Sean Carroll

in truth, only atoms and the void

Space Emerging from Quantum Mechanics

Posted on July 18, 2016 by Sean Carroll

The other day I was amused to find a quote from Einstein, in 1936, about how hard it would be to quantize gravity: "like an attempt to breathe in empty space." Eight decades later, I think we can still agree that it's hard.

So here is a possibility worth considering: rather than quantizing gravity, maybe we should try to gravitize quantum mechanics. Or, more accurately but less evocatively, "find gravity inside quantum mechanics." Rather than starting with some essentially classical view of gravity and "quantizing" it, we might imagine starting with a quantum view of reality from the start, and find the ordinary three-dimensional space in which we live somehow emerging from quantum information. That's the project that ChunJun (Charles) Cao, Spyridon (Spiros) Michalakis, and I take a few tentative steps toward in a new paper.

We human beings, even those who have been studying quantum mechanics for a long time, still think in terms of a classical concepts. Positions, momenta, particles, fields, space itself. Quantum mechanics tells a different story. The quantum state of the universe is not a collection of things distributed through space, but something called a wave function. The wave function gives us a way of calculating the outcomes of measurements: whenever we measure an observable quantity like the position or momentum or spin of a particle, the wave function has a value for every possible outcome, and the probability of obtaining that outcome is given by the wave function squared. Indeed, that's typically how we construct wave functions in practice. Start with some classical-sounding notion like "the position of a particle" or "the amplitude of a field," and to each possible value we attach a complex number. That complex number, squared, gives us the probability of observing the system with that observed value.

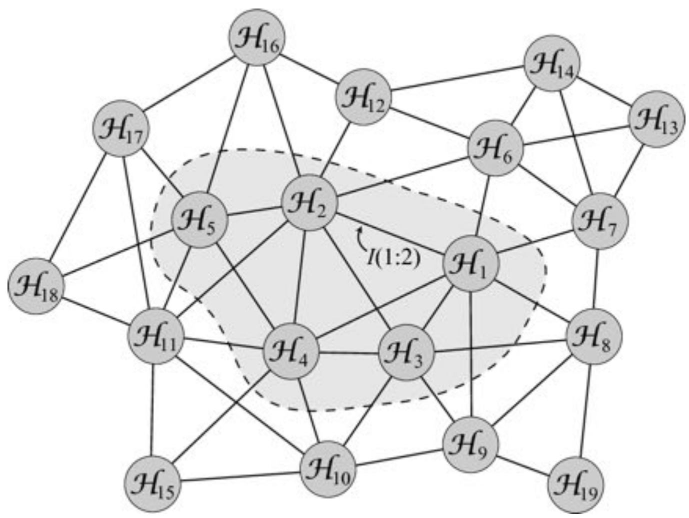
Mathematically, wave functions are elements of a mathematical structure called Hilbert space. That means they are vectors — we can add quantum states together (the origin of superpositions in quantum mechanics) and calculate the angle ("dot product") between them. (We're skipping over some technicalities here, especially regarding complex numbers — see e.g. *The Theoretical Minimum* for more.) The word "space" in "Hilbert space" doesn't mean the good old three-dimensional space we walk through every day, or even the four-dimensional spacetime of relativity. It's just math-speak for "a collection of things," in this case "possible quantum states of the universe."

Hilbert space is quite an abstract thing, which can seem at times pretty removed from the tangible phenomena of our everyday lives. This leads some people to wonder whether we need to supplement ordinary quantum mechanics by additional new variables, or alternatively to imagine that wave functions reflect our knowledge of the world, rather than being representations of reality. For purposes of this post I'll take the straightforward view that quantum mechanics says that the real world is best described by a wave function, an element of Hilbert space, evolving through time. (Of course time could be emergent too … something for another day.)

Here's the thing: we can construct a Hilbert space by starting with a classical idea like "all possible positions of a particle" and attaching a complex number to each value, obtaining a wave function. All the conceivable wave functions of that form constitute the Hilbert space we're interested in. But we don't *have* to do it that way. As Einstein might have said, God doesn't do it that way. Once we make wave functions by quantizing some classical system, we have states that live in Hilbert space. At this point it essentially doesn't matter where we came from; now we're in Hilbert space and we've left our classical starting point behind. Indeed, it's well-known that very different classical theories lead to the same theory when we quantize them, and likewise some quantum theories don't have classical predecessors at all.

The real world simply is quantum-mechanical from the start; it's not a quantization of some classical system. The universe is described by an element of Hilbert space. All of our usual classical notions should be derived from that, not the other way around. Even space itself. We think of the space through which we move as one of the most basic and irreducible constituents of the real world, but it might be better thought of as an approximate notion that emerges at large distances and low energies.

So here is the task we set for ourselves: start with a quantum state in Hilbert space. Not a random or generic state, admittedly; a particular kind of state. Divide Hilbert space up into pieces — technically, factors that we multiply together to make the whole space. Use quantum information — in particular, the amount of entanglement between different parts of the state, as measured by the mutual information — to define a "distance" between them. Parts that are highly entangled are considered to be nearby, while unentangled parts are far away. This gives us a graph, in which vertices are the different parts of Hilbert space, and the edges are weighted by the emergent distance between them.



We can then ask two questions:

1. When we zoom out, does the graph take on the geometry of a smooth, flat space with a fixed number of dimensions? (Answer: yes, when we put in the right kind of state to start with.)
2. If we perturb the state a little bit, how does the emergent geometry change? (Answer: space curves in response to emergent mass/energy, in a way reminiscent of Einstein's equation in general relativity.)

It's that last bit that is most exciting, but also most speculative. The claim, in its most dramatic-sounding form, is that gravity (spacetime curvature caused by energy/momentum) isn't hard to obtain in quantum mechanics — it's automatic! Or at least, the most natural thing to expect. If geometry is defined by entanglement and quantum information, then perturbing the state (e.g. by adding energy) naturally changes that geometry. And if the model matches onto an emergent field theory at large distances, the most natural relationship between energy and curvature is given by Einstein's equation. The optimistic view is that gravity just pops out effortlessly in the classical limit of an appropriate quantum system. But the devil is in the details, and there's a long way to go before we can declare victory.

Here's the abstract for our paper:

Space from Hilbert Space: Recovering Geometry from Bulk Entanglement

ChunJun Cao, Sean M. Carroll, Spyridon Michalakis

> We examine how to construct a spatial manifold and its geometry from the entanglement structure of an abstract quantum state in Hilbert space. Given a decomposition of Hilbert space *H* into a tensor product of factors, we consider a class of "redundancy-constrained states" in *H* that generalize the area-law behavior for entanglement entropy usually found in condensed-matter systems with gapped local Hamiltonians. Using mutual information to define a distance measure on the graph, we employ classical multidimensional scaling to extract the best-fit spatial dimensionality of the emergent geometry. We then show that entanglement perturbations on such emergent geometries naturally give rise to local modifications of spatial curvature which obey a (spatial) analog of Einstein's equation. The Hilbert space corresponding to a region of flat space is finite-dimensional and scales as the volume, though the entropy (and the maximum change thereof) scales like the area of the boundary. A version of the ER=EPR conjecture is recovered, in that perturbations that entangle distant parts of the emergent geometry generate a configuration that may be considered as a highly quantum wormhole.

Like almost any physics paper, we're building on ideas that have come before. The idea that spacetime geometry is related to entanglement has become increasingly popular, although it's mostly been explored in the holographic context of the AdS/CFT correspondence; here we're working directly in the "bulk" region of space, not appealing to a faraway boundary. A related notion is the ER=EPR conjecture of Maldacena and Susskind, relating entanglement to wormholes. In some sense, we're making this proposal a bit more specific, by giving a formula for distance as a function of entanglement. The relationship of geometry to energy comes from something called the Entanglement First Law, articulated by Faulkner et al., and used by Ted Jacobson in a version of entropic gravity. But as far as we know we're the first to start directly from Hilbert space, rather than assuming classical variables, a boundary, or a background spacetime. (There's an enormous amount of work that has been done in closely related areas, obviously, so I'd love to hear about anything in particular that we should know about.)

We're quick to admit that what we've done here is extremely preliminary and conjectural. We don't have a full theory of anything, and even what we do have involves a great deal of speculating and not yet enough rigorous calculating.

Most importantly, we've assumed that parts of Hilbert space that are highly entangled are also "nearby," but we haven't actually derived that fact. It's certainly what *should* happen, according to our current understanding of quantum field theory. It might seem like entangled particles can be as far apart as you like, but the contribution of particles to the overall entanglement is almost completely negligible — it's the quantum vacuum itself that carries almost all of the entanglement, and that's how we derive our geometry.

But it remains to be seen whether this notion really matches what we think of as "distance." To do that, it's not sufficient to talk about space, we also need to talk about time, and how states evolve. That's an obvious next step, but one we've just begun to think about. It raises a variety of intimidating questions. What is the appropriate Hamiltonian that actually generates time evolution? Is time fundamental and continuous, or emergent and discrete? Can we derive an emergent theory that includes not only curved space and time, but other quantum fields? Will those fields satisfy the relativistic condition of being invariant under Lorentz transformations? Will gravity, in particular, have propagating degrees of freedom corresponding to spin-2 gravitons? (And only one kind of graviton, coupled universally to energy-momentum?) Full employment for the immediate future.

Perhaps the most interesting and provocative feature of what we've done is that we start from an assumption that the degrees of freedom corresponding to any particular region of space are described by a *finite-dimensional* Hilbert space. In some sense this is natural, as it follows from the Bekenstein bound (on the total entropy that can fit in a region) or the holographic principle (which limits degrees of freedom by the area of the boundary of their region). But on the other hand, it's completely contrary to what we're used to thinking about from quantum field theory, which generally assumes that the number of degrees of freedom in any region of space is infinitely big, corresponding to an infinite-dimensional Hilbert space. (By itself that's not so worrisome; a single simple harmonic oscillator is described by an infinite-dimensional Hilbert space, just because its energy can be arbitrarily large.) People like Jacobson and Seth Lloyd have argued, on pretty general grounds, that any theory with gravity will locally be described by finite-dimensional Hilbert spaces.

That's a big deal, if true, and I don't think we physicists have really absorbed the consequences of the idea as yet. Field theory is embedded in how we think about the world; all of the notorious infinities of particle physics that we work so hard to renormalize away owe their existence to the fact that there are an infinite number of degrees of freedom. A finite-dimensional Hilbert space describes a very different world indeed. In many ways, it's a much simpler world — one that should be easier to understand. We shall see.

Part of me thinks that a picture along these lines — geometry emerging from quantum information, obeying a version of Einstein's equation in the classical limit — pretty much *has* to be true, if you believe (1) regions of space have a finite number of degrees of freedom, and (2) the world is described by a wave function in Hilbert space. Those are fairly reasonable postulates, all by themselves, but of course there could be any number of twists and turns to get where we want to go, if indeed it's possible. Personally I think the prospects are exciting, and I'm eager to see where these ideas lead us.

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36 Responses to *Space Emerging from Quantum Mechanics*

Ben Goren *says:*
July 18, 2016 at 9:06 am

Exciting stuff — thanks for letting us peek over your shoulder like this!

This seems to me, as a layperson, very closely related to something I've been thinking a fair bit about recently.

If we are to take seriously that everything is waves, then there's something big missing from all the for-the-lay-audience explanations of QM. Take the double-slit experiment: the electron wave diffracts past the slits; when it reaches the detector, magic happens. It might be the collapse of the wave function or the branching of the universe or whatever…

…but *all* those explanations only make sense if the detector is, itself, a monolithic classical (if not Platonic) entity.

But if it's instead an assemblage of waves…then "detection" can *only* mean the interaction of one of the electron waves in the detector with the diffracted electron wave — a phosphorus atom absorbs the electron and spits it back out with a photon, a charge is induced in an element in the CCD array, whatever.