Space-Time Coding in Frequency-Selective Multiple Access Channels

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Abstract—This paper explores the extensions of the Alamouti code to frequency selective channels. The Lindskog-Paulraj (LP) extension to frequency selective channels as well as the Diggavi, Al-Dhahir and Calderbank extension to multiple users will also be covered. The schemes will be shown both in D-domain and its corresponding version using finite block length matrices. Finally, simulations are done using a modified form of MLSE to make execution of the tests feasible.

I. INTRODUCTION

There are many ways to communicate using a MIMO system, one of which is using Space-Time Block Codes (STBCs). One of the advantages of STBCs is that they do not require CSI at the transmitter. This is an advantage because in a time-varying channel, the coherence time of the channel only needs to be long enough for transmitter to send a training sequence and the data; in systems requiring CSIT, there is the additional time to measure and relay the channel coefficients to the transmitter as well. In addition, sending the coefficients (which are prone to estimation error) to the transmitter decreases the overall data rate. While systems with CSIT have inherently higher capacity, their advantage also fades with increasing SNR [1]. Thus defining:

\[ r = \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = Hs + n \tag{2} \]

where \( s = [s_1 \ s_2]^T \), \( n = [n_1 \ n_2]^T \), and

\[ H = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \]

Also, because conjugation of \( n_2 \) does not change its noise statistics, the conjugation was left out of \( n \) and \( r \).

One of the important features of Alamouti’s scheme is that the matrix \( H \) is orthogonal: that is, \( H^H \mathbf{H} = ||h||_F^2 \mathbf{I} \)

where \( ||h||_F^2 \) is the Frobenius norm of the MISO channel \( h = [h_1 \ h_2] \). This outcome is exploited by multiplying the received vector by \( \mathbf{H}^H \):

\[ z = H^H r = H^H Hs + H^H n = ||h||_F^2 s + \tilde{n} \tag{3} \]

where \( \tilde{n} = H^H n \). This results in the highly desirable outcome of decoupling the two received symbols, eliminating intersymbol interference (ISI). Comparing this to the output of a maximal-ratio combiner (MRC) SIMO system [4] over two symbols, they both look like (3). Although due to transmit power restrictions, the Alamouti scheme has half the signal power of the SIMO MRC meaning half the array gain, the most important consequence of using the Alamouti scheme is that it achieves full diversity gain of 2 like the SIMO case.

II. D-Domain

The D-domain allows for compact notation of the response of the channel, and gives good insights on the behaviour of the system. To simplify notation in frequency selective channels, the delay operator, \( D^i \), is used. This operator represents a convolution with \( \delta[n-i] \), that is, a delay of the signal by \( i \) samples. Thus a signal passing through a channel with impulse
response \( h[n] = \sum_{i=0}^{\infty} h_i \delta[n-i] \) can be represented as multiplication with the polynomial in \( D \) of \( h(D) = \sum_{i=0}^{\infty} h_i D^i \). The complex conjugate of \( h(D) \) is \( (h(D))^* = h^*(D^{-1}) \), which is anti-causal.

A. Single User Frequency Selective Channel

A ubiquitous phenomenon that was not taken into consideration in Alamouti’s scheme was multipath propagation of the signal. If the Alamouti scheme was used in a frequency selective channel, there would be ISI between pairs of symbols transmitted due the delay spread of the channel. To address this issue, Lindskog and Paulraj developed a modified version of Alamouti’s scheme (called the LP scheme) that also decouples the two transmitted streams from each other at the receiver in a frequency selective channel [3].

To consider the effects in a frequency selective channel, the symbols cannot be considered as being sent one at a time, but rather as two data streams. It is assumed that the received has CSI, which in this case means the impulse responses of the two channels, \( h[n] = [h_1[n], h_2[n]] \), are known at the receiver.

The LP scheme mirrors the Alamouti scheme, with the difference being that instead of \( H \) having constant elements \( h_i \), it now has polynomials elements \( h_i(D) \). That is, equation (2) now becomes:

\[
\begin{align*}
 r[n] &= \begin{bmatrix} h_1(D) & h_2(D) \\ h_2(D) & -h_1(D) \end{bmatrix} \begin{bmatrix} s_1[n] \\ s_2[n] \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \\
&= H(D)s[n] + n[n]
\end{align*}
\]

The orthogonality of this scheme is also maintained similar to Alamouti, but in this case the remaining product on the diagonal is a polynomial in \( D \). That is, \( H^H(D^{-1})H(D) = (h_1(D^{-1})h_1(D) + h_2(D^{-1})h_2(D))I \). And thus multiplying (4) by \( H^H(D^{-1}) \), gives:

\[
\begin{align*}
 z[n] &= H^H(D^{-1})r[n] \\
&= H^H(D^{-1})H(D)s[n] + H^H(D^{-1})n[n] \quad (5)
\end{align*}
\]

Similar to Alamouti’s scheme, the effect of multiplying by \( H^H(D^{-1}) \) is that of a matched filter in that it decouples the two streams \( s_1[n] \) and \( s_2[n] \), leaving two parallel streams.

For the two users, the received antenna case in (7), the goal is to decouple the two users’ signals to mitigate the interference between the users. A zero forcing method is to multiply \( r \) in (7) by

\[
\begin{align*}
 W(D) &= \begin{bmatrix} \|H_{2,2}(D)\|^2 I_2 & -H_{1,2}(D)H_{2,2}(D^{-1}) \\ -H_{2,1}(D)H_{1,2}(D^{-1}) & \|H_{1,1}(D)\|^2 I_2 \end{bmatrix} \\
&= \begin{bmatrix} \Delta_{H_{2,2}} & 0 \\ 0 & \Delta_{H_{1,1}} \end{bmatrix} \begin{bmatrix} s_1[n] \\ s_2[n] \end{bmatrix} + \begin{bmatrix} \tilde{n}_1[n] \\ \tilde{n}_2[n] \end{bmatrix} \quad (8)
\end{align*}
\]

where

\[
\begin{align*}
\Delta_{H_{2,2}} &= \|H_{2,2}(D)\|^2 (H_{1,1}(D) - H_{1,2}(D)H_{2,2}(D^{-1})H_{2,1}(D)) \\
\Delta_{H_{1,1}} &= \|H_{1,1}(D)\|^2 (H_{2,2}(D) - H_{2,1}(D)H_{1,2}(D^{-1})H_{1,2}(D))
\end{align*}
\]

The important conclusion to be made is that the LP scheme improves the Alamouti scheme to achieve full diversity in a frequency selective channel.

B. Frequency-Selective Multiple Access Channel

The focus of this paper is space-time coding in multi-user frequency selective channels, specifically the multiple access channel (MAC or reverse link). The paper by Diggavi, Al-Dhahir and Calderbank [5] extends the LP scheme to work in the reverse link over multiple receive antennas and multiple users. Developing the signal model for this case, first (4) must be extended to the case where there are multiple antennas. Assuming that the LP transmit scheme is applied, for each receive antenna there is a matrix \( H_i(D) \) of the form defined in (4). For the \( M_R = 2 \) case, the result is:

\[
\begin{bmatrix} r_1[n] \\ r_2[n] \end{bmatrix} = \begin{bmatrix} H_{1,1}(D) & H_{1,2}(D) \\ H_{2,1}(D) & H_{2,2}(D) \end{bmatrix} \begin{bmatrix} s_1[n] \\ s_2[n] \end{bmatrix} + \begin{bmatrix} n_1[n] \\ n_2[n] \end{bmatrix} \quad (6)
\]

Finally to get the signal model for multiple users, the signal vector and effective channel of (6) is concatenated with those of other users. For \( M_R = 2 \) and two users, (6) becomes:

\[
\begin{bmatrix} r_1[n] \\ r_2[n] \end{bmatrix} = \begin{bmatrix} H_{1,1}(D) & H_{1,2}(D) \\ H_{2,1}(D) & H_{2,2}(D) \end{bmatrix} \begin{bmatrix} s_1[n] \\ s_2[n] \end{bmatrix} + \begin{bmatrix} n_1[n] \\ n_2[n] \end{bmatrix} \quad (7)
\]

where \( H_{i,j}(D) \) is the corresponding effective channel matrix in antenna \( i \) due to user \( j \)’s signal.

C. MAC Zero Forcing Solution

The algebraic property that is emphasized in [5] is that the set of \( H \) matrices of the form (4), call this set \( Q \), is closed under multiplication, and form a multiplicative group. Define \( \|V(D)\|^2 = V(D)V^H(D^{-1}) \) for \( V \in Q \).

For the two user, two receive antenna case in (7), the goal is to decouple the two users’ signals to mitigate the interference between the users. A zero forcing method is to multiply \( r \) in (7) by

\[
\begin{align*}
 W(D) &= \begin{bmatrix} \|H_{2,2}(D)\|^2 I_2 & -H_{1,2}(D)H_{2,2}(D^{-1}) \\ -H_{2,1}(D)H_{1,2}(D^{-1}) & \|H_{1,1}(D)\|^2 I_2 \end{bmatrix} \\
&= \begin{bmatrix} \Delta_{H_{2,2}} & 0 \\ 0 & \Delta_{H_{1,1}} \end{bmatrix} \begin{bmatrix} s_1[n] \\ s_2[n] \end{bmatrix} + \begin{bmatrix} \tilde{n}_1[n] \\ \tilde{n}_2[n] \end{bmatrix} \quad (8)
\end{align*}
\]

where

\[
\begin{align*}
\Delta_{H_{2,2}} &= \|H_{2,2}(D)\|^2 (H_{1,1}(D) - H_{1,2}(D)H_{2,2}(D^{-1})H_{2,1}(D)) \\
\Delta_{H_{1,1}} &= \|H_{1,1}(D)\|^2 (H_{2,2}(D) - H_{2,1}(D)H_{1,2}(D^{-1})H_{1,2}(D))
\end{align*}
\]

The important consequence of this is that \( s_1 \) and \( s_2 \) are now decoupled, with \( \tilde{r}_1[n] \) corresponding to \( s_1 \) and \( \tilde{r}_2[n] \) corresponding to \( s_2 \). Also, it is shown in [5] that \( \Delta_{H_{1,1}} \in Q \); that is, they are of the form (4). This means that the methods of II-A hold, and the receiver can detect each user as if they were the sole users of the channel. When doing this, however,
the noise terms for each user are no longer white and must be
whitened using a whitening filter, such as:

\[
U_1(D) = \left[ \|H_{2,2}(D)\| \sqrt{\|H_{1,2}(D)\|^2 + \|H_{2,2}(D)\|^2} \right]^{-1} I_2
\]

\[
U_2(D) = \left[ \|H_{1,1}(D)\| \sqrt{\|H_{1,1}(D)\|^2 + \|H_{1,2}(D)\|^2} \right]^{-1} I_2
\]

where \(U_i(D)\) is used on \(\tilde{r}_i\).

III. Finite Block Matrices

While D-domain processing allows for compact notation and
deeper understanding of the problem, implementing the
solutions is typically done using finite block-length matrices
through their structured manipulations[5].

First the single-user analog of the LP scheme will be
mentioned and then the multi-user one will be developed for
use in frequency selective channels.

The channel has memory \(\nu\) and its impulse response has
length \(\nu+1\). Note that in these block matrices, the data streams
are grouped into \(N\) symbol blocks, and are zero padded with
\(\nu\) zeros after the symbol.

A. Single-User

The LP transmit diversity scheme was originally specified
in D-domain format [3], and in [5], it specified in finite block
length matrix form.

\[
\begin{bmatrix}
    y_1 \\
    y_2
\end{bmatrix}^T = \begin{bmatrix} H_1 & H_2 \end{bmatrix} I_{zs} \begin{bmatrix} s_1 \\
    s_2 \\
    J_N s_2^T
\end{bmatrix} + \begin{bmatrix} n_1 \\
    n_2
\end{bmatrix}^T
\]

(11)

where \(H_i\) is a square \((N+\nu)\)-dimensional, lower triangular
Toeplitz matrix with the first column corresponding to the
impulse response of the \(i^{th}\) channel, \(I_{zs}\) is the zero stuffing
matrix that corresponds to \(i^{th}\) data block and \(n_i\) is an \((N+\nu)\)-dimensional vector

Defining \(r\) in a similar way as in the D-domain by conjugating and reversing \(y_2\):

\[
r = \begin{bmatrix} y_1 \\
    J_{N+\nu} y_2^T
\end{bmatrix} = \begin{bmatrix} H_1 I_{zs} \\
    J_{N+\nu} H_2 I_{zs} J_N
\end{bmatrix} \begin{bmatrix} s_1 \\
    s_2 \\
    J_N s_2
\end{bmatrix} + \begin{bmatrix} n_1 \\
    n_2
\end{bmatrix}
\]

\[\triangleq Hs + n\] (12)

It is possible to use a maximum likelihood detector at this
stage, and achieve full diversity at this stage, but it is possible
to exploit the nature of the space time code, like done in the
Alamouti and LP schemes. In both of these schemes, (3,5) we
multiplied the respective \(r\) by a matched filter \(H^H\). For the \(H\)
in the single user finite block matrix case, the matched filter
is also \(H^H\). Multiplying through gives:

\[
z = H^H r = \begin{bmatrix} H_{eqv} & 0 \end{bmatrix} \begin{bmatrix} s_1 \\
    s_2
\end{bmatrix} + H^H n
\]

(13)

where \(H_{eqv} = (H_1 I_{zs})^H H_1 I_{zs} + (H_2 I_{zs})^H H_2 I_{zs}\).

Clearly the user’s two data streams \(s_1\) and \(s_2\) are decoupled,
each only being multiplied by \(H_{eqv}\). Using an ML detector
at this stage is less complex because there is no interstream
interference.

B. Multi-User

In order to implement the D-domain multi-user interference
mitigation filters using finite block matrices, (12) will need to
be modified. It is shown in [5] that if the zero stuffing operation
is moved to the input blocks and instead of \(J_{N+\nu}\), a different
permutation matrix \(P\) is used that does row-reversal, but also
moves the last \(\nu\) symbols to the end, the result is

\[
r^{(c)} = \begin{bmatrix} y_1 \\
    Py_2^T
\end{bmatrix} = \begin{bmatrix} H_1^{(c)} & H_2^{(c)} \\
    H_2^{(c)H} & -H_1^{(c)H}
\end{bmatrix} \begin{bmatrix} I_{zs} s_1 \\
    I_{zs} s_2
\end{bmatrix} + \begin{bmatrix} n_1 \\
    n_2
\end{bmatrix}
\]

\[\triangleq H^{(c)} s + \tilde{n}\] (14)

where \(H_i^{(c)}\) is the square \((N+\nu)\)-dimensional circulant form
of \(H_i\); that is, a Toeplitz matrix with the first column being the
impulse response of channel \(i\) appended with \(N-1\) zeros, and
the first row being equal to the first column, but reversed and
shifted to the right by one. This is for the single user, single
receive antenna case. The finite block length matrix parallel of
In the two user, two antenna case, this becomes

\[
r = \begin{bmatrix} r_1^{(c)} \\
    r_2^{(c)}
\end{bmatrix} = \begin{bmatrix} H_1^{(c)} & H_2^{(c)} \\
    H_2^{(c)H} & -H_1^{(c)H}
\end{bmatrix} \begin{bmatrix} I_{zs} s_1 \\
    I_{zs} s_2
\end{bmatrix} + \begin{bmatrix} n_1 \\
    n_2
\end{bmatrix}
\]

(15)

C. MAC Zero Forcing Solution

Similar to the D-domain case, define \(Q^{(c)}\) be the set of \(H\)
matrices of the form (14). This set of circulant matrices, like
the set \(Q\) in the D-domain, is closed under multiplication, and
forms a multiplicative group. For \(H_6 Q^{(c)}\), define \(H^{(c)}^{-1} Q^{(c)}\)
as

\[
H^{(c)}^{-1} = \begin{bmatrix} I_2 \otimes I_{2(N+\nu)} & -H_1^{(c)H} H_2^{(c)H} \\
    -H_2^{(c)H} H_1^{(c)H} & I_{2(N+\nu)}
\end{bmatrix}
\]

(16)

Using these definitions, the zero forcing filter in the D-domain
becomes

\[
W = \begin{bmatrix} I_{2(N+\nu)} & -H_1^{(c)H} H_2^{(c)H} \\
    -H_2^{(c)H} H_1^{(c)H} & I_{2(N+\nu)}
\end{bmatrix}
\]

(17)

and thus multiplying by \(r^{(c)}\) gives

\[
\tilde{r} = \begin{bmatrix} \tilde{r}_1 \\
    \tilde{r}_2
\end{bmatrix} \triangleq W r
\]

\[= \begin{bmatrix} \tilde{H}_1 & 0 \\
    0 & \tilde{H}_2
\end{bmatrix} \begin{bmatrix} s_1 \\
    s_2
\end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\
    \tilde{n}_2
\end{bmatrix}
\]

(18)

where

\[
\tilde{H}_1 = H_1^{(c)} - H_1^{(c)H} H_2^{(c)} H_2^{(c)H} - H_1^{(c)H} H_2^{(c)H} \\
\tilde{H}_2 = H_2^{(c)} - H_2^{(c)H} H_1^{(c)} H_1^{(c)H} - H_2^{(c)H} H_1^{(c)H}
\]

(19)
Thus the correspondence between the D-domain and finite block matrix domain has been shown, and it is clear that by using the decorrelating filter $\mathbf{W}$ the users do not experience interference between eachother.

IV. SIMULATIONS

ML detection was initially intended to be the method of simulation. It was known to be a difficult search, but due to the unique features of this channel, which will be discussed, standard implementations could not be used properly. Instead an MLSE-based method will be used to estimate the signals with a fair amount of accuracy. The simulations are done using the following assumptions: $N = 16$ and $\nu = 3$. The channel is a four-tap Rayleigh fading channel, with each tap having average power of 1. A modulation scheme of QPSK will be used. Higher modulation schemes or longer delays produced memory allocation problems in the MLSE algorithm in MATLAB.

In the single-user case, after multiplying the signal by its matched filter, the two streams became clearly decoupled from eachother. The matrix was block diagonal as in (13). The remaining blocks, while still circulant, were non-causal. That is, the signals at beginning of the block were dependent on the final ones. Although it seems logical, because of the MIMO matched filter we apply, it would be non-causal. The MLSE function in MATLAB, mlseeq, however, does not allow a non-causal channel impulse response. In order to make the system causal to allow ML detection, $\nu$ zeros are prepended to the input vector $\mathbf{r}$, and the impulse response of the equivalent channel is shifted to make it causal. This allows the MLSE algorithm to work as if the channel was causal. The results are shown in the figure below.

In the multi-user case, many problems were experienced that made using a non-exhaustive ML search difficult. From (18), it is seen that the overall channel can decouple, leaving two effective channels with matrix $\tilde{\mathbf{H}}_i$, corresponding to user $i$. These two effective channel matrices are of the same form as the single user case, in that they have the form of (14), but with one difference that caused problems: the impulse response is no longer naturally FIR; that is, it is only FIR because the matrix has finite length, otherwise it would be longer.

Upon inspection of (19), three important points can be noticed: these circulant matrices are filters, multiplying two of them together means cascading them, and inverting them means finding the inverse of the filter. The channel matrices $\mathbf{H}^{(c)}_{i,j}$ correspond to FIR filters, with memory $\nu$. When two signals are convolved, their support effectively becomes the sum of the support of each signal, like when we multiplied by the matched filter in the single user case. In the multi-user case, the second term of (19) is what causes the problem. The inverse of an FIR filter, by its very nature is IIR (assuming it has memory), and hence the $\mathbf{H}^{(c)-1}_{1,1}$ is non-zero for all its samples. Fortunately, the coefficients of $\mathbf{H}^{(c)-1}_{1,1}$ have a small modulus. Thus when we multiply $\mathbf{H}^{(c)}_{i,j}$ by its matched filter, $\mathbf{H}^{H}_{i,j}$, the user’s streams are decoupled, and the impulse response of the stream is dominated by the effect of the first term in (19), meaning we can truncate the impulse response of the effect channel of the each stream and apply MLSE to obtain a reasonable estimate of the channel. Clearly using more samples of the impulse response will have lower probability of error, but there is a memory allocation limitation imposed by the traceback length of the MLSE method, making longer channel impulse responses slower and more memory consuming. For the purposes of this project, we will truncate the impulse response at to have length $(2\nu + 1) + 1$, after applying the trick of making it causal like in the single user case. This gives the BER curve seen in Figure 1.

Looking at the curves the most immediate thing that is noticed is the degrade in performance for the multi-user case. This is due to the truncation done on the impulse response of the channels to be able to use the MLSE function. In true cases of using ML detection this should not be the case. Also there seems to be a noise floor for higher SNR, and this could be associated with edge effects at the beginning and the end of the blocks which could be decreased if more zero padding was done on the signal, at the expense of a slower data rate.

V. CONCLUSIONS

This project explored one way of extending the Alamouti STBC to frequency selective channels. Although there are numerous ways of doing this, such as [6], a few key points were found using the methods in [3] and [5]. First, the side effect of matched filtering a LP code, in the original single user case and in the multi-user case, is that it extends the channel impulse response. Second, the elegance of correlation between D-domain methods and finite block methods was seen. And finally, the performance limitations of ML detection were experienced, and a method of using the MLSE method to estimate the transmitted signal given a non-causal signal was developed.
REFERENCES