1. (Content and Primitive Part)
Let \( R \) be a UFD. The content \( c(p(x)) \) of a polynomial \( p(x) \in R[x] \) is defined to be the gcd of its coefficients (defined up to associates). The primitive part \( \text{prim}(p(x)) \) is defined to be \( p(x)/c(p(x)) \) (which makes sense as \( c(p(x)) \) divides \( p(x) \)).

(a) Show \( c(p_1(x)p_2(x)) = c(p_1(x))c(p_2(x)) \) for polynomials \( p_1(x), p_2(x) \). Use this to verify the statement made in class that if \( p(x) = r_1 \ldots r_n a_1(x) \ldots a_n(x) \) for \( r_i \in R \) irreducible and \( a_i(x) \in R[X] \) irreducible with \( \deg a_i(x) > 0 \), then \( c(p(x)) = r_1 \ldots r_n \).

(b) Show \( \text{prim}(p_1(x)p_2(x)) = \text{prim}(p_1(x))\text{prim}(p_2(x)) \).

2. Notation as in the previous question. We have seen that \( R[x] \) is a UFD and hence we can talk about the gcd of elements in \( R[x] \). Show:

(a) \( c(\gcd(p_1(x), p_2(x))) = \gcd(c(p_1(x)), c(p_2(x))). \)

(b) \( \text{prim}(\gcd(p_1(x), p_2(x))) = \gcd(\text{prim}(p_1(x)), \text{prim}(p_2(x))). \)

Write up solutions to the following exercises from Dummit and Foote.

- Section 8.1: 1b, 1d, 2c, 3, 8, 12