School Choice with Unequal Outside Options∗

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Abstract

Students with identical valuations for public schools but unequal outside options have different opportunity costs of revealing their preferences. Consequently, manipulable mechanisms need not resolve conflicting preferences in a Pareto-improving manner. We show that when they do not, welfare improvements for students with outside options come at the expense of students without outside options. This result strengthens the argument that strategy-proof mechanisms “level the playing field.” Our model predicts that students without outside options are more likely to strategize, consistent with recent findings in empirical studies of education markets.

Keywords: Matching, School Choice, Strategy-proof, Inequality, Outside options
JEL Classification Codes: D47, D82, I24

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1 Introduction

School-age children in the U.S. and in many other countries are entitled to public education. This, however, does not mean that all students will have equal access to the same public school seats. Because some schools are more popular and over-demanded, school districts design non-monetary assignment mechanisms to ration seats. The purpose of this note is to provide a simple argument showing that some (but not all) school assignment mechanisms shape competition for seats in such a way that a student’s chances of assignment to a given public school vary with constraints that he faces in markets that are outside the control of the designer. In particular, strategy-proof mechanisms generate the same probability distribution over possible allocations for students with a common ranking over public schools, regardless of differential access to an outside option. However, manipulable mechanisms can translate asymmetry in students’ outside options into welfare improvement (over the strategy-proof allocation) for students with access to an outside option, accompanied by welfare loss for students without.

We model asymmetry in access to private schools as an outside option that is included in the choice set of some students, but not all.\footnote{The term “outside option” is sometimes used in the literature to denote the option of declining an assigned seat. We assume that this option is available to all students after public school assignment takes place. In this note, we use the term “outside option” to refer to seats in schools that are not part of the centralized public school system. These seats are therefore not under the control of the designer, and need not be available to all students.} In this setting, two students with identical valuations for public school options can have different valuations for the outcome of being unassigned by the mechanism. With such asymmetry in outside options, the ex ante Pareto improvement (over the strategy-proof allocation) from the Boston mechanism predicted by Abdulkadiroglu et al. (2011) need not be realized. The central take-away from this paper is that, with asymmetry in outside options, a manipulable choice mechanism always improves outcomes for students with outside options, while potentially reducing welfare for students without outside options. In that sense, we may call manipulable mechanisms regressive. A corollary to this result is that manipulable assignment mechanisms endogenously segregate the public school system by increasing the fraction of less constrained students who go to the best public schools.

We argue that, within a certain class of school choice mechanisms that includes as a special case Abdulkadiroglu et al. (2011), henceforth ACY, allocations produced by strategy-proof mechanisms do not vary with student characteristics that are orthogonal to their rankings over public schools. For example, high-income students may be more likely to prefer the most highly demanded public school because it is situated in their neighborhood, and this will be reflected in allocations produced by strategy-proof mechanisms. However, the fact that these students are also more likely to prefer private school over some public schools does not affect their ordinal
ranking of public schools, hence it is not reflected in strategy-proof allocations. By contrast, manipulable mechanisms are sensitive to asymmetry in outside options between two otherwise identical students.

Economists often find strategy-proof mechanisms desirable for reasons such as reduction in participation costs, preventing agents from making strategic errors (Abdulkadiroglu et al. (2006), Pathak and Sonmez (2008)) and the fact that the resulting outcome does not depend on agents' higher-order beliefs (Vickrey (1961); Wilson (1985); Li (2017)). Our analysis provides a new argument in favor of strategy-proof mechanisms: Strategy-proofness neutralizes the effect of inequality in outside options, thus providing all market participants with an equal opportunity to receive the most popular public resources.

1.1 Related Work

The most relevant papers to ours are Pathak and Avery (2015), Calsamiglia and Guell (2017), and Calsamiglia et al. (2017). Like our paper, each of these papers considers how parallel markets may play a role in determining assignment outcomes for a centralized public school mechanism.

Pathak and Avery (2015) study parallel centralized markets when households make joint residential and school choices. In contrast to our paper, their focus is not on incentive properties of the particular school choice mechanisms used. Calsamiglia and Guell (2017) empirically study the Boston mechanism as it is implemented in Barcelona. Their findings include evidence that naive behavior is associated with higher income and education levels of parents. This finding is consistent with the predictions of the model in the current paper, as we would generally expect higher-income students to have more access to private-school outside options. Calsamiglia et al. (2017) study sorting and segregation under the Boston mechanism as a result of the presence of private schools, in a theoretical model that endogenizes school quality as a function of the peer quality of the student population of the school. By contrast, we focus on welfare implications, and our model is more stripped down to highlight the interaction of outside options with incentive features of the mechanism. Furthermore, our results characterize manipulability within a more general class of school choice mechanisms. The results in Calsamiglia et al. (2017) may be seen as complementary to ours, as segregation by income in the presence of peer quality may have additional welfare implications.

Abdulkadiroglu et al. (2006) and Pathak and Sonmez (2008) argue in favor of deferred acceptance based on the argument that it levels the playing field, albeit with a different prediction about behavior: In their model, less privileged students are behavioral agents who are less sophisticated, thus they are less likely to strategize.\footnote{In a new paper, Babaioff et al. (2018) show that this result relies on schools’ strict priorities – with coarse priorities, the results can go either way.}
In our model, constrained students are those who are more likely to strategize. Of course, these two explanations are not mutually exclusive and behavior in practice could arise as a mix of the two. In section 5 we discuss empirical evidence, and find that most available evidence is aligned with our model’s prediction that wealthier parents are less likely to strategize.

The rest of the paper is organized as follows. In section 2, we study a simple economy to illustrate the main insight of the paper. Section 3 generalizes the simply economy of the previous section by introducing our model and then our main results. In section 4 we study two other school choice mechanisms and connect our findings to the notion of strategy-proof in the large. In section 5 we discuss the empirical relevance of outside options, as well as role of strategy-proofness in providing equality of access. Section 6 concludes.

2 An Illustrative Example

We start with a simple example that illustrates the main insight of the paper. Let us consider a setting with three public schools, \{s_1, s_2, s_3\}, one private school \(s_p\), and a continuum mass 1 of students. Suppose each student is unconstrained with probability 1/2, and constrained with probability 1/2. Students know their own types, but have probabilistic knowledge of other students’ types. All schools have capacity \(q = 1/3\), and they have no priorities over students so they break ties uniformly at random. Unconstrained students prefer \(s_1\) to \(s_p\) to \(s_2\) to \(s_3\) and constrained students prefer \(s_1\) to \(s_2\) to \(s_3\) to \(s_p\). This could be simply because unconstrained students can afford private school, but constrained students cannot. As a result, unconstrained students’ rank-order list for public schools is \(s_1 \succeq \emptyset \succeq s_2 \succeq s_3\), while for constrained students, the rank-order list is \(s_1 \succeq s_2 \succeq s_3 \succeq \emptyset\). We restrict attention to the case where constrained students have the same cardinal values: their value from attending \(s_i\) is \(v_i\), where \(v_1 \geq v_2 \geq v_3\). We assume \(v_1 = 1\) and \(v_3 = 0\), and simply let \(v_2 = v\), where \(0 \leq v \leq 1\).

We now compare outcomes under two well-known school choice mechanisms: (student-proposing) deferred acceptance and Boston mechanism (see Appendix A for the definitions of these two mechanisms). Under deferred acceptance, all students report truthfully, which leads to the probability distribution over outcomes of Table 1.

<table>
<thead>
<tr>
<th></th>
<th>(s_1)</th>
<th>(s_2)</th>
<th>(s_3)</th>
<th>(s_p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconstrained</td>
<td>1/3</td>
<td>0</td>
<td>0</td>
<td>2/3</td>
</tr>
<tr>
<td>Constrained</td>
<td>1/3</td>
<td>2/3</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1: Outcomes under deferred acceptance with random tie-breaking.
What will happen under the BM? It is clear that under the Boston mechanism, unconstrained students have no incentive to lie (since they only want $s_1$). Constrained students, however, can be truthful or not. In this example, since $s_3$ is available to everyone, each constrained student effectively has two pure strategies available to her: to be truthful and report $s_1 \succeq s_2 \succeq s_3 \succeq \emptyset$, or to be strategic and report $s_2 \succeq s_1 \succeq s_3 \succeq \emptyset$. For clarity of exposition, let $v \geq 1/2$. Then, solving for the symmetric (Bayesian) Nash equilibrium, we can show that the probability of reporting truthfully is:

$$p = \frac{1 - v}{1 + v}.$$

Therefore, as long as $v \geq 1/2$, constrained students have incentive to play the non-truthful strategy with positive probability. That is because if all students play the truthful strategy, then each constrained student has an incentive to deviate and get assigned to $s_2$ with probability 1.

The key point is that when constrained students assign a positive probability to the non-truthful strategy, they decrease their likelihood of going to $s_1$, which, in turn, increases the likelihood of unconstrained students to be offered a seat at $s_1$. On the other hand, in any symmetric equilibrium, constrained students can never increase their likelihood of going to $s_2$ because they are not competing for those seats with unconstrained students. As a result, the competition among constrained students only increases their likelihood of going to $s_3$ at the cost of decreasing their likelihood of going to $s_1$. In the end, manipulability makes unconstrained students weakly better off, and constrained students weakly worse off.

To make things even more clear, let us assume $v = 0.6$, which leads to $p = 1/4$. This leads to the following ex ante assignment:

<table>
<thead>
<tr>
<th></th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
<th>$s_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconstrained</td>
<td>8/15</td>
<td>0</td>
<td>0</td>
<td>7/15</td>
</tr>
<tr>
<td>Constrained</td>
<td>2/15</td>
<td>2/3</td>
<td>1/5</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2: Outcomes under Boston mechanism with random tie-breaking.

Comparing the outcomes of Boston mechanism and deferred acceptance, one can see that constrained students are strictly worse off, and unconstrained students are strictly better off. There are some key observations in this example:

- If all of the constrained students could ‘collude’ and report truthfully, they could get the same outcome as under deferred acceptance – a 1/3 chance of going to $s_1$. Nevertheless, the Boston mechanism creates an incentive for each constrained student to deviate from such collusion to get a higher chance of
being assigned a seat at \( s_2 \), so the truthful strategy profile cannot be sustained in equilibrium.

- A striking scenario occurs when constrained students are close to indifferent between the two schools, or \( v = 1 - \epsilon \), for an arbitrarily small \( \epsilon > 0 \). The truthful strategy profile cannot be an equilibrium under the Boston mechanism, because under the truthful strategy profile, any of the constrained students has incentive to deviate, report \( s_2 \) as her top choice and be assigned to \( s_2 \) with probability 1. This competition continues to the point that in equilibrium as \( \epsilon \to 0 \), we have \( p \to 0 \). Consequently, unconstrained students go to \( s_1 \) with probability \( 2/3 \), and constrained students go to \( s_1 \) with zero probability, and to \( s_2 \) with probability \( 2/3 \) and to \( s_3 \) with probability \( 1/3 \).

- In this example, the manipulability of the Boston mechanism gives unconstrained students a higher chance than constrained students to attend the most popular school, which endogenously segregates the school system according to access to the private school: Under the Boston mechanism, more unconstrained students will go to the most popular school, and more constrained students will go to the least popular school.

3 Model

We now consider a model of school choice in which each student knows his own outside option, but only has probabilistic knowledge of other students’ outside options. Suppose there is a continuum mass 1 of students. A student is described by his type \( \theta \in \Theta = \{\text{unconstrained, constrained}\} \), distributed according to \( p(\theta = \text{unconstrained}) = \eta \) and \( p(\theta = \text{constrained}) = 1 - \eta \) for some \( \eta > 0 \) which is common knowledge. For ease of notation, we will use \( u \) and \( c \) to denote unconstrained and constrained students, respectively.

There is also a set \( S = \{s_1, s_2, \ldots, s_M\} \) of public schools, where \( M \geq 3 \). Each school \( j \) has capacity \( 0 < q_j < 1 \), and there is one private school, \( s_p \), with infinite capacity. We assume schools have no priorities over students and break ties randomly. This assumption captures part of the real world but not all of it: in practice, schools have coarse priorities, but within a specific group (e.g. students who are within the school’s walk-zone) ties are broken randomly.

We assume student \( i \) has vNM utility value of \( v^i_j \) when he attends school \( j \), where \( v^i = [v^i_1, v^i_2, \ldots, v^i_M, v^i_p] \) is the valuation vector of student \( i \). Each student \( i \) draws a valuation vector \( v^i \) from a finite set \( \mathcal{V} = \{(v_1, v_2, \ldots, v_m, v_p) \in [0, 1]^{M+1} | v_1 > v_2 > \cdots v_l > v_p > v_{l+1} > \cdots > v_M \} \). This means that all students agree on their ordinal preferences, but they may have different cardinal preferences. The probability of a valuation vector \( v^i \in \mathcal{V} \) is \( f(v^i) \), where \( \sum_{v \in \mathcal{V}} f(v) = 1 \). We assume \( f(\cdot) \) is common
knowledge. To make sure that the least popular public school is not irrelevant, we assume \( \sum_{j \in S \setminus S_M} q_j < 1 \). In addition, since every student is entitled to public education by law, we assume \( \sum_{j \in S} q_j \geq 1 \).

The difference between unconstrained and constrained students is that unconstrained students have access to the private school, while constrained students do not. Therefore, an unconstrained student’s truthful rank-order list of the public schools is \( s_1 \succ s_2 \succ \cdots s_l \succ \emptyset \succ s_{l+1} \succ \cdots \succ s_M \) (since they prefer their outside option to other public schools) and a constrained student’s truthful ranking is \( s_1 \succ s_2 \succ \cdots \succ s_M \succ \emptyset \).

A strategy is a mapping \( \sigma : \Theta \rightarrow \Delta(\Pi) \), where \( \Pi \) is the set of all rank-order lists of \( S \) (potentially with truncation) and \( \Delta(\Pi) \) is the set of probability distributions over \( \Pi \). We focus on symmetric strategies in which students (of the same type) follow the same strategy.

An (ex ante) assignment is a matrix \( X = [X(\theta,j)] \), for \( \theta \in \Theta \) and \( j \in S \). An assignment describes the allocation of students to public schools. In particular, for any school \( s \) it assigns a probability \( X(u,s) \) to unconstrained students and a probability \( X(c,s) \) to constrained students which represents the ex ante probability for unconstrained and constrained students to go to school \( s \), respectively. The capacity constraints require that \( \eta X(u,j) + (1 - \eta) X(c,j) \leq q_j \) for all \( j \in S \). An assignment mechanism (or simply, a mechanism) is a systematic procedure that results in an assignment.

**Student-proposing Deferred Acceptance Mechanism.** It is well-known that this mechanism is strategy-proof. Hence, unconstrained students would report \( s_1 \succ s_2 \succ \cdots s_l \succ \emptyset \succ s_{l+1} \succ \cdots \succ s_M \) (since they prefer their outside option to other public schools) and the constrained students would report \( s_1 \succ s_2 \succ \cdots \succ s_M \succ \emptyset \). Assuming that schools break ties in a symmetric way, the deferred-acceptance mechanism generates the following assignment matrix:

\[
X_{DA} = \begin{bmatrix}
q_1 & q_2 & \cdots & q_l & 0 & 0 & \cdots & 0 \\
q_1 & q_2 & \cdots & q_l & \frac{q_{l+1}}{1-\eta} & \frac{q_{l+2}}{1-\eta} & \cdots & 1 - \sum_{j=1}^{l} q_j - \sum_{j=l+1}^{M-1} \frac{q_j}{1-\eta}
\end{bmatrix}
\]

We consider the class of symmetric and monotone assignment mechanisms that are non-wasteful. Let us call a mechanism monotone if ranking a school higher does not decrease your chance of being admitted there. Also let us call a mechanism non-wasteful if no student who would have preferred an unassigned public school seat is unassigned to that seat. Also let us call a mechanism symmetric if it has a symmetric tie-breaking rule. An assignment mechanism is called standard if it is monotone, non-wasteful and symmetric.

To state the main theorem, we need one more definition.

**Definition 1** A student \( i \) always prefers an assignment mechanism \( A \) to an
assignment mechanism B, if he gets a weakly higher expected utility under any symmetric equilibrium of the mechanism A than under any symmetric equilibrium of the mechanism B.

We are now ready to state the main result. We prove this theorem in Appendix B.

**Theorem 1** A student \( i \) always prefers the Boston mechanism to deferred acceptance if and only if he is unconstrained.

Theorem 1 shows that the result presented in ACY will not go through for all students; only students who are unconstrained are guaranteed to be better off under the Boston mechanism. In the proof of the theorem, we show that this claim is true for any manipulable standard mechanism. Our example in section 2, on the other hand, shows that there might be plausible cases in which constrained students are strictly worse off under manipulable mechanisms. One may ask: Is it the case that constrained students are always worse off under the Boston mechanism? The following example shows that the answer to this question is no.

**Example 1** Suppose there are three public schools with capacity \( 1/3 \) each, and suppose all students value those schools at \( v_1 = 1, v_2 = 0.9 \) and \( v_3 = 0 \), and a private school that unconstrained students value at \( v_p^u = 0.9 - \epsilon \) for some \( \epsilon > 0 \). Let \( \eta = 2/3 \). Then, for sufficiently small \( \epsilon \), a symmetric equilibrium of the Boston mechanism is for unconstrained students to report \( s_1 \succ s_2 \succ \emptyset \), and for constrained students to report \( s_2 \succ s_1 \succ s_3 \succ \emptyset \). Note that under these strategies, unconstrained students go to \( s_1 \) with probability \( 1/2 \) and to \( s_p \) with probability \( 1/2 \), while constrained students go to \( s_2 \) with probability \( 1/3 \). For sufficiently small \( \epsilon \), no deviation can make any student better off.

On the other hand, under deferred acceptance, all students go to \( s_1 \) with probability \( 1/3 \) and to \( s_2 \) with probability \( 1/3 \). Constrained students go to \( s_3 \) with probability \( 1/3 \), while unconstrained students go to the private school with probability \( 1/3 \). It is then easy to check that all students are strictly better off under the Boston mechanism.

While our opening example in section 2 shows that constrained students are strictly worse off under the Boston mechanism, the previous example shows that this is not a necessary consequence. Whether constrained students prefer deferred acceptance to the Boston mechanism depends on the parameters of the school choice problem. Asymmetry in access to outside options can therefore change conclusions about welfare improvements previously associated with manipulable school choice mechanisms by, for instance, Abdulkadiroglu et al. (2011), who show that in a world without meaningful outside options the Boston mechanism makes all students weakly better off.
One may be curious to see whether there are some plausible conditions under which constrained students always prefer the deferred-acceptance mechanism to the Boston mechanism. To introduce one such condition, we first introduce the notion of a ‘single-minded’ student; in words, a student is single-minded if he only wishes to attend the best public school or else prefers the private school.

**Definition 2** A student $i$ is single-minded iff $v^i_1 \geq v^i_p \geq v^i_2 \geq \cdots \geq v^i_M$.

The following theorem identifies one condition under which constrained students always prefer deferred acceptance.

**Theorem 2** Suppose unconstrained students are single-minded, and all constrained students have the same valuation vectors. Then, constrained students always prefer the deferred-acceptance mechanism to the Boston Mechanism.

**Proof.** When unconstrained students are single-minded, they will always report truthfully. Therefore, their ex ante probability of going to $s_1$ is at least $q_1$.

Next, note that all constrained students will play the same strategy, because we are studying the symmetric NE and they all have the same valuation vector. Now, if all constrained students report $s_1$ as their top choice with probability 1, they all get a $q_1$ chance of going to $s_1$, and by symmetry, the outcome would be same as deferred acceptance. Suppose, on the contrary, that in the symmetric NE, constrained students assign a non-zero chance to a rank-order lists which does not put $s_1$ at the top, as it was the case in our illustrative example. Then, the probability that constrained students go $s_1$ would be strictly less than $q_1$ and consequently, they are all strictly worse off. □

The above theorem states that if we assume that constrained students have homogeneous intensity of preferences, then single-minded unconstrained students are all better off under the Boston mechanism, while constrained students are all worse off.

We would like to emphasize that the assumption of same valuations for constrained students is essentially shutting down the channel by which Boston mechanism enhances efficiency. Nevertheless, this is not a knife-edge result. One can in principle use the above theorem and a continuity argument to show that for “small enough” variations in preferences, the same insight goes through. In other words, the Boston mechanism hurts constrained students by forcing them to compete more within themselves and less with unconstrained students, and as long as the preference-signaling gains of the mechanism are smaller than the cost, constrained students are worse off.

An observation worth pointing out is that if all of the constrained students play the truthful strategy, they are all better off. This behavior, nonetheless, cannot be sustained in equilibrium because each constrained student has an incentive to
deviate. This creates competition within the group of constrained students for the seats of those schools that unconstrained students are not interested in. When all constrained students assign a positive probability to the non-truthful strategy, they are competing more among themselves, while competing less with unconstrained students for the best public school seats.

This result suggests that students with better outside options are more likely to attend the most popular public schools, when the mechanism is not strategy-proof. Hence, a direct prediction of our model is that manipulability can segregate students according to the constraints they face outside the public school system; that is, ceteris paribus, there will be more unconstrained students and less constrained students in the best public school. The following corollary of the Theorem 1 formalizes this observation.

**Corollary 1 (Segregation)** In any symmetric Nash equilibrium produced by the Boston mechanism, the fraction of unconstrained students who attend the best public school is weakly higher than their population share \( \eta \), and the fraction of constrained students who attend the best public school is weakly lower than their population share \( 1 - \eta \). In addition, these weak inequalities hold strictly for some parameters.

## 4 Unequal Outside Options in Other Mechanisms

So far we have focused on the Boston mechanism and deferred acceptance, since they are the main two mechanisms used in practice and discussed in the literature. Nevertheless, the main insights goes through for any manipulable versus strategy-proof mechanism.

Moreover, in our continuum model, manipulable mechanisms that are strategy-proof in the large (SP-L), as defined in Azevedo and Budish (2013), are also insensitive to the existence of private options. To illustrate this point, we now discuss two other mechanisms, one of which is similar to the Boston mechanism and manipulable in the large, and another mechanism which is manipulable in small economies, but strategy-proof in the large.

### 4.1 Choice-augmented deferred-acceptance

The choice-augmented deferred-acceptance (CADA) mechanism, a modification of the standard deferred acceptance, has been recently introduced in Abdulkadiroğlu et al. (2015). The CADA mechanism is similar to deferred acceptance, with an additional step in which each student also submits the name of a “target” school. A school then favors students who named that school as their target by breaking

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3The idea of targeting a specific school is similar to the idea of signaling in dating markets in Lee and Niederle (2015)
ties in favor of them. The mechanism is strategy-proof with respect to reporting the rank-order lists, but the targeting stage involves strategic behavior.\textsuperscript{4} We will now show that, in the context of our opening example, students with limited choice sets are strictly worse off under the choice-augmented deferred acceptance compared to the standard deferred acceptance.

In reporting their rank-order lists, unconstrained students report $s_1 \succeq \emptyset \succeq s_2 \succeq s_3$ and constrained students report $s_1 \succeq s_2 \succeq s_3 \succeq \emptyset$.

In choosing the target school, unconstrained students introduce $s_1$. For constrained students, recall that we assumed $1 = v_1 \geq v_2 = v \geq 1/2$. In symmetric NE, constrained students play a mixed strategy, where with probability $p$ they target $s_1$ and with the remaining probability they target $s_2$. They will never target $s_3$ because they are guaranteed a seat in that school anyways. Solving for the equilibrium $p$, one can show that $p = \frac{1 - v}{1 + v}$, which is exactly the same probability that constrained students assigned to the truthful strategy profile of the Boston mechanism. Therefore, the outcome of the CADA coincides with the outcome of the Boston mechanism, damaging students with limited choice sets. The following corollary summarizes this discussion.

\textbf{Corollary 2} A student $i$ prefers the CADA to the deferred-acceptance mechanism if and only if he is unconstrained. In addition, if unconstrained students are single-minded and constrained students have the same valuation vectors, then constrained students always prefer deferred acceptance to the CADA.

\section*{4.2 Probabilistic serial mechanism}

Another mechanism for the allocation of indivisible items to agents was introduced in Bogomolnaia and Moulin (2001). The mechanism, which is known as the probabilistic serial (PS) mechanism or the “eating algorithm”, works as follows: \textit{Agents report a rank-order list. Time runs continuously from 0 to 1. At every point in time, each student eats her reported favorite school with speed one among those that have not been completely eaten up. At time $t = 1$, each student is endowed with probability shares of schools. The probabilistic serial assignment is defined as the resulting probability shares.}\textsuperscript{5}

It is well-known that the PS mechanism is not strategy-proof in small economies (Kojima and Manea (2010)). Therefore, if we have a finite number of agents, then the unconstrained students always prefer the PS mechanism to deferred acceptance. However, PS is strategy-proof in our setting. To see why, note that agents are infinitesimal so no single agent’s strategy can change the total shares “eaten” from

\textsuperscript{4}The idea of ‘signaling’ preference to the other side of the market has been also discussed in dating markets; see Lee and Niederle (2015) for details.

\textsuperscript{5}The definition of this mechanism is taken from Budish et al. (2013).
any school. Now suppose on the contrary that there exists some equilibrium in which some agents are not truthful. For any agent \( i \) who is playing a non-truthful strategy, deviating to the truthful strategy is a weakly dominant strategy because his eating speed is fixed at one, so by deviation, he gets more share of his more preferred school, while losing equal shares from less preferred schools. Since PS is strategy-proof in our economy, it is ex ante equivalent to deferred acceptance.

5 Discussion

**A. Equality of opportunity and strategy-proofness.** It is no surprise that students who have more options available to them have weakly better outcomes under any mechanism that does not require participants to commit to their public school assignment. After all, if they are assigned to a less desirable school, they can always exit the public school system. However, our model makes a stronger prediction: when a manipulable mechanism is used, a student with better outside options is better off within the public school system.

It seems plausible for the social planner to prefer an allocation mechanism that satisfies some notion of equality of opportunity, in the sense that unconstrained students should get the same lottery within the public school system, among those schools that they prefer to their outside options. But this can only happen if all students get the exact same share for popular public schools, essentially generating the same assignment as the assignment generated by strategy-proof mechanisms such as deferred acceptance.

**B. Do school markets have a meaningful extensive margin?** It is typically hard to observe directly how many students in a school district participate in the public school assignment process, while preferring private education or charter schools over some of the available public schools. However, empirical facts based on school choice data suggest that in many school markets, there is a subset of students who are on the margin. Calsamiglia and Guell (2014) estimate that, in Barcelona, where the Boston mechanism is used for primary school assignment, 13.4% of students who do not receive an offer from their most-preferred public school end up enrolling in a school outside the public school system. Abdulkadiroglu et al. (2006) observe that, in Boston, before the change from Boston mechanism to deferred acceptance, between 10 and 14% of elementary, middle, and high school students withdrew from the public school system after having initially applied. Abdulkadiroglu et al. (2015) document that among NYC’s public middle school applicants, 2.5% enroll in private school after receiving their assignment. Finally, He (2017)}

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6This is indeed expected. Che and Kojima (2010) and Azevedo and Budish (2013) show that the PS mechanism is strategy-proof in the limit.
finds that more than 10% of students in Beijing chose their outside option over the school prescribed by the mechanism.

Our modeling device of asymmetry in outside options can be interpreted as a reduced-form representation of any differences in (exogenous determinants of) students’ choice sets. Of particular interest to us is asymmetry in conditions in a parallel market that is not under the control of the designer of the public school choice system. This is the example that we built the narrative of this paper around: differential access to private schools. This could happen because of differences in budget or borrowing constraints, but the required setup, with one type of students opting in and another opting out of public education if it is of sufficiently low quality, could also be the result of income effects when education is a normal good.

Additional examples with empirical relevance are easily produced. They include differences in access to information about the quality or application processes of schools outside the centralized system, to education markets other than the one for which the school choice system is designed, or to school seats that are allocated in a second round in the allocation process. Differences in distance to charter schools (which tend to have separate application systems) and neighborhood priorities within the public school system (e.g. in Boston or New Haven) are two other examples of asymmetric outside options.

C. Does available empirical evidence on strategic behavior and unequal outside options align with predictions from our model? (He, 2017) finds that, in Beijing, wealthier parents are less likely to strategize. Agarwal and Somaini (2016) estimate that students with free lunch status (a proxy for low-income families) are not more likely than students with paid lunch status to adopt a naive strategy. Similarly, Calsamiglia and Guell (2014) find that students whose parents are less highly educated are not more likely to be naive. The latter two observations are consistent with the idea that disadvantaged families are less strategically sophisticated (as described in Abdulkadiroglu et al. (2006) and Pathak and Sonmez (2008)), but only if families of a higher socio-economic status are also adopting a strategy that appears to be naive because they prefer their outside option over some public schools.

D. Justifying quotas and priorities. In practice, schools can and do impose quotas and do not always break ties uniformly at random. Quotas and priorities

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7Many school districts have such second rounds. e.g., Denver (Abdulkadiroglu et al., 2016), and parents may be able to influence later rounds through their social network.
8Kapor et al. (2016) state that in New Haven, “each student has a default school that he will be placed in if he does not receive a placement through the lottery.”
9See Kominers and Sonmez (2013); Budish et al. (2013); Kamada and Kojima (2014); Akbarpour and Nikzad (2015) for the role of diversity quotas in matching markets, and in particular school choice.
can serve to counteract the segregating forces of manipulable mechanisms that we described in this paper. But they can also severely restrict the extent to which preferences are reflected in final outcomes (Calsamiglia and Guell, 2014). Ideally, quotas and priorities would therefore be carefully deliberated. Our results suggest that quotas and priorities may be implemented to counteract the segregating forces of a manipulable assignment mechanism, and should therefore receive careful consideration if the mechanism itself is fixed to be manipulable. On the other hand, there may be a weaker justification for the same quota rules under strategy-proof mechanisms.

6 Conclusion

We have shown that in school choice settings with asymmetric choice sets, manipulable mechanisms such as the Boston mechanism skew outcomes in favor of students who face fewer constraints in other markets, potentially at the expense of more constrained students. In choosing between mechanisms to be implemented in practice, this feature of the Boston mechanism (manipulable mechanisms) can be added to the list of arguments in support of deferred acceptance (strategy-proof mechanisms), and should be weighed against potential benefits of the Boston mechanism such as those set out in Abdulkadiroglu et al. (2011) and Featherstone and Niederle (2008).

More broadly, our discussion shows that there are plausible circumstances under which asymmetric access to substitutes outside the control of the designer of a centralized matching market can have distributional consequences that are of particular interest in settings where publicly-funded resources are being allocated.
References


A  Definitions

Here we define Boston Mechanism and Deferred Acceptance.\footnote{Definitions are obtained from Abdulkadiroglu et al. (2005).}

A.1  Boston mechanism

The Boston mechanism assigns students as follows:

Step 0. Create a priority ordering of students uniformly at random.

Step 1. For each school, consider the students who have listed it as their first choice and assign seats to these students in priority order until either no seats remain or no student remains who has listed it as first choice.

Step k. For each school with seats still available, consider the students who have listed it as their $k^{th}$ choice and assign seats to these students in priority order until either no seats remain or no student remains who has listed it as $k^{th}$ choice.

A.2  Deferred acceptance

The Gale Shapley/Deferred Acceptance mechanism assigns students as follows:

Step 0. Create a priority ordering of students uniformly at random.

Step 1. Each student “proposes” to her first choice. Each school tentatively assigns its seats to its proposers one at a time in their priority order. Any remaining proposers are rejected.

Step k. Each student who was rejected in the previous step proposes to her next choice if one remains. Each school considers the students it has been holding together with its new proposers and tentatively assigns its seats to these students one at a time in priority order. Any remaining proposers are rejected.

The algorithm terminates when no student proposal is rejected, and each student is assigned her final tentative assignment.

B  Proof of Theorem 1

We prove that, compared to deferred acceptance, unconstrained students are always weakly better off under any (potentially manipulable) standard mechanism. Fix the mechanism. Let $\sigma_u^*(v)_{v \in V}$ and $\sigma_c^*(v)_{v \in V}$ be the symmetric equilibrium strategies of the unconstrained and constrained students, respectively. Let $\pi_j^u(v^i)$ be the probability that an unconstrained student with valuation $v^i$ goes to school $j$ in the Nash equilibrium. Define $\pi_j^c(v^i)$ similarly for constrained students. Our goal is to show that for an unconstrained student $i$: 
\[ \sum_{j=1}^{l} \pi_j^u(v^i)v_j^i \geq \sum_{j=1}^{l} q_j v_j^i. \] (1)

The left-hand side and the right-hand side are the expected utilities of unconstrained students under the Boston mechanism and deferred acceptance, respectively. Recall that \( s_l \) was the last public school that an unconstrained student \( i \) preferred to his outside option.

Now, using a similar strategy as ACY’s, suppose unconstrained students follow a different strategy and ‘mimic’ the population: for an unconstrained student \( i \) with valuation vector \( v^i \), with probability \( \eta f(v) \) they play the strategy \( \sigma^*_u(v) \) and with probability \( (1 - \eta) f(v) \) they play the strategy \( \sigma^*_c(v) \). In playing \( \sigma^*_c(v) \), unconstrained students drop schools with value less than their outside option from the list. The probability of going to school \( j \) under this strategy is at least:

\[ \sum_{v \in \mathcal{V}} \left( \eta \pi_j^u(v) + (1 - \eta) \pi_j^c(v) \right) f(v) = q_j \] (2)

It is not too hard to see why this inequality holds. The left-hand side is the total number of students assigned to a school, in expectation, and in the equilibrium of our continuum economy, this should be equal to the capacity of the school.

The utility of an constrained students from this new strategy is at least:

\[ \sum_{i=1}^{l} v_j^i \left( \sum_{v \in \mathcal{V}} (\eta \pi_j^u(v) + (1 - \eta) \pi_j^c(v)) f(v) \right) = \sum_{i=1}^{l} v_j^i q_j \] (3)

This is exactly their utility under deferred acceptance. Note that this is a lower bound on their utility, since by dropping those schools with value less than the outside option, they potentially increase their chances of going to schools they like.

This shows that an unconstrained student’s utility under this new strategy is at least equal to his utility under deferred acceptance. Clearly, they must be weakly better off under the original equilibrium strategy \( \sigma^*_u \), or else they could deviate to this new strategy that we just constructed. Hence, unconstrained students are weakly better off under any standard mechanism such as the Boston mechanism than under deferred acceptance. To complete the proof, note that if a student is not unconstrained, then our opening example shows that he can be worse off under a symmetric equilibrium produced by the Boston mechanism (compared to deferred acceptance equilibrium) so he will not always prefer the Boston mechanism. This completes the proof.