Solutions should be complete and concisely written. Please, use a separate sheet (or set of sheets) per each problem. Staple sheets referring to the same problem, and write your name on each sheet.

You are welcome to discuss problems with your colleagues, but should write and submit your own solution.

In some cases, multiple homework options will be proposed (and indicated as ‘Option 1’, ‘Option 2’, etc.). You are welcome to work on all the problems proposed (solutions will be posted), but should submit only those corresponding to one ‘Option’.

**Exercises on characteristic functions**

Solve Exercises [3.3.10], [3.3.20], [3.3.21], in Amir Dembo’s lecture notes.

**An exercise on weak convergence of measures**

Let \( \Omega = \{0, 1\}^\mathbb{N} \) be the set of (infinite) binary sequences \( \omega = (\omega_1, \omega_2, \omega_3, \ldots) \), and consider the topology generated by the following basis of neighborhoods of \( \omega \in \Omega \):

\[
N_\ell(\omega) = \{ \xi \in \Omega : \xi_1^\ell = \omega_1^\ell \},
\]

with \( \ell \in \mathbb{N} \) (here we use the notation \( \omega_1^\ell = (\omega_1, \ldots, \omega_\ell) \)). Let \( \mathcal{B}_\Omega \) be the Borel \( \sigma \)-algebra associated to this topology.

For \( n \) even, let \( A_n \) be the set of sequences defined as follows

\[
A_n = \{ \omega \in \Omega : \sum_{i=1}^n \omega_i = n/2, \ \omega_i = 0 \text{ for all } i > n \}.
\]

Consider the sequence of probability measures \( \{\nu_n\}_{n \in 2\mathbb{N}} \), with \( \nu_n \) the uniform distribution over \( A_n \), i.e.

\[
\nu_n(\{\omega\}) = \begin{cases} 
\left(\frac{n}{2}\right)^{-1} & \text{if } \omega \in A_n, \\
0 & \text{otherwise}.
\end{cases}
\]

1. Show that, for each \( n \), \( \nu_n \) is indeed a measure over \( \mathcal{B}_\Omega \).

2. What is the weak limit of \( \nu_n \) as \( n \to \infty \)? Prove your answer.

In solving the last point you can assume the following

**Fact 1.** Let \( h : \Omega \to \mathbb{R} \) be a continuous function. Then \( h \) is uniformly continuous in the following sense. There exists a function \( \delta : \mathbb{N} \to \mathbb{R} \), \( \ell \mapsto \delta(\ell) \), with \( \lim_{\ell \to \infty} \delta(\ell) = 0 \), such that, for any \( \omega \in \Omega \), \( \omega' \in N_\ell(\omega) \), we have \( |h(\omega') - h(\omega)| \leq \delta(\ell) \).