Quantum Mechanics: Week 3 overview

Stanford University

Week 3

Monday	Tuesday	Wednesday	Thursday	Friday
Recap Bell, CHSH, KS Q. Game: CHSH Measurement Problem Decoherence 1 Density matrix Q. Eraser Interpretations (start).	Interpretations (Relativity) No-cloning Continuous Obs. Generalized S.E. Schrödinger eqn. (SE) Time-Indep. SE,	S.E. H atom PEP, standard model Tunneling Uncertainty principle Q. Computing Intro Q. Crypto.	Q. Logic gates Q. Teleportation GHZ Jouza's alg. (P vs NP) Q. Compl. P=NP with non-linear.	Harm. Osc. QFT GR, BH String theory.
	Afternoon topics (tbd)			
	Solving TISE, (Relativity) (secret stuff)	Q. Complexity (?) Why QM? (?) MZ inter. LIGO, Q.E.		
	Outside readings			
Scarani Ch 5,9				
	Possible problems			
Day 9 problems/Day 11	[none] / Day 11 problems (possibly more)	[none] possibly optional	TBD	

Week 3 potential items

- Relativity (complete).
- Summary CHSH, KS, significance.
- Quantum game CHSH (afternoon problem).
- GHZ (problem, Mermin, Q. Game).
- Measurement problem.
- Schrödinger's cat.
- Decoherence 1
- Density matrix
- Decoherence 2
- Quantum Eraser, demo
- Interpretations
- Uncertainty principle
- Spin: Standard model
- PEP
- Why QM? (new).
- Q. Complexity (new)
- Continuous observables
- Gen. S.E.
- Maxwell's $\rightarrow$ S.E.
- Solving S.E.
- Spectra, H atom (demo).
- Tunneling (STM)
- Harmonic Oscillator
- Q. Computing, introduction
- Q. Cryptography.
- Q. Teleportation.
- No cloning, monogamy.
- Jouza's algorithm.
- Grover's algorithm.
- Other quantum algorithms.
- Black hole info paradox.
- Advanced topics: QFT, QG, etc.

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Recap polarization Operators Expectation values Multiparticle states (tensor product) Hermitian operators, Postulates of QM, EPR intro. Resolution of identity	Bell inequality Probability, jpd. Measurement problem Mixtures Interpretations	Interpretations (cont.) Interp. probability Mixtures Density matrix Decoherence CHSH	Contextuality Kochen-Specker More experiments GHZ PR box	Principles of QM Computational basis. Logic gates

Afternoon topics (tbd)

Basics of spin, PEP		H atom		

Outside readings

Scarani Ch 1, Susskind Ch 1,2				

Possible problems

Projections, polarization	GHZ	GHZ		

All your polarization bases are belong to us

Eigenvalue:	$+1$	-1	Operator
HV basis:	$\|H\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\|V\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$	$\hat{S}_{HV} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
PM basis:	$\|P\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	$\|M\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	$\hat{S}_{PM} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
RL basis:	$\|R\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$	$\|L\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$	$\hat{S}_{RL} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

Most general state:

$$|\theta, \phi\rangle = \cos\theta|H\rangle + e^{i\phi}\sin\theta|V\rangle = \begin{pmatrix} \cos\theta \\ e^{i\phi}\sin\theta \end{pmatrix}$$

Operators in general:

$$\hat{\Omega} = \sum_i \omega_i |\omega_i\rangle\langle\omega_i|$$

$$Prob(\text{to find } \omega \text{ when in state } \psi) = |\langle\omega|\psi\rangle|^2.$$

All your polarization bases are belong to us

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Finding probabilities for specific outcomes:

$$Prob(\text{to find } \omega \text{ when in state } \psi) \;=\; |\langle\omega|\psi\rangle|^2.$$

Finding the expectation value of an observable Ω when in state ψ:

$$\langle\hat{\Omega}\rangle \;=\; \langle\psi|\hat{\Omega}|\psi\rangle. \quad \text{Why?} \quad = \langle\psi| \left[\sum_i \omega_i |\omega_i\rangle\langle\omega_i|\right] |\psi\rangle = \sum_i \omega_i \langle\psi|\omega_i\rangle\langle\omega_i|\psi\rangle = \sum_i \omega_i p(\omega_i).$$

Resolution of the identity: $\hat{1} \;=\; \sum_i |\lambda_i\rangle\langle\lambda_i|.$

For example, to expand a state vector into the RL basis we apply the identity as expressed in that basis:

$$\hat{1}|\psi\rangle \;=\; |R\rangle\langle R|\psi\rangle + |L\rangle\langle L|\psi\rangle = \psi_R|R\rangle + \psi_L|L\rangle,$$

where ψ_R and ψ_L are the components extending along R and L respectively.

Physical realization: photon polarization	2 dimensional Hilbert space $\mathcal{H}$			Physical realization: electron spin	
$|\psi\rangle = \cos\theta|H\rangle + e^{i\phi}\sin\theta|V\rangle$	HV	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	z	$|\psi\rangle = \cos\frac{\theta}{2}|+\rangle + e^{i\phi}\sin\frac{\theta}{2}|-\rangle$
	PM	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	x	
	RL	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$	y	

1: State

The state, including all you can know about it, is represented mathematically by a normalized ket $|\psi\rangle$ in Hilbert space.

2: Observables

A physical observable is represented mathematically by a Hermitian operator A that acts on kets on the Hilbert space $\mathcal{H}$.

3: Measurement

The probability of obtaining the eigenvalue a_n in a measurement of the observable A on the system in state $|\psi\rangle$ is $\mathcal{P}_{a_n} = |\langle a_n|\psi\rangle|^2$ where $|a_n\rangle$ is the normalized eigenvector of A corresponding to the eigenvalue a_n.

4: Time Evolution

The state $|\psi\rangle$ of the system at time t_1 is related to the state $|\psi'\rangle$ of the system at time t_2 by a unitary operator U which depends only on the times t_1 and t_2,

$$|\psi(t_2)\rangle \;=\; \hat{U}(t_2, t_1)|\psi(t_1)\rangle.$$

Day 12 (3.3) Plan

Main topics: measurement problem, interpretations, decoherence, quantum eraser. (other?)
Interpretations, points:
von-Neumann/WIgner:

- Pro: The mathematics of QM is clear. Atoms, and things made up of atoms are quantum. Thus, atom, detector, retina, etc. should all be in a superposition. The mind (consciousness) is presumably something different. We may not know, but vN (and Roger Penrose for that matter) might argue that we will eventually understand it. And there are arguments that it should be a non-algorithmic process as well (e.g. epiphanies). Thus, it might be that this is the non-linear source of wavefunction collapse.
- Con: Mentioned before. This leads to a dualist view of nature.
- Wigner's friend. Wigner steps back in and asks question. To him, the wavefunction collapsed at that point, or did it? Perhaps he asks the student what he felt right before he asked the question. The student would no doubt say that she saw the light flash. So he must conclude that the student collapsed it (or else he falls into solipsism that everything perceived is a figment of your imagination). Thus, Wigner concluded, the first conscious being collapsed the wavefunction.
- Interesting suggestion. NPR had an author talking about those with ? syndrome. Where feel you are dead. Would these people be able to collapse a wavefunction? Somehow you would have them obtain

WPI, and later reveal the result. There are experiments that indicate that neural processing occurs well before conscious awareness. Minds can react to things prior to the person being consciously aware.