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Policymakers frequently use price regulations as a response to inequality in the markets they control. In this paper, we examine the optimal structure of such policies from the perspective of mechanism design. We study a buyer-seller market in which agents have private information about both their valuations for an indivisible object and their marginal utilities for money. The planner seeks a mechanism that maximizes agents' total utilities, subject to incentive and market-clearing constraints. We uncover the constrained Pareto frontier by identifying the optimal trade-off between allocative efficiency and redistribution. We find that competitive-equilibrium allocation is not always optimal. Instead, when there is inequality across sides of the market, the optimal design uses a tax-like mechanism, introducing a wedge between the buyer and seller prices, and redistributing the resulting surplus to the poorer side of the market via lump-sum payments. When there is significant same-side inequality that can be uncovered by market behavior, it may be optimal to impose price controls even though doing so induces rationing.

KEYWORDS: Mechanism design, redistribution, inequality, welfare theorems.

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1. INTRODUCTION

POLICYMAKERS FREQUENTLY USE PRICE REGULATIONS as a response to inequality in the markets they control. Local housing authorities, for example, often institute rent control to improve housing access for low-income populations. State governments, meanwhile, use minimum wage laws to address inequality in labor markets. And in the only legal marketplace for kidneys—the one in Iran—there is a regulated price floor in large part because the government is concerned about the welfare of organ donors, who tend to come from low-income households. But to what extent are these sorts of policies the right approach—and if they are, how should they be structured? In this paper, we examine this question from the perspective of optimal mechanism design.[1]

Price controls introduce multiple allocative distortions: they drive total trade below the efficient level; moreover, because they necessitate rationing, some of the agents who trade may not be the most efficient ones. Yet at the same time, price controls can shift surplus to poorer market participants. Additionally, as we highlight here, they can help identify poorer individuals through their behavior. Thus, a policymaker who cannot observe who is most in need (or who cannot redistribute wealth directly) may opt for carefully constructed price controls—effectively, maximizing the potential of the marketplace itself to serve as a redistributive tool. Our main result shows that optimal redistribution through markets can be obtained through a simple combination of lump-sum transfers and rationing.

Our framework is as follows. There is a market for an indivisible good, with a large number of prospective buyers and sellers. Each agent has quasi-linear preferences and is characterized by a pair of values: a value for the good (v^K) and a marginal value for money (v^M), the latter of which we think of as capturing the reduced-form consequences of agents' wealth—or, more broadly, social and economic circumstances (we discuss the precise meaning of v^M and the interpretation of our model in Section 1.1).[2] A market designer chooses a mechanism that allocates both the good and money to maximize the sum of agents' utilities, subject to market-clearing, budget-balance, and individual-rationality constraints. Crucially, we also require incentive-compatibility: the designer knows the distribution of agents' characteristics but does not observe individual agents' values directly; instead, she must infer those values through the mechanism. We show that each agent's behavior is completely characterized by the ratio of her two values (v^K/v^M), that is, her *rate of substitution*. As a result, we can rewrite our two-dimensional mechanism design problem as a unidimensional problem with an objective function equal to the weighted sum of agent's utilities, with each agent receiving a welfare weight equal to her expected value for money conditional on her rate of substitution, given the distribution of values on her side of the market.

[1]For rent control in housing markets, see, for example, van Dijk (2019) and Diamond, McQuade, and Qian (2019). For discussion of minimum wages at the state level, see, for example, Rinz and Voorheis (2018) and the recent report of the National Conference of State Legislatures (2019). For discussion of the price floor in the Iranian kidney market, see Akbarpour, Fatemi, and Matoorian (2019). Price regulations and controls are also common responses to inequality in pharmaceutical markets (see, e.g., Mrazek (2002)), education (see, e.g., Deming and Walters (2017)), and transit (see, e.g., Cohen (2018)).

[2]Our setup implicitly assumes that the market under consideration is a small enough part of the economy that the gains from trade do not substantially change agents' wealth levels. In fact, utility can be viewed as approximately quasi-linear from a perspective of a single market when it is one of many markets—the so-called "Marshallian conjecture" demonstrated formally by Vives (1987). More recently, Weretka (2018) showed that quasi-linearity of per-period utility is also justified in infinite-horizon economies when agents are sufficiently patient.

In principle, mechanisms in our setting can be quite complex, offering a (potentially infinite) menu of prices and quantities (i.e., transaction probabilities) to agents. Nonetheless, we find that there exists an optimal menu with a simple structure. Specifically, we say that a mechanism offers a *rationing option* if agents on a given side of the market can choose to trade with some strictly interior probability. We prove that the optimal mechanism needs no more than a total of two distinct rationing options on both sides of the market. Moreover, if at the optimum some monetary surplus is generated and passed on as a lump-sum transfer, then at most one rationing option is needed. In this case, one side of the market is offered a single posted price, while the other side can potentially choose between trading at some price with probability 1, or trading at a more attractive price (higher for sellers; lower for buyers) with probability less than 1 (i.e., with some risk of being rationed).

The simple form of the optimal mechanism stems from our large-market assumption. Any incentive-compatible mechanism can be represented as a pair of lotteries over quantities, one for each side of the market. Hence, the market-clearing constraint reduces to an equal-means constraint—the average quantity sold by sellers must equal the average quantity bought by buyers. It then follows that the optimal value is obtained by concavifying the buyer- and seller-surplus functions at the market-clearing trade volume. Since the concave closure of a one-dimensional function can always be generated by a binary lottery, we can derive optimal mechanisms that rely on implementing a small number of distinct trading probabilities.

Given our class of optimal mechanisms, we examine which combinations of lump-sum transfers and rationing are optimal as a function of the characteristics of market participants. We focus on two forms of inequality that can be present in the market. *Cross-side inequality* measures the average difference between buyers' and sellers' values for money, while *same-side inequality* measures how much dispersion in values for money (on a given side of the market) can be identified by observing individuals' rates of substitution. We find that cross-side inequality determines the direction of the lump-sum payments—the surplus is redistributed to the side of the market with a higher average value for money. Meanwhile, same-side inequality determines the use of rationing.

Concretely, under certain regularity conditions, we prove the following results: When same-side inequality is not too large, the optimal mechanism is "competitive," that is, it offers a single posted price to each side of the market and clears the market without relying on rationing. Even so, however, the designer may impose a wedge between the buyer and seller prices, redistributing the resulting surplus as a lump-sum transfer to the side of the market that is "poorer," in the sense that it has a higher average v^M. The degree of cross-side inequality determines the magnitude of the wedge—and hence determines the size of the lump-sum transfer. When same-side inequality is substantial, meanwhile, the optimal mechanism may offer non-competitive prices and rely on rationing to clear the market. Finally, there is an asymmetry in the way rationing is used on the buyer and seller sides—a consequence of a simple observation that, everything else being equal, the decision to trade identifies sellers with the lowest ratio of v^K to v^M (i.e., "poorer" sellers, with a relatively high v^M in expectation) and buyers with the highest ratio of v^K to v^M (i.e., "richer" buyers, with a relatively low v^M in expectation).

On the seller side, rationing allows the designer to redistribute to the "poorest" sellers by raising the price that those sellers receive (conditional on trade) above the market-clearing level. In such cases, the designer uses the redistributive power of the market directly: willingness to sell at a given price can be used to identify—and effectively subsidize—sellers that have higher values for money in expectation. Rationing in this way is socially optimal when (and only when) it is the very poorest sellers who trade, which requires the volume of trade to be relatively small. This happens, for example, in markets where there are relatively few buyers. Often, the optimal mechanism on the seller side takes the simple form of a single price raised above the market-clearing level.

By contrast, at any given price, the decision to trade identifies buyers with *lower* values for money in expectation. Therefore, unlike in the seller case, it is never optimal to have buyer-side rationing at a single price. Instead, if rationing is optimal, the designer must offer at least two prices: a high price at which trade happens for sure—which attracts buyers with high willingness to pay, who are richer on average—and a low price with rationing at which poorer buyers may wish to purchase. The decision to choose the lower price identifies buyers that are likely to be poor; the market then effectively subsidizes these buyers by providing the good at a low price (possibly 0) with positive probability. Using the redistributive power of the market for buyers, then, is only possible when sufficiently many (rich) buyers choose the high price, so that the low price attracts only the very poorest buyers. In particular, for rationing on the buyer side to make sense, the volume of trade must be sufficiently high. We show that there are markets in which having a high volume of trade, and hence buyer rationing, is always suboptimal, regardless of the imbalance in the sizes of the sides of the market. In fact, we argue that—in contrast to the seller case—buyer rationing can only become optimal under relatively narrow conditions.

Our results may help explain the widespread use of price controls and other market-distorting regulations in settings with inequality. Philosophers and policymakers often speak of markets as having the power to "exploit" participants through prices.[3] The possibility that prices could somehow take advantage of individuals who act according to revealed preference seems fundamentally unnatural to an economist.[4] Yet our framework suggests at least one sense in which the idea can take on a precise economic meaning: as inequality among market participants increases, the competitive equilibrium price can shift in response to some market participants' relatively stronger desire for money, leaving more of the surplus with the other agents. At the same time, however, our approach suggests that the proper social response to this problem is not banning or eliminating markets, but rather designing market-clearing mechanisms in ways that directly attend to inequality. Policymakers can "redistribute through the market" by choosing market-clearing mechanisms that give up some allocative efficiency in exchange for increased equity.

We emphasize that it is *not* the point of this paper to argue that markets are a superior tool for redistribution relative to more standard approaches that work through the tax system. Rather, we think of our "market design" approach to redistribution as complementary to public finance at the central government level. Indeed, many local regulators are responsible for addressing inequality in individual markets, without access to macro-economic tools; our framework helps us understand how those regulators should set policy.

The remainder of this paper is organized as follows. Section 1.1 explains how our approach relates to the classical mechanism design framework and the welfare theorems. Section 1.2 reviews the related literature in mechanism design, public finance, and other areas. Section 2 lays out our framework. Then, Section 3 builds up the main intuitions and terminology for our approach by analyzing a simplified version of the model. In Sections 4

[3]For discussion of issues of equity and coercion in market-based allocation, see, for example, Satz (2010) and Sandel (2012).

[4]Roth (2015) has reviewed these philosophical and economic arguments.

and 5, we identify optimal mechanisms in the general case, and then examine how those optimal mechanisms depend on the level and type of inequality in the market. Section 6 discusses policy implications; and Section 7 presents concluding remarks.

1.1. *Interpretation of the Model and Relation to Welfare Theorems*

Two important consequences of wealth distribution for market design are that (i) individuals' preferences may vary with their wealth levels, and (ii) social preferences may naturally depend on individuals' wealth levels (typically, with more weight given to less wealthy or otherwise disadvantaged individuals). The canonical model of mechanism design with transfers assumes that individuals have quasi-linear preferences—ruling out wealth's consequence (i) for individual preferences. Moreover, in a less obvious way, quasi-linearity, along with the Pareto optimality criterion, rules out wealth's consequence (ii) for social preferences by implying that any monetary transfer between agents is neutral from the point of view of the designer's objective (i.e., utility is perfectly transferable). In this way, the canonical framework fully separates the question of maximizing total surplus from distributional concerns—all that matters are the agents' rates of substitution between the good and money, conventionally referred to as agents' *values*.

Our work exploits the observation that while quasi-linearity of individual preferences (consequence (i)) is key for tractability, the assumption of perfectly transferable utility (consequence (ii)) can be relaxed. By endowing agents with two-dimensional values (v^K, v^M), we keep the structure of individual preferences the same while allowing the designer's preferences to depend on the distribution of money among agents. In our framework, the rate of substitution v^K/v^M still describes individual preferences, while the "value for money" v^M measures the contribution to social welfare of transferring a unit of money to a given agent—it is the "social" value of money for that agent, which could depend on that agent's monetary wealth, social circumstances, or status.

Thus, our marginal values for money serve the role of Pareto weights—we make this analogy precise in Appendix A.1, where we show a formal equivalence between our two-dimensional value model and a standard quasi-linear model with one-dimensional types and explicit Pareto weights.[5]

Of course, when the designer seeks a market mechanism to maximize a weighted sum of agents' utilities, an economist's natural response is to look to the welfare theorems. The first welfare theorem guarantees that we can achieve a Pareto-optimal outcome by implementing the competitive-equilibrium mechanism.[6] The second welfare theorem predicts that we should moreover be able to achieve any split of surplus among the agents by redistributing endowments prior to trading (which in our simple model would just take the form of redistributing monetary holdings). Thus, the welfare theorems suggest that allowing for Pareto weights in the designer's objective function should not create a need to adjust the market-clearing rule—competitive pricing should remain optimal.

The preceding argument fails, however, when the designer faces incentive-compatibility (IC) and individual-rationality (IR) constraints. Indeed, while the competitive-equilibrium mechanism is feasible in our setting, arbitrary redistribution of endowments

[5] All appendixes are presented in the Supplemental Material (Dworczak, Kominers, and Akbarpour (2021)).

[6] Given that our model features a two-sided market with private information, it is natural to wonder why existence of an efficient mechanism does not contradict the Myerson and Satterthwaite (1983) impossibility. This is because we have assumed a continuum of traders. Under the interpretation of a single buyer and a single seller, the continuum assumption implies ex ante (rather than ex post) budget-balance and market-clearing constraints.

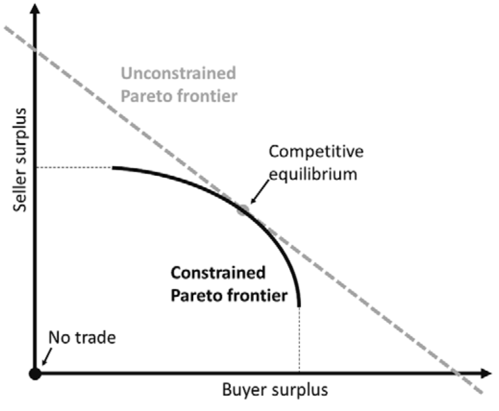

FIGURE 1.—The Pareto curve without the IC and IR constraints ("unconstrained," dotted line) and with them ("constrained," solid line)

is not: It would, in general, violate both IR constraints (if the designer took more from an agent than the surplus that agent appropriates by trading) and IC constraints (agents would not reveal their values truthfully if they expected the designer to decrease their monetary holdings prior to trading). This point is illustrated in Figure 1: While the hypothetical (unconstrained) Pareto frontier is linear because of the quasi-linearity of individual preferences, the actual (constrained) frontier is concave, and the two frontiers coincide only at the competitive-equilibrium outcome. This means that IC and IR introduce a trade-off between efficiency and redistribution, violating the conclusion of the second welfare theorem.[7] For example, giving sellers more surplus than in competitive equilibrium requires raising additional revenue from the buyers which—given our IC and IR constraints—can only be achieved by limiting supply.

The preceding argument provides an economic justification for price controls. But of course, price controls create a deadweight loss and lead to allocative inefficiencies from rationing. Thus, price controls make sense only when the gains from increased equity outweigh the losses due to allocative inefficiency. For instance, when sellers are sufficiently poor relative to buyers, the social value of a transfer of surplus from buyers to sellers can be greater than the associated allocative loss.

Put differently, the classic idea that competitive-equilibrium pricing maximizes welfare relies on an implicit assumption that the designer places the same welfare weight on all agents in the market. Thus, the standard economic intuitions in support of competitive-equilibrium pricing become unreliable as the dispersion of wealth in a society expands.

1.2. *Related Work*

Price Controls

It is well-known in economics (as well as in the public discourse) that a form of price control (e.g., a minimum wage) can be welfare-enhancing if the social planner has a preference for redistribution. That observation was made in the theory literature at least as

[7]This trade-off is of course not exclusive to our model; indeed, it can be seen as a cornerstone of the theory of public finance (see, e.g., Kaplow (2008), as well as the discussion in Section 1.2). In the mechanism design context, meanwhile, Williams (1987) showed how to optimally trade-off (second-best) efficiency in the Myerson and Satterthwaite (1983) framework to achieve different splits of surplus between buyers and sellers.

early as 1977, when Weitzman showed that fully random allocation (an extreme form of price control corresponding to setting a price at which all buyers want to buy) can be better than competitive pricing (a "market" price) when the designer cares about redistribution. Even before that, Tobin (1970) developed a framework for thinking about which goods we might want to distribute by rationing for equity reasons.[8]

Any form of price control has allocative costs, which can sometimes outweigh the equity gains. For instance, Glaeser and Luttmer (2003) found that in New York City, rent-control has led a substantial percentage of apartments to be misallocated with respect to willingness to pay. And Freeman (1996) examined when minimum wages have (and have not) succeeded in raising the welfare of low-paid workers. On the theory side, Bulow and Klemperer (2012) showed that—somewhat paradoxically—it is possible for demand and supply curves to be such that a price control decreases the welfare of *all* market participants.

Even when price controls are welfare-enhancing in the short run, they might be harmful in the long run depending on the elasticity of supply. Rent-control policies, for instance, can reduce supply of housing (Diamond, McQuade, and Qian (2019)). Our model makes an extreme assumption—taking long-run supply to be fixed—and thus matches most closely to settings in which long-run supply is not especially responsive to short-run price changes.[9]

Mechanism and Market Design

Our principal divergence from classical market models—the introduction of heterogeneity in marginal values for money—has a number of antecedents, as well. Condorelli (2013) asked a question similar to ours, working in an object allocation setting in which agents' willingness to pay is not necessarily the characteristic that appears in the designer's objective; he provided conditions for optimality of non-market mechanisms in that setting.[10] Huesmann (2017) studied the problem of allocating an indivisible item to a mass of agents with low and high income, and non-quasi-linear preferences.[11] More broadly, the idea that it is more costly for low-income individuals to spend money derives from capital market imperfections that impose borrowing constraints on low-wealth individuals; such constraints are ubiquitous throughout economics (see, e.g., Loury (1981), Aghion and Bolton (1997), Esteban and Ray (2006)).

[8]The idea of using public provision of goods as a form of redistribution—which is inefficient from an optimal taxation perspective—has also been examined (see, e.g., Besley and Coate (1991), Blackorby and Donaldson (1988), Gahvari and Mattos (2007)).

[9]Our approach can also be relevant when short-run equity is a chief concern; for instance, Taylor (2020) built on our analysis here to provide equity arguments for short-run price controls on essential goods during the COVID-19 pandemic.

[10]Although our framework is different across several dimensions, the techniques Condorelli (2013) employed to handle ironing in his optimal mechanism share kinship with the way we use concavification to solve our problem. Meanwhile, Loertscher and Muir (2020) used related tools to provide a complementary argument for why non-competitive pricing may arise in practice—showing that in private markets, non-competitive pricing may be the optimal behavior of a monopolist seller, so long as resale cannot be prevented.

[11]Subsequent to our work, several papers have built on our approach: Kang and Zheng (2019) characterized the set of constrained Pareto-optimal mechanisms for allocating a good and a bad to a finite set of asymmetric agents, with each agent's role—a buyer or a seller—determined endogenously by the mechanism. Kang (2020) studied a one-sided allocation problem under redistributive preferences in which agents can purchase the good in an exogenous private market. Reuter and Groh (2021) also analyzed a one-sided version of our model but with a finite number of goods and finitely many agents.

In our model, (implicit) wealth heterogeneity motivates attaching non-equal Pareto weights to agents. An alternative approach to capturing wealth disparities is to introduce heterogeneous budget constraints, as in the work of Che and Gale (1998), Fernandez and Gali (1999), Che, Gale, and Kim (2012), and Pai and Vohra (2014). The literature on mechanism design with budget constraints differs fundamentally from ours in terms of the objective function—it focuses on allocative efficiency, rather than redistribution. Additionally, due to the nonlinearities associated with "hard" budget constraints, agents' types in the budget-constrained setting cannot be reduced to a unidimensional object in the way that our agents' types can. (Using the terminology of this paper: under budget constraints, agents' values for money can be seen as being equal to 1 if their budgets are not exceeded, and ∞ otherwise.) Nevertheless, the work on mechanism design under budget constraints points to instruments similar to those we identify here: rationing and (internal) cash transfers. And while the structure of the results under budget constraints is different, some of the core intuitions correspond. For example, Che, Gale, and Kim (2012), unlike us, concluded that the planner should *always* use rationing in the constrained-efficient mechanism—but this is because in the Che, Gale, and Kim (2012) setting, there are always agents for whom the budget constraints bind at the optimal mechanism; these agents effectively receive a higher shadow weight in the designer's objective, leading to a version of what we call high same-side inequality.

More broadly, we find that suitably designed market mechanisms (if we may stretch the term slightly beyond its standard usage) can themselves be used as redistributive tools. In this light, our work also has kinship with the growing literature within market design that shows how variants of the market mechanism can achieve fairness and other distributional goals in settings that (unlike ours) do not allow transfers (see, e.g., Budish (2011) and Prendergast (2020, forthcoming) for recent examples).

Redistributive Approaches from Public Finance

The nonuniform welfare weights approach we take is a classic idea in public finance (see, e.g., Diamond and Mirrlees (1971), Atkinson and Stiglitz (1976), Saez and Stantcheva (2016))—and as in public finance, we use this approach to study the equity–efficiency trade-off. Conceptually, however, we are asking a different and complementary question from much of public finance: we seek to understand the equity–efficiency trade-off in a relatively small individual market, taking inequality as given. Our designer cannot change agents' endowments directly—she can only structure the rules of trade; this is in contrast to most public finance models in which agents' incomes are determined endogenously and can be directly affected through the tax code.

Because of our market design approach, the redistribution question in our context is especially tractable, and the optimal mechanism can often be found in closed form—unlike in very general models such as that of Diamond and Mirrlees (1971), in which only certain properties of optimal mechanisms can be derived. More precisely, because goods in our setting are indivisible and agents have linear utility with unit demand, agents' behavior in our model is described by a "bang-bang" solution, rather than the first-order conditions that are used in much of modern public finance. This underlies the simple structure of our optimal mechanism because it limits the amount of information that the designer can infer about agents from their equilibrium behavior. As a consequence, we can assess how the structure of the optimal mechanism depends on the type and degree of inequality in the market.

Two other distinctions, albeit less central ones, are worth mentioning: First, our model includes a participation constraint for agents, which is absent from many public finance frameworks; this is especially relevant when we are designing an individual market because agents can always choose not to trade. Second, agent types in our model reflect only valuations for the good, rather than inputs to production such as productivity or ability.

Of course, public finance has already thought about whether optimal taxation should be supplemented by market-based rationing. Guesnerie and Roberts (1984), for instance, investigated the desirability of anonymous quotas (i.e., quantity control and subsequent rationing) when only linear taxation is feasible; they showed that when the social cost of a commodity is different from the price that consumers face, small quotas around the optimal consumption level can improve welfare.

For the case of labor markets specifically, Allen (1987), Guesnerie and Roberts (1987), and Boadway and Cuff (2001) have shown that with linear taxation, some form of minimum wage can be welfare-improving. Those authors mostly studied the efficiency of the minimum wage under the assumption of perfect competition. Cahuc and Laroque (2014), on the other hand, considered a monopolistic labor market in which firms set the wages and found that, for empirically relevant settings, minimum wages are not helpful.

Lee and Saez (2012), meanwhile, showed that minimum wages can be welfare-improving—even when they reduce employment on the extensive margin—so long as rationing is efficient, in the sense that those workers whose employment contributes the least to social surplus leave the labor market first. At the same time, Lee and Saez (2012) found that minimum wages are never optimal in their setting if rationing is uniform. As our analysis highlights, the latter conclusion derives in part from the fact that Lee and Saez (2012) looked only at a small first-order perturbation around the equilibrium wage. Our results show that when rationing becomes significant (as opposed to a small perturbation around the equilibrium), it influences the incentives of agents to sort into different trading behaviors (in our setting: no trade, choosing the rationing option, or trade at a competitive price). Thus, in our setting, endogenous sorting allows the planner to identify the poorest traders through their behavior.

Additionally, our results give some guidance as to when rationing is optimal: In the setting of Lee and Saez (2012), the inefficiency of rationing is second-order because Lee and Saez (2012) assumed efficient sorting; this is why rationing in the Lee and Saez (2012) model is always optimal. In our setting, we use uniform rationing, which creates a first-order inefficiency—which redistribution can only counterbalance when same-side inequality is high.

Moreover, unlike in our model, much of public finance operates in one-sided markets. An exception is the work of Scheuer (2014), who studied taxation in a two-sided market.[12] Agents in the Scheuer (2014) model have two-dimensional types: a baseline skill level and a taste for entrepreneurship; after the realization of their private types, agents can choose whether to be entrepreneurs or workers. In our setting, by contrast, buyers and sellers are identifiable ex ante, and their choice is whether to trade. Scheuer (2014) proved that the optimal tax schedules faced by workers and entrepreneurs are different; this resembles our finding that buyers and sellers may face different prices.

[12] Scheuer and Werning (2017) also studied taxation in a two-sided market, although their context is very different from ours: specifically, an assignment model in which firms decide how much labor to demand as a function of their productivity levels, and workers decide how much labor to provide depending on their ability. Scheuer and Werning (2017) were concerned with the taxation of superstars, and thus their model exhibits super-modularity in the assignment, which leads to assortative matching and makes the economics of the problem quite different from ours.

Despite the similarities just described, our work is substantively different from that of Scheuer (2014) both technically and conceptually. Perhaps most importantly, Scheuer (2014) explicitly ruled out bunching (rationing), which turns out to be a key feature of the optimal mechanism that we focus on. As a result, we have to develop different methods, and the Scheuer (2014) results do not extend to our setting.

2. FRAMEWORK

We study a two-sided buyer-seller market with inequality. There is a unit mass of *owners*, and a mass μ of *non-owners* in the market for a good K. All agents can hold at most one unit of K but can hold an arbitrary amount of money M. Owners possess one unit of good K; non-owners have no units of K. Because of the unit-supply/demand assumption, we refer to owners as *(prospective) sellers* (S), and to non-owners as *(prospective) buyers* (B).

Each agent has values v^K and v^M for units of K and M, respectively. If (x^K, x^M) denotes the holdings of K and M, then an agent with type (v^K, v^M) receives utility

$$v^K x^K + v^M x^M.$$

The pair (v^K, v^M) is distributed according to a joint distribution $F_S(v^K, v^M)$ for sellers, and $F_B(v^K, v^M)$ for buyers. The designer knows the distribution of (v^K, v^M) on both sides of the market, and can identify whether an agent is a buyer or a seller, but does not observe individual realizations of values.

The designer is utilitarian and aims to maximize the total expected utility from allocating both the good and money. The designer selects a trading mechanism that is "feasible," in the sense that it satisfies incentive-compatibility, individual-rationality, budget-balance, and market-clearing constraints. (We formalize the precise meaning of these terms in our context soon; we also impose additional constraints in Section 3, which we subsequently relax.)

We interpret the parameter v^M as representing the marginal utility that society (as reflected by the designer) attaches to giving an additional unit of money to a given agent. We refer to agents with high v^M as being "poor" (or "in need"). The interpretation is that such agents have a higher marginal utility of money because of their lower wealth or adverse social circumstances. Analogously, we refer to agents with low v^M as "rich" (or "wealthy"). Heterogeneity in v^M implies that utility is not fully transferable. Indeed, transferring one unit of M from an agent with $v^M = 2$ to an agent with $v^M = 5$ increases total welfare (the designer's objective) by 3. This is in contrast to how money is treated in standard mechanism design frameworks that assume fully transferable utility—when all agents value good M equally, the allocation of money is irrelevant for total welfare.

In a market in which good K can be exchanged for money, a parameter that fully describes the behavior of any individual agent is the *marginal rate of substitution* r between K and M, that is, $r = v^K/v^M$. Rescaling the utility of any agent does not alter his or her preferences: The behavior of an agent with values $(10, 1)$ does not differ from the behavior of an agent with values $(20, 2)$. As a consequence, by observing agents' behavior in the market, the designer can at most hope to infer agents' rates of substitution.[13] We denote by $G_j(r)$ the cumulative distribution function of the rate of substitution induced by the joint distribution $F_j(v^K, v^M)$, for $j \in \{B, S\}$. We let $\underline{r}_j$ and $\bar{r}_j$ denote the lowest and the highest

[13]This claim is non-obvious when arbitrary mechanisms are allowed—but in Section 4 we demonstrate that there is a formal sense in which the conclusion holds.

values of r in the support of G_j, respectively. Unless stated otherwise, we assume throughout that the equation $\mu(1-G_B(r)) = G_S(r)$ has a unique solution, implying existence and uniqueness of a competitive equilibrium with strictly positive volume of trade.

Even though the designer cannot learn v^K and v^M separately, the rate of substitution is informative about both parameters. In particular, fixing the value for the good v^K, a buyer with higher willingness to pay $r = v^K/v^M$ must have a lower value for money v^M; consequently, the correlation between r and v^M may naturally be negative. For example, under many distributions, a buyer with willingness to pay 10 is more likely to have a low v^M than a buyer with willingness to pay 5. In this case, our designer will value giving a unit of money to a buyer with rate of substitution 5 more than to a buyer with rate of substitution 10.

Formally, we observe that the designer's preferences depend on the rate of substitution r through two terms: the (normalized) utility, which we denote by $U_j(r)$, and the expected value for money conditional on r, which we denote by $\lambda_j(r)$, for $j \in \{B, S\}$. For example, the expected contribution of a buyer with allocation (x^K, x^M) to the designer's objective function can be written as

$$\begin{aligned}\mathbb{E}^B_{(v^K,v^M)}\left[v^K x^K + v^M x^M\right] &= \mathbb{E}^B_{(v^K,v^M)}\left[v^M\left[\frac{v^K}{v^M}x^K + x^M\right]\right]\\ &= \mathbb{E}^B_r\big[\underbrace{\mathbb{E}^B\left[v^M \mid r\right]}_{\lambda_B(r)}\underbrace{\left[rx^K + x^M\right]}_{U_B(r)}\big].\end{aligned} \tag{2.1}$$

Equality (2.1) allows us to reinterpret the problem as one where the designer maximizes a standard utilitarian welfare function with *Pareto weights* $\lambda_j(r)$ equal to the expected value for money conditional on a given rate of substitution r on side j of the market:

$$\lambda_j(r) = \mathbb{E}^j\left[v^M \mid \frac{v^K}{v^M} = r\right]$$

(see Appendix A.1 for further details); this highlights the difference between our model and the canonical mechanism design framework, which implicitly assumes that $\lambda_j(r)$ is constant in j and r. In both our setting and the classical framework, r determines the behavior of agents—but in our model, r also provides information that the designer can use to weight agents' utilities in the social objective.

3. SIMPLE MECHANISMS

In this section, we work through a simple application of our general framework, building intuitions and terminology that are useful for the full treatment we give in Section 4. In order to highlight the economic insights, in this section we impose two major simplifications: We assume that (1) the designer is limited to a simple class of mechanisms that only allows price controls and lump-sum transfers (in a way we formalize soon), and (2) the agents' rates of substitution are uniformly distributed.[14] We show in Section 4 that the simple mechanisms we focus on are in fact optimal among all mechanisms satisfying incentive-compatibility, individual-rationality, market-clearing, and budget-balance constraints. Moreover, all of the qualitative conclusions we draw in this section extend to

[14]Working with uniform distributions for now simplifies the analysis and allows us to deliver particularly sharp results.

general distributions as long as appropriate regularity conditions hold, as we show in Section 5.

Throughout this section, we assume that $\lambda_j(r)$ is continuous and non-increasing. The assumption that $\lambda_j(r)$ is non-increasing is of fundamental importance to our analysis: it captures the idea discussed earlier that the designer associates higher willingness to pay with lower *expected* value for money. The techniques we develop in Section 4 can be applied for any $\lambda_j(r)$; however, we focus on the non-increasing case because we view it to be the most important one from an economic perspective.[15]

3.1. *Measures of Inequality*

We begin by introducing two measures of inequality that are central to our analysis. For $j \in \{B, S\}$, we define

$$\Lambda_j := \mathbb{E}^j\big[v^M\big] \tag{3.1}$$

to be buyers' and sellers' *average values for money*.

DEFINITION 1: We say that there is *cross-side inequality* if buyers' and sellers' average values for money differ, that is, if $\Lambda_S \neq \Lambda_B$.

DEFINITION 2: We say that there is *same-side inequality on side* $j \in \{B, S\}$ (or just *j-side inequality*) if λ_j is not identically equal to Λ_j. Same-side inequality is *low* on side j if $\lambda_j(\underline{r}_j) \leq 2\Lambda_j$; it is *high* if $\lambda_j(\underline{r}_j) > 2\Lambda_j$.

Cross-side inequality allows us to capture the possibility that agents on one side of the market are on average poorer than agents on the other side of the market. Meanwhile, same-side inequality captures the dispersion in $\lambda_j(r)$, the conditional expected values for money within each side of the market. To see this, consider the sellers: Under the assumption that $\lambda_S(r)$ is non-increasing, a seller with the lowest rate of substitution $\underline{r}_S$ is the poorest seller that can be identified based on her behavior in the marketplace—that is, she has the highest conditional expected value for money. Seller-side inequality is low if the poorest-identifiable seller has a conditional expected value for money that does not exceed the average value for money by more than a factor of 2. The opposite case of high seller-side inequality implies that the poorest-identifiable seller has a conditional expected value for money that exceeds the average by more than a factor of 2. (The fact that, as we show in the sequel, the threshold of 2 delineates qualitatively different solutions to the optimal design problem may seem surprising; we provide intuition for why 2 is the relevant threshold in Appendix A.2.)

We emphasize that same-side inequality does *not* simply measure the dispersion in v^M. Rather, it measures the dispersion in the "best estimates" of v^M that the designer can form by observing r. Consequently, high dispersion in v^M is necessary but not sufficient for high same-side inequality. Same-side inequality additionally requires that the effects of dispersion in v^M are not dominated by differences in tastes that are also reflected in agents' rates of substitution. For a simple example, suppose that individuals A and B have willingness to pay for a bottle of milk of \$5 and \$0, respectively. Intuitively, we are more likely to conclude that individual B does not like milk than that she must be very poor. In

[15]Mathematically, this assumption is also fairly natural: Generating an increasing $\lambda_j(r)$ would require a very strong positive correlation between v^K and v^M.

contrast, suppose that individuals A and B are diagnosed with the same serious disease, and that A is willing to pay \$50,000 for treatment, while B is only willing to pay \$1000. In this case, it seems more plausible to infer that B's low willingness to pay is likely to be a consequence of an adverse socioeconomic situation, as reflected by a high v^M.

3.2. *Decomposition of the Design Problem*

In our model, the only interaction between the buyer and seller sides of the market arises from the facts that (1) the market has to clear, and (2) the designer must maintain budget-balance. Fixing both the quantity traded Q and the revenue R, our problem therefore decomposes into two one-sided design problems. To highlight our key intuitions, we thus solve the design problem in three steps:

1. **Optimality on the seller side**—We identify the optimal mechanism that acquires Q objects from sellers while spending at most R (for any Q and R).
2. **Optimality on the buyer side**—We identify the optimal mechanism that allocates Q objects to buyers while raising at least R in revenue (again, for any Q and R).
3. **Cross-side optimality**—We identify the optimal market-clearing mechanism by linking our characterizations of seller- and buyer-side solutions through the optimal choices of Q and R.

In Appendix B.9, we show how the results in this section follow as special cases of the more general results we establish in Sections 4 and 5.

3.3. *Single-Price Mechanisms*

At first, we allow the designer to choose only a single price p_j for each side of the market.[16] A given price determines supply and demand—and if there is excess supply or demand, then prospective traders are rationed uniformly at random until the market clears (reflecting the designer's inability to observe the traders' values directly). Moreover, the price has to be chosen in such a way that the designer does not need to subsidize the mechanism; if there is monetary surplus, that surplus is redistributed as a lump-sum transfer.

One familiar example of a single-price mechanism is the *competitive* mechanism, which, for a fixed quantity Q, is defined by setting the price p_j^{C} that clears the market:

$$G_S(p_S^{\mathsf{C}}) = Q \quad \text{or} \quad \mu(1 - G_B(p_B^{\mathsf{C}})) = Q$$

for sellers and buyers, respectively. Here, the word "competitive" refers to the fact that the ex post allocation is determined entirely by agents' choices based on their individual rates of substitution. In contrast, a "rationing" mechanism allocates the object with interior probability to some agents, with the ex post allocation determined partially by a lottery.

In a two-sided market, the *competitive-equilibrium* mechanism is defined by a single price p^{CE} that clears both sides of the market at the same (equilibrium) quantity:

$$G_S(p^{\mathsf{CE}}) = \mu(1 - G_B(p^{\mathsf{CE}})).$$

The competitive-equilibrium mechanism is always feasible; moreover, it is optimal when $\lambda_j(r)$ is constant in r and j, that is, when the designer does not have redistributive preferences.[17]

[16]Here and hereafter, when we refer to a "price," we mean a payment conditional on selling or obtaining the good, net of any lump-sum payment or transfer.

[17]As explained in Section 1.1, this follows from the first welfare theorem.

Optimality on the Seller Side

We first solve the seller-side problem, determining the designer's optimal mechanism for acquiring Q objects while spending at most R. We assume that $QG_S^{-1}(Q) \leq R$, as otherwise there is no feasible mechanism.

We note first that the designer cannot post a price below $G_S^{-1}(Q)$ as then there would not be enough sellers willing to sell to achieve the quantity target Q. However, the designer can post a higher price and ration with probability $Q/G_S(p_S)$. Thus, any seller willing to sell at p_S gains utility $p_S - r$ (normalized to units of money) with probability $Q/G_S(p_S)$. Because each unit of money given to a seller with rate of substitution r is worth $\lambda_S(r)$ in terms of social welfare, the net contribution of such a seller to welfare is $(Q/G_S(p_S))\lambda_S(r)(p_S - r)$. Finally, with a price p_S, buying Q units costs $p_S Q$; if this cost is strictly less than R, then the surplus can be redistributed as a lump-sum payment to all sellers. Since all sellers share lump-sum transfers equally, the marginal social surplus contribution of each unit of money allocated through lump-sum transfers is equal to the average value for money on the seller side, Λ_S. Summarizing, the designer solves

$$\max_{p_S \in [G_S^{-1}(Q), R/Q]} \left\{ \frac{Q}{G_S(p_S)} \int_{\underline{r}_S}^{p_S} \lambda_S(r)(p_S - r)\, dG_S(r) + \Lambda_S(R - p_S Q) \right\}. \tag{3.2}$$

Uniform rationing has three direct consequences for social welfare: (i) allocative efficiency is reduced; (ii) the mechanism uses more money to purchase the objects from sellers, leaving a smaller amount, $R - p_S Q$, to be redistributed as a lump-sum transfer; and (iii) those sellers who trade receive a higher price. From the perspective of welfare, the first two effects are negative and the third one is positive; the following result describes the optimal resolution of this trade-off.

PROPOSITION 1: *When seller-side inequality is low, it is optimal to choose $p_S = p_S^{\mathrm{C}}$ (i.e., the competitive mechanism is optimal). When seller-side inequality is high, there exists a non-decreasing function $\bar{Q}(R) \in [0, 1)$ (strictly positive for high enough R) such that rationing at a price $p_S > p_S^{\mathrm{C}}$ is optimal if and only if $Q \in (0, \bar{Q}(R))$. Setting $p_S = p_S^{\mathrm{C}}$ (i.e., using the competitive mechanism) is optimal otherwise.*

Proposition 1 shows that when the designer is constrained to use a single price, competitive pricing is optimal (on the seller side) whenever seller-side inequality is low; meanwhile, under high seller-side inequality, rationing at a price above market-clearing becomes optimal when the quantity to be acquired is sufficiently low. As we show in Section 4, the simple mechanism described in Proposition 1 is in fact optimal among all incentive-compatible, individually-rational, budget-balanced, market-clearing mechanisms.

The key intuition behind Proposition 1 is that the decision to trade always identifies sellers with low rates of substitution: at any given price, sellers with low rates of substitution are weakly more willing to trade. By our assumption that $\lambda_S(r)$ is non-increasing, we know that sellers with low rates of substitution are the poorest sellers that can be identified based on market behavior. Consequently, the trade-off between the effects (ii)—reducing lump-sum transfers—and (iii)—giving more money to sellers who trade—is always resolved in favor of effect (iii): By taking a dollar from the average seller, the designer decreases surplus by the *average* value for money Λ_S, while giving a dollar to a seller who wants to sell at price p increases surplus by $\mathbb{E}^S[\lambda_S(r) \mid r \leq p] \geq \Lambda_S$ in expectation. However, to justify rationing, the net redistributive effect has to be stronger than the negative effect (i) on allocative efficiency. When same-side inequality is low, the conditional value for money $\mathbb{E}^S[\lambda_S(r) \mid r \leq p]$ is not much higher than the average value Λ_S, even at low prices, so the net redistributive effect is weak. Thus, the negative effect of (i) dominates, meaning that the competitive price is optimal. When same-side inequality is high, however, the redistributive benefit from rationing can dominate the cost of allocative inefficiency; Proposition 1 states that this happens precisely when the volume of trade is sufficiently low. The intuition for why the optimal mechanism depends on the quantity of goods acquired is straightforward: When the volume of trade is low, only the sellers with the lowest rates of substitution sell—therefore, market selection is highly effective at targeting transfers to the agents who are most likely to be poor. In contrast, when the volume of trade is high, the decision to trade is relatively uninformative about sellers' conditional values for money, weakening the net redistributive effect.

The threshold value of the volume of trade $\bar{Q}(R)$ depends on the revenue target R—when the budget constraint is binding, there is an additional force pushing towards the competitive price because that price minimizes the cost of acquiring the target quantity Q. It is easy to show that $\bar{Q}(R) > 0$ for any R that leads to strictly positive lump-sum transfers (i.e., when the budget constraint is slack). At the same time, it is never optimal to ration when Q approaches 1 because if nearly all sellers sell, then the market does not identify which sellers are poorer in expectation.

Optimality on the Buyer Side

We now turn to the buyer-side problem, normalizing $\mu = 1$ for this subsection as μ plays no role in the buyer-side optimality analysis. We assume that $QG_B^{-1}(1-Q) \geq R$, as otherwise there is no mechanism that allocates Q objects to buyers while raising at least R in revenue.

Similarly to our analysis on the seller side, we see that the designer cannot post a price above $G_B^{-1}(1-Q)$, as otherwise there would not be enough buyers willing to purchase to achieve the quantity target Q. However, the designer can post a lower price and ration with probability $Q/(1-G_B(p_B))$. Moreover, if the mechanism generates revenue strictly above R, the surplus can be redistributed to all buyers as a lump-sum transfer. Thus, analogously to (3.2), the designer solves

$$\max_{p_B \in [R/Q, G_B^{-1}(1-Q)]} \left\{ \frac{Q}{1-G_B(p_B)} \int_{p_B}^{\bar{r}_B} \lambda_B(r)(r-p_B)\, dG_B(r) + \Lambda_B(p_B Q - R) \right\}.$$

Uniform rationing by setting $p_B < p_B^{\mathsf{C}}$ has three direct consequences: (i) allocative efficiency is reduced, (ii) the mechanism raises less revenue, resulting in a smaller amount $(p_B Q - R)$ being redistributed as a lump-sum transfer, and (iii) the buyers that end up purchasing the good each pay a lower price. Like with the seller side of the market, the first two effects are negative and the third one is positive. Yet the optimal trade-off is resolved differently, as our next result shows.

PROPOSITION 2: *Regardless of buyer-side inequality, it is optimal to set* $p_B = p_B^{\mathsf{C}}$; *that is, the competitive mechanism is always optimal.*

Proposition 2 shows that it is never optimal to ration the buyers at a single price below the market-clearing level—standing in sharp contrast to Proposition 1, which shows that rationing the sellers at a price above market-clearing can sometimes be optimal.

The economic forces behind Propositions 1 and 2 highlight a fundamental asymmetry between buyers and sellers with respect to the redistributive power of the market: Whereas willingness to sell at any given price identifies sellers that have low rates of substitution and hence are poor in expectation, the buyers who buy at any given price are those that have *higher* rates of substitution and are hence relatively *rich* in expectation (recall that $\lambda_B(r)$ is non-increasing). Effects (ii) and (iii) on the buyer side thus result in taking a dollar from an average buyer with value for money Λ_B and giving it to a buyer (in the form of a price discount) with a conditional expected value for money $\mathbb{E}^B[v^M \mid r \geq p] \leq \Lambda_B$. Thus, even ignoring the allocative inefficiency channel (i), under a single price the net redistributive effect of (ii) and (iii) decreases surplus.

3.4. *Two-Price Mechanisms*

We now extend the analysis of Section 3.3 by allowing the designer to introduce a second price on each side of the market. The idea is that the designer may offer a "competitive" price at which trade is guaranteed, and a "non-competitive" price that is more attractive (higher for sellers; lower for buyers) but induces rationing. Individuals self-select by choosing one of the options, enabling the designer to screen the types of the agents more finely than with a single price. For example, on the buyer side, the lowest-r buyers will not trade, medium-r buyers will select the rationing option, and highest-r buyers will prefer to trade for sure at the "competitive" price.

Optimality on the Seller Side

As we noted in the discussion of Proposition 1, the simple single-price mechanism for sellers is in fact optimal among all feasible mechanisms. Thus, the designer cannot benefit from introducing a second price for sellers—at least under the uniform distribution assumption we have made in this section.[18]

Optimality on the Buyer Side

As we saw in discussing Proposition 2, at any single price, the buyers who trade have a lower expected value for money than the buyer population average Λ_B; hence, lowering a (single) price redistributes money to a subset of buyers with lower contribution to social welfare. However, if the designer has the option to introduce a second price, she can potentially screen the buyers more finely. Suppose that the buyers can choose to trade at p_B^H with probability 1 or at p_B^L with probability $\delta < 1$ (thus being rationed with probability $1-\delta$). Then, buyers with willingness to pay above p_B^L but below $r_\delta := (p_B^H - \delta p_B^L)/(1-\delta)$ choose the rationing option while buyers with higher willingness to pay (above r_δ) choose the "competitive-price" option. Volume of trade is $1 - \delta G_B(p_B^L) - (1-\delta)G_B(r_\delta)$ and revenue is $p_B^L \delta(G_B(r_\delta) - G_B(p_B^L)) + p_B^H(1 - G_B(r_\delta))$. Thus, to compute the optimal p_B^H, p_B^L, and δ, the designer solves

$$\max_{p_B^H \geq p_B^L, \delta} \left\{ \begin{array}{l} \delta \displaystyle\int_{p_B^L}^{r_\delta} \lambda_B(r)\big(r - p_B^L\big)\, dG_B(r) + \int_{r_\delta}^{\bar{r}_B} \lambda_B(r)\big(r - p_B^H\big)\, dG_B(r) \\ \quad + \Lambda_B\big(p_B^L \delta\big(G_B(r_\delta) - G_B\big(p_B^L\big)\big) + p_B^H\big(1 - G_B(r_\delta)\big) - R\big) \end{array} \right\}$$

[18] In Section 4, we extend the results to a general setting and show that a second price may be optimal on the seller side for some distributions—nevertheless, the intuitions and conditions for optimality of rationing remain the same.

subject to the market-clearing and revenue-target constraints

$$1 - \delta G_B(p_B^L) - (1-\delta)G_B(r_\delta) = Q,$$
$$p_B^L \delta\big(G_B(r_\delta) - G_B(p_B^L)\big) + p_B^H\big(1 - G_B(r_\delta)\big) \geq R.$$

We say that there is *rationing at the lower price* p_B^L if $\delta < 1$ and $G_B(r_\delta) > G_B(p_B^L)$, that is, if a non-zero measure of buyers choose the lottery. With this richer class of mechanisms, we obtain the following result.

PROPOSITION 3: *When buyer-side inequality is low, it is optimal not to offer the low price* p_B^L *and to choose* $p_B^H = p_B^{\mathsf{C}}$ (*i.e., the competitive mechanism is optimal*). *When buyer-side inequality is high, there exists a non-increasing function* $\underline{Q}(R) \in (0, 1]$, *strictly below* 1 *for low enough* R, *such that rationing at the low price is optimal if and only if* $Q \in (\underline{Q}(R), 1)$. *Setting* $p_B^H = p_B^{\mathsf{C}}$ (*and not offering the low price* p_B^L) *is optimal for* $Q \leq \underline{Q}(R)$.

We show in Section 4 that the mechanism described in Proposition 3 is in fact optimal among all incentive-compatible, individually-rational, budget-balanced, market-clearing mechanisms.

The result of Proposition 3 relies on the fact that the decision to choose the rationing option identifies buyers who are poorer in expectation, as long as p_B^H, p_B^L, and δ are chosen so that

$$\mathbb{E}^B\big[v^M \mid r \in [p_B^L, r_\delta]\big] > \Lambda_B,$$

making the net redistributive effect positive. However, this effect is strong enough to overcome allocative inefficiency only if inequality is substantial and sufficiently many (rich-in-expectation) buyers choose the high price; a large volume of trade ensures this because it implies that most buyers choose to buy for sure (i.e., r_δ is relatively small). In such cases, our mechanism optimally redistributes by giving a price discount to buyers with higher-than-average value for money.

The revenue target R influences the threshold volume of trade $\underline{Q}(R)$ above which rationing becomes optimal: If the designer needs to raise a lot of revenue, then rationing becomes less attractive. The threshold $\underline{Q}(R)$ is strictly below 1 whenever the optimal mechanism gives a strictly positive lump-sum transfer. Even so, $\underline{Q}(R)$ is never equal to 0—when almost no agent buys, those who do buy must be relatively rich in expectation, and thus rationing would (suboptimally) redistribute to wealthier buyers.

3.5. *Cross-Side Optimality*

Having found the optimal mechanisms for buyers and sellers separately under fixed Q and R, we now derive the optimal mechanism with Q and R determined endogenously.

PROPOSITION 4: *When same-side inequality is low on both sides of the market, it is optimal to set* $p_B \geq p_S$ *such that the market clears, that is,* $G_S(p_S) = \mu(1 - G_B(p_B))$, *and redistribute the resulting revenue as a lump-sum payment to the side of the market* $j \in \{B, S\}$ *that has higher average value for money* Λ_j.

When same-side inequality is low, rationing on either side is suboptimal for any volume of trade and any revenue target (Propositions 1 and 3); hence, rationing is also suboptimal in the two-sided market. However, in order to address cross-side inequality, the mechanism may introduce a tax-like wedge between the buyer and seller prices in order to raise revenue that can be redistributed to the poorer side of the market. Intuitively, the size of the wedge (and hence the size of the lump-sum transfer) depends on the degree of cross-side inequality. For example, when there is no same-side inequality and $\Lambda_S \geq \Lambda_B$, prices satisfy

$$p_B - p_S = \left(\frac{\Lambda_S - \Lambda_B}{\Lambda_S}\right)\frac{1 - G_B(p_B)}{g_B(p_B)}. \tag{3.3}$$

Now, we suppose instead that there is high seller-side inequality. We know from Proposition 1 that rationing the sellers becomes optimal when the volume of trade is low. A sufficient condition for low volume of trade is that there are few buyers relative to sellers; in this case, rationing the sellers becomes optimal in the two-sided market.

PROPOSITION 5: *When seller-side inequality is high and $\Lambda_S \geq \Lambda_B$, if μ is low enough, then it is optimal to ration the sellers by setting a single price above the competitive-equilibrium level.*

The assumption $\Lambda_S \geq \Lambda_B$ in Proposition 5 is important: If instead buyers were poorer than sellers on average, then the optimal mechanism might prioritize giving a lump-sum payment to buyers over redistributing among sellers. In that case, the optimal mechanism would minimize expenditures on the seller side—and as posting a competitive price is the least expensive way to acquire a given quantity Q, rationing could be suboptimal.

As we saw in Proposition 3, rationing the buyers in the one-sided problem can be optimal if the designer introduces both a high price at which buyers can buy for sure and a discounted price at which buyers are rationed—however, for rationing to be optimal, we also require a high volume of trade. As it turns out, there are two-sided markets in which the optimal volume of trade is always relatively low, so that buyer rationing is suboptimal even under severe imbalance between the sizes of the two sides of the market—in contrast to Proposition 5.

PROPOSITION 6: *If seller-side inequality is low and $\underline{r}_B = 0$, then the optimal mechanism does not ration the buyers.*

To understand Proposition 6, suppose for now that the designer does not take seller welfare into account and has access to some supply of the good. Then, by Proposition 3, for rationing to be optimal on the buyer side, it must be that there is high buyer-side inequality and the volume of trade is above a threshold; however, we show that when $\underline{r}_B = 0$, the *optimal* volume of trade is always lower than that threshold. For intuition, note that under $\underline{r}_B = 0$ and high volume of trade, revenue from the buyer side must be low. As a result, under rationing, buyers with low willingness to pay r (equivalently, with high expected value for money) are more likely to receive the good, but at the same time they receive small or no lump-sum transfers. Yet for buyers with r close to $\underline{r}_B = 0$, money is far more valuable than the good. Thus, it is better to raise the price and limit the volume of trade—this increases revenue, thereby increasing the lump-sum transfer. In fact, under high buyer-side inequality and $\underline{r}_B = 0$, it is optimal to choose a relatively low volume of trade (below the level required for rationing to become optimal) even if the designer could acquire the goods at zero cost.

When the supply of the good is determined optimally in a two-sided market by also accounting for seller surplus, the preceding reasoning is reinforced as long as seller-side inequality is low. Indeed, we know that in this case the designer will not distort the market-clearing rule on the seller side but may use a lump-sum transfer. If $\Lambda_S > \Lambda_B$, so that the lump-sum transfer is given to sellers, the value of raising revenue is even higher, which could only further decrease the optimal volume of trade.

The reasoning just described is still valid when $\underline{r}_B$ is above 0 but not too large. However, rationing the buyers in the two-sided market may be optimal when all buyers' willingness to pay is high, as formalized in the following result.

PROPOSITION 7: *If there is high buyer-side inequality*, $\Lambda_B \geq \Lambda_S$, *and*

$$\underline{r}_B - \bar{r}_S \geq \frac{1}{2}(\bar{r}_B - \underline{r}_S), \tag{3.4}$$

then there is some $\epsilon > 0$ *such that it is optimal to ration the buyers for any* $\mu \in (1, 1+\epsilon)$.

The condition (3.4) in Proposition 7 is restrictive: It requires that the lower bound on buyers' willingness to pay is high relative to sellers' rates of substitution *and* relative to the highest willingness to pay on the buyer side. To understand the role of that condition, recall that, by Proposition 3, a necessary and sufficient condition for buyer rationing in the presence of high buyer-side inequality is that a sufficiently large share of buyers trade.[19] The condition (3.4) ensures that the optimal mechanism maximizes volume of trade because (i) there are large gains from trade between any buyer and any seller ($\underline{r}_B$ is larger than $\bar{r}_S$), and (ii) it is suboptimal to limit supply to raise revenue ($\underline{r}_B$ is large relative to $\bar{r}_B$). When $\mu \in (1, 1+\epsilon)$ (there are slightly more potential buyers than sellers), maximal volume of trade means that almost all buyers buy, and hence rationing becomes optimal.

4. OPTIMAL MECHANISMS—THE GENERAL CASE

In this section, we show how the insights we obtained in Section 3 extend to our general model. We demonstrate that even when the designer has access to arbitrary (and potentially complex) mechanisms, there is an optimal mechanism that is quite simple, with only a few trading options available to market participants. Then, in Section 5, we show that our results about optimal market design under inequality continue to hold for general distributions of rates of substitution.

We assume that the designer can choose any trading mechanism subject only to four natural constraints: (1) incentive-compatibility (the designer does not observe individuals' values), (2) individual-rationality (each agent weakly prefers the outcome of the mechanism to the status quo), (3) market-clearing (the volume of goods sold is equal to the volume of goods bought), and (4) budget-balance (the designer cannot subsidize the mechanism).

By the Revelation Principle, it is without loss of generality to look at direct mechanisms in which agents report their values and are incentivized to do so truthfully. This leads us to the following formal definition of a feasible mechanism.

DEFINITION 3: A *feasible mechanism* (X_B, X_S, T_B, T_S) consists of $X_j : [\underline{v}_j^K, \bar{v}_j^K] \times [\underline{v}_j^M, \bar{v}_j^M] \to [0, 1]$ and $T_j : [\underline{v}_j^K, \bar{v}_j^K] \times [\underline{v}_j^M, \bar{v}_j^M] \to \mathbb{R}$ for $j \in \{B, S\}$ that satisfy the following

[19]The budget constraint is slack in this case because the assumption $\underline{r}_B > \bar{r}_S$ guarantees that any mechanism yields strictly positive revenue.

conditions for all types (v^K, v^M) and potential false reports $(\hat{v}^K, \hat{v}^M)$:

$$X_B(v^K, v^M)v^K - T_B(v^K, v^M)v^M \geq X_B(\hat{v}^K, \hat{v}^M)v^K - T_B(\hat{v}^K, \hat{v}^M)v^M, \tag{IC-B}$$

$$-X_S(v^K, v^M)v^K + T_S(v^K, v^M)v^M \geq -X_S(\hat{v}^K, \hat{v}^M)v^K + T_S(\hat{v}^K, \hat{v}^M)v^M, \tag{IC-S}$$

$$X_B(v^K, v^M)v^K - T_B(v^K, v^M)v^M \geq 0, \tag{IR-B}$$

$$-X_S(v^K, v^M)v^K + T_S(v^K, v^M)v^M \geq 0, \tag{IR-S}$$

$$\int_{\underline{v}_B^K}^{\bar{v}_B^K} \int_{\underline{v}_B^M}^{\bar{v}_B^M} X_B(v^K, v^M)\mu \, dF_B(v^K, v^M) = \int_{\underline{v}_S^K}^{\bar{v}_S^K} \int_{\underline{v}_S^M}^{\bar{v}_S^M} X_S(v^K, v^M) \, dF_S(v^K, v^M), \tag{MC}$$

$$\int_{\underline{v}_B^K}^{\bar{v}_B^K} \int_{\underline{v}_B^M}^{\bar{v}_B^M} T_B(v^K, v^M)\mu \, dF_B(v^K, v^M) \geq \int_{\underline{v}_S^K}^{\bar{v}_S^K} \int_{\underline{v}_S^M}^{\bar{v}_S^M} T_S(v^K, v^M) \, dF_S(v^K, v^M). \tag{BB}$$

We can now formally define optimal mechanisms.

DEFINITION 4: A feasible mechanism (X_B, X_S, T_B, T_S) is *optimal* if it maximizes the *total value*

$$\begin{aligned} \mathrm{TV} := {} & \int_{\underline{v}_B^K}^{\bar{v}_B^K} \int_{\underline{v}_B^M}^{\bar{v}_B^M} \big[X_B(v^K, v^M)v^K - T_B(v^K, v^M)v^M\big]\mu \, dF_B(v^K, v^M) \\ & + \int_{\underline{v}_S^K}^{\bar{v}_S^K} \int_{\underline{v}_S^M}^{\bar{v}_S^M} \big[-X_S(v^K, v^M)v^K + T_S(v^K, v^M)v^M\big] \, dF_S(v^K, v^M) \end{aligned} \tag{VAL}$$

among all feasible mechanisms.

In our model, in general, direct mechanisms should allow agents to report their two-dimensional types, as in Definition 3. However, as we foreshadowed in Section 3, and as we show formally in Appendix A.1, it is without loss of generality to assume that agents only report their rates of substitution. Intuitively, reporting rates of substitution suffices because those rates fully describe individual agents' preferences. (The mechanism could elicit information about both values by making agents indifferent between reports—but we show that this can only happen for a measure-zero set of types, and thus cannot raise the surplus achieved by the optimal mechanism.) Abusing notation slightly, we write $X_j(v^K/v^M)$ for the probability that an agent of type (v^K, v^M) trades object K, and $T_j(v^K/v^M)$ for the net change in the holdings of money. Moreover, again following the same reasoning as in Section 3, we can simplify the designer's objective function to

$$\begin{aligned} \mathrm{TV} = {} & \int_{\underline{r}_B}^{\bar{r}_B} \lambda_B(r)\big[X_B(r)r - T_B(r)\big]\mu \, dG_B(r) \\ & + \int_{\underline{r}_S}^{\bar{r}_S} \lambda_S(r)\big[-X_S(r)r + T_S(r)\big] \, dG_S(r), \end{aligned} \tag{VAL'}$$

where, as in Section 2, G_j is the induced distribution of the rate of substitution $r = v^K/v^M$, and $\lambda_j(r)$ is the expectation of the value for money conditional on the rate of substitution r,

$$\lambda_j(r) = \mathbb{E}^j\left[v^M \;\middle|\; \frac{v^K}{v^M} = r\right].$$

We assume that $G_j(r)$ admits a density $g_j(r)$ fully supported on $[\underline{r}_j, \bar{r}_j]$.

4.1. *Derivation of Optimal Mechanisms*

We now present and prove our main technical result. While a mechanism in our setting can involve offering a menu of prices and quantities (i.e., transaction probabilities) for each rate of substitution r, we nevertheless find that there is always an optimal mechanism with a relatively simple form.

To state our theorem, we introduce some terminology that relates properties of direct mechanisms to more intuitive properties of their indirect implementations: If an allocation rule $X_j(r)$ takes the form $X_B(r) = \mathbf{1}_{\{r \geq p\}}$ for buyers or $X_S(r) = \mathbf{1}_{\{r \leq p\}}$ for sellers (for some p), then we call the corresponding mechanism a *competitive mechanism*, reflecting the idea that the (ex post) allocation in the market depends only on agents' behavior. An alternative to a competitive mechanism is a *rationing mechanism* that (at least sometimes) resorts to randomization to determine the final allocation: We say that side j of the market is *rationed* if $X_j(r) \in (0, 1)$ for a non-zero measure set of types r. Rationing for type r can always be implemented by setting a price p that is acceptable to r and then excluding r from trading with some probability. If $n = |\operatorname{Im}(X_j) \setminus \{0, 1\}|$, then we say that the mechanism offers n (distinct) *rationing options* to side j of the market; then,

$$\left|\operatorname{Im}(X_B) \setminus \{0, 1\}\right| + \left|\operatorname{Im}(X_S) \setminus \{0, 1\}\right|$$

is the total number of rationing options offered in the market.[20] Finally, fixing a mechanism (X_B, X_S, T_B, T_S), we let $\underline{U}_j$ be the minimum utility among all types on side j on the market, expressed in units of money. Then, if $\underline{U}_j > 0$, we say that the mechanism *gives a lump-sum payment* to side j—the interpretation is that all agents on side j of the market receive a (positive) monetary lump-sum transfer of $\underline{U}_j$.[21]

THEOREM 1: *Either*:

- *there exists an optimal mechanism that offers at most two rationing options in total and does not give a lump-sum payment to either side* (*i.e.*, $\underline{U}_S = \underline{U}_B = 0$), *or*
- *there exists an optimal mechanism that offers at most one rationing option in total and gives a lump-sum payment to the side of the market that has a higher average value for money* (*i.e.*, *the side* j *with higher* Λ_j).

Theorem 1 narrows down the set of candidate solutions to a class of mechanisms indexed by eight parameters: four prices, two rationing probabilities, and a pair of lump-sum payments.[22] In particular, Theorem 1 implies that optimal redistribution can always be achieved by the use of lump-sum transfers and rationing. Moreover, if lump-sum redistribution is used, then rationing takes a particularly simple form: it is only used on one side of the market, and consists of offering a single rationing option. When lump-sum

[20]Here, $\operatorname{Im}(X_j)$ denotes the image of the function X_j, and $|A|$ denotes the cardinality of the set A.

[21]For this interpretation to be valid, we implicitly assume that prices belong to the range $[\underline{r}_j, \bar{r}_j]$ (which is without loss of generality). For example, if buyers' willingness to pay lies in $[1, 2]$ and the price in the market is 0 with all buyers trading, then $\underline{U}_B = 1$; we can equivalently set the price to 1 and think of buyers as receiving a lump-sum transfer of 1 each.

[22]The mechanism is effectively characterized by five parameters, as lump-sum payments are pinned down by a binding budget-balance condition and the property that one of the lump-sum payments is 0. Also, the market-clearing condition for good K reduces the degrees of freedom on prices by 1.

redistribution is not used, rationing could take a more complicated form, with either a single rationing option on each side of the market, or a competitive mechanism on one side, and two rationing options on the other side.

Except for the case in which two rationing options (and hence three prices) may be needed on one side of the market (which we can rule out with certain regularity conditions that we explore later), the simple two-price mechanism considered in Section 3 is sufficient to achieve the fully optimal market design under arbitrary forms of inequality.

4.2. *Proof of Theorem 1*

In this section, we explain the proof of Theorem 1, while relegating a number of details to Appendix B.

First, we simplify the problem by applying the canonical method developed by Myerson (1981), allowing us to express feasibility of the mechanism solely through the properties of the allocation rule and the transfer received by the worst type (the now-standard proof is omitted).

CLAIM 1: *A mechanism (X_B, X_S, T_B, T_S) is feasible if and only if*

$$X_B(r) \text{ is non-decreasing in } r, \tag{B-Mon}$$

$$X_S(r) \text{ is non-increasing in } r, \tag{S-Mon}$$

$$\int_{\underline{r}_B}^{\bar{r}_B} X_B(r)\mu\, dG_B(r) = \int_{\underline{r}_S}^{\bar{r}_S} X_S(r)\, dG_S(r), \tag{MC'}$$

$$\int_{\underline{r}_B}^{\bar{r}_B} J_B(r)X_B(r)\mu\, dG_B(r) - \mu\underline{U}_B \geq \int_{\underline{r}_S}^{\bar{r}_S} J_S(r)X_S(r)\, dG_S(r) + \underline{U}_S, \tag{BB'}$$

where $J_B(r) := r - \frac{1-G_B(r)}{g_B(r)}$ and $J_S(r) := r + \frac{G_S(r)}{g_S(r)}$ denote the virtual surplus functions, and $\underline{U}_B, \underline{U}_S \geq 0$.

Second, using the formulae in Claim 1 and integrating by parts, we can show that the objective function (VAL') also only depends on the allocation rule:

$$\mathrm{TV} = \mu\Lambda_B\underline{U}_B + \int \Pi_B^\Lambda(r)X_B(r)\mu\, dG_B(r) + \Lambda_S\underline{U}_S + \int \Pi_S^\Lambda(r)X_S(r)\, dG_S(r), \tag{OBJ'}$$

where

$$\Pi_B^\Lambda(r) := \frac{\displaystyle\int_r^{\bar{r}_B} \lambda_B(r)\, dG_B(r)}{g_B(r)}, \tag{4.1}$$

$$\Pi_S^\Lambda(r) := \frac{\displaystyle\int_{\underline{r}_S}^{r} \lambda_S(r)\, dG_S(r)}{g_S(r)}. \tag{4.2}$$

We refer to Π_j^Λ as the *inequality-weighted information rents* of side j. In the special case of transferable utility, that is, when $\lambda_j(r) = 1$ for all r, the Π_j^Λ reduce to the usual information rent terms, that is, $G_S(r)/g_S(r)$ for sellers, and $(1 - G_B(r))/g_B(r)$ for buyers.

Third, finding the optimal mechanism is hindered by the fact that the monotonicity constraints (B-Mon) and (S-Mon) may bind ("ironing" may be necessary, as shown by Myerson (1981)); in such cases, it is difficult to employ optimal control techniques. We instead represent allocation rules as mixtures over quantities; this allows us to optimize in the space of distributions and make use of the concavification approach.[23] Because G_S has full support (it is strictly increasing), we can represent any non-increasing, right-continuous function $X_S(r)$ as $X_S(r) = \int_0^1 \mathbf{1}_{\{r \le G_S^{-1}(q)\}}\, dH_S(q)$, where H_S is a distribution on $[0, 1]$. Similarly, we can represent any non-decreasing, right-continuous function $X_B(r)$ as $X_B(r) = \int_0^1 \mathbf{1}_{\{r \ge G_B^{-1}(1-q)\}}\, dH_B(q)$. Economically, our representation means that we can express a feasible mechanism in the quantile (i.e., quantity) space. To buy quantity q from the sellers, the designer has to offer a price of $G_S^{-1}(q)$ because then exactly sellers with $r \le G_S^{-1}(q)$ sell. An appropriate randomization over quantities (equivalently, prices) will replicate an arbitrary feasible quantity schedule X_S. Similarly, to sell quantity q to buyers, the designer has to offer a price $G_B^{-1}(1-q)$, at which exactly buyers with $r \ge G_B^{-1}(1-q)$ buy. Thus, it is without loss of generality to optimize over H_S and H_B rather than X_S and X_B in (OBJ').[24]

Fourth, we arrive at an equivalent formulation of the problem: Maximizing

$$\mu \int_0^1 \left(\int_{G_B^{-1}(1-q)}^{\bar{r}_B} \Pi_B^{\Lambda}(r)\, dG_B(r) \right) dH_B(q) + \int_0^1 \left(\int_{\underline{r}_S}^{G_S^{-1}(q)} \Pi_S^{\Lambda}(r)\, dG_S(r) \right) dH_S(q) + \mu \Lambda_B \underline{U}_B + \Lambda_S \underline{U}_S \tag{4.3}$$

over $H_S, H_B \in \Delta([0, 1])$, $\underline{U}_B, \underline{U}_S \ge 0$, subject to the constraints that

$$\mu \int_0^1 q\, dH_B(q) = \int_0^1 q\, dH_S(q), \tag{4.4}$$

$$\mu \int_0^1 \left(\int_{G_B^{-1}(1-q)}^{\bar{r}_B} J_B(r)\, dG_B(r) \right) dH_B(q) - \mu \underline{U}_B \ge \int_0^1 \left(\int_{\underline{r}_S}^{G_S^{-1}(q)} J_S(r)\, dG_S(r) \right) dH_S(q) + \underline{U}_S. \tag{4.5}$$

Crucially, with the lottery representation of the mechanism, the market-clearing condition (MC') states that the expected quantity must be the same under the buyer- and the seller-side lotteries (4.4), and that the objective function is an expectation of a certain function of the realized quantity with respect to the pair of lotteries.

[23] Myerson (1981) also used a concavification argument in his ironing procedure; the derivation we use can be seen as an adaption of his technique to our setting.

[24] Formally, considering all distributions H_B and H_S is equivalent to considering all feasible right-continuous X_B and X_S. The optimal schedules can be assumed right-continuous because a monotone function can be made continuous from one side through a modification of a measure-zero set of points which thus does not change the value of the objective function (OBJ').

Fifth, we can incorporate the constraint (4.5) into the objective function using a Lagrange multiplier $\alpha \geq 0$. Defining

$$\phi_B^{\alpha}(q) := \int_{G_B^{-1}(1-q)}^{\bar{r}_B} \big(\Pi_B^{\Lambda}(r) + \alpha J_B(r)\big)\, dG_B(r) + (\Lambda_B - \alpha)\underline{U}_B,$$

$$\phi_S^{\alpha}(q) := \int_{\underline{r}_S}^{G_S^{-1}(q)} \big(\Pi_S^{\Lambda}(r) - \alpha J_S(r)\big)\, dG_S(r) + (\Lambda_S - \alpha)\underline{U}_S,$$

the problem becomes one of maximizing the expectation of an additive function over two distributions, subject to an equal-means constraint. Thus, for any fixed volume of trade, our problem becomes mathematically equivalent to a pair of "Bayesian persuasion" problems (one for each side of the market) with a binary state in which the market-clearing condition (4.4) corresponds to the Bayes-plausibility constraint. We can thus employ concavification (see Aumann and Maschler (1995) and Kamenica and Gentzkow (2011)) to solve the problem. Let $\mathrm{co}(\phi)(q)$ denote the *concave closure* of $\phi(q)$, that is, the pointwise smallest concave function that lies above $\phi(q)$.

LEMMA 1: *Suppose that there exist $\alpha^\star \geq 0$, $\underline{U}_S^\star \geq 0$, $\underline{U}_B^\star \geq 0$, and distributions $H_S^\star$ and $H_B^\star$ such that*

$$\begin{aligned}&\int_0^1 \phi_S^{\alpha^\star}(q)\, dH_S^\star(q) + \mu \int_0^1 \phi_B^{\alpha^\star}(q)\, dH_B^\star(q) \\ &\quad = \max_{Q\in[0,\min\{\mu,1\}],\underline{U}_B,\underline{U}_S\geq 0} \big\{\mathrm{co}\big(\phi_S^{\alpha^\star}\big)(Q) + \mu\, \mathrm{co}\big(\phi_B^{\alpha^\star}\big)(Q/\mu)\big\},\end{aligned} \tag{4.6}$$

with constraints (4.4) and (4.5) holding with equality. Then, $\underline{U}_S^\star$, $\underline{U}_B^\star$, $H_S^\star$, and $H_B^\star$ correspond to an optimal mechanism.

Conversely, if $\underline{U}_S^\star$, $\underline{U}_B^\star$, $H_S^\star$, and $H_B^\star$ correspond to an optimal mechanism, then we can find $\alpha^\star$ such that conditions (4.4)–(4.6) hold, with (4.5) being satisfied as an equality.

Because the optimal $H_j^\star$ found through Lemma 1 concavifies a one-dimensional function $\phi_j^{\alpha^\star}(q)$ while satisfying a linear constraint (4.5), Carathéodory's theorem implies that it is without loss of generality to assume that the lottery induced by each $H_j^\star$ has at most three realizations; this implies that the corresponding allocation rule X_j has at most three jumps, and hence the mechanism offers at most two rationing options on each side of the market.[25] To arrive at the conclusion of Theorem 1 that at most two rationing options are used in total, we exploit the fact that two constraints (market-clearing and budget-balance) are *common* across the two sides of the market—looking at both sides of the market simultaneously then allows us to further reduce the dimensionality of the solution by avoiding the double-counting of constraints implicit in solving for the buyers' and sellers' optimal lotteries separately. Moreover, if additionally a strictly positive lump-sum payment is used for side j, then the Lagrange multiplier $\alpha^\star$ must be equal to Λ_j; hence,

[25] Samuelson (1984) examined a two-person bargaining setting in which the seller has strictly better information about the potential value of the transaction than the buyer, and found that two-price mechanisms maximize the seller's surplus subject to buyer's ex ante participation constraint. Although Samuelson (1984) relied on a different proof technique, the underlying mathematical principle behind his result is similar to ours because his problem corresponds to maximizing a linear objective function subject to a single linear constraint.

because $\underline{U}_j$ enters the Lagrangian (4.6) with a coefficient $(\Lambda_j - \alpha)$, the Lagrangian is constant in $\underline{U}_j$ in this case. Starting from a mechanism offering two rationing options, we can find a mechanism with one rationing option such that (4.4) holds and the budget constraint (4.5) is satisfied as an inequality. Then, we can increase $\underline{U}_j$ to satisfy (4.5) as an equality; the alternate solution still maximizes the Lagrangian, and is thus optimal, yielding the characterization we state in the second part of Theorem 1. The proof of Lemma 1, as well as the formal proofs of the preceding claims, are presented in Appendixes B.1 and B.2, respectively.

Lemma 1 contains the key mathematical insight that allows us to relate the shape of $\phi_j^{\alpha}(q)$ to the economic properties of the optimal mechanism: When $\phi_j^{\alpha}(q)$ is concave, the optimal lottery is degenerate—corresponding to a competitive mechanism. When $\phi_j^{\alpha}(q)$ is convex, it lies below its concave closure, and thus the optimal lottery is non-degenerate, leading to rationing.

5. OPTIMAL DESIGN UNDER INEQUALITY

In this section, we use the characterization of optimal mechanisms we derived in Section 4 to extend the conclusions of Section 3 to a large class of distributions satisfying certain regularity conditions.

5.1. *Preliminaries*

We maintain the key assumption that $\lambda_j(r)$ is non-increasing in r (and continuous); we also impose some regularity conditions. First, we assume that the densities g_j of the distributions G_j of the rates of substitution are strictly positive and continuously differentiable (in particular continuous) on $[\underline{r}_j, \bar{r}_j]$, and that the virtual surplus functions $J_B(r)$ and $J_S(r)$ are non-decreasing. We make the latter assumption to highlight the role that inequality plays in determining whether the optimal mechanism makes use of rationing: With non-monotone virtual surplus functions, rationing (more commonly known in this context as "ironing") can arise as a consequence of revenue-maximization motives implicitly present in our model due to the budget-balance constraint. We need an even stronger condition to rule out ironing due to irregular local behavior of the densities g_j. To simplify notation, let $\bar{\lambda}_j(r) = \lambda_j(r)/\Lambda_j$ for all r and j (we normalize so that $\bar{\lambda}_j$ is equal to 1 in expectation), and define

$$\Delta_S(p) := \frac{\int_{\underline{r}_S}^{p} [\bar{\lambda}_S(\tau) - 1] g_S(\tau)\, d\tau}{g_S(p)}, \tag{5.1}$$

$$\Delta_B(p) := \frac{\int_{p}^{\bar{r}_B} [1 - \bar{\lambda}_B(\tau)] g_B(\tau)\, d\tau}{g_B(p)}. \tag{5.2}$$

ASSUMPTION 1: *The functions $\Delta_S(p) - p$ and $\Delta_B(p) - p$ are strictly quasi-concave.*

Unless otherwise specified, we impose Assumption 1 for the remainder of our analysis. A sufficient condition for Assumption 1 to hold is that the functions $\Delta_j(p)$ are (strictly) concave. Intuitively, concavity of $\Delta_j(p)$ is closely related to non-increasingness of $\lambda_j(r)$ (these two properties become equivalent when G_j is uniform, which is why Assumption 1 did not appear in Section 3). A non-increasing $\lambda_j(r)$ reflects the belief of the market designer that agents with lower willingness to pay (lower r) are "poorer" on average, that is, that those agents have a higher conditional expected value for money. When $\lambda_j(r)$ is assumed to be non-increasing, concavity of Δ_j rules out irregular local behavior of g_j. Each function $\Delta_j(p)$ is 0 at the endpoints $\underline{r}_j$ and $\bar{r}_j$, and non-negative in the interior. There is no same-side inequality if and only if $\Delta_j(p) = 0$ uniformly for all p.

5.2. *Addressing Cross-Side Inequality with Lump-Sum Transfers*

In this section, we show that lump-sum transfers are an optimal response of the market designer when cross-side inequality is significant, and that rationing is suboptimal when same-side inequality is low.

THEOREM 2: *Suppose that same-side inequality is low on both sides of the market. Then, the optimal mechanism is a competitive mechanism (with prices p_B and p_S).*

A competitive-equilibrium mechanism, $p_B = p_S = p^{\mathsf{CE}}$, is optimal if and only if

$$\Lambda_S \Delta_S(p^{\mathsf{CE}}) - \Lambda_B \Delta_B(p^{\mathsf{CE}}) \geq \begin{cases} (\Lambda_S - \Lambda_B)\dfrac{1 - G_B(p^{\mathsf{CE}})}{g_B(p^{\mathsf{CE}})} & \text{if } \Lambda_S \geq \Lambda_B, \\ (\Lambda_B - \Lambda_S)\dfrac{G_S(p^{\mathsf{CE}})}{g_S(p^{\mathsf{CE}})} & \text{if } \Lambda_B \geq \Lambda_S. \end{cases} \tag{5.3}$$

When condition (5.3) fails, we have $p_B > p_S$; prices are determined by the market-clearing condition $\mu(1 - G_B(p_B)) = G_S(p_S)$ and, in the case of an interior solution,[26]

$$p_B - p_S = \begin{cases} -\dfrac{1}{\Lambda_S}\left[\Lambda_S \Delta_S(p_S) - \Lambda_B \Delta_B(p_B) - (\Lambda_S - \Lambda_B)\dfrac{1 - G_B(p_B)}{g_B(p_B)}\right] \\ \text{if } \Lambda_S \geq \Lambda_B, \\ -\dfrac{1}{\Lambda_B}\left[\Lambda_S \Delta_S(p_S) - \Lambda_B \Delta_B(p_B) - (\Lambda_B - \Lambda_S)\dfrac{G_S(p_S)}{g_S(p_S)}\right] \\ \text{if } \Lambda_B \geq \Lambda_S. \end{cases} \tag{5.4}$$

The mechanism gives a lump-sum payment to the sellers (resp. buyers) when $\Lambda_S > \Lambda_B$ (resp. $\Lambda_B > \Lambda_S$).

Theorem 2 is a generalization of Proposition 4 of Section 3. As we explained in Section 3, rationing is suboptimal when same-side inequality is low because the positive redistributive effects of rationing are too weak to overcome the allocative inefficiency that it induces. However, the optimal mechanism will often redistribute *across* the sides of the market if the difference in average values for money is sufficiently large; redistribution in this case takes the form of a tax-like wedge between the buyer and seller prices, which finances a lump-sum transfer to the poorer side of the market.[27]

[26] When no such interior solution exists, one of the prices is equal to the bound of the support: either $p_B = \underline{r}_B$ or $p_S = \bar{r}_S$.

[27] If lump-sum transfers are not available, then rationing can sometimes arise as a second-best way of redistributing across the market. We undertake a more general analysis of the case without lump-sum transfers in a follow-up paper (Akbarpour ® Dworczak ® Kominers (2021)), which also allows heterogeneous objects.

Condition (5.3), which separates optimality of competitive equilibrium from optimality of lump-sum redistribution, depends on same-side inequality (through the term Δ_j) because introducing a price wedge has redistributive consequences also within a given side of the market. However, in the special case of no same-side inequality, Assumption 1 is automatically satisfied, (5.3) cannot hold unless $\Lambda_B = \Lambda_S$, and (5.4) boils down to (3.3), so that the wedge between the prices is proportional to the size of the cross-side inequality.

The proof of Theorem 2 relies on techniques developed in Section 4. A competitive mechanism corresponds to a one-step allocation rule, which in turn corresponds to a degenerate lottery over quantities. A degenerate lottery is optimal exactly when the objective function—that is concavified in the optimal solution—is concave to begin with. Therefore, the key to the proof of Theorem 2 is showing that the Lagrangian $\phi_j^{\alpha^\star}(q)$ is concave under the assumption of low same-side inequality.

5.3. *Addressing Same-Side Inequality with Rationing*

A disadvantage of the competitive mechanism is that it is limited in how much wealth it can redistribute to the poorest agents. Indeed, market-clearing imposes bounds on equilibrium prices, and lump-sum transfers are shared equally by all agents on a given side of the market. When same-side inequality is low, a lump-sum transfer is a fairly effective redistributive channel. However, when same-side inequality is high, the conclusion of Theorem 2 may fail, as already demonstrated in Section 3. Here, we generalize the preceding insights through a series of results. The first two results highlight and generalize the asymmetries between buyers and sellers identified in Section 3; the third result (an extension of Proposition 5) gives sufficient conditions supporting seller-side rationing; the fourth and fifth results (extensions of Propositions 6 and 7) give sufficient conditions opposing and supporting buyer-side rationing, respectively.

THEOREM 3: 1. *For rationing to be optimal on the buyer side, the optimal volume of trade must be sufficiently large*: $Q \geq \underline{Q}_B > 0$ *for some* $\underline{Q}_B$ *that does not depend on seller characteristics. Moreover, there must be a non-zero measure of buyers who trade with probability* 1.

2. *For rationing to be optimal on the seller side, the optimal volume of trade must be sufficiently small*: $Q \leq \bar{Q}_S < 1$ *for some* $\bar{Q}_S$ *that does not depend on buyer characteristics. Moreover, there must be a non-zero measure of sellers who trade with probability* 0.

Theorem 3 highlights the asymmetry between buyers and sellers with respect to the redistributive properties of trading mechanisms. A competitive mechanism selects sellers who are poorest in expectation and buyers who are richest in expectation (due to our assumption that $\lambda_j(r)$ is non-increasing). Rationing on the seller side relies on identifying poor sellers directly via their decision to trade, and is successful only if relatively rich sellers are excluded from trading; this requires a relatively low volume of trade. In contrast, to identify relatively poor buyers, a mechanism must offer two prices and attract sufficiently many rich buyers to the high "competitive" price (thus requiring a relatively large volume of trade). The intuitions just described are confirmed by the following result.

THEOREM 4: 1. *It is never optimal to ration buyers at a single price—that is, an optimal* $X_B^\star$ *cannot have* $\mathrm{Im}(X_B^\star) \subseteq \{0, x\}$ *for any* $x < 1$.

2. *If either* (i) *the optimal mechanism gives a lump-sum payment to the sellers, or* (ii) $J_S(G_S^{-1}(q))$ *and* $G_S^{-1}(q) - \Delta_S(G_S^{-1}(q))$ *are convex in* q, *then rationing on the seller side* (*if optimal*) *takes the form of offering a single price above the competitive level.*

Rationing at a single price is never optimal on the buyer side, as it would essentially amount to giving a price discount to the buyers who are relatively rich. In contrast, it is often optimal to ration the sellers at a single price. The assumptions required by the second part of Theorem 4 are restrictive because with general distributions it is difficult to predict how the optimal mechanism will be influenced by the budget-balance constraint (the Lagrange multiplier α is endogenous and influences the shape of the function $\phi_j^\alpha(q)$); condition (i) addresses this difficulty by directly assuming that the optimal mechanism gives a lump-sum transfer to the sellers (this pins down a unique candidate for a Lagrange multiplier α), while the alternative condition (ii) gives conditions on the primitives under which the form of the mechanism does not depend on how tight the budget constraint is (the key properties of $\phi_j^\alpha(q)$ do not depend on the choice of α). Condition (ii) is satisfied when G_S is uniform.

Next, we identify sufficient conditions for rationing to be optimal.

THEOREM 5: *Suppose that $\Lambda_S \geq \Lambda_B$ and seller-side inequality is high. Then, if μ is low enough* (*i.e., there are few buyers relative to sellers*), *the optimal mechanism rations the sellers.*

Theorem 5 is a generalization of Proposition 5. When seller-side inequality is high, a low volume of trade is not only necessary but also sufficient for rationing to become optimal. Mathematically, this is because the function $\phi_S^{\alpha^\star}(q)$ is convex for low q when seller-side inequality is high. Because the volume of trade is bounded above by the mass of buyers μ, a low μ guarantees that $\phi_S^{\alpha^\star}(q)$ lies below its concave closure at the optimal volume of trade Q (see Lemma 1); this in turn implies that the optimal mechanism must correspond to a non-degenerate lottery $H_S^\star$ over quantities (which is equivalent to rationing).

As we already explained in Section 3, there is no analog of Theorem 5 for buyers—it is possible for rationing to be suboptimal regardless of the size imbalance in the market. This happens in particular when the poorest buyers that can be identified by their market behavior have very low willingness to pay—in this case, providing the good at a "below-competitive" price (rationing) is always dominated by a competitive mechanism that redistributes money. We confirm this by extending Proposition 6.

THEOREM 6: *Suppose that seller-side inequality is low, and $\underline{r}_B = 0$. Then, if either* (i) *the optimal mechanism gives a lump-sum payment to the buyers, or* (ii) $G_B^{-1}(q) - \Delta_B(G_B^{-1}(q))$ *and $J_B(G_B^{-1}(q))$ are convex in q, then the optimal mechanism does not ration the buyers.*

Condition (i) *holds when $\bar{r}_S \geq \bar{r}_B$ and either* (a) $\Lambda_B \geq 2\Lambda_S$, *or* (b) *there is no seller-side inequality and $\Lambda_B \geq \Lambda_S$. Condition* (ii) *holds when G_B is uniform.*

The economic intuition for Theorem 6 is analogous to that underlying Proposition 6. Here, we briefly explain the proof. When buyer-side inequality is high, the function $\phi_B^{\alpha^\star}(q)$ is convex for high enough q, and thus rationing would be optimal for the buyer side if the volume of trade were large enough. However, under the assumption that $\underline{r}_B = 0$, we are able to show that $\phi_B^{\alpha^\star}(q)$ is non-increasing whenever it is convex (the additional assumptions allow us to establish this property without necessarily knowing the value of the Lagrange multiplier $\alpha^\star$). Therefore, from the perspective of buyer welfare, it is never optimal to choose a volume of trade in the region in which rationing would become optimal—the maximum buyer welfare is attained at a volume of trade lower than the one required for rationing. The assumption of low seller-side inequality implies that the optimal volume of trade overall is even lower than the optimal volume from the perspective of buyer welfare alone.

Theorem 6 crucially relies on the existence of buyers with low willingness to pay. When all buyers value the good significantly, and it is relatively easy to ensure a large supply, assigning the good in a lottery at a "below-competitive" price may become optimal; we demonstrate this by extending Proposition 7 to general distributions.

THEOREM 7: *Suppose that there is high buyer-side inequality and* $\Lambda_B \geq \Lambda_S$. *Then, there exists a constant* M *such that whenever* $\underline{r}_B - \bar{r}_S \geq M$, *it is optimal to ration the buyers for any* $\mu \in (1, 1+\epsilon)$, *for some* $\epsilon > 0$.[28]

The intuition behind Theorem 7 is the same as that for Proposition 7: The assumption of a large gap between buyer and seller rates of substitution guarantees that it is optimal to sell all the goods supplied by sellers; with a slight size imbalance in the market, this means that almost all buyers buy, and hence rationing becomes optimal under high buyer-side inequality.

6. IMPLICATIONS FOR POLICY

As we noted in the Introduction, policymakers are actively engaged in redistributive market policies such as price controls that connect with the results described here. Economic analysis often opposes such regulations because they lead to allocative inefficiency. Yet our work suggests that such policies can in fact be part of the optimal design. Of course, our model is an extreme abstraction in many ways. Nevertheless, as we now describe, our basic assumptions are reasonable approximations of some markets—and our framework provides at least intuitions for some others.

One real-world market that fits our model particularly well is the *Iranian kidney market*—the only cash market for kidneys in the world. In Iran, prospective kidney buyers and sellers register in a centralized market, mediated by the government. Humans have two kidneys, but need just one functional kidney to survive. Kidney buyers (i.e., end-stage renal disease patients) thus have unit demand for kidneys—which of course are indivisible goods. Prospective sellers quite literally have unit supply.[29] The price of a kidney is fixed by the government, and as of July 2019, it is equivalent to 18 months of the minimum wage in Iran (see Akbarpour, Fatemi, and Matoorian (2019)). Note that the pool of prospective buyers is completely separated from the pool of prospective sellers; moreover, essentially every individual who is not a buyer can potentially be a seller. Arguably, each individual's willingness to sell a kidney correlates strongly with characteristics that inform social preferences—such as wealth or financial hardship. Because these characteristics exhibit significant dispersion in Iranian society, the assumption of high seller-side inequality is plausible for the Iranian kidney market. Cross-side inequality, however, is relatively low, because the wealth distribution of the set of potential sellers (i.e., all citizens) is close to that of the buyers. In addition, since kidney patients are a tiny fraction of the population, the number of potential buyers is substantially less than the number of potential sellers. Therefore, Theorem 5 suggests that the policy of rationing the sellers at a single price corresponds to the optimal way to transfer surplus to the poorest kidney sellers.

A second application that fits the outlines of our model is the *rental real estate market*; sellers in the rental market are the landlords, and the buyers are prospective renters.

[28]In particular, $M \leq \frac{1}{g_B(\underline{r}_B)} + \frac{1}{g_S(\bar{r}_S)}$, which is finite by our assumption that the densities g_j are strictly positive and continuous.

[29]While kidneys are not quite homogeneous due to blood-type differences, in Iran each blood-type submarket clears independently of the others.

Of course, the rental real estate market has heterogeneous objects, which our model abstracts from. And changes in prices may induce income effects, which we also do not account for, since we assume agents have fixed marginal utilities for money. Nevertheless, our work provides some intuition for how we might think about addressing inequality in rental housing. In general, the sellers tend to be wealthier than the buyers (indeed, they own real estate equity). Moreover, there is tremendous buyer-side inequality. In addition, the quantity of trade is high, since nearly all buyers need housing.[30] Thus, the assumptions of Theorem 6 are unlikely to hold in the rental real estate market. As Theorems 3 and 7 suggest, rationing the buyers may be optimal: so long as most buyers rent at a high price, it might be optimal to provide a "lower-quality" rental option at a lower price—but the quality differential must be such that wealthier buyers do not want to mimic the poor buyers and buy at the low price. And indeed, some cities have faced problems in making public housing too prevalent and high-quality, since then even wealthy people claim public housing, reducing poorer people's access.[31] In such cases, rationing can be suboptimal, as it fails to identify poor buyers—see the reasoning behind our Proposition 3 and Theorem 3. Meanwhile, our model suggests that lump-sum transfers might be an effective strategy for addressing cross-side inequality in the rental market; such a policy could be implemented through a tax on rental transactions that is rebated as a tax credit to anyone who does not own housing, as suggested by Diamond, McQuade, and Qian (2019).[32]

Our findings also provide some intuitions for *labor markets*. For example, our results suggest that rationing policies (e.g., local minimum wage policies) are most effective when there is high same-side inequality among potential workers, since transferring additional surplus to workers more than compensates for the allocative inefficiency. Thus, we might think that minimum wages make more sense for low-skilled jobs for which people from a wide variety of incomes can in principle supply labor, and the reservation wage may be strongly correlated with v^M. To address cross-side inequality (e.g., in the case of rideshare, where drivers are on average poorer than riders), lump-sum transfers are typically superior to rationing-based solutions. Of course, we should be cautious in extrapolating our results into labor market contexts because labor markets typically fail a number of our assumptions. Perhaps most pertinently, labor markets literally determine people's incomes, yet our model rules out income effects by assuming fixed marginal utility of money. In addition, lump-sum transfers are difficult to implement in labor markets because the complete set of potential workers may be challenging to define, much less to target with transfer policies. Thus, for our results to provide more than just intuitions for labor markets, we need to think about closed, small economies such as work-study programs on university campuses.[33]

In general, Theorem 3, Theorem 6, and Theorem 7 suggest that rationing on the buyer side may only be justified for "essential" goods such as housing and healthcare: goods that are highly valued by all potential buyers; for which buyers' willingness to pay is highly in-

[30]There is of course also an extensive margin on the buyer side—prospective renters who might have the outside option of purchasing real estate.

[31]This problem exists in Amsterdam, for instance, where nearly 25% of households that live in public housing have incomes above the median (van Dijk (2019)).

[32]That said, if these sorts of tax credits are available as an instrument, it is possible that broader, more efficient tax-based redistributive instruments may be available as well.

[33]But of course, even university labor markets have intensive and extensive labor supply decisions that put pressure on both the indivisibility assumption of our model and the assumption that the sets of buyers and sellers are fixed ex ante.

formative of their values for money v^M; and that will induce a high quantity of trade.[34] But of course—as with housing, already described—many essential goods are large enough to induce income effects, so we must be careful to interpret our results as just providing intuition for these markets, rather than a precise characterization of when rationing is optimal in practice. Additionally, our results on rationing rely on the indivisibility assumption; hence, they do not carry over directly to contexts like food aid, in which quantities are (almost) continuously divisible—although see our discussion of divisible goods in the next section.

Our results on lump-sum transfers are of course impractical for markets in which lump-sum transfers are infeasible (as with open labor markets, already described). That said, lump-sum transfers are natural in contexts in which the buyer and seller populations can be clearly defined according to characteristics that are either costly to acquire or completely exogenous—for example, if the only potential sellers are those who own land in a given area, or if the only eligible sellers are military veterans (as in some labor markets). Likewise, lump-sum transfers make sense when there is a licensing requirement or other rule that prevents agents from entering the market just to claim the transfer, or when the transfer can be made to an outside authority (e.g., a charity) that benefits the target population.

7. DISCUSSION AND CONCLUSION

Regulators often introduce price controls that distort markets' allocative role in order to effect redistribution. Our work provides some justification for this approach, by showing that carefully structured price controls can indeed be an optimal response to inequality among market participants. The key observation, as we highlight here, is that properly designed price controls can identify poorer individuals through their behavior, using the marketplace itself as a redistributive tool.

Moreover, at least for a simple goods market, we can characterize the form that price controls should take. Our main result shows that optimal redistribution through markets can be obtained through a simple combination of lump-sum transfers and rationing. When there is substantial inequality between buyers and sellers, the optimal mechanism imposes a wedge between buyer and seller prices, passing on the resulting surplus to the poorer side of the market. When there is significant inequality on a given side of the market, meanwhile, the optimal mechanism may impose price controls even though doing so induces rationing. The form of rationing differs across the two sides: on the seller side, rationing at a single price can be optimal because willingness to sell identifies "poorer" sellers on average; by contrast, rationing the buyer side can only be optimal when several prices are offered, since buying at relatively high prices identifies agents who are likely to be wealthier.

Our paper does not examine how redistribution through markets interacts with macro-level redistribution. Theoretically, if macro-level redistribution already achieves the socially desired income distribution, then there is no scope for a market regulator with the same preferences to try to redistribute further by distorting the market allocation. Thus, we might naturally think that there is less need of market-level redistributive mechanisms in countries that either have large amounts of macro-level redistribution or simply have

[34]We discuss redistributive allocation of essential goods in more detail in a follow-up paper (Akbarpour ® Dworczak ® Kominers (2021)).

endogenously lower inequality—perhaps, that is, we should expect less need for market-based redistribution in a country like Sweden than in the United States. However, even in countries with low levels of inequality like Sweden, market interventions like rent control are prevalent; this could simply be because those societies have a higher preference for redistribution overall. Additionally, as our work highlights, market-based redistributive mechanisms can help screen for unobservable heterogeneity in the values for money that are not reflected in income. Finally, because our mechanisms are individually rational, they provide incentives for all agents to participate, whereas extremely progressive income taxation might raise extensive margin concerns, for example, with wealthy individuals seeking tax havens. In any event, it remains an open question how much and when micro- and macro-level redistributive approaches are complements or substitutes.

The extent to which the mechanisms we derive here are valuable could also depend on policymakers' preferences and political economy concerns: Market-level redistribution might be particularly natural in a context with divided government—either where the executive has a stronger preference for redistribution than the legislature, or where local policymakers have stronger preferences for redistribution than the central government.

The specific mechanisms we identify depend heavily on our assumptions—indivisibility of objects, unit demand, and linearity of agents' utilities. We nevertheless expect that the core economic intuitions should carry over to settings with some of our assumptions relaxed. For example, in a model with a divisible good and utility that is concave in the quantity of the good but linear in money, we would expect rationing in the mechanism to be replaced with offering below-efficient quantities in order to screen for agents that are likely to be poorer. Likewise, if we were to instead relax quasi-linearity in money to allow for wealth effects (and hence endogenous Pareto weights that depend on the agents' allocations and transfers), there would most likely still be opportunities to improve welfare through price wedges and/or rationing, but the scale of the optimal intervention would be decreased because each unit of redistribution would also shift agents' Pareto weights closer to equality.

That said, our framework abstracts from several practical considerations that are important in real-world settings. For instance, if there is an aftermarket (i.e., if agents can engage in post- or outside-of-mechanism trades), then the mechanisms we consider might no longer be incentive-compatible (or budget-balanced).[35] In addition, the generic form of our optimal solution is a randomized mechanism, which can negatively affect the utilities of risk-averse agents and lead to wasted pre-market investments; both of these concerns are particularly salient in contexts with inequality, as the poor often have both less tolerance for day-to-day income variance and less ability to undertake upfront investments. Understanding how to redistribute through markets while accounting for these sorts of additional design constraints may be worth examining in future research.

More broadly, there may be value in further reflecting on how underlying macroeconomic issues like inequality should inform market design. And we hope that the modeling approach applied here—allowing agents to have different marginal values of money—may prove useful for studying inequality in other microeconomic contexts.

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[35] On the other hand, as shown, for example, by Che, Gale, and Kim (2012), a carefully regulated aftermarket may be used to implement an outcome involving a combination of rationing and transfers.

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