Math 52 - Fall 2017 - Midterm Exam I

Name: _________________________________

Student ID: ____________________________

Signature: ______________________________

Instructions:

• Print your name and student ID number and write your signature to indicate that
you accept the Honor Code.

• There are 8 problems (from 1 to 8) on the pages numbered from 1 to 14. Please check
that the version of the exam you have is complete, and correctly stapled.

• Read each question carefully. In order to receive full credit, please show all of your
work and justify your answers except on multiple choice and true/false.

• You DO NOT need to give your answers in simplified form (e.g. you can leave sums
of numbers or fractions without combining them).

• You have 2 hours. This is a closed-book, closed-notes exam. No calculators or
other electronic aids will be permitted. If you finish early, you must hand your exam
paper to a member of teaching staff.

• You may use scratch paper for your own calculations which will not be turned in. If
you need extra room for your solutions, use a colored page labeled with your name
and the problem number. WRITE IN THE TEST BOOK FOR THAT PROBLEM
THAT YOUR WORK CONTINUES ON AN EXTRA COLOR PAGE.

• You have only until Thursday, November 2nd, to request any regrade considera-
tions. Please write email to the course instructor justifying your request.
Problem 1. In these problems \( f(x, y) \) or \( f(x, y, z) \) could be any continuous function of two or three variables. Circle TRUE if the statement is always true. Circle FALSE if there is a counterexample. You do not need to prove or justify your answer on the exam. You will only be graded on the circled response.

a) \[ \int_0^6 \int_2^4 \int_1^3 f(x, y, z) \, dx \, dy \, dz = \int_1^6 \int_0^3 \int_2^4 f(x, y, z) \, dz \, dx \, dy \]

TRUE    FALSE

b) The volume of the solid contained in the cylinder \( x^2 + y^2 = 4 \), below the surface \( z = 8 + xy \) and above the surface \( z = -x^2 - y^2 \) can be calculated by
\[ \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (8 + xy) \, dy \, dx \]

TRUE    FALSE

c) \[ \int_0^1 \int_x^1 f(x, y) \, dy \, dx = \int_0^1 \int_y^1 f(x, y) \, dx \, dy \]

TRUE    FALSE

d) \[ \int_0^1 \int_0^2 f(x, y) \, dx \, dy = \int_{-1}^{0} \int_{-2}^{0} f(x, y) \, dx \, dy \]

TRUE    FALSE

e) \[ \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} f(x, y, z) \, dz \, dy \, dx \]
\[ = \int_0^1 \int_0^\pi \int_0^{2\pi} f(\rho \cos \theta \sin \phi, \rho \sin \theta \sin \phi, \rho \cos \phi) \, d\theta \, d\phi \, d\rho \]

TRUE    FALSE
Problem 2. An pool is 40m long by 20m wide. The depth at one end of the 40m length is 1m, and 5m at the other end. The pool gets deeper at a constant rate (linear) as you go across the pool.

a) Write a Riemann sum by using a partition of the surface of the pool into four rectangles which gives an UNDERESTIMATE of the volume of the pool. Explain clearly your choices of partition and function values.

b) Write a Riemann sum by using a partition of the surface of the pool into four rectangles which gives an OVERESTIMATE of the volume of the pool. Explain clearly your choices of partition and function values.
Problem 3. Let $R$ be the region inside the triangle whose vertices are the points $(0, 0)$, $(3, 0)$, and $(2, 1)$.

a) Sketch the two dimensional region $R$ and then describe the region with inequalities in terms of $x$ and $y$.

b) Suppose $f(x, y)$ is the height function of a building whose base is this triangle in the $(x, y)$ plane. Write an iterated integral or sum of iterated integrals representing the volume of the building with respect to $dx\,dy$ (first integrating with respect to $x$ then with respect to $y$). (Your answer should be an integral, not a number.)
c) Suppose $f(x,y)$ is the height function of a building whose base is this triangle in the $(x,y)$ plane. Write an iterated integral or sum of iterated integrals representing the volume of the building with respect to $dy \, dx$ (first integrating with respect to $y$ then with respect to $x$). (Your answer should be an integral, not a number.)
Problem 4. Let $S$ be the solid bounded by the paraboloid $z = x^2 + y^2 - 4$ and the plane $z = 3$.

a) Write a triple integral (or sum of triple integrals) using the ordering $dz \, dx \, dy$ which represents the volume of $S$ (you do not need to evaluate the integral yet).
b) Write a triple integral (or sum of triple integrals) using the ordering \(dx \, dy \, dz\) which represents the volume of \(S\) (you do not need to evaluate the integral yet).
c) Write a triple integral (or sum of triple integrals) in *cylindrical coordinates* \((r, \theta, z)\) representing the volume of \(S\) (in any ordering you choose). (you do not need to evaluate the integral yet)
d) Choose any one of your triple integral set-ups from parts (a), (b), or (c) to calculate the volume of $S$. 
Problem 5. Suppose you have a flat shape with constant density 1 described by the inequalities
\[ 0 \leq x \leq 2 \quad \text{and} \quad 0 \leq y \leq x^2 \]
a) Sketch the region and label the corners.

b) Calculate the mass of the object
c) Calculate the centroid \((x, y)\) of the object.

[Make sure your answer makes sense as a balancing point.]
Problem 6. Determine whether the following integrals are positive, negative, or zero. You will only be graded on which answer you circle, so you do not need to include your justification here.

a) \[ \int_{-1}^{1} \int_{0}^{1} x \, dy \, dx \]
   POSITIVE ZERO NEGATIVE

b) \[ \int_{-2}^{2} \int_{-1}^{1} x^2 + y^2 \, dx \, dy \]
   POSITIVE ZERO NEGATIVE

c) \[ \int_{-2}^{2} \int_{0}^{y^2} xy \, dx \, dy \]
   POSITIVE ZERO NEGATIVE

d) \[ \int_{-1}^{1} \int_{\sqrt{1-y^2}}^{0} \int_{0}^{z} z \, dz \, dx \, dy \]
   POSITIVE ZERO NEGATIVE

e) \[ \int_{0}^{1} \int_{0}^{2\pi} \int_{0}^{\pi} \rho^2 \sin \phi \, d\phi \, d\theta \, d\rho \]
   POSITIVE ZERO NEGATIVE
Problem 7. Calculate the volume of the region between the sphere of radius 1 and the sphere of radius 2 and inside the upward cone $z = \sqrt{x^2 + y^2}$. 
Problem 8. Consider the solid ABOVE $z = 0 \ (z \geq 0)$, BELOW $z = y + 2 \ (z \leq y + 2)$, AND BELOW the paraboloid $z = 4 - x^2 - y^2 \ (z \leq 4 - x^2 - y^2)$, shown below.

a) Set up a triple integral or sum of triple integrals for the volume of the solid as an iterated integral with the order $dx \ dz \ dy$. [DO NOT EVALUATE THE INTEGRAL]
b) BONUS EXTRA CREDIT: Set up a triple integral for the volume of the solid as an iterated integral with the order $dz\ dy\ dx$. [DO NOT EVALUATE THE INTEGRAL]. Some extra views of the same shape are shown below.