A quick note on doubling time.

-Michael Rosenfeld

Let's assume a rate of population annual increase of r. What is td, or the time to doubling?

$$(1.0r)^{td} = 2$$

take the log of both sides

$$\ln\left[(1.0r)^{td}\right] = \ln(2)$$

Use the property of logs that $\ln(x^a) = a\ln(x)$

$$td\left[\ln(1.0r)\right] = \ln(2)$$

The value of $\ln(2) \approx 0.7$ (actually it is 0.693, but 0.7 is commonly used here for simplicity)

$$td\left[\ln(1.0r)\right] = 0.7$$

$$td = \frac{0.7}{\ln(1.0r)}$$

So this is simple enough to calculate, but we can make it simpler by approximating $\ln(1.0r)$ using the taylor series expansion that

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} .....$$

When x is small as it is here, the first term of the taylor series expansion dominates, so:

$$\ln(1.0r) \approx \frac{r}{100}$$

So, substituting in:

$$td = \frac{0.7}{\ln(1.0r)} \approx \frac{0.7}{r/100} = \frac{70}{r}$$

So, if the population grows 2% per year, doubling time is 70/2=35 years.