B Web Appendix: Proofs and extensions.

B.1 Proofs of results about block correlated markets.

This subsection provides proofs for Propositions A1, A2, A3 and A4, and the proof of Lemma A1.

Proof of Proposition A1 (Equilibrium with no signals)**.** Consider some agent preference profile $\theta \in \Theta$. We will compare two strategies for firm f given its profile of preferences θ_f: strategy σ_f of making an offer to its top worker, and strategy σ'_f of making an offer to its nth ranked worker, $n > 1$. We have $\sigma_f(\theta) = \theta_f^1 \equiv w$ and $\sigma'_f(\theta) = \theta_f^n \equiv w^n$. We will show that for any anonymous strategies σ_{-f} of opponent firms $-f$, these two strategies yield identical probabilities of f being matched, so that f optimally makes its offer to its most preferred worker. The proposition straightforwardly follows.

Denote a permutation that changes the ranks of w and w^n in a firm preference list (or profile of firm preference lists) as

$$\rho : (..., w, ..., w^n, ...) \longrightarrow (..., w^n, ..., w, ...).$$

We now construct preference profile $\theta' \in \Theta$ from θ as follows:

- firm f preferences are the same as in $\theta : \theta'_f = \theta_f$,
- workers w and w^n are exchanged in the preference lists of firms $-f \,:\, \forall f' \in -f$, we have $\theta'_{f'} = \rho(\theta_{f'})$
- worker w and worker w^n preference profiles are exchanged: $\theta'_w = \theta_{w^n},\ \theta'_{w^n} = \theta_w$, and
- $\theta_{w'} = \theta'_{w'}$ for any other $w' \in \mathcal{W} \setminus \{w, w^n\}$.

Define function $m_f : (\Sigma_w)^W \times (\Sigma_f)^F \times \Theta \rightarrow \mathbf{R}$ as the probability of firm f being matched as a function of agent strategies and types. Since firm $-f$ strategies are anonymous we have

$$\sigma_{-f}(\theta'_{-f}) = \sigma_{-f}(\rho(\theta_{-f})) = \rho\left(\sigma_{-f}(\theta_{-f})\right)$$

Therefore, the probability of firm f', $f' \in -f$, making an offer to worker w for profile θ equals the probability of making an offer to worker w^n for profile θ'. Moreover, since we exchange worker w and w^n preference lists for profile θ', whenever it is optimal for worker w to accept firm f offer for profile θ, it is optimal for worker w^n to accept firm f's offer for profile θ'. Therefore,

$$m_f(\sigma_f, \sigma_{-f}, \theta) = m_f(\sigma'_f, \sigma_{-f}, \theta')$$

In other words, given θ_f, for each θ_{-f} there exists θ'_{-f} such that the probability of f's offer to θ_f^1 being accepted when opponent preferences are θ_{-f} equals the probability of f's offer to θ_f^n being accepted when opponent preferences are θ'_{-f}.[29] Moreover θ'_{-f} is different for different θ_{-f} by construction. Since θ_{-f} and θ'_{-f} are equally likely, we have

$$E_{\theta_{-f}} m_f(\sigma_f, \sigma_{-f}, \theta \,|\, \theta_f) = E_{\theta_{-f}} m_f(\sigma'_f, \sigma_{-f}, \theta \,|\, \theta_f)$$

and

$$E_\theta m_f(\sigma_f, \sigma_{-f}, \theta) = E_\theta m_f(\sigma'_f, \sigma_{-f}, \theta).$$

That is, the expected probability of getting a match from firm f's top choice equals the expected probability of getting a match from firm f's nth ranked choice. Since the utility from obtaining a top match is greater, the strategy of firm f of making an offer to its top worker is optimal. □

Proof of Proposition A2 (Binary nature of firm optimal offer)**.** Consider firm f from some block $\mathcal{F}_b$, $b \in \{1, ..., B\}$ that has realized preference profile $\theta^* \in \Theta_f$ and that receives signals from the set of workers $\mathcal{W}^\mathcal{S} \subset \mathcal{W}$. Denote worker S_f as w and select arbitrary other worker $w' \in \mathcal{W}^\mathcal{S}$. We first prove that the expected payoff to f from making an offer to worker w is strictly greater than the expected payoff from making an offer to worker w'. We denote the strategies of firm f that correspond to these actions as $\sigma_f(\theta^*, \mathcal{W}^\mathcal{S}) = w$ and $\sigma'_f(\theta^*, \mathcal{W}^\mathcal{S}) = w'$.

Workers use symmetric best-in-block strategies and firms have best-in-block beliefs. Specifically, firm f believes that it is the top firm within block $\mathcal{F}_b$ in the preference lists of workers w and w'. Denote the set of all possible agents' profiles consistent with firm f beliefs as[30]

$$\bar{\Theta} \equiv \{\theta \in \Theta \,|\, \theta_f = \theta^* \text{ and } f = \max_{\theta_w}(f' \in \mathcal{F}_{b'}) \text{ for each } w \in \mathcal{W}^\mathcal{S}\}$$

As in the proof of Proposition A1, we denote a permutation that changes the ranks of w

[29] In this context, θ_{-f} is a preference profile for all agents – both workers and firms – other than f.

[30] For the case of one block of firms, firm f beliefs also exclude preference profiles where firm f is a top firm for those workers that did not send signal to firm f.

$$\bar{\Theta} \equiv \{\theta \in \Theta \,|\, \theta_f = \theta^*,\ f = \max_{\theta_w}(f' \in \mathcal{F}_{b'}) \text{ for each } w \in \mathcal{W}^\mathcal{S},\ \text{ and } f \neq \max\nolimits_{\theta_w}(f' \in \mathcal{F}_{b'}) \text{ for each } w \in \mathcal{W} \backslash \mathcal{W}^\mathcal{S}\}.$$

For simplicity, we assume that there are at least two blocks. All the derivations are also valid without change for the case of one block.

and w' in a firm preference list (or profile of firm preference lists) as

$$\rho : (..., w, ...w', ...) \to (..., w', ...w, ...).$$

We now construct preference profile $\theta' \in \Theta$ from θ^* as follows:

- firm f preferences are the same as in θ^*: $\theta'_f = \theta^*$,
- workers w and w' are exchanged in the preference lists of firms $-f$: $\forall f' \in -f$, we have $\theta'_{f'} = \rho(\theta_{f'})$,
- worker w and worker w' preference profiles are exchanged $\theta'_w = \theta_{w'}$, $\theta'_{w'} = \theta_w$, and
- for any other $w^0 \in \mathcal{W} \backslash \{w, w'\}$, $\theta_{w^0} = \theta'_{w^0}$.

Since firm f 's preference list is unchanged and since w,$w' \in \mathcal{W}^{\mathcal{S}}$, profile θ' belongs to $\bar{\Theta}$. Since strategies of firms $-f$ are anonymous, then for any $f' \in -f$ and for any $\mathcal{W}^{\mathcal{S}}_{f'} \subset \mathcal{W}$ we have

$$\sigma_{f'}(\rho(\theta_{f'}), \rho(\mathcal{W}^{\mathcal{S}}_{f'})) = \rho\left(\sigma_{f'}(\theta_{f'}, \mathcal{W}^{\mathcal{S}}_{f'})\right).$$

Worker w and w' send their signals to firm f under both profile θ and θ'. Therefore, they do not send their signals to firms $-f$, i.e. $\rho(\mathcal{W}^{\mathcal{S}}_{f'}) = \mathcal{W}^{\mathcal{S}}_{f'}$. Since $\theta'_f = \rho(\theta_f)$ we have

$$\sigma_{f'}(\theta'_{f'}, \mathcal{W}^{\mathcal{S}}_{f'}) = \rho\left(\sigma_{f'}(\theta_{f'}, \mathcal{W}^{\mathcal{S}}_{f'})\right).$$

This means that the probability of firm f' making an offer to worker w for profile θ equals the probability of making an offer to worker w' for profile θ'. Moreover, since we exchange worker w and w' preference lists for profile θ', whenever it is optimal for worker w to accept firm $f's$ offer under profile θ, it is optimal for worker w' to accept an offer from firm f' under profile θ'. Since firm types are independent, the probability of firm f being matched when it uses strategy σ_f for profile θ equals the probability of firm f being matched when it uses strategy σ'_f for profile θ' :

$$m_f(\sigma_f, \sigma_{-f}, \theta) = m_f(\sigma'_f, \sigma_{-f}, \theta').$$

Therefore, for each $\theta \in \bar{\Theta}$ there exists $\theta' \in \bar{\Theta}$ such that the probability that firm f gets an offer from worker w equals the probability that firm f gets an offer from worker w'. Moreover, profile θ' is different for different θ by our construction. Since θ and θ' are equally likely,

$$E_\theta m_f(\sigma_f, \sigma_{-f}, \theta \,|\, \theta \in \bar{\Theta}) = E_\theta m_f(\sigma'_f, \sigma_{-f}, \theta \,|\, \theta \in \bar{\Theta}).$$

Therefore, the expected probability that firm f gets a match if it makes an offer to some worker in $\mathcal{W}^S$ is the same across all workers in $\mathcal{W}^S$. But within this set, a match with S_f offers the greatest utility, so the expected payoff to f from making an offer to S_f is strictly greater than the payoff from making an offer to any other worker in $\mathcal{W}^S$.

A similar construction is valid for the workers in set $\mathcal{W}\backslash\mathcal{W}^S$. That is, the probability that firm f's offer is accepted is the same across all workers in $\mathcal{W}\backslash\mathcal{W}^S$. Hence, firm f prefers making an offer to its most valuable worker, T_f, than to any other worker in $\mathcal{W}\backslash\mathcal{W}^S$.[31] $\square$

Proof of Proposition A3 (Optimality of Cutoff Strategies)**.** If workers use best-in-block strategies and firms have best-in-block beliefs, the optimal choice of firm f for each set of received signals is either S_f or T_f (or some lottery between them) (see Proposition A2). In light of this, we break the proof into two parts. First we show that the identities of workers that have sent a signal to firm f influence neither the expected payoff of making an offer to S_f nor the expected payoff of making an offer to T_f, conditional on the total number of signals received by f remaining constant. Second we prove that if it is optimal for firm f to choose S_f when it receives signals from some set of workers, then it still optimal for firm f to choose S_f if the number of received signals does not change and S_f has a smaller rank (S_f is more valuable to f).

Let us consider some firm f from block $\mathcal{F}_b$, $b \in \{1, ..., B\}$ and some realization θ^* of its preference list. Assume that it is optimal for firm f to make an offer to S_f if it receives a set of signals $\mathcal{W}^S \subset \mathcal{W}$. We want to show that if firm f receives the set of signals $\mathcal{W}^{S'}$ such that $S_f(\theta^*, \mathcal{W}^S) = S_f(\theta^*, \mathcal{W}^{S'})$ and $\left|\mathcal{W}^{S'}\right| = \left|\mathcal{W}^S\right|$, it is still optimal for firm f to make an offer to S_f. For simplicity, we only consider the case when $\mathcal{W}^S$ and $\mathcal{W}^{S'}$ differ only in one signal. (The general case then follows straightforwardly.) That is, there exist worker w and worker w' such that w belongs to set $\mathcal{W}^S$, but not to set $\mathcal{W}^{S'}$; while w' belongs to $\mathcal{W}^{S'}$, but not to $\mathcal{W}^S$. We consider two firm f strategies for realization of signals $\mathcal{W}^S$ and $\mathcal{W}^{S'}$.

$$\sigma_f(\theta^*, \cdot) = S_f(\theta^*, \cdot)$$
$$\sigma'_f(\theta^*, \cdot) = T_f(\theta^*, \cdot).$$

We denote the set of possible agents' profiles that are consistent with firm f having received

[31] It is certainly possible that $T_f = S_f$. In this case the statement of the proposition is still valid. Firm f believes that it is T_f's top firm within block $\mathcal{F}_b$ and firm f prefers making an offer to $T_f = S_f$ rather than to any other worker in $\mathcal{W}$.

signals from $\mathcal{W}^S$ and $\mathcal{W}^{S'}$ as[32]

$$\begin{aligned}\bar{\Theta}^S &\equiv \{\theta \in \Theta \,|\, \theta_f = \theta^* \text{ and } f = \max_{\theta_w}(f' \in \mathcal{F}_{b'}) \text{ for each } w \in \mathcal{W}^S\} \\ \bar{\Theta}^{S'} &\equiv \{\theta \in \Theta \,|\, \theta_f = \theta^* \text{ and } f = \max_{\theta_w}(f' \in \mathcal{F}_{b'}) \text{ for each } w \in \mathcal{W}^{S'}\}\end{aligned}$$

correspondingly. We now construct a bijection between $\bar{\Theta}^S$ and $\bar{\Theta}^{S'}$. Denote a permutation that changes the ranks of w and w' in a firm preference profile as

$$\rho : (..., w, ...w', ...) \longrightarrow (..., w', ...w, ...).$$

For any profile $\theta \in \bar{\Theta}^S$ we construct profile $\theta' \in \Theta$ as follows:

- firm f preferences are the same as in θ: $\theta'_f = \theta^*$,
- the ranks of workers w and w' are exchanged in the preference lists of firms $-f$: $\forall f' \in -f,\ \theta'_f = \rho(\theta_f)$,
- the preference lists of worker w and worker w' are exchanged: $\theta'_w = \theta_{w'},\ \theta'_{w'} = \theta_w$, and
- for any other $w^0 \in \mathcal{W} \backslash \{w, w'\}$, $\theta_{w^0} = \theta'_{w^0}$.

Since this construction leaves the preference list of firm f unchanged, and since workers w and w' swap preference lists, we have that if $\theta \in \bar{\Theta}^S$, then $\theta' \in \bar{\Theta}^{S'}$. By construction, profile θ' is different for different θ. Finally, since the cardinality of sets $\bar{\Theta}^S$ and $\bar{\Theta}^{S'}$ are the same, the above correspondence is a bijection.

Since firm $-f$ strategies are anonymous, for any $f' \in -f$ and $\mathcal{W}^S_{f'} \subset \mathcal{W}$

$$\sigma_{f'}(\rho(\theta_{f'}), \rho(\mathcal{W}^S_{f'})) = \rho\left(\sigma_{f'}(\theta_{f'}, \mathcal{W}^S_{f'})\right).$$

This means that the probability of firm f' making an offer to worker w for any profile θ equals the probability of firm f' making an offer to worker w' for corresponding profile θ'. Moreover, since we exchange worker w and w' preference lists for profile θ', whenever it is optimal for worker w to accept firm f offer for profile θ, it is optimal for worker w' to accept firm f's offer for profile θ'. Since firms types are independent, the probability of firm f being matched when it uses strategy $\sigma_f(\theta^*, \cdot)$ for profile θ equals the probability of firm f being matched when it uses strategy $\sigma_f(\theta^*, \cdot)$ for profile θ':

$$m_f(\sigma_f, \sigma_{-f}, \theta) = m_f(\sigma_f, \sigma_{-f}, \theta').$$

[32]See footnote 30 for the definition of firm beliefs for the case of one block.

Similarly, for strategy $\sigma'_f(\theta^*, \cdot)$ we have

$$m_f(\sigma'_f, \sigma_{-f}, \theta) = m_f(\sigma'_f, \sigma_{-f}, \theta').$$

Since our construction is a bijection between $\bar{\Theta}^S$ and $\bar{\Theta}^{S'}$, and since θ and θ' are equally likely, we have

$$\begin{aligned} E_\theta m_f(\sigma_f, \sigma_{-f}, \theta \,|\, \theta &\in \bar{\Theta}^S) = E_\theta m_f(\sigma_f, \sigma_{-f}, \theta' \,|\, \theta' \in \bar{\Theta}^{S'}) \\ E_\theta m_f(\sigma'_f, \sigma_{-f}, \theta \,|\, \theta &\in \bar{\Theta}^S) = E_\theta m_f(\sigma'_f, \sigma_{-f}, \theta' \,|\, \theta' \in \bar{\Theta}^{S'}). \end{aligned}$$

Therefore, if firm f optimally makes an offer to S_f (T_f) when it has received set of signals $\mathcal{W}^S$, it also should optimally make an offer to S_f (T_f), which is the same worker, for the set of signals $\mathcal{W}^{S'}$.

We now prove that if firm f optimally chooses $S_f(\theta^*, \mathcal{W}^S)$ when it receives signals from $\mathcal{W}^S$, then it should still optimally choose $S_f(\theta^*, \mathcal{W}^{S'})$ for set of signals $\mathcal{W}^{S'}$, if the number of received signals is the same $\left|\mathcal{W}^{S'}\right| = \left|\mathcal{W}^S\right|$ and $S_f(\theta^*, \mathcal{W}^{S'})$ has a smaller rank, that is, when the signaling worker is more valuable to f. We consider set $\mathcal{W}^{S'}$ that differs from $\mathcal{W}^S$ only in the best (for firm f) worker and the difference between the ranks of top signaled workers equals one. (The general case follows straightforwardly.) That is,

$$\begin{gathered} w \in \mathcal{W}^S / S_f(\theta^*, \mathcal{W}^S) \Leftrightarrow w \in \mathcal{W}^{S'} / S_f(\theta^*, \mathcal{W}^{S'}) \quad \text{and} \\ rank_f(S_f(\theta^*, \mathcal{W}^{S'})) = rank_f(S_f(\theta^*, \mathcal{W}^S)) - 1. \end{gathered}$$

The construction in the first part of the proof works again in this case. Using sets of profiles and a correspondence similar to the one above, we can show that the probabilities of firm f being matched with S_f (T_f) are the same for $\mathcal{W}^S$ and $\mathcal{W}^{S'}$. Observe that if firm f's offer to T_f is accepted, naturally firm f gets the same payoff for sets $\mathcal{W}^S$ and $\mathcal{W}^{S'}$. If firm f's offer to S_f is accepted, firm f gets strictly greater payoff for set $\mathcal{W}^{S'}$ compared to set $\mathcal{W}^S$, because by definition $S_f(\theta^*, \mathcal{W}^{S'})$ has smaller rank than $S_f(\theta^*, \mathcal{W}^S)$. Hence, if it is optimal for firm f to make an offer to $S_f(\theta^*, \mathcal{W}^S)$ when it receives set of signals $\mathcal{W}^S$, it is optimal for firm f to make an offer to $S_f(\theta^*, \mathcal{W}^{S'})$ when firm f receives set of signals $\mathcal{W}^{S'}$.

Combined, the two statements we have just proved allow us to conclude that if firms $-f$ use anonymous strategies, firm f's optimal strategy can be represented as some cutoff strategy.[33] □

[33] Note that there can be other optimal strategies. If firm f is indifferent between making an offer to S_f and making an offer to T_f for some set of signals, firm f could optimally make its offer to S_f or to T_f for any set of signals conditional on maintaining the same rank of the most preferred signaling worker and cardinality of signals received.

Proof of Proposition A4 (Strategic complements under block correlation). Consider some firm f from some block $\mathcal{F}_b$, $b \in \{1, ..., B\}$. We consider two strategy profiles, σ_{-f} and σ'_{-f}, for firms $-f$ that vary only in the strategy for firm f' . For simplicity, we assume that $\sigma'_{f'}$ differs from $\sigma_{f'}$ only for some profile $\bar{\theta}_{f'}$ and some set of received signals $\overline{\mathcal{W}^S_{f'}}$

$$\begin{aligned} \sigma_{f'}(\bar{\theta}_{f'}, \overline{\mathcal{W}^S_{f'}}) &= \alpha S_{f'} + (1-\alpha) T_{f'} \\ \sigma'_{f'}(\bar{\theta}_{f'}, \overline{\mathcal{W}^S_{f'}}) &= \alpha' S_{f'} + (1-\alpha') T_{f'} \end{aligned}$$

such that $\alpha' > \alpha$. Formally, this means σ'_f is not a cutoff strategy, because a cutoff strategy requires the same behavior for any profile of preferences (anonymity) when firms receive the same number of signals. We will prove the statement using our simplifying assumption about strategies for firms $-f$, and the extension to the full proposition follows from iterated application of this result.

Consider some realized firm f preference profile $\theta^*_f \in \Theta$ and some set of signals $\mathcal{W}^S \subset \mathcal{W}$. We want to show that firm f's payoff from making an offer to T_f (weakly) decreases whereas firm f's payoff from making an offer to S_f (weakly) increases when firm f' responds more to signals, i.e. plays strategy $\sigma'_{f'}$ instead of $\sigma_{f'}$. That is,

$$\begin{aligned} I)\ E_\theta(\pi_f(T_f, \sigma_{-f}, \theta) \,|\, \theta_f &= \theta^*_f, \mathcal{W}^S_f = \mathcal{W}^S) \geq E_\theta(\pi_f(T_f, \sigma'_{-f}, \theta) \,|\, \theta_f = \theta^*_f, \mathcal{W}^S_f = \mathcal{W}^S) \\ II)\ E_\theta(\pi_f(S_f, \sigma_{-f}, \theta) \,|\, \theta_f &= \theta^*_f, \mathcal{W}^S_f = \mathcal{W}^S) \leq E_\theta(\pi_f(S_f, \sigma'_{-f}, \theta) \,|\, \theta_f = \theta^*_f, \mathcal{W}^S_f = \mathcal{W}^S). \end{aligned}$$

Since firm f's offer can only be either accepted or declined, the above statements are equivalent to

$$\begin{aligned} I)\ E_\theta(m_f(T_f, \sigma_{-f}, \theta) \,|\, \theta_f &= \theta^*_f, \mathcal{W}^S_f = \mathcal{W}^S) \geq E_\theta(m_f(T_f, \sigma'_{-f}, \theta) \,|\, \theta_f = \theta^*_f, \mathcal{W}^S_f = \mathcal{W}^S) \\ II)\ E_\theta(m_f(S_f, \sigma_{-f}, \theta) \,|\, \theta_f &= \theta^*_f, \mathcal{W}^S_f = \mathcal{W}^S) \leq E_\theta(m_f(S_f, \sigma'_{-f}, \theta) \,|\, \theta_f = \theta^*_f, \mathcal{W}^S_f = \mathcal{W}^S). \end{aligned}$$

That is, we wish to show that the probability of being matched to T_f weakly decreases, and the probability of being matched to S_f weakly increases.

We first prove *I)* first. Define the sets of agent profiles that lead to the increase and decrease in the probability of getting a match given the change in firm f' strategy as

$$\begin{aligned} \bar{\Theta}_+ &\equiv \{\theta \in \Theta \,|\, \theta_f = \theta^*_f, \mathcal{W}^S_f = \mathcal{W}^S \text{ and } m_f(T_f, \sigma_{-f}, \theta) < m_f(T_f, \sigma'_{-f}, \theta)\} \\ \bar{\Theta}_- &\equiv \{\theta \in \Theta \,|\, \theta_f = \theta^*_f, \mathcal{W}^S_f = \mathcal{W}^S \text{ and } m_f(T_f, \sigma_{-f}, \theta) > m_f(T_f, \sigma'_{-f}, \theta)\} \end{aligned}$$

correspondingly. If set $\bar{\Theta}_+$ is empty, the statement has been proved. Otherwise, select arbitrary $\theta \in \bar{\Theta}_+$ and denote $T_f \equiv w$. Since in this case, f''s strategy change pivotally reduces competition to f's offer to w, we must have $T_{f'}(\bar{\theta}_{f'}, \overline{\mathcal{W}^S_{f'}}) = w$ and $S_{f'}(\bar{\theta}_{f'}, \overline{\mathcal{W}^S_{f'}}) = w' \neq w$, and

$$\begin{aligned}
\sigma_{f'}(\bar{\theta}_{f'}, \overline{\mathcal{W}^S_{f'}}) &= \alpha w' + (1-\alpha)w \\
\sigma'_{f'}(\bar{\theta}_{f'}, \overline{\mathcal{W}^S_{f'}}) &= \alpha' w' + (1-\alpha')w.
\end{aligned}$$

Note that it cannot be that firm f is from a higher ranked block than firm f' , i.e. $f' \in \mathcal{F}_{b'}$ where $b' > b$. If f were from a higher ranked block, an offer from firm f' is always worse than the offer of firm f and could not influence the probability that firm f obtains a match. Therefore, firm f is from a block that is weakly worse than $\mathcal{F}_{b'}$, i.e. $b' \leq b$.

Note that under θ, worker w has sent a signal neither to firm f nor to firm f'. This will allow us to construct element $\theta' \in \bar{\Theta}_-$. Consider a permutation that changes the ranks of w and w' in a firm preference profile

$$\rho : (..., w, ...w', ...) \longrightarrow (..., w', ...w, ...).$$

For any profile $\theta \in \bar{\Theta}_+$ we construct profile $\theta' \in \Theta$ as follows:

- $\theta'_f = \theta^*_f$
- the ranks of workers w and w' are exchanged in the preference lists of firms $-f$: for each firm $f' \in -f$, $\theta'_{f'} = \rho(\theta_{f'})$
- worker w and worker w' preference profiles are exchanged: $\theta'_w = \theta_{w'}$, $\theta'_{w'} = \theta_w$, and
- for any other $w^0 \in \mathcal{W} \setminus \{w, w'\}$, $\theta_{w^0} = \theta'_{w^0}$.

Note that under θ and θ', firm f has the same preferences θ^*_f and receives the same set of signals.

Since firm strategies are anonymous we have that

$$\begin{aligned}
\sigma_{f'}(\theta'_{f'}, \mathcal{W}^{S'}_{f'}) &= \sigma_{f'}(\rho(\theta_{f'}), \rho(\mathcal{W}^S_{f'})) \text{ (by our construction)} \\
&= \alpha\rho(w') + (1-\alpha)\rho(w) \quad \text{(by anonymity)} \\
&= \alpha w + (1-\alpha)w'
\end{aligned}$$

and similarly

$$\sigma'_{f'}(\theta'_{f'}, \mathcal{W}^{S'}_{f'}) = \alpha' w + (1-\alpha')w'.$$

We will now argue that $\theta' \in \bar{\Theta}_-$. Since $\theta \in \bar{\Theta}_+$, the strategy change for firm f' reduces the likelihood of firm f being matched with worker w (when f makes T_f an offer under profile θ). Under profile θ', firm f' makes an offer to worker w more frequently when using strategy $\sigma'_{f'}$ rather than $\sigma_{f'}$. Furthermore, worker w prefers firm f' to firm f under profile θ'. (We have already shown that f' cannot be in a lower ranked block than f. If firm f' is in a higher ranked block $\mathcal{F}_{b'}$, $b > b'$, worker w always prefers firm f' to firm f. If firm f and firm f' are from the same block, $b = b'$, worker w prefers f to f', since worker w sends a signal to firm f' under profile θ').

To finish our proof, we must also investigate the behavior of a firm that receives worker w's signal for profile θ, say firm f_y. If firm f_y makes an offer to worker w for profile θ, since the change of firm f' strategy changes firm f's payoff, firm f_y must be lower ranked than both firms f and f' in worker w's preferences. Hence, firm f_y's offer cannot change the action of worker w. If worker w' sends her signal to firm f_y then firm f_y either makes an offer to worker w' or to worker T_{f_y}, which are both different from worker w.

Hence, firm f_y does not influence the behavior of the agents in question, and the overall probability that firm f's offer to worker w is accepted is smaller when firm f' uses strategy $\sigma'_{f'}$ rather than $\sigma_{f'}$. That is, $\theta' \in \bar{\Theta}_-$.

Note that the above construction gives different profiles in $\bar{\Theta}_+$ for different profiles of $\bar{\Theta}_-$. Hence, our construction is an injective function from $\bar{\Theta}_+$ to $\bar{\Theta}_-$, so $|\bar{\Theta}_-| \geqslant |\bar{\Theta}_+|$.[34] Since profiles θ and θ' are equally likely, we have

$$E_\theta(m_f(T_f, \sigma_{-f}, \theta) \,|\, \theta_f = \theta_f^*, \mathcal{W}_f^S = \mathcal{W}^S) \geq E_\theta(m_f(T_f, \sigma'_{-f}, \theta) \,|\, \theta_f = \theta_f^*, \mathcal{W}_f^S = \mathcal{W}^S).$$

We now prove inequality *II)*. That is, we will show that if firm f' responds more to signals, the probability of firm f being matched to S_f (upon making S_f an offer) weakly increases. If firm f, $f \in \mathcal{F}_b$, receives a signal from worker w it believes it is the best firm in block $\mathcal{F}_b$ according to worker w's preferences. That is, worker w prefers the offer of firm f to an offer from any other firm f' from any block $\mathcal{F}_{b'}$ with $b' \geq b$. Therefore, the change of the behavior of any firm f' from block $\mathcal{F}_{b'}$, $b' \geq b$, does not influence firm f's payoff.

If we consider some firm f' from group $\mathcal{F}_{b'}$, $b' < b$, it can draw away worker w's offer from firm f only if it makes an offer to worker w. However, firm f' makes an offer to worker w, conditionally on firm f receiving a signal from worker w, only when worker w is $T_{f'}$. However, if firm f' responds more to signals, it makes an offer to its T_f more rarely. This means that firm f' draws worker w away from firm f less often. Therefore, the probability that firm f's offer is accepted by S_f increases:

$$E_\theta(m_f(S_f, \sigma_{-f}, \theta) \,|\, \theta_f = \theta_f^*, \mathcal{W}_f^S = \mathcal{W}^S) \leq E_\theta(m_f(S_f, \sigma'_{-f}, \theta) \,|\, \theta_f = \theta_f^*, \mathcal{W}_f^S = \mathcal{W}^S).$$

[34] One may show by example that this is not, in general, a bijection.

As a corollary of *I)* and *II)*, if firm f' increases its cutoff point for some set of signals, firm f will also optimally (weakly) increase its cutoff points. The above logic is valid for the change of cutoff points for any set of signals of the same size and any profile of preferences, so the statement of the proposition immediately follows. □

Proof of Lemma A1. We prove the first statement first. Let us consider firm f cutoff strategies σ_f and σ'_f such that σ'_f has weakly greater cutoffs. We consider two sets of preference profiles

$$\bar{\Theta}_+ \equiv \{\theta \in \Theta \mid m(\sigma_f, \sigma_{-f}, \theta) < m(\sigma'_f, \sigma_{-f}, \theta)\}$$
$$\bar{\Theta}_- \equiv \{\theta \in \Theta \mid m(\sigma_f, \sigma_{-f}, \theta) > m(\sigma'_f, \sigma_{-f}, \theta)\}.$$

For each profile θ from set Θ^+, it must be the case that without firm f's offer, T_f has an offer from another firm and worker S_f does not:

$$m(\sigma'_f, \sigma_{-f}, \theta) - m(\sigma_f, \sigma_{-f}, \theta) = 1. \tag{B.1.1}$$

Similarly, if profile θ is from set Θ^-, it must be the case that without firm f offer, S_f has an offer from another firm, and T_f does not

$$m(\sigma'_f, \sigma_{-f}, \theta) - m(\sigma_f, \sigma_{-f}, \theta) = -1. \tag{B.1.2}$$

We will now show that $|\bar{\Theta}_+| \geq |\bar{\Theta}_-|$. Equations (B.1.1) and (B.1.2), along with the fact that each $\theta \in \Theta^+ \cup \Theta^-$ occurs equally likely, will then be enough to prove the result.

Let us denote $T_f = w'$ and $S_f = w$. We construct function $\psi : \Theta \to \Theta$ as follows. Let $\psi(\theta)$ be the profile in which workers have preferences as in θ, but firms $-f$ all swap the positions of workers w' and w in their preference lists. If profile θ belongs to $\bar{\Theta}_-$, without firm f's offer, worker w has an offer from another firm, and worker w' does not. Therefore, when preferences are $\psi(\theta)$, without firm f's offer the following two statements must be true: *i)* worker w' ***must*** have another offer and *ii)* worker w ***cannot*** have another offer.

To see *i)*, note that under θ, worker w sends a signal to firm f, so his outside offer must come from some firm f' who has ranked him first. Under profile $\psi(\theta)$, firm f' ranks worker w' first. If worker w' has not sent a signal to firm f', then by anonymity, w' gets the offer of firm f'. If worker w' has signaled to firm f', worker w' again gets firm f''s offer.

To see *ii)*, suppose to the contrary that under $\psi(\theta)$, worker w does in fact receive an offer from some firm $f' \neq f$. Since worker w sends a signal to firm f, worker w must be $T_{f'}$ under $\psi(\theta)$, so that worker w' is $T_{f'}$ under θ. But then by anonymity w' receives the offer of firm f' under θ, a contradiction.

From i) and ii), we have

$$\theta \in \bar{\Theta}_{-} \Rightarrow \psi(\theta) \in \bar{\Theta}_{+}.$$

Since function ψ is injective, we have $|\bar{\Theta}_{+}| \geq |\bar{\Theta}_{-}|$.

In order to prove the second statement note that the expected number of matches of each worker increases when firm f responds more to signals. Using the construction presented above, one can show that whenever worker w "loses" a match with firm f for profile θ (worker w is T_f) it is possible to construct profile θ' when worker w obtains a match (worker w is S_f). The function that matches these profiles is again injective. Moreover, worker w values more greatly the match with firm f when she has signaled it (S_f) rather when she is simply highest ranked (T_f). Therefore, the ex-ante utility of worker w increases when firm f responds more to signals. □

B.2 Market Structure and the Value of a Signaling Mechanism — Proofs and Extensions

This set of results pertains to Section 7: Market Structure and the Value of a Signaling Mechanism. In this section, we denote as $u(j)$ the utility of a firm from matching with its jth ranked worker.

The first proposition states that when preferences over workers are sufficiently flat, then in any non-babbling equilibrium firms always respond to signals.

Proposition B1. *Under the assumption that*

$$u(W) > \tfrac{W}{F}\left(1 - \left(1 - \tfrac{1}{W}\right)^{F}\right) u(1) \tag{B.2.1}$$

there is a unique non-babbling equilibrium in the offer game with signals. Each worker sends her signal to her top firm. Each firm f makes an offer to S_f if it receives at least one signal; otherwise, firm f makes an offer to T_f.

Proof. We will show that under condition (B.2.1) even if S_f is the worst ranked worker in firm f preferences, firm f still optimally makes her an offer.

Proposition 2 shows that if firms $-f$ respond more to signals, i.e. increase their cutoffs, it is also optimal for firm f to respond more to signals. Therefore, if firm f optimally responds to signals when no other firm does, it will certainly optimally respond to signals when other firms respond. Hence, it will be enough to consider the incentives of firm f when firms $-f$ do not respond to signals and always make an offer their top ranked workers.

Let us consider some realized profile of preferences of firm f and denote T_f as w. If firms $-f$ do not respond to signals, then some firm among $-f$ makes an offer to worker w with probability $q = \frac{1}{W}$. Therefore, the probability that the offer of firm f to worker w is accepted equals

$$(1-q)^{F-1} + ... + C_{F-1}^{j} q^{j} (1-q)^{F-1-j} \frac{1}{j+1} + ... + q^{F-1} \frac{1}{F} \tag{B.2.2}$$

where $C_x^y = \frac{x!}{y!(x-y)!}$. Intuitively, j firms among the other $F-1$ firms simultaneously make an offer to worker w with probability $C_{F-1}^{j} q^{j} (1-q)^{F-1-j}$. Therefore, firm f is matched with worker w only with probability $\frac{1}{j+1}$ because worker w's preferences are uniformly distributed. The sum over all possible j from 0 to $F-1$ gives us the overall probability of firm f's offer being accepted. We can simplify this expression as follows:

$$\sum_{j=0}^{F-1} C_{F-1}^{j} q^{j} (1-q)^{F-1-j} \frac{1}{j+1} \tag{B.2.3}$$

$$= \sum_{j=0}^{F-1} \frac{(F-1)!}{j!(F-1-j)!} q^{j} (1-q)^{F-1-j} \frac{1}{j+1} \tag{B.2.4}$$

$$= \sum_{j=0}^{F-1} \frac{1}{Fq} \frac{F!}{(j+1)!(F-(1+j))!} q^{j+1} (1-q)^{F-(1+j)} \tag{B.2.5}$$

$$= \frac{1}{Fq} \sum_{j=1}^{F} \frac{F!}{j!(F-j)!} q^{j} (1-q)^{F-j} \tag{B.2.6}$$

$$= \frac{1}{Fq} \left(\sum_{j=0}^{F} \frac{F!}{j!(F-j)!} q^{j} (1-q)^{F-j} - (1-q)^{F} \right) \tag{B.2.7}$$

$$= \frac{1}{Fq} \left(1 - (1-q)^{F}\right) = \frac{W}{F} \left(1 - \left(1 - \frac{1}{W}\right)^{F}\right). \tag{B.2.8}$$

Alternatively, if firm f makes an offer to its top signaling worker, its offer is accepted with probability one. Therefore, it is optimal for the firm to make an offer to the signaling worker only if $u(W) > \frac{W}{F}\left(1 - \left(1 - \frac{1}{W}\right)^{F}\right) u(1)$. We conclude that under assumption B.2.1 there is no other non-babbling symmetric equilibrium in the offer game with signals. □

The following proposition characterizes equilibria in the multi-period offer game.

Proposition B2. *Consider the following assumptions on agent utility functions and the discount factor.*

$$u(W) > \frac{W}{F}\left(1 - \left(1 - \frac{1}{W}\right)^{F}\right) u(1)$$
$$u(W) > \delta u(1), \; v(W) > \delta v(1)$$

Then

1. *There is a unique symmetric sequential equilibrium in the offer game with no signals and L periods of interaction: each firm makes an offer to its most preferred worker and each worker accepts its best offer in each period.*

2. *There is a unique symmetric, sequential, non-babbling (in each period) equilibrium in*

the offer game with signals and L periods of interaction: In period 0, each worker sends her signal to her most preferred firm. In periods $l = 1, ..., L$, each firm makes an offer at to its top signaling worker among workers remaining in the market; otherwise the firm makes an offer to its top ranked worker among those in the market. Each worker accepts the best available offer in each period.

Proof. Consider the offer game with no signals and L periods of interaction. We will apply backward induction, examining first the final stage of the game. Since the final stage of the game is identical to a one period offer game with no signals, in the unique symmetric equilibrium of the subgame, each firm makes an offer to its top ranked worker and each worker accepts best available offer.

Assumptions $u(W) > \delta u(1)$ and $v(W) > \delta v(1)$ guarantee that there is no incentive to hold offers or make dynamically strategic offers. Since firms $-f$ use symmetric anonymous strategies at stage $L-1$ and stage L, the only optimal strategy of firm f at stage $L-1$ is to make an offer to T_f. Each worker who receives at least one offer in stage $L-1$ optimally accepts the best available offer immediately. Similar logic applies to the other stages.

Now consider the offer game with signals and L periods of interaction. The symmetry of the strategies of workers $-w$ and the anonymity of firm strategies guarantee that the equilibrium probability that a firm makes an offer to worker w (across any of the L periods) conditional on receiving a signal from w (and also conditional on not receiving her signal) is the same for all firms. Therefore, workers optimally send their signals to their most preferred firm in period 0.

Observe that signals play a meaningful role for firms only in the first period. Since $u(W) > \delta u(1)$ and $u(W) > \frac{W}{F}\left(1-\left(1-\frac{1}{W}\right)^F\right)u(1)$, each firm f makes a period 1 offer to S_f if it received at least one signal. Since $v(W) > \delta v(1)$, workers accept the best available offers immediately. In period 2, each remaining firm either received no signals or else saw its offers rejected in period 1. Thereafter firm offers to their most preferred remaining workers prevail, as the logic of backward induction in the offer game with no signals and many periods applies to periods 2 through L. □

Proof of Proposition 5. We first calculate an explicit formula for the increase in the expected number of matches from the introduction of the signaling mechanism.

Lemma B1. *Consider a market with W workers and $F > 2$ firms. The expected number of matches in the offer game with no signals equals*

$$m^{NS}(F, W) = W\left(1-\left(1-\tfrac{1}{W}\right)^F\right). \tag{B.2.9}$$

The expected number of matches in the offer game with signals equals

$$m^S(F,W) = F\left(\begin{array}{c} 1-(\frac{F-1}{F})^W + \frac{W(F-1)^{2W-2}}{F^W(F-2)^{W-1}}\left(1-\frac{F-1}{W}\left(1-(\frac{F-2}{F-1})^W\right)\right)* \\ *\left(1-\left(1-\frac{1}{W}(\frac{F-2}{F-1})^{W-1}\right)^{F-1}\right)\end{array}\right). \qquad \text{(B.2.10)}$$

Proof of Lemma B1. Let us first calculate the expected number of matches in the pure coordination game with no signals. Proposition A1 establishes that the unique symmetric non-babbling equilibrium when agents use anonymous strategies is as follows. Each firm makes an offer to its top worker and each worker accepts the best offer among those available. We have already calculated the probability of firm f being matched to its top worker in Proposition B1. The probability of this event is

$$\frac{W}{F}\left(1-\left(1-\frac{1}{W}\right)^F\right)$$

.Therefore, the expected total number of matches in the game with no signals equals

$$m^{NS}(F,W) = W\left(1-\left(1-\frac{1}{W}\right)^F\right) \qquad \text{(B.2.11)}$$

Let us now calculate the expected number of matches in the offer game with signals. Proposition B1 derives agent strategies in the unique symmetric non-babbling equilibrium in the pure coordination game with signals. Each worker sends her signal to her top firm and each firm makes its offer to its top signaling worker if it receives at least one signal, otherwise it makes an offer to its top ranked worker.

We first calculate ex-ante probability of being matched by some firm f. We denote the set of workers that send her signal to firm f as $\mathcal{W}_f^S \subset \mathcal{W}$. If firm f receives at least one signal, $|\mathcal{W}_f^S| > 0$, it is guaranteed a match because each worker sends her signal to her top firm. If firm f receives no signals, it makes an offer to its top ranked worker T_f. This worker accepts firm f's offer only if this offer is the best one among those she receives. Let us denote the probability that T_f accepts firm f's offer (under the condition that firm f receives no signals) as

$$P_{T_f,|\mathcal{W}_f^S|=0} \equiv P(T_f \text{ accepts firm } f'\text{s offer}|\left|\mathcal{W}_f^S\right| = 0).$$

The ex-ante probability that firm f is matched then equals

$$Prob\_match_f(F,W) = P(|\mathcal{W}_f^S| > 0) * 1 + P(|\mathcal{W}_f^S| = 0) * P_{T_f,|\mathcal{W}_f^S|=0}. \qquad \text{(B.2.12)}$$

If firm f receives no signals, $|\mathcal{W}_f^S| = 0$, it makes an offer to T_f, which we will call worker

w. Worker w receives an offer from its top ranked firm, say firm f_0, conditional on firm f receiving no signals, $|\mathcal{W}_f^S| = 0$, with probability equal to

$$\begin{aligned} G &= P(|\mathcal{W}_{f_0}^S| = 1||\mathcal{W}_f^S| = 0) * 1 + ... + P(|\mathcal{W}_{f_0}^S| = W||\mathcal{W}_f^S| = 0) * \tfrac{1}{W} \quad \text{(B.2.13)} \\ &= \sum_{j=0}^{W-1} C_{W-1}^j \left(\tfrac{1}{F-1}\right)^j (1 - \tfrac{1}{F-1})^{W-j-1} \tfrac{1}{j+1}. \quad \text{(B.2.14)} \end{aligned}$$

Intuitively, firm f_0 receives a signal from a particular worker with probability $\frac{1}{F-1}$ (note that firm f receives no signals). Then, if firm f_0 receives signals from j other workers, worker w receives an offer from firm f_0 with probability $\frac{1}{j+1}$. Similarly to equation (B.2.3) the expression for G simplifies to

$$G = \tfrac{F-1}{W} \left(1 - (1 - \tfrac{1}{F-1})^W\right). \quad \text{(B.2.15)}$$

Firm f can be matched with worker w only if worker w does not receive an offer from its top firm, which happens with probability $1 - G$. If worker w does not receive an offer from her top firm $-$ firm f_0 $-$ firm f competes with other firms that have received no signals from workers. The probability that some firm f' among firms $\mathcal{F}\backslash\{f, f_0\}$ receives no signals conditional on the fact that worker w sends her signal to firm f_0 and firm f receives no signals ($|\mathcal{W}_f^S| = 0$) equals $r = (1 - \frac{1}{F-1})^{W-1}$. Note that the probability that firm f' does not receive a signal from a worker equals $1 - \frac{1}{F-1}$, because firm f receives no signals. There are also only $W - 1$ workers that can send a signal to firm f', because worker w sends her signal to firm f_0.

Therefore, the probability that some firm f' among firms $\mathcal{F}\backslash\{f, f_0\}$ receives no signals and makes an offer to worker w, conditional on the fact that worker w sends her signal to firm f_0, equals $\frac{r}{W}$. Therefore, the probability that worker w prefers the offer of firm f to other offers (conditional on the fact that firm f receives no signals and worker w sends her signal to firm f_0) equals[35]

$$\sum_{j=0}^{F-2} C_{F-2}^j \left(\tfrac{r}{W}\right)^j (1 - \tfrac{r}{W})^{F-2-j} \tfrac{1}{j+1} = \tfrac{W}{(F-1)r} \left(1 - \left(1 - \tfrac{r}{W}\right)^{F-1}\right). \quad \text{(B.2.16)}$$

The probability that worker w accepts firm f's offer then equals

$$P_{T_f, |\mathcal{W}_f^S|=0} = (1 - G) \left(\tfrac{W}{(F-1)r} \left(1 - \left(1 - \tfrac{r}{W}\right)^{F-1}\right)\right).$$

Taking into account that firm f receives no signals with probability $P(|\mathcal{W}_f^S| = 0) = (1-\frac{1}{F})^W$,

[35]Note that the maximum number of offers worker w could get equals to $M - 1$ as it does not receive an offer from its top firm f_0.

the probability of firm f being matched in the offer game with signals is then

$$\begin{aligned} Prob\_match_f(F,W) &= 1-(1-\tfrac{1}{F})^W + (1-\tfrac{1}{F})^W * P_{T_f,|\mathcal{W}_f^S|=0} \\ &= 1-(1-\tfrac{1}{F})^W + (1-\tfrac{1}{F})^W \tfrac{W}{(F-1)r} * \\ &\quad \left(1-\tfrac{F-1}{W}\left(1-(1-\tfrac{1}{F-1})^W\right)\right) * \left(1-\left(1-\tfrac{r}{W}\right)^{F-1}\right) \end{aligned} \tag{B.2.17}$$

where $r = (1-\frac{1}{F-1})^{W-1}$. The expected total number of matches in the offer game with signals equals $m^S(F,W) = F * Prob\_match_f(F,W)$. □

Lemma B1 establishes the expected total number of matches in the offer game with and without signals. Let us first fix W and calculate where the increase in the expected number of matches from the introduction of the signaling mechanism, $V(F,W) = m^S(F,W) - m^{NS}(F,W)$, attains its maximum. In order to obtain the proposition, we consider large markets (markets where the number of firms and the number of workers are large) and we use Taylor's expansion formula:

$$(1-a)^b = \exp(-ab + O(a^2b)). \tag{B.2.18}$$

where $O(a^2b)$ is a function that is smaller than a constant for large values of a^2b. Setting $x \equiv \frac{F}{W}$, the expected number of matches in the offer game with no signals can be approximated as

$$m^{NS}(F,W) = W\left(1-\left(1-\tfrac{1}{W}\right)^F\right) = W(1-e^{-x+O(x/W)}).$$

Let us now consider the expected number of matches in the offer game with signals. Using the result of Lemma B1 we get

$$m^S(F,W) = Wx\left(1-e^{-1/x+O\left(1/(x^2W)\right)} + A * B\right)$$

where

$$\begin{aligned} A &= \left(1-\tfrac{F-1}{W}\left(1-(\tfrac{F-2}{F-1})^W\right)\right) \text{ and} \\ B &= \tfrac{W(F-1)^{2W-2}}{F^W(F-2)^{W-1}}\left(1-\left(1-\tfrac{1}{w}(\tfrac{F-2}{F-1})^{W-1}\right)^{F-1}\right). \end{aligned}$$

We first calculate an approximation of A for large markets. Using (B.2.18) we have that

$$1-(1-\tfrac{1}{F-1})^W = 1-e^{-x+O(1/(x^2W))}$$

and

$$A = 1 - x\left(1 - e^{-1/x + O(1/(x^2W))}\right) + O(1/(xW)).$$

We now calculate an approximation of B for large markets:

$$\begin{aligned}
\frac{W(F-1)^{2W-2}}{F^W(F-2)^{W-1}} &= \frac{W}{F}\left(\frac{F-1}{F}\right)^{W-1}\left(\frac{F-1}{F-2}\right)^{W-1} \\
&= \frac{1}{x}e^{-(W-1)/F + O(1/(x^2W))}e^{(W-1)/(F-1) + O(1/(x^2W))} \\
&= \frac{1}{x}e^{O(1/(x^2W))}.
\end{aligned}$$

Also, we have that

$$\begin{aligned}
\left(1 - \left(1 - \frac{Z}{W}\right)^{F-1}\right) &= 1 - e^{-Z(F-1)/W + O(x/W)} \\
&= 1 - e^{-Zx + O(x/W)}
\end{aligned}$$

where $Z = (\frac{F-2}{F-1})^{W-1} = e^{-1/x + O(1/(x^2W))}$. Finally, we have

$$\begin{aligned}
B &= \frac{W(F-1)^{2W-2}}{F^W(F-2)^{W-1}} * \left(1 - \left(1 - \frac{1}{W}(\frac{F-2}{F-1})^{W-1}\right)^{F-1}\right) \\
&= \frac{1}{x}e^{O(1/(x^2W))}(1 - e^{-xe^{-1/x} + O(x/W)}).
\end{aligned}$$

Putting it all together, we have

$$\begin{aligned}
V(F,W) &= Wx\left(\begin{array}{c} 1 - e^{-1/x + O(1/W)} + \left(1 - x\left(1 - e^{-1/x + O(1/W)}\right) + O(1/W)\right) * \\ *\frac{1}{x}e^{O(1/W)}(1 - e^{-xe^{-1/x} + O(1/W)}) \end{array}\right) - \\
&\quad -W(1 - e^{-x + O(1/W)}) \\
&= W\left(x - xe^{-1/x} + \left(1 - x\left(1 - e^{-1/x}\right)\right)(1 - e^{-xe^{-1/x}}) - 1 + e^{-x}\right) + O(1) \\
&= W\alpha(x) + O(1)
\end{aligned}$$

where $\alpha(x)$ is a positive quasi-concave function that attains maximum at $x_0 \simeq 1.012113$. Therefore, for fixed W, $V(F,W)$ attains its maximum value at $F = x_0W + O(1)$.

Similar to the previous derivation, we can fix F and calculate the value of W where

$V(F,W)$ attains its maximum:

$$\begin{aligned} V(F,W) &= F\left(\begin{array}{c} 1-e^{-1/x+O(1/W)}+\left(1-x\left(1-e^{-1/x+O(1/W)}\right)+O(1/W)\right) \\ *\frac{1}{x}e^{O(1/W)}(1-e^{-xe^{-1/x}+O(1/W)}) \end{array}\right) \\ &\quad -\frac{F}{x}(1-e^{-x+O(1/F)}) \\ &= F\left(1-e^{-1/x}+\left(1-x\left(1-e^{-1/x}\right)\right)\frac{1}{x}(1-e^{-xe^{-1/x}})-\frac{1}{x}\left(1-e^{-x}\right)\right)+O(1) \\ &= F\beta(x)+O(1) \end{aligned}$$

where $\beta(x)$ is a positive quasi-concave function that attains maximum at $x_{00} \simeq 0.53074$. Therefore, for fixed F, $V(F,W)$ attains its maximum value at $W = y_0 F + O(1)$, where $y_0 = 1/x_{00} = 1.8842$. □

B.3 Extension: Signaling with Many Positions and Many Signals

In this section we consider a model of matching markets in a symmetric environment that is similar to the one in Sections 3 and 4. The difference is that each firm now has the capacity to hire up to L workers, and each worker may send up to K identical costless private signals. We assume that the number of signals each worker may send is less than the number of firms, $K < F$, and each worker can send at most one signal to a particular firm.

We assume that firm utilities are additive, i.e. firm f with preferences θ_f over individual workers values a match with a subset of workers $\mathcal{W}_0 \subset \mathcal{W}$ as $u(\theta_f, \mathcal{W}_0) = \sum_{w\in\mathcal{W}_0} u(\theta_f, w)$, where $u(\theta_f, \cdot)$ is a von-Neumann Morgenstern utility function. Worker w with preference list θ_w values a match with firm f as $v(\theta_w, f)$. We keep all assumptions of Sections 3 and 4 regarding agent utilities $u(\cdot,\cdot)$ and $v(\cdot,\cdot)$.

The timing and strategies of agents of the offer game without signals can be adopted from Section 3:

1. Agents' preferences are realized. In the case of a signaling mechanism, each worker sends up to K private, identical, costless signals to firms. Signals are sent simultaneously, and are observed only by firms who have received them.

2. Each firm makes an offer to at most L workers; offers are made simultaneously.

3. Each worker accepts at most one offer from the set of offers she receives.

Once again, sequential rationality ensures that workers will always select the best available offer. Hence, we take this behavior for workers as given and focus on the reduced game consisting of the first two stages.

A mixed strategy for worker w when deciding wether and to whom to send signals is a map from the set of all possible preference lists to the set of distributions over subsets of firms of size K or less that we denote as $\Delta(2^{\mathcal{F}_{\mathrm{K}}})$, i.e. $\sigma_w : \Theta_w \to \Delta(2^{\mathcal{F}_{\mathrm{K}}})$. Similarly, a mixed strategy of firm f is a map from the set of all possible preference lists, Θ_f, and the set of all possible combinations of received signals, $2^{\mathcal{W}}$, to the set of distributions over subsets of workers of size L or less, which we denote as $\Delta(2^{\mathcal{W}_{\mathrm{L}}})$. That is, $\sigma_f : \Theta_f \times 2^{\mathcal{W}} \to \Delta(2^{\mathcal{W}_{\mathrm{L}}})$.

Preferences of both firms and workers are independently and uniformly chosen from all possible preference orderings. Similarly to Sections 3 and 4 we define σ_W, σ_F, Σ_w,Σ_f, π_w, and π_f. The definition of sequential equilibrium and anonymous strategies can also be adopted in an analagous manner.

We first consider an offer game without signals. If firms use anonymous strategies, the chances of hiring any worker, conditional on making her an offer, are the same. Therefore, each firm optimally makes its offers to the L highest-ranked workers on its preference list. This is the unique symmetric equilibrium of the offer game without signals when firms use anonymous strategies (see Proposition B4).

We now turn to the analysis of the offer game with signals. In any symmetric equilibrium in which workers send signals and signals are interpreted as a sign of interest by firms and hence increase the chance of receiving an offer, each worker sends her K signals to her K most preferred firms (see Proposition B5). As in the case of one signal and each firm only having one position, we pin down the behavior of workers in equilibrium: workers send their signals to their highest ranked firms, and workers accept the best available offer. We now examine offers of firms in the second stage of the game, taking the strategies of workers and beliefs of firms about interpreting signals as given.[36]

In Section 4 each worker could send up to one signal, and each firm had $L = 1$ positions to fill. Then, when all other firms used anonymous strategies, firm f decided between making an offer to f's most preferred worker T_f (or T_f^1) and f's most preferred worker in the subset of signaled workers S_f (or S_f^1). Now, when all other firms use anonymous strategies, firm f can make up to L offers. When deciding whom to make the first offer, firm f, once more, decides between the most preferred worker T_f (or T_f^1) and the most prefered worker among those who sent a signal S_f (or S_f^1) where that decision may depend on the total number of signals received. So, if firm f received $|\mathcal{W}^S|$ signals and uses a cutoff strategy with corresponding cutoff $j_{|\mathcal{W}^S|}$, then f makes an offer to S_f^1 if and only if the rank of S_f^1 is lower or equal than $j_{|\mathcal{W}^S|}$. If firm f made an offer to S_f^1, then, for the second position, the firm decides between

[36]Note that in any non-babbling symmetric equilibrium, all information sets for firms are realized with positive probability. Hence, the beliefs of firms are determined by Bayes' Law: if a firm receives a signal from a worker, it believes that it is on of the kth top firms, $k \in \{1, ..., K\}$, in the workers' preference list and the probability of having rank k is identical across ranks $\{1, ..., K\}$.

T_f^1 and S_f^2 the most preferred worker among those that sent a signal to whom firm f has not made an offer yet. Furthermore, firm f will use the *same* cutoff strategy as before: Firm f still received $|\mathcal{W}^S|$ signals and hence will make an offer to S_f^2 compared to T_f^1 if and only if the rank of S_f^1 is lower than $j_{|\mathcal{W}^S|}$.

If the firm made its first offer to T_f^1, then for the second offer, firm f decides between T_f^2 and S_f^1, where f can use a new cutoff strategy, since the alternative to a signaling worker is now T_f^2, the overall second most preferred worker, and not T_f^2. We can show that in equilibrium, the cutoff for T_f^2 will be greater than for T_f^1 for any number if received signals (see Proposition B6). We can now define the notion of cutoff strategies for this setting.

Definition B1 (Cutoff Strategies in Case of Many Positions and Multiple Signals)**.** Strategy σ_f is a *cutoff strategy* for firm f if there are L vectors $J^l = (j_1^l, ..., j_W^l)$, $l = 1, ..., L$ such that for any $\theta_f \in \Theta_f$ and any set $\mathcal{W}^S$ of workers who sent a signal to firm f we have the following: For any number m of offers already made, let the most preferred worker to whom firm f has not yet made an offer be T_f^r of rank $1 \leq r < L$ and let the most preferred worker who sent a signal and to whom f has not yet made an offer be S_f^q of rank $1 \leq q < L$, where $m = q + r - 2$. Then firm f makes its next offer to

$$\begin{cases} S_f^q & \text{if } \text{rank}_{\theta_f}(S_f^q) \leq j^r_{|\mathcal{W}^S|} \\ T_f^r & \text{otherwise.} \end{cases}$$

We call $(J^1, \ldots, J^L)$ *a cutoff matrix* that has cutoff vectors for each of the top L ranked workers as its components. Note that the probability of a firm's offer being accepted by any worker who has signaled to it is the same as in a symmetric equilibrium. Similarly, the probability of a firm's offer being accepted by any worker who has not signaled to the firm is also the same across such workers (see Lemma B2).

Using an argument similar to the case of one position and one signal, we show that cutoff strategies are optimal strategies for firms (see Proposition B7). We can also impose a partial order on the cutoff strategies as in Section 4. However, strategies of firms are no longer necessarily strategic complements. When other firms respond more to signals, this decreases the payoff from making an offer to both workers who have and workers who have not signaled to the firm. This is because receiving a signal does not guarantee acceptance in case an offer is tendered to that worker. We can, however, assure the existence of symmetric mixed strategy equilibrium.

Theorem B1 (Equilibrium Existence)**.** *There exists a symmetric equilibrium of the offer game with signals where 1) workers send their signals to top K firms, and 2) firms play symmetric cutoff strategies.*

We now address the welfare implications from the introduction of a signaling mechanism. Proposition B3 and Theorem B2 formally restate our welfare results from previous chapters for the case when firms have many positions and workers can send multiple signals. The logic of their proofs again begins with an incremental approach: we first study the effect of a single firm increasing its cutoff, that is, responding more to signals. We then rank various signaling equilibria in terms of their outcomes. Finally, we show how the introduction of a signaling mechanism impacts our three measures of welfare.

Proposition B3 (Welfare Across Equilibria)**.** *Consider any two symmetric cutoff strategy equilibria where in one equilibrium firms have greater cutoffs. Compared to the equilibrium with lower cutoffs, in the equilibrium with greater cutoffs we have the following:*

- *the expected number of matches is weakly greater,*
- *workers have weakly higher expected payoffs, and*
- *firms have weakly lower expected payoffs.*

Theorem B2 (Welfare Impact of a Signaling Mechanism)**.** *Consider any non-babbling symmetric equilibrium of the offer game with signals. Then the following three statements hold.*

i. The expected number of matches is strictly greater than in the unique equilibrium of the offer game with no signals.

ii. The expected welfare of workers is strictly greater than in the unique equilibrium of the offer game with no signals.

iii. The welfare of firms may be greater or smaller than in the unique equilibrium of the offer game with no signals.

Proofs: Signaling with Many Positions and Many Signals

In addition to providing proofs for the above results, this section introduces Propositions B4-B7 and Lemma B2 which help establish the main findings.

Proposition B4. *The unique equilibrium of the offer game with no signals when firms use anonymous strategies and workers accept the best available offer is* $\sigma_f(\theta_f) = (\theta_f^1, ..., \theta_f^L)$ *for all* $f \in \mathcal{F}$ *and* $\theta_f \in \Theta_f$.

Proof. The proof repeats the argument of Proposition 1. □

Proposition B5. *In any symmetric non-babbling equilibrium of the offer game with signals each worker sends signals to her K top firms.*

Proof. Select an arbitrary worker. Firms use symmetric anonymous strategies, signals are identical, and the worker can send at most one signal to a given firm. Hence, from the worker's perspective the probability of getting an offer from a firm depends only on whether the worker has sent a signal to this firm or not. Similar to the argument of the proof of Proposition 4 the probability of getting an offer from a firm that receives the worker's signal is greater than the probability of getting an offer from a firm that does not receive the worker's signal. Since this probability does not depend on the identity of the firm in a symmetric equilibrium we conclude that the worker optimally sends her signals to her K top firms. □

Proposition B6. *Suppose firms $-f$ use anonymous strategies and workers send their signals to their top K firms. Then firm f makes offers to its $L^{NS} \in \{0, ..., L\}$ top workers who have signaled to it and to its $L^S = L - L^{NS}$ top workers who have not signaled to it in any non-babbling symmetric sequential equilibrium.*

Proof. Note that firms use anonymous strategies, workers send their signal to their top K firms, and workers accept the best available offer. We first prove a lemma that states that from point of view of firm f, the probability that workers who have and have not signaled to it accept its offer depends only on the number of signals firm f receives.

Lemma B2. *Suppose firms $-f$ use anonymous strategies and workers send their signals to their top K firms. Consider two events, A and B. Event A is that firm f receives the set of signals $\mathcal{W}^S$. Event B is that firm f receives the set of signals $\check{\mathcal{W}}^S$, where $|\mathcal{W}^S| = |\check{\mathcal{W}}^S|$. Then*

- *the probability that worker $w \in \mathcal{W}^S$ accepts firm f's offer conditional on event A equals the probability that worker $w' \in \check{\mathcal{W}}^S$ accepts firm f offer conditional on event B;*
- *the probability that worker $w \in \mathcal{W} \backslash \mathcal{W}^S$ accepts firm f's offer conditional on event A equals the probability that worker $w' \in \mathcal{W} \backslash \check{\mathcal{W}}^S$ accepts firm f offer conditional on event B.*

Proof. Let us consider firm f with realized preference profile $\theta_f^* \in \Theta_f$ that receives signals from the set of workers $\mathcal{W}^S$. We first prove that the probability that a worker from $\mathcal{W}^S$ accepts firm f's offer conditional on event A equals the probability that a worker from $\check{\mathcal{W}}^S$ accepts firm f's offer conditional on event B.

Note that firm f believes that it is one of the top K firms in worker preference list if it receives an offer from her. Let us denote the set of possible agent profiles consistent with firm f beliefs in both events as

$$\Theta^A \equiv \{\theta \in \Theta | \theta_f = \theta^*_f \text{ and } rank_{\theta_{w^s}}(f) \in \{1, ..., K\} \text{ for each } w^s \in \mathcal{W}^S\}$$

$$\Theta^B \equiv \{\theta \in \Theta | \theta_f = \theta^*_f \text{ and } rank_{\theta_{w^s}}(f) \in \{1, ..., K\} \text{ for each } w^s \in \check{\mathcal{W}}^S\}$$

Since firm f receives the same number of signals for both events, i.e. $|\mathcal{W}^S| = |\check{\mathcal{W}}^S|$, for each worker $w_a \in \mathcal{W}^S$ we pair some worker $w'_a \in \check{\mathcal{W}}^S$, $a = 1, ..., |\mathcal{W}^S|$. Let us denote $rank_{\theta_{w_a}}(f) = k_a$ and $rank_{\theta_{w'_a}}(f) = k'_a$. Therefore, $k_a, k'_a \in \{1, ..., K\}$ for each a. We denote a permutation that changes k_a and k'_a's positions in a worker's preference list as

$$\rho^{w_a} : (..., k_a, ..., k'_a, ...) \to (..., k'_a, ..., k_a, ...).$$

We also denote a permutation that changes the ranks of w_a and w'_a for every a in a firm preference lists as

$$\rho^f : (..., w_a, ..., w'_a, ...) \to (..., w'_a, ..., w_a, ...).$$

Beginning with arbitrary profile of preferences $\theta \in \Theta^A$, we construct a profile of preferences θ' as follows:

- we do not change firm f preference list, i.e. $\theta'_f = \theta^*_f$,
- the ranks of workers w_a and w'_a are exchanged in the preference lists of firms $-f$ for each a: for each firm $f' \in -f$, $\theta'_f = \rho^f(\theta_f)$,
- firms in positions k_a and k'_a in worker w_a and worker w'_a preference profiles are exchanged for each a:
$$\theta'_{w_a} = \rho^{w_a}(\theta_{w_a}),\ \theta'_{w'_a} = \rho^{w_a}(\theta_{w'_a}), \quad \text{and}$$
- for any other $w^0 \in \mathcal{W} \backslash (\mathcal{W}^S \bigcup \check{\mathcal{W}}^S)$, $\theta_{w^0} = \theta'_{w^0}$.

Since firm f 's preference list is unchanged, $\theta'_f = \theta^*$, and firm f receives signals from the set $\check{\mathcal{W}}^S$ for profile θ', this profile belongs to Θ^B. Since firm $-f$ strategies are anonymous for any $f' \in -f$ and for any $\mathcal{W}^S_{f'} \subset \mathcal{W}$, we have that

$$\sigma_{f'}(\rho^f(\theta_{f'}), \rho^f(\mathcal{W}^S_{f'})) = \rho^f \left(\sigma_{f'}(\theta_{f'}, \mathcal{W}^S_{f'})\right).$$

Workers in $\mathcal{W}^S$ and $\check{\mathcal{W}}^S$ send their signals to the same firms among $-f$ for both profiles θ

and θ'. Therefore, i.e. $\rho^f(\mathcal{W}^S_{f'}) = \mathcal{W}^S_{f'}$. Since $\theta'_f = \rho^f(\theta_f)$ we have that

$$\sigma_{f'}(\theta'_{f'}, \mathcal{W}^S_{f'}) = \rho\left(\sigma_{f'}(\theta_{f'}, \mathcal{W}^S_{f'})\right)$$

This means that the probability of firm f' making an offer to worker $w_a \in \mathcal{W}^S$ for profile θ equals the probability of making an offer to a worker in $w'_a \in \check{\mathcal{W}}^S$ for profile θ'. Moreover, since we exchange worker w_a and w'_a preference lists for profile θ', whenever it is optimal for worker w_a to accept firm f offer for profile θ, it is optimal for worker w'_a to accept firm f's offer for profile θ'.

Since firm types are independent the probability of firm f being matched when it makes an offer to w_a for profile θ equals the probability of firm f being matched when it makes an offer to worker w'_a for profile θ'. Therefore, for each $\theta \in \Theta^A$ there exists $\theta' \in \Theta^B$ such that the probability that firm f gets an offer from worker w_a equals the probability that firm f gets an offer from worker w'_a. Moreover, profile θ' is different for different θ by the construction. Therefore, we have constructed a bijection between sets Θ^A and Θ^B. Since θ and θ' are equally probable, the likelihood that firm f 's offer is accepted by worker w_a in the event A equals the probability that firm f's offer is accepted by worker w'_a in the event B.

An analagous construction works for the proof of the second statement that involves workers in sets $\mathcal{W}\backslash\mathcal{W}^S$ and $\mathcal{W}\backslash\check{\mathcal{W}}^S$. Therefore, the probability that worker $w \in \mathcal{W}\backslash\mathcal{W}^S$ accepts firm f offer conditional on event A equals the probability that worker $w' \in \mathcal{W}\backslash\check{\mathcal{W}}^S$ accepts firm f offer conditional on event B. □

The statement of the proposition follows directly from the lemma. Since the probability that the worker who has sent a signal to firm f accepts its offer is independent of the identity of the worker, firm f prefers to make offers to its top workers among those who signaled to it. Similarly, firm f prefers to make offers to its top workers among those who has not signaled to it. Finally, firm f prefers to make all L offers. □

Proposition B7. *Suppose workers send their signals to their top K firms. Then for any strategy σ_f of firm f, there exists a cutoff strategy that provides f with a weakly higher expected payoff than σ_f for any anonymous strategies σ_{-f} of opponent firms $-f$.*

Proof. Let us consider two sets of workers that firm f might receive $\mathcal{W}^S$ and $\check{\mathcal{W}}^S$ such that $\mathcal{W}^S$=$\check{\mathcal{W}}^S$. Firm f makes an offer to workers $\mathcal{W}_{offer} = \mathcal{W}^S_{offer} \bigcup \mathcal{W}^{NS}_{offer}$ such that$\mathcal{W}^{NS}_{offer} \subset \mathcal{W}^S$ and $\mathcal{W}^{NS}_{offer} \subset \mathcal{W}\backslash\mathcal{W}^S$ in equilibrium. Lemma B2 proves that identities of workers who have sent a signal to firm f do not influence the probability that workers accept the firm's offer provided that the total number of signals firm f receives is constant. Therefore, if workers $\mathcal{W}^S_{offer}$ are among $\check{\mathcal{W}}^S$, i.e. $\mathcal{W}^S_{offer} \subset \check{\mathcal{W}}^S$, it is still optimal for firm f to make its offers to workers $\mathcal{W}_{offer}$.

Let us again consider two sets of signals with the same power, i.e. $\mathcal{W}^S$ and $\check{\mathcal{W}}^S$ such that $\mathcal{W}^S$=$\check{\mathcal{W}}^S$. However, these sets differ now in one worker: there exist $w \in \mathcal{W}^S$ and $w' \in \check{\mathcal{W}}^S$ such that $\mathcal{W}^S \backslash w$=$\check{\mathcal{W}}^S \backslash w'$. Moreover, firm f prefers worker w' to worker w, i.e. $rank_{\theta_f}(w') > rank_{\theta_f}(w)$. As a consequence of Lemma B2, if firm f makes an offer to worker w when it receives the set of signals $\mathcal{W}^S$ in equilibrium, it should make an offer to w' when it receives the set of signals $\check{\mathcal{W}}^S$. Let us consider the case when sets $\mathcal{W}^S$ and $\check{\mathcal{W}}^S$ differ in more than one worker. There are some workers in $\check{\mathcal{W}}_0 \subset \check{\mathcal{W}}^S$ who are better than workers in $\mathcal{W}_0 \subset \mathcal{W}^S$ who receive an offer from firm f when it recieves signals from $\mathcal{W}^S$. Similar argument shows that firm f should then optimally make an offer to $\check{\mathcal{W}}_0$ when it receives signals from $\check{\mathcal{W}}^S$.

The two arguments presented above allows us to conclude that if firm $-f$ use anonymous strategies, firm f's optimal strategy could be represented as some cutoff strategy. □

Proof of Theorem B1.

The proof repeats the steps of the proof of Theorem 3. □

Lemma B3. *Assume firms use cutoff strategies and workers send their signals to their top K firms. Fix the strategies of firms $-f$ as σ_{-f}. Let firm f's strategy σ_f differ from σ'_f only in that σ'_f has greater cutoffs (responds more to signals). Then we have*

$$E_\theta(m(\sigma'_f, \sigma_{-f}, \theta)) \geq E_\theta(m(\sigma_f, \sigma_{-f}, \theta))$$
$$E_\theta(\pi_w(\sigma'_f, \sigma_{-f}, \theta)) \geq E_\theta(\pi_w(\sigma_f, \sigma_{-f}, \theta))$$

where $m(\cdot)$ denotes the total number of matches.

Proof. Let us consider firm f cutoff strategies σ_f and σ'_f such that σ'_f has weakly greater cutoffs for profile θ_f:

$$\begin{aligned} \sigma_f(\theta_f, \mathcal{W}^S_f) &= \mathcal{W}^S_{offer} \bigcup \mathcal{W}^{NS}_{offer} \\ \sigma'_{f'}(\theta_f, \mathcal{W}^S_f) &= \check{\mathcal{W}}^S_{offer} \bigcup \check{\mathcal{W}}^{NS}_{offer} \end{aligned}$$

In order to preserve anonymity firm f also should have the corresponding increase in cutoff strategies for any profile of preferences and any set of received signals of the same power.

Firm f responds more to signals for profile θ_f means that $\mathcal{W}^S_{offer} \subset \check{\mathcal{W}}^S_{offer} \subset \mathcal{W}^S_f$ and $\check{\mathcal{W}}^{NS}_{offer} \subset \mathcal{W}^{NS}_{offer} \subset \mathcal{W} \backslash \mathcal{W}^S_f$. Proposition B6 shows that $|\mathcal{W}^S_{offer} \bigcup \mathcal{W}^{NS}_{offer}| = |\check{\mathcal{W}}^S_{offer} \bigcup \check{\mathcal{W}}^{NS}_{offer}| = L$. We consider only the case when $\mathcal{W}^S_{offer} \backslash \check{\mathcal{W}}^S_{offer} = w^S$ and $\check{\mathcal{W}}^{NS}_{offer} \backslash \mathcal{W}^{NS}_{offer} = w^{NS}$. More general case directly follows.

We denoter two sets of preference profiles

$$\Theta_+ \equiv \{\theta \in \Theta | \; m(\sigma_f, \sigma_{-f}, \theta) < m(\sigma'_f, \sigma_{-f}, \theta)\}$$
$$\Theta_- \equiv \{\theta \in \Theta | \; m(\sigma_f, \sigma_{-f}, \theta) > m(\sigma'_f, \sigma_{-f}, \theta)\}$$

For each profile θ from set Θ^+ it must be the case that without firm f offer w^{NS} has an offer from another firm, and worker w^S does not

$$m(\sigma'_f, \sigma_{-f}, \theta) - m(\sigma_f, \sigma_{-f}, \theta) = 1. \tag{B.3.1}$$

Similarly, if profile θ is from set Θ^-, it must be the case that without firm f offer w^S has an offer from another firm and w^{NS} does not

$$m(\sigma'_f, \sigma_{-f}, \theta) - m(\sigma_f, \sigma_{-f}, \theta) = -1. \tag{B.3.2}$$

We will now show that $|\Theta_+| \geq |\Theta_-|$. Equations (B.3.1) and (B.3.2) along with the fact that each $\theta \in \Theta_+ \cup \Theta_-$ happens equally likely will then be enough to prove the result.

If profile θ belongs to Θ_-, without firm f's offer, worker w^S has an offer from another firm, name this firm f', and worker w^{NS} does not. We construct function $\psi : \Theta \to \Theta$ as follows. Let us considerLet $\psi(\theta)$ be the profile such that

- firms swap the positions of workers w^{NS} and w^S in their preference lists.
- if both w^S and w^{NS} send signals to firm f' for profile θ their preferences remain unchanged
- if woker w^S (w^{NS}) sends her signal to firm f' but worker w^{NS} (w^S) does not for profile θ, find a firm f_y such that worker w^S(w^{NS}) does not send her signal to firm f_y, and worker w^{NS} (w^S)does. Exchange the positions of firm f' and firm f_y in worker w^{NS} and worker w^S preference lists.

Note that firm f_y exists because each worker sends exactly K signals in any non-babling symmetric equilibrium. We need the latter modification because each worker can send several signals, and the fact that worker w^Ssends her signal to firm f does not guarantee that she does not send another signal to firm f'.

If profile θ belongs to Θ_-, without firm f's offer, worker w^S has an offer from firm f', and worker w^{NS} does not. Therefore, when preferences are $\psi(\theta)$, without firm f's offer the following two statements should be true i) worker w^{NS} ***must*** have another offer and ii) worker w^S ***cannot*** have another offer.

To see i), note that under θ, worker w^S his outside offer comes from firm f'. Under $\psi(\theta)$ worker w^{NS} take position of worker w^S in firm f' preference list, and worker w^{NS} sends a signal to firm f' for profile $\psi(\theta)$ whenever worker w^S sends a signal to firm f' for profile θ. Anonymity of firm strategies guarantee that firm f' makes an offer to worker w^{NS}.

To see ii), suppose to the contrary that under $\psi(\theta)$, worker w does in fact receive an outside offer from some firm f''. This cannot be firm f'. Otherwise worker w^{NS} should get an offer from firm f' for profile θ by anonymity. This cannot be firm f_y because worker w^{NS} would get an offer from firm f_y for profile θ.

The main idea of the construction preserves the logic of Theorem 4. Specifically, if a worker receives firm's offer when she does not send a signal to the firm, she will definitely receives an offer if she sends a signal to the firm.

From *i*) and *ii*), we have

$$\theta \in \Theta_- \Rightarrow \psi(\theta) \in \Theta_+.$$

Since function ψ is injective, we have $|\Theta_+| \geq |\Theta_-|$.

In order to prove the second statement note that the expected number of matches of each worker increases when firm f responds more to signals. Using the construction presented above, one could show whenever worker w looses a match with firm f for profile θ (worker w ranks firm f low) it is possible to construct profile θ' when worker w obtains the match (worker w ranks firm f high). The function that matches these profiles is again injective. Moreover, worker w values more the match with high ranked firms. Therefore, ex-ante utility of worker w increases when firm f responds more to signals. $\square$

Proof of Proposition B3.

The result that the expected number of matches and the expected welfare of workers is higher in the equilibrium with higher cutoffs is an immediate consequence of Lemma B3.

In order to show that firms have lower expected payoffs in the equilibrium with greater cutoffs we first consider the following situation. We take some firm f such that its strategy σ_f differs from σ'_f only in that σ'_f has weakly greater cutoffs. Let us consider some firm $f' \in -f$. For each profile of preferences $\theta_{f'}$ and a set of signals $\mathcal{W}^S$, firm f' either makes an offer to $S_{f'}(\theta_{f'}, \mathcal{W}^S)$ or $T_{f'}(\theta_{f'}, \mathcal{W}^S)$. If firm f responds more to signals this decreases the probability that both $T_{f'}$ and $S_{f'}$ accept firm f' offer. Therefore, the expected payoff of firm

$f' \in -f$ weakly decreases when firm f responds more to signals.

$$E_\theta(\pi_{f'}(\sigma_f, \sigma_{-f}, \theta)) \geq E_\theta(\pi_{f'}(\sigma'_f, \sigma_{-f}, \theta)).$$

Let us now consider two symmetric equilibria where firms play cutoff strategies $\tilde{\sigma}$ and $\bar{\sigma}$ correspondingly such that $\tilde{\sigma} \geq \bar{\sigma}$. From the definition of an equilibrium strategy we have:

$$E_\theta[\pi_f(\bar{\sigma}_f, \bar{\sigma}_{-f}, \theta)] \geq E_\theta[\pi_f(\tilde{\sigma_f}, \bar{\sigma}_{-f}, \theta)]$$

Using the result proved above we proceed with

$$E_\theta[\pi_f(\tilde{\sigma_f}, \bar{\sigma}_{-f}, \theta)] \geq E_\theta[\pi_f(\tilde{\sigma_f}, \tilde{\sigma}_{-f}, \theta)]$$

Therefore

$$E_\theta[\pi_f(\bar{\sigma}_f, \bar{\sigma}_{-f}, \theta)] \geq E_\theta[\pi_f(\tilde{\sigma}_f, \tilde{\sigma}_{-f}, \theta)]$$

□

Proof of Theorem B2.

Denote firm strategies in the unique equilibrium of the offer game with no signals as σ_F^0. Now consider a symmetric equilibrium of the offer game with signals where agents use strategies (σ_F, σ_W). If agents employ strategies (σ_F^0, σ_W), the expected number of matches and the welfare of workers equal the corresponding parameters in the offer game with no signals. Therefore, the result that the expected number of matches and the expected welfare of workers in a symmetric equilibrium in the offer game with signals are *weakly* greater than the corresponding parameters in the unique equilibrium of the offer game with no signals is a consequence of sequential application of Lemma B3. The result for worker and firm welfare, and the argument that the comparison is strict are analagous to those in Theorem 4. □