Lecture 3 Assignment

Problem 3.1 the Principle of Relativity \((\text{in groups})(\text{may be done in class})\)

Two overlapping free-float frames are in uniform relative motion. On the following list, check “Y” (yes) the quantities that must necessarily be the same as measured in the two frames. Check “N” (no) the quantities that are not necessarily the same as measured in the two frames.

a) Y N  time it takes for light to go one meter of distance in a vacuum.
b) Y N  spacetime interval between two events.
c) Y N  kinetic energy of an electron.
d) Y N  value of the mass of an electron.
e) Y N  value of the magnetic field at a given point.
f) Y N  distance between two events.
g) Y N  structure of the DNA molecule.
h) Y N  What numeral is displayed by a digital clock at one event.

Problem 3.2: things that move faster than light \((\text{in groups})\)

Can “things” or “messages” move faster than light? Does relativity really say “No” to this possibility? Explore these questions further using the following examples.

a) \textbf{The Scissors Paradox}. A very long straight rod, inclined at an angle \(\theta\) to the x-axis, moves downward with uniform speed \(v_{\text{rod}}\) as shown in the figure. Find the speed \(v_A\) of the point of intersection \(A\) of the lower edge of the stick with the x-axis. Can this speed be greater than the speed of light? Can the motion of point \(A\) be used to transmit a message faster than light from someone at the origin to someone far out on the x-axis? (See next part).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{scissors_paradox.png}
\caption{The Scissors Paradox}
\end{figure}

a2) \textbf{A Morbid Tale: Relativistic Guillotine}. \textit{A student in an earlier class suggested this variant of the previous gedankenexperiment. I am not that morbid, but it does highlight the issues involved in an excellent manner.}

The story goes as follows: There once was a kingdom with unparalleled engineering skills. A revolution occurred in this kingdom, and both the king and queen are sentenced to be executed. However, they are a light-year apart (ok, perhaps this is in the future), but they want to use the same blade to execute both parties. Thus they create a very large guillotine, the blade is at a slight angle (\(\theta\) as in the previous part) and spans the distance between the two heads. Thus the setup is identical to that in the previous part, except now there are two events (king’s execution which happens first and the queen’s a while later). The blade is dropped from a height such that it is traveling downward at constant velocity, \(v\), during these two events.

There is a group of loyalists that are poised to start a counter-revolution positioned near each monarch. The group near the queen await word to find out if the King has been spared before they challenge the revolutionaries. If the blade near the King visibly reacts (shakes a little say) as the blade reaches the King, then they will go into hiding for they know the King has been executed. If the blade does not react, then the King has been spared and they will fight the revolutionaries. Can you determine whether they will revolt or hide? Could this scheme work? Explain.
b) **Transmission of a Hammer Pulse.** Suppose the same rod is initially at rest in the laboratory with the point of intersection initially at the origin. The region of the rod centered at the origin is struck sharply with a downward blow of a hammer. The point of intersection moves to the right. Can this motion of the point of intersection be used to transmit a message faster than the speed of light?

c) **Searchlight Messenger?** A very powerful searchlight is rotated rapidly in such a way that its beam sweeps out a flat plane. Observers $A$ and $B$ are at rest on the plane and each the same distance from the searchlight but not nearer to each other. How far from the searchlight must $A$ and $B$ be in order that the searchlight beam will sweep from $A$ to $B$ faster than a light signal could travel from $A$ to $B$? Before they took their positions, the two observers were given the following instructions:

To A: “When you see the searchlight beam, fire a bullet at $B$.”

To B: “When you see the searchlight beam, duck because $A$ has fired a bullet at you.”

Under these circumstances, has a warning message traveled from $A$ to $B$ with a speed faster than that of light?

d) **Oscilloscope Writing Speed.** The manufacturer of an oscilloscope claims a writing speed (the speed with which the bright spot moves across the screen) in excess of the speed of light. Is this possible? [Think about how the bright spot is created in a CRT tube, draw a picture of this process.]

**Problem 3.3 Four times the speed of light?**

This in Spacetime Physics 3-15

We look westward across the United States and see the rocket approaching us at four times the speed of light. [How can this be? We did not say the rocket moves faster than light; we said only that we see it moving faster than light.]

Here is what happens: The rocket streaks under the Golden Gate Bridge in San Francisco, emitting a flash of light that illuminates the rocket, the bridge, and the surroundings. At time $\Delta t$ later the rocket threads the Gateway Arch in St. Louis. The top figure is a visual summary of measurements from our continent-spanning latticework of clocks taken at this moment.

Now the rocket continues toward us as we stand in New York City. The center figure summarizes data taken as the first flash is about to enter our eye. Flash 1 shows us the rocket passing under the Golden Gate Bridge. An instant later flash 2 shows us the rocket passing through the Gateway Arch.

\[ \text{Rocket at St. Louis} \]

\[
\begin{align*}
S.F. & \quad S.L. & \quad \text{flash 1} & \quad N.Y. \\
\quad & \quad v \Delta t & \quad (1 - \frac{v}{c})c \Delta t & \\
\text{emit flash 1} & \quad \text{emit flash 2} & \\
\end{align*}
\]

\[ \text{Rocket Headed East} \]

\[
\begin{align*}
S.F. & \quad S.L. & \quad \text{flash 2} & \quad \text{flash 1} & \quad N.Y. \\
\quad & \quad v \Delta t & \quad (1 - \frac{v}{c})c \Delta t & \\
\text{emit flash 1} & \quad \text{emit flash 2} & \\
\end{align*}
\]

a) Answer the following questions using the symbols from the last two figures. The images carried by the two flashes show the rocket how far apart in space? What is the time lapse between our reception of these two images? Therefore, what is the apparent speed of the approaching rocket we see? For what speed $v$ of the rocket does the apparent speed of approach equal four times the speed of light? For what rocket speed do we see the approaching rocket to be moving at 99 times the speed of light?
Problem 3.4 The distinguished Prof. F part 1 (in groups)

A few years ago I wrote a paper on the inconsistency of relativistic mass, though not accepted to the American Journal of Physics it did get some exposure (cited in wikipedia) and I received many comments. Often these took the form of, ‘hey I liked your paper, please read my attached paper on my whacked out version of relativity’ or some such. One particular comment developed into a lengthy correspondence. Basically this professor at a European university and a researcher at CERN (hence someone of reputable credentials you would think) attached his 50 some odd page paper to his comment with ‘let me know what you think’. Well, I was not going to bother reading it, especially since in the abstract he claims that, though time dilation is real, length contraction (and relativistic simultaneity) are not and everyone has been misled into believing Einstein’s work. Well, I couldn’t be bothered wasting time on such ideas until he posted a 3 page paper summarizing his argument. Three pages I can read, and I was flabbergasted. Not only did he make an elementary mathematical error which he refused to accept he continued to defend his viewpoint after several email exchanges.

So why am I mentioning this? Well, there are two good problems here and in this exercise we will cover the first. (Perhaps later I will ask you to find the elementary mistake in his paper). It took many back and forth emails for me to try to ascertain his interpretation of relativity. I intended to provide him with a simple gedankenexperiment to show him the fallacy of his thinking – but he was unwavering. First let me introduce the gedankenexperiment and have you fill in the blanks for what you think occurs and then I will state his position and have you determine what is wrong with it.

Consider the following gedankenexperiment: A very fast spaceship travels at 87% the speed of light between Earth and a planet circling Alpha Centauri ≈ 4ly away. There are two observers in the Earth frame on each planet who have previously synchronized their clocks (call them observer A (Earth) and B(Alpha Centauri)). Two more observers ride on board the rocket (observers C). When the rocket passes Earth (call this event 1) all clocks are set to 0 (the Earth observer and the rocket observers). When the rocket passes the second planet (event 2) the three observers can be considered to be at the same spatial point at this instant. At event 2 what do the clocks read as observed in each frame. That is,

\[\begin{align*}
a) & \text{Observer B sees clock B to read } y. \\
\text{Observer B sees clock C to read } y. \\
\text{Observers C see clock C to read } y. \\
\text{Observers C see clock B to read } y. \\
b) & \text{Well after much obfuscation I was able to get this professor to state his answer to this question. I will not trouble you with why he believes the following but just state his result,} \\
\text{Observer B sees clock B to read 4.6 y.} \\
\text{Observer B sees clock C to read 2.3 y.} \\
\text{Observers C see clock C to read 4.6 y.} \\
\text{Observers C see clock B to read 2.3 y.}
\end{align*}\]

Can this be correct? I want you to explain what results in this universe and our own in terms of the following extension of the gedankenexperiment. Unbeknownst to the rocket travelers a small container containing a lethal poison has been placed in observer C1’s spacesuit that is triggered to be opened by the on board clock (clock C) when it strikes the time 4 years. The trigger is precise, it occurs at one single event when the needle of the clock points to 4 y. On Earth, someone has discovered the plot to kill C1 and radios to Alpha Centauri to use its radio controller to disable the clock. Thus when the rocket passes Alpha Centauri (event 2) observer B flicks a switch that disables the rockets clock freezing the needle at its current position. If done in time, C1 might be saved but it depends on what clock C reads at event 2.

From the distinguished professors answer explain what happens as seen in both frames when the ship passes (event 2) and is programmed to stop and land on Alpha Centauri where all three observers are next to each other at rest. Explain why special relativity (your answer in a) is consistent and that our friends idea leads to a logical inconsistency.

c) It may seem that we physicists like to make dark, morbid problems. However, there is no more glaring logical inconsistency as that found in the previous part. You can make an even more glaring discrimination by substituting the poison capsule in the previous problem with a bomb that destroys the ship. Explain what is observed from both perspectives at event 2 in this case. [If you ever want to test for a logical inconsistency, use a bomb –it is very decisive and leaves little doubt].

d) Explain what idea or principle is behind these inconsistencies. Why is your explanation correct and our friends not?

If you finish early: Do Spacetime Physics 5-7, 4-2, L-7
Short questions (in groups)(may be done in class)

Problem 3.0: Spacetime diagram for an accelerating object. (in groups)(may be done in class)

a) Draw a spacetime diagram ($ct$ vs $x$) for the frame in which an object, initially at rest, begins to accelerate in the positive $x$ direction at a constant positive rate at time $t = 0$.
b) Draw the same scenario but from another frame moving at $c/2$ in the positive $x$-direction with respect to the frame in the previous part. The frame is defined such that the object is momentarily at rest at the origin of this frame.

3S.1

Create plots for the following scenarios in spacetime.

1. Object traveling at $c/2$ in the negative $x$ direction.
2. Starship Enterprise traveling at warp 3 ($=3c$).
3. Object undergoing simple harmonic motion. (Say a block on the end of a spring which has been pulled and then released).
4. Particle traveling in a circle (requires a 3D plot).
5. Particle undergoing constant acceleration, starting from rest.
6. Particle moving at constant speed up until time $t = 0$ when it hits a wall and stops.
7. An object thrown straight up in the air, near the Earth's surface.
3S.2: Identify the motion

Examine the following plots of objects worldlines in spacetime. Describe what the motion is like, whether it is valid physically, and if not why. (Think of what the worldline is actually describing and see if you can determine what exactly is being violated with its motion).