag{33}$$

F. Level Crossing Rate

Rice [14] first observed that the Level Crossing Rate (LCR) of the envelope of a fading model at a threshold level r_{th} could be expressed as

$$N(r_{th}) = \int_0^\infty \dot{r} f_{r,\dot{r}}(r_{th},\dot{r})d\dot{r}, \tag{34}$$

where $\dot{r}$ is the time derivative of the fading envelope. In the case where the specular components arrive perpendicular to the direction of motion (they do not undergo Doppler fading) and the diffuse component consists of isotropic $2-D$ scattering, it is seen that the fading envelope and its time derivative are independent, i.e. $f_{r,\dot{r}}(r_{th},\dot{r}) = f_r(r_{th})f_r(\dot{r})$.

In this scenario, the LCR for the Rician fading envelope is known to be

$$N_{Rice}(r_{th}) = \sqrt{\frac{\pi}{2}}\times\sqrt{\frac{\bar{P}_r}{K+1}}f_D f_{Rice}(r_{th}), \tag{35}$$

where f_D is the maximum Doppler frequency. Hence, the LCR for the TWDP channel is directly given by

$$N_{TWDP}(r_{th}) = \sqrt{\frac{\pi}{2}}\times\sqrt{\frac{\bar{P}_r}{K+1}}f_D f_{TWDP}(r_{th}). \tag{36}$$

The Average Outage Duration (AOD) is a metric that indicates how long the channel is in a fade level below a certain threshold, and is conveniently defined as the quotient between the cdf and the LCR, i.e. $AOD(r_{th}) = \Pr(r < r_{th})/N(r_{th})$. Hence, the AOD for the TWDP fading is given by

$$AOD_{TWDP}(r_{th}) = \sqrt{\frac{2(K+1)}{\pi\bar{P}_r}}\frac{F_{TWDP}(r_{th})}{f_D f_{TWDP}(r_{th})}. \tag{37}$$

IV. Application: Performance of Digital Modulation over TWDP Channels

The average probability of symbol error P_s of a fading channel is given by [16] as

$$P_s(\bar{\gamma}) = \int_0^\infty P_{AWGN}(\gamma) f_\gamma(\gamma) d\gamma, \tag{38}$$

where $P_{AWGN}(\gamma)$ is the probability of symbol error of an AWGN channel with SNR γ. For a DPSK modulation scheme, we have $P_{AWGN}(\gamma) = \frac{1}{2}\exp(-\gamma)$, and the BER using (38) can be obtained in closed-form as

$$\begin{aligned} P_{s,TWDP}(\bar{\gamma}) &= \frac{1}{2}\mathcal{M}_{TWDP}(-1;\, K, \Delta) \\ &= \tfrac{1}{2}\tfrac{1+K}{1+K+\bar{\gamma}} \exp\left(\tfrac{-K\bar{\gamma}}{1+K+\bar{\gamma}}\right) I_0\left(\tfrac{K\bar{\gamma}\Delta}{1+K+\bar{\gamma}}\right). \end{aligned} \tag{39}$$

We observe that the BER for the TWDP fading can be seen as the BER for the Rician case modulated by a term that depends on the modified Bessel function. The Bessel function term is always greater than one except for the case when $\Delta = 0$; hence, being a monotonically increasing function, the error increases as Δ increases. Specifically, if $K\bar{\gamma}/(1+K+\bar{\gamma}) > 4$ the error worsens by a factor greater than 10 as Δ increases to 1.

For other modulation schemes, the alternate Gaussian $Q-$function representation can be employed to arrive at the average probability of error, i.e. $P_{AWGN}(\gamma) = \alpha Q(\sqrt{2g\gamma})$. The resultant expression is an integral of a smooth finite integrand over finite limits which can efficiently be computed by numerical quadrature schemes. For the BPSK modulation scheme for instance, $\alpha = 1,\ g = 1$. The average probability of error of a BPSK modulation in a TWDP channel is,

$$P_{s,TWDP}(\bar{\gamma}) = \frac{1}{\pi}\int_{\phi=0}^{\pi/2} \mathcal{M}_{TWDP}\left(\frac{-1}{\sin^2(\phi)};\, K, \Delta\right) d\phi. \tag{40}$$

V. Numerical Results

In this section, we provide numerical results for the performance metrics calculated previously. Monte Carlo (MC) simulations have been included in order to check the validity of the derived expressions.

The pdf of the TWDP model is represented in Figs. 2 and 3, for different values of the parameters K and Δ, and keeping $\bar{P}_r$ constant. As K increases, the envelope values tend to be more concentrated for a given Δ; conversely, as Δ increases then the envelope values tend to be more spread out.

Fig. 4 presents the AF metric of the TWDP distribution. As K increases, the channel is more benign and AF is reduced. However, in the Hyper-Rayleigh regime ($\Delta \approx 1$) there is minimal improvement in the AF with increasing K, tending asymptotically to 0.5 as $K \to \infty$.

The LCR normalized to f_D is represented in Fig. 5, for different values of K and D. Interestingly, we observe a larger number of crossings for low values of the threshold envelope in the Hyper-Rayleigh regime.

Finally, the BER performance of BPSK modulation i.e. evaluated in Figs. 6 and 7, for different values of K and Δ. It is seen that when the TWDP model operates in the Hyper-Rayleigh regime, its BER drops.

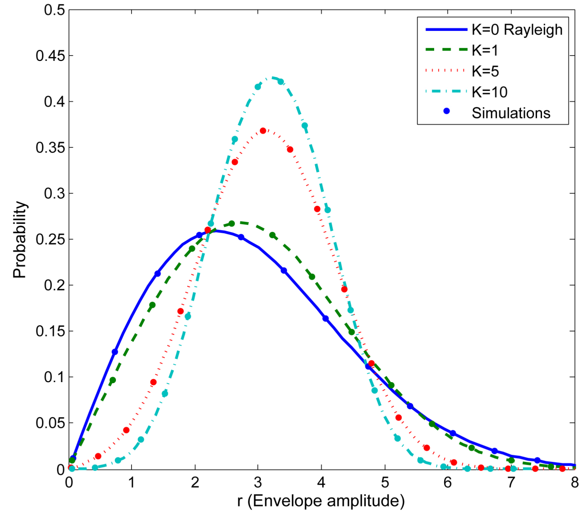

Figure 2. Probability density function of the TWDP model for different values of K and $\Delta = 0.5$. Markers correspond to MC simulations.

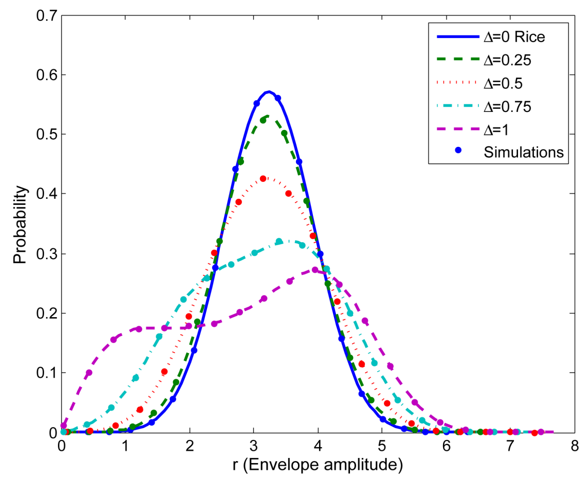

Figure 3. Probability density function of the TWDP model for different values of Δ and $K = 10$. Markers correspond to MC simulations.

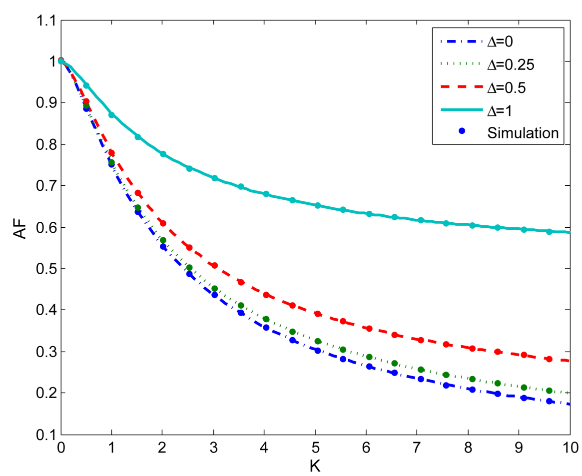

Figure 4. Amount of Fading as a function of K for different values of Δ. Markers correspond to MC simulations.

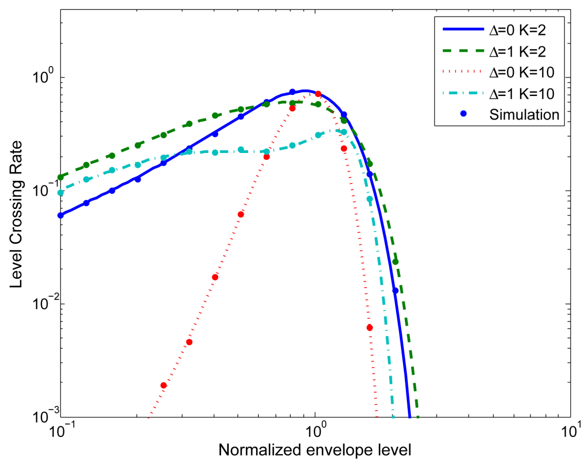

Figure 5. Level crossing rate of TWDP fading for different values of K, Δ and f_D. Markers correspond to MC simulations.

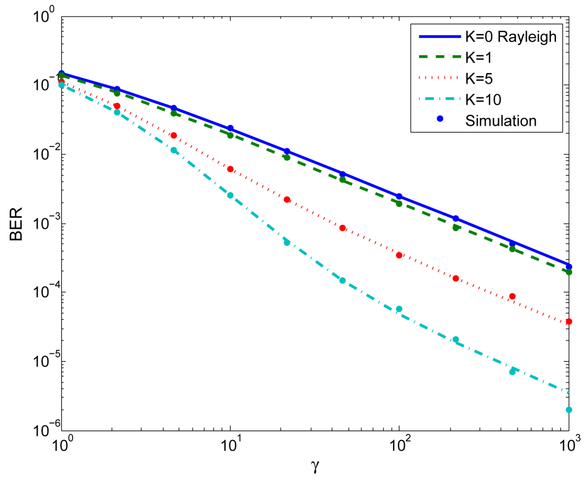

Figure 6. BER of BPSK modulation scheme in TWDP fading for different values of K and $\Delta = 0.5$. Markers correspond to MC simulations.

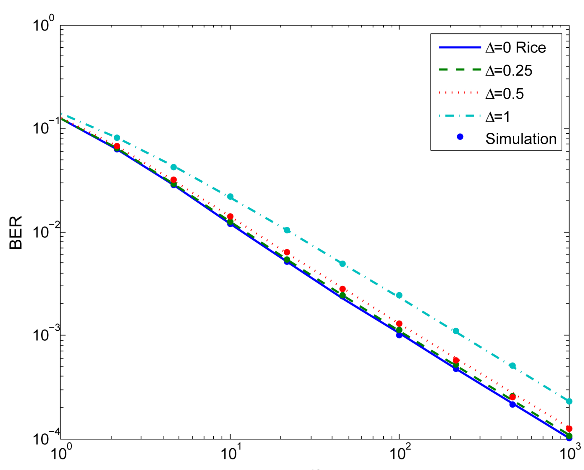

Figure 7. BER of BPSK modulation scheme in TWDP fading for different values of Δ and $K = 10$. Markers correspond to MC simulations.

VI. Conclusion

We provided a new look at the TWDP fading model, which characterizes fading more severe than Rayleigh fading. By observing that the TWDP fading conditioned on the difference in phase between the two LoS components results in Rician fading, any linear metric of the TWDP fading can be expressed in terms of a simple finite integral of the corresponding metric of the Rice fading model. This simple yet powerful approach has allowed us to derive a closed-form expression for the MGF of the TWDP fading model for the first time in the literature. We also provided very simple expressions for the most relevant performance metrics of systems experiencing TWDP fading.

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