Strategic Trading in Informationally Complex Environments*

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Abstract

We study trading behavior and the properties of prices in informationally complex markets. Our model is based on the single-period version of the linear-normal framework of Kyle (1985). We allow for essentially arbitrary correlations among the random variables involved in the model: the value of the traded asset, the signals of strategic traders and competitive market makers, and the demand from liquidity traders. We show that there always exists a unique linear equilibrium, characterize it analytically, and illustrate its properties in a series of examples. We then use this characterization to study the informational efficiency of prices as the number of strategic traders becomes large. If liquidity demand is positively correlated (or uncorrelated) with the asset value, then prices in large markets aggregate all available information. If liquidity demand is negatively correlated with the asset value, then prices in large markets aggregate all information except that contained in liquidity demand.

Keywords: Information aggregation, rational expectations equilibrium, efficient market hypothesis, market microstructure, strategic trading.

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1 Introduction

Whether and how dispersed information enters into market prices is one of the central questions of information economics. A key difficulty in answering this question is the strategic behavior of informed traders. A trader who has private information about the value of an asset has an incentive to trade in the direction of that information. However, the more he trades, the more he reveals his information, and the more he moves the prices closer to the true value of an asset. Thus, to maximize his profits, an informed trader may stop short of fully revealing his information, and so the informational efficiency of market prices may fail.

Therefore, an important and natural question is when we should expect market prices to in fact reflect all information available to market participants. One stream of literature considers trading in dynamic environments, with informed traders having multiple opportunities for trading.[1] In these settings, in each period, traders may have an incentive to withhold some of their information in order not to eliminate their profits. However, over time, traders will gradually reveal all of their information, and in many (although not all) cases, by the end of trading, market prices will in fact aggregate all available information.

One issue with the case of dynamic trading is that while at the end, market prices accurately reflect all available information, that is generally not the case during most of the time the market is in operation—and thus much of the trading may happen at prices that are far away from the ones that would prevail if all private information was publicly available to all market participants. Therefore, another important stream of research abstracts away from the time dimension and repeated trading in markets, and considers instead an alternative intuition for when market prices may accurately reflect information: when the number of informed traders is large, and each one of them is informationally small. In that case, each of the informed traders has limited impact on market prices, but their aggregate behavior does in fact reflect the aggregate information available in the market. As a result, market prices are close to those that would prevail if all private information were publicly available, and all trades happen at those prices.

Non-strategic explorations of this intuition go back to Hayek (1945), Grossman (1976), and Radner (1979).[2] Subsequently, a line of research (which we discuss in more detail in Section 2) has considered strategic foundations for this intuition, studying strategic behavior of informed agents in finite markets, and then considering the properties of prices as the number of these agents becomes large. This stream of work, however, imposes very strict assumptions on how information is distributed among the agents, typically assuming that the signals of informed agents are symmetrically

[1] See, e.g., Hellwig (1982), Kyle (1985), Dubey et al. (1987), Wolinsky (1990), Back (1992), Foster and Viswanathan (1996), Back et al. (2000), Ostrovsky (2012), and Golosov et al. (2014), among others.

[2] Other foundational papers in the rich literature on Rational Expectations Equilibrium and related non-strategic solution concepts include Kreps (1977), Hellwig (1980), Allen (1981), and Anderson and Sonnenschein (1982); for surveys of the literature, see Jordan and Radner (1982), Allen and Jordan (1998), and Glycopantis and Yannelis (2005). These papers focus on equilibrium existence and the amount of information aggregated and communicated by prices in equilibrium, but only consider environments with either an infinite number of infinitesimally small traders, or with a finite number of traders who essentially ignore the impact they have on market prices and behave non-strategically.

distributed, or satisfy other related restrictions so that in equilibrium, the strategies of all informed traders are identical (see Section 2). In practice, however, the distribution of information in the economy can be much more complex. Some agents may be strictly more informed than others. Groups of agents may have access to different sources of information, so that the correlations of signals within a group are very different from correlations across groups (and the sizes of the groups may be different, and the correlations of signals between different groups may be different as well). Some agents may be informed about the fundamental value of the security, while others may be uninformed about the fundamentals but possess some "technical" information about the market or other traders. And of course all such possibilities may be present in a market at the same time.

Our paper makes two main contributions.

First, we present an analytically tractable framework that makes it possible to study trading in such informationally complex environments. Our model is based on the single-period version of the model of Kyle (1985). As in that paper, an important assumption that makes our model analytically tractable is the assumption of joint normality of random variables involved: the true value of the traded asset, the signals of strategic traders, the signals of competitive market makers, and the demand coming from liquidity traders. Beyond that assumption, however, we impose essentially no restrictions on the joint distribution of these variables, making it possible to model informationally rich situations such as those described above. In this framework, we show that there always exists a unique linear equilibrium, which can be computed in closed form.

Second, we explore the informational properties of equilibrium prices as the number of informed agents becomes large. We assume that there are several types of agents, with each agent of a given type receiving the same information (possibly affected by idiosyncratic noise), and fix the matrix of correlations of signals across the types (and other random variables in the model). We then allow the numbers of agents of every type to grow (without restricting the rates of growth in any way; e.g., the number of agents of one type may grow much faster than the number of agents of another type). We find that the informational properties of prices in these large markets depend on the informativeness of the demand from liquidity traders. If the demand from liquidity traders is uncorrelated with the true value of the asset or is positively correlated with it (conditional on other signals), then prices in large markets aggregate all available information. If, however, liquidity demand is negatively correlated with the true value of the asset, then prices in large markets aggregate all available information except that contained in liquidity demand. Crucially, in both cases, as markets become large, the information possessed by the strategic traders is fully aggregated and fully incorporated into market prices, for very general (multidimensional and asymmetric) information structures. We should note, however, that the presence of exogenous liquidity demand plays an important role in our information aggregation results: it makes trading possible by providing a source of profits for the strategic traders.[3]

We also illustrate our model with several examples. One example shows that under fairly

[3]In fact, our information aggregation results rely on a slightly stronger assumption that the variance of liquidity demand is positive conditional on the realizations of the signals of the strategic traders and the market maker. See Section 6 and Footnote 20 for the discussion.

simple asymmetric information structures, an informed trader may choose to trade "against" his information, i.e., sell the asset when his signal implies that the expected value of the asset is positive, and vice versa. Two examples explore the profitability of "technical" trading, and show that a trader may be able to make substantial positive expected profit even if he has no information about the value of the asset, provided there is at least one other ("fundamental") trader who does, and provided that the technical trader has information about the demand from liquidity traders or about the mistakes of the fundamental trader. Our last set of examples shows how equilibrium trading and outcomes depend on the amount of private information available to the market maker (beyond the aggregate market demand), and in particular illustrates that having a market maker observe a particular signal is not equivalent to having that signal observed publicly.

This distinction plays an important role for the next result of the paper, which characterizes the informational properties of prices in a "hybrid" case: some information is available only to a small number of traders ("scarce" information), while some other information is available to a large number of traders ("abundant" information). As the number of traders having access to abundant information becomes large, the equilibrium converges to the one that would obtain if these traders were not present in the market at all, and instead their information was observed by the market maker (but not by the remaining strategic traders, who continue to observe scarce information).

Finally, to investigate the driving force behind our main information aggregation result, we consider a simpler model in which there are no liquidity traders, and in which the sensitivity of prices to aggregate quantity is fixed (instead of being endogenously determined by a Bayesian market maker). We present the model in the language of Cournot competition, but note that it is isomorphic to a model of trading with a mechanical (rather than Bayesian) market maker. We find that in this simpler model, information dispersed among the strategic agents gets fully aggregated in the limit as their numbers grow—just as in the main model of the paper.

The remainder of this paper is organized as follows. In Section 2, we discuss related literature. In Section 3, we present the main model. In Section 4, we state and prove our first main result, on the existence and uniqueness of linear equilibrium, and characterize this equilibrium analytically. In Section 5, we illustrate our result with several examples. In Section 6, we present our second main result, on information aggregation in large markets. In Section 7, we explore the hybrid case in which some information is scarce and some is abundant. In Section 8, we study the simpler model of Cournot competition. Section 9 concludes.

2 Related Literature

The literature on strategic foundations of information aggregation and revelation in markets goes back to Wilson (1977), who considers an auction-based model in which multiple partially informed agents bid on a single object. Other work in this tradition includes Milgrom (1981), Pesendorfer and Swinkels (1997), Kremer (2002), and Reny and Perry (2006). These papers find that under various suitable conditions, information does get aggregated (and revealed in winning bids) when the number of bidders becomes large. However, these results depend critically on strong symmetry assumptions on the bidders' signals and strategies.

Another related stream of literature, going back to Kyle (1989), considers equilibria in demand and supply functions, where bidders specify how many units of an asset they demand or supply for each possible price level, and then the market maker picks the price that clears the market.[4] Most papers in this tradition also require a very high degree of symmetry among the trading agents, typically assuming that these agents are ex ante identical, receive symmetrically distributed information, and employ identical strategies in equilibrium. Notable exceptions are recent papers by Rostek and Weretka (2012), who replace the symmetry assumption with a weaker assumption of "equicommonality" on the matrix of correlations of agents' values; Rostek and Yoon (2014), who go beyond equicommonality and provide sufficient conditions on the (potentially asymmetric) matrix of correlations for the existence of linear equilibrium; and Babus and Kondor (2013), who assume a symmetric matrix of correlations of agents' values, but allow for asymmetries in the graph of possible trading relationships (thus resulting in asymmetric trading behavior of agents in equilibrium). There are important differences between our model and the settings of Rostek and Weretka (2012), Rostek and Yoon (2014), and Babus and Kondor (2013). First, in our model, while agents generally receive different signals, their valuations for the security are the same, while in the settings of the above papers, the valuations are allowed to differ. Correspondingly, while the focus in our paper is on whether prices fully aggregate and reveal information, in the above papers the focus is on whether prices are "privately revealing." Second, the trading mechanisms are different: in our model, agents submit quantity orders, while in the above papers, agents submit demand and supply curves. So while the questions are related, our results and those of Rostek and Weretka (2012), Rostek and Yoon (2014), and Babus and Kondor (2013) are not directly comparable.

The stream of literature most closely related to our paper is the work building on Kyle (1985). In that literature, one or more strategic traders, fully or partially informed about the value of the traded asset, are present in the market. These strategic traders submit market orders to centralized market makers. There are also liquidity traders who submit exogenously determined market orders. The market makers set the price of the asset equal to their Bayesian estimate of its value, given their prior information, the knowledge of strategic traders' strategies, and the observed order flow. Our paper borrows much of its analytical framework from this literature. The key difference is that while many of the papers in this area consider both static and dynamic models of trading but place restrictive assumptions on the information structure, our paper places virtually no restrictions on the information structure (beyond joint normality), and focuses on the one-period model of trading and on the informational properties of prices as the number of strategic traders becomes large.

In the original model of Kyle (1985), there is only one informed trader, who knows the value of the asset. Admati and Pfleiderer (1988), Holden and Subrahmanyam (1992), Foster and Viswanathan (1996), and Back et al. (2000) study generalizations of Kyle (1985) in which multiple informed traders are either all fully informed about the value of the asset, or receive imperfect signals about

[4]See Vives (2008) for a textbook treatment of that literature.

the value, in which case different traders may observe different signals, but the distribution of these signals across the traders is symmetric (as are the traders' strategies). Caballé and Krishnan (1994) and Pasquariello (2007) consider multi-asset versions of the one-period model with multiple traders, but still maintain the assumption of symmetry of information among the traders. Dropping the assumption of normality of the underlying random variables, Bagnoli et al. (2001) provide conditions for the existence and uniqueness of linear equilibria in models with multiple strategic traders whose (possibly imperfect) signals about the value of the asset are distributed symmetrically.[5]

Several papers go beyond the case of fully symmetric distributions of strategic traders' signals. Foster and Viswanathan (1994) consider a model with two strategic traders in which one trader is strictly more informed than the other. Dridi and Germain (2009) study a model in which the signals of strategic traders are independent conditionally on the true value of the security, but may have different precisions. Colla and Mele (2010) consider a model in which strategic traders are located on a circle, with the correlations of signals being stronger for traders who are closer to each other (in this model, as in the Rostek and Weretka (2012) model discussed above, all traders use identical strategies in equilibrium).

Bernhardt and Miao (2004) consider a dynamic model with a very general information structure, allowing, as our paper does, for essentially arbitrary covariance matrices of traders' signals.[6] However, while Bernhardt and Miao (2004) characterize necessary and sufficient conditions for linear equilibria (analogous to Steps 1 and 2 in the proof of Theorem 1 in our paper, but in a multi-period setting), and use these conditions to study the properties of such equilibria analytically and numerically in some specific examples, they do not provide general results on equilibrium existence or uniqueness and do not provide general closed-form equilibrium characterizations. Whether such results and closed-form characterizations can be established for a general multi-period setting is an open question.

There are also a number of papers building on Kyle's (1985) framework in which the information structure is not limited to strategic traders observing signals about the value of the asset. In Jain and Mirman (1999), the market maker receives a separate informative signal about the value of the asset, in addition to simply observing the aggregate order flow. In Rochet and Vila (1994) and Foucault and Lescourret (2003), some of the strategic traders observe informative signals about the amount of liquidity demand.[7] These features of the information structure are naturally incorporated in our general model. Hence, our equilibrium existence and uniqueness result, as

[5]See also Nöldeke and Tröger (2001, 2006) for the analysis of the role of the normality assumption for the existence of linear equilibria in the Kyle (1985)-style models with multiple strategic traders who receive perfect signals about the value of the asset.

[6]There are several differences between the models. Bernhardt and Miao (2004) consider a model with an arbitrary number of trading periods, while we restrict attention to one period. On the other hand, unlike Bernhardt and Miao (2004), we allow liquidity demand to be correlated with the value of the asset and/or the signals of informed traders. We also allow the market maker to observe signals correlated with the value of the security, the demand from noise traders, and/or the signals of informed traders. Finally, we do not impose any special structure on how the informed traders' signals are related to the value of the asset (and other random variables in the model), beyond joint normality.

[7]Röell (1990), Sarkar (1995), and Madrigal (1996) also consider related models in which some agents observe signals about liquidity demand.

well as the characterization we derive, provide a unified approach with closed-form solutions to various models that include these features. In Section 5, we provide several examples illustrating the flexibility of our general model, and its ability to naturally incorporate such features as the market maker receiving a signal about the value of the asset and the strategic traders observing signals about liquidity demand, among others.

In Section 8, we study information aggregation in a model of Cournot competition. The literature on information aggregation under Cournot competition as the number of firms becomes large goes back to Li (1985) and Palfrey (1985). These papers consider environments in which all firms' signals about the true state of the world are symmetrically distributed. In contrast, our information aggregation result holds for essentially arbitrary variance-covariance matrices of firms' signals.[8] Our focus in Section 8 is on information aggregation as the number of firms becomes large, and the parallels between this information aggregation result and the main information aggregation result in the paper. Thus, we do not explore in depth the connections between equilibrium outcomes under Cournot competition (in which the slope of the demand curve is fixed) and in our main model based on the framework of Kyle (in which the slope of the demand curve is determined endogenously) for a fixed, finite number of strategic traders. For the case of symmetric distributions of signals, these connections (along with the connections to equilibrium outcomes in a model of demand-function competition in the spirit of Kyle (1989)) are explored by Bergemann et al. (2015).

3 Model

There is a security traded in the market, whose value v is not initially known to market participants. There are n strategic traders, $i = 1, \ldots, n$. Prior to trading, each strategic trader i privately observes a multidimensional signal $\theta_i \in \mathbb{R}^{k_i}$, where $k_i \geq 1$ is the dimensionality of the signal. For convenience, we denote by $\theta = (\theta_1; \theta_2; \cdots; \theta_n)$ the vector[9] summarizing the signals of all strategic traders. The dimensionality of vector θ is $K = \sum_{i=1}^{n} k_i$. There is also a market maker, who privately observes signal $\theta_M \in \mathbb{R}^{k_M}$, $k_M \geq 0$ (when $k_M = 0$, the market maker does not receive any signals, as in the standard Kyle (1985) model).[10,11] Finally, there are liquidity traders, whose exogenously given random demand, denoted by u, is in general not directly observed by either the strategic traders or the market maker.

The key assumption that makes the model analytically tractable is that all of the random vari-

[8] As in Li (1985) and Palfrey (1985), we also assume that the firms' marginal costs of production are constant. Vives (1988) shows that full information aggregation in large Cournot markets is not necessarily obtained when marginal costs of production are increasing, even in the fully symmetric case.

[9] We denote the row vector with elements $x_1, \ldots, x_k$ by $(x_1, \ldots, x_k)$, and the column vector with the same elements by $(x_1; \ldots; x_k)$. All vectors are column vectors unless specified otherwise.

[10] Strictly speaking, θ_i and θ_M are random variables whose realizations are in $\mathbb{R}^{k_i}$ and $\mathbb{R}^{k_M}$.

[11] The multidimensionality of the traders' and the market maker's signals allows our model to incorporate complex relationships among their information sets: for instance, one trader can observe strictly more information than another trader; one trader can observe the union of two other traders' signals; some information may be common to several players while some other information is not; and so forth. We illustrate the richness of the model with several examples in Section 5.

ables mentioned above—v, θ, θ_M, and u—are jointly normally distributed. Specifically, we assume that the vector $\mu = (v; \theta; \theta_M; u)$ is drawn randomly from the multivariate normal distribution with expected value 0 and variance Ω. The assumption that the expected value of vector μ is equal to zero is simply a normalization that allows us to simplify the notation. We also assume that every variance-covariance matrix for signal θ_i of strategic trader i and the variance-covariance matrix of the marker maker's signal θ_M are full rank. This assumption is without loss of generality; it simply eliminates redundancies in each trader's signals. Note that we do not place a full rank restriction on matrix Ω itself: for instance, two different strategic traders are allowed to have perfectly correlated signals. The only substantive restrictions that we place on matrix Ω are as follows.

Assumption 1 At least one strategic trader receives at least some information about the value of the security, beyond that contained in the market maker's signal. Formally:

$$Cov(v, \theta|\theta_M) \neq 0. \tag{1}$$

Assumption 2 The market maker does not perfectly observe the demand from liquidity traders. Formally:

$$Var(u|\theta_M) > 0. \tag{2}$$

3.1 Trading and Payoffs

After observing his signal θ_i, each strategic trader i submits his demand $d_i(\theta_i)$ to the market. In addition, the realized demand from liquidity traders, u, is also submitted to the market. The market maker observes her signal θ_M and the total demand $D = \sum_{i=1}^n d_i(\theta_i) + u$, and subsequently sets the price of the security, $P(\theta_M, D)$, based on these observations. Securities are traded at this price $P(\theta_M, D)$ (with each strategic trader getting his demand $d_i(\theta_i)$, liquidity traders getting u, and the market maker taking the position of size $-D$ to clear the market). At a later time, the true value of the security is realized, and each strategic trader i obtains profit $\pi_i = d_i(\theta_i) \cdot (v - P(\theta_M, D))$.

3.2 Linear Equilibrium

Our solution concept is essentially the same as that in Kyle (1985): linear equilibrium. Definition 1 below formalizes the notion of equilibrium, while Definition 2 states what it means for an equilibrium to be linear.

Definition 1 *A profile of demand functions $d_i(\cdot)$ and pricing rule $P(\cdot, \cdot)$ form an* equilibrium *if*

(i) on the equilibrium path, the price P set by the market maker is equal to the expected value of the security conditional on θ_M and D, given the primitives and the demand functions $d_i(\cdot)$; and

(ii) for every player i, for every realization of signal θ_i, the expected payoff from submitting demand $d_i(\theta_i)$ is at least as high as the expected payoff from submitting any alternative demand

d_i', *given the realization of signal* θ_i, *the pricing rule* $P(\cdot,\cdot)$ *and the profile of strategies of other players* $(d_j(\cdot))_{j\neq i}$.[12]

Definition 2 *Equilibrium* $(\{d_i(\cdot)\}_{i=1,\dots,n}, P(\cdot,\cdot))$ *is* linear *if functions* d_i *and pricing rule* P *are linear functions of their arguments, i.e.,* $d_i(\theta_i) = \alpha_i^T \theta_i$ *for some* $\alpha_i \in \mathbb{R}^{k_i}$ *and* $P(\theta_M, D) = \beta_M^T \theta_M + \beta_D D$ *for some* $\beta_M \in \mathbb{R}^{k_M}$ *and* $\beta_D \in \mathbb{R}$.[13]

4 Equilibrium Existence and Uniqueness

We can now state and prove our first main result.

Theorem 1 *There exists a unique linear equilibrium.*

The proof of Theorem 1 is in Appendix A. The notation used in the proof, as well as in some of the subsequent sections, is given in Section 4.1 below.

The proof consists of five steps. The first two steps are fairly standard, and are essentially the same as in the earlier literature on linear-normal equilibria: they show that if all strategic traders follow linear strategies, then the pricing rule resulting from Bayesian updating is also linear; and that if all strategic traders other than trader i follow linear strategies, and the market maker is also using a linear pricing rule (with a positive coefficient β_D on aggregate demand D), then the best response of trader i is also linear and is uniquely determined by the other traders' strategies and the pricing rule. The substantively novel parts of the proof are the next three steps. First, we show that the conditions derived in the first two steps allow us to express all parameters of the pricing rule and the traders' strategies as functions of "market depth" $\gamma = 1/\beta_D$. Next, using that derivation, we show that the entire system of equations from the first two steps collapses into one quadratic equation in γ. Finally, we prove that this quadratic equation has exactly one positive root, which concludes the proof.

[12]Our interpretation of condition (i) is similar to that of Kyle (1985): it is a reduced-form way of representing the outcome of Bertrand competition among multiple market makers. In that interpretation, Kyle (1985) assumes that all market makers observe the total order flow and nothing else. In our case, all market makers observe the total order flow D and the signal θ_M, and nothing else. (For an alternative way of modeling competition among liquidity-supplying market makers, in which the market makers post price schedules and make positive profits in equilibrium, see Biais et al. (2000, 2013).)

Another, technical difference from the equilibrium notion of Kyle (1985) is that in our case, condition (i) is required to hold only on the equilibrium path. In the standard Kyle (1985) model and many of its generalizations, every observation of the market maker can be rationalized as being on the equilibrium path, and thus this qualifier is not needed. In our case, it is in general possible that for some strategy profiles $d_i(\cdot)$, only some realizations of aggregate demand D can be observed by the market maker if the strategic traders follow those strategies. In such cases, by analogy with perfect Bayesian equilibrium, our definition restricts the beliefs of the market maker on the equilibrium path, where they are pinned down by Bayes rule, and does not restrict them off the equilibrium path. For an example in which not all realizations of aggregate demand are observed in equilibrium, consider the following market. Value $v \sim N(0,1)$. There is one strategic trader with signal θ_1 who observes the value perfectly: $\theta_1 = v$. The demand of liquidity traders is $u = -v$. Then in the unique linear equilibrium, the demand of the strategic trader is equal to the value of the security, and the aggregate demand is thus always equal to zero. See Section 1 of the Online Appendix for details.

[13]In principle, we could consider a more general definition of linear equilibrium and allow the strategies and the pricing rule to potentially have nonzero intercepts. However, one can show that in our setting, linear equilibria with nonzero intercepts do not exist. See Section 2 of the Online Appendix for a formal proof of this statement.

4.1 Notation

We decompose the covariance matrix Ω of the vector $(v;\theta_1;\ldots;\theta_n;\theta_M;u)$ as follows:

$$\left(\begin{array}{c|ccc|c|c}
\sigma_{vv} & \Sigma_{v1} & \cdots & \Sigma_{vn} & \Sigma_{vM} & \sigma_{vu} \\
\hline
\Sigma_{1v} & \Sigma_{11} & \cdots & \Sigma_{1n} & \Sigma_{1M} & \Sigma_{1u} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
\Sigma_{nv} & \Sigma_{n1} & \cdots & \Sigma_{nn} & \Sigma_{nM} & \Sigma_{nu} \\
\hline
\Sigma_{Mv} & \Sigma_{M1} & \cdots & \Sigma_{Mn} & \Sigma_{MM} & \Sigma_{Mu} \\
\hline
\sigma_{uv} & \Sigma_{u1} & \cdots & \Sigma_{un} & \Sigma_{uM} & \sigma_{uu}
\end{array}\right).$$

In this matrix, every σ represents a (scalar) variance or covariance of the asset value and/or the demand of liquidity traders, and every Σ represents a (generally non-scalar) covariance matrix of an element of vector $(v;\theta_1;\ldots;\theta_n;\theta_M;u)$ with another element. We also introduce notation for the covariance matrices of the entire vector of strategic traders' signals, $\theta = (\theta_1;\ldots;\theta_n)$, with itself and with other elements of vector μ. Specifically:

$$\Sigma_{\theta\theta} = Var\,(\theta) = \begin{pmatrix} \Sigma_{11} & \cdots & \Sigma_{1n} \\ \vdots & \ddots & \vdots \\ \Sigma_{n1} & \cdots & \Sigma_{nn} \end{pmatrix}, \quad \Sigma_{\theta M} = Cov\,(\theta, \theta_M) = \begin{pmatrix} \Sigma_{1M} \\ \vdots \\ \Sigma_{nM} \end{pmatrix},$$

$$\Sigma_{\theta v} = Cov\,(\theta, v) = \begin{pmatrix} \Sigma_{1v} \\ \vdots \\ \Sigma_{nv} \end{pmatrix}, \quad \Sigma_{\theta u} = Cov\,(\theta, u) = \begin{pmatrix} \Sigma_{1u} \\ \vdots \\ \Sigma_{nu} \end{pmatrix}.$$

In addition, we use the following matrices:

$$\Sigma_{diag} = \begin{pmatrix} \Sigma_{11} & 0 & 0 & 0 \\ 0 & \Sigma_{22} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \Sigma_{nn} \end{pmatrix},$$

$$\begin{aligned}
\Lambda &= \Sigma_{diag} + \Sigma_{\theta\theta} - \Sigma_{\theta M}\Sigma_{MM}^{-1}\Sigma_{\theta M}^{T}, \\
A_u &= \Lambda^{-1}(\Sigma_{\theta u} - \Sigma_{\theta M}\Sigma_{MM}^{-1}\Sigma_{Mu}), \\
A_v &= \Lambda^{-1}(\Sigma_{\theta v} - \Sigma_{\theta M}\Sigma_{MM}^{-1}\Sigma_{Mv}).
\end{aligned}$$

(We show in the proof of Theorem 1 that matrix Λ is invertible.)

4.2 Closed-Form Solution

The proof of Theorem 1 is constructive, producing the following expressions for the parameters of interest.

Depth $\gamma = -\left(b + \sqrt{b^2 - 4ac}\right)/2a$, where

$$a = -A_v^T \Sigma_{diag} A_v,$$
$$b = A_v^T \left(2\Sigma_{diag} + \Lambda\right) A_u + \Sigma_{uM} \Sigma_{MM}^{-1} \Sigma_{Mv} - \sigma_{uv},$$
$$c = Var(A_u^T \theta - u|\theta_M).$$

(The proof shows that $a < 0$, $c > 0$, and thus $\gamma > 0$.) Equilibrium pricing rule and strategies are then as follows:

$$\beta_D = \frac{1}{\gamma},$$
$$\beta_M = \Sigma_{MM}^{-1} \left(\Sigma_{Mv} - \Sigma_{\theta M}^T A_v\right) - \beta_D \Sigma_{MM}^{-1} \left(\Sigma_{Mu} - \Sigma_{\theta M}^T A_u\right),$$
$$\alpha = \frac{1}{\beta_D} A_v - A_u.$$

These expressions are simplified in the case $k_M = 0$, when the market maker does not observe any private signals (other than the aggregate demand D).[14] In that case,

$$a = -A_v^T \Sigma_{diag} A_v,$$
$$b = A_v^T \left(2\Sigma_{diag} + \Lambda\right) A_u - \sigma_{uv},$$
$$c = Var(A_u^T \theta - u),$$

where

$$\Lambda = \Sigma_{\theta\theta} + \Sigma_{diag},$$
$$A_u = \Lambda^{-1} \Sigma_{\theta u},$$
$$A_v = \Lambda^{-1} \Sigma_{\theta v}.$$

These expressions are further simplified if, in addition, the demand from liquidity traders, u, is uncorrelated with the other random variables in the model. Then $b = 0$ and $\gamma = \sqrt{\frac{\sigma_{uu}}{A_v^T \Sigma_{diag} A_v}}$, and so

$$\beta_D = \sqrt{\frac{A_v^T \Sigma_{diag} A_v}{\sigma_{uu}}} \quad \text{and} \quad \alpha = \sqrt{\frac{\sigma_{uu}}{A_v^T \Sigma_{diag} A_v}} A_v.$$

5 Examples

In this section, we give several examples that illustrate the general framework presented above and also help develop intuition for the information aggregation results of Sections 6 and 7. We first present a simple yet seemingly counterintuitive example in which a trader informed about the value of the security trades in the direction opposite to that value. Next, we study what happens when one of the strategic traders is informed about the demand of liquidity traders. We conclude

[14] Strictly speaking, our proof does not apply directly to the case $k_M = 0$ since, for example, it uses the inverse of the covariance matrix of θ_M. However, one can drop all terms related to θ_M from the proof and immediately obtain the proof for that case. Alternatively, one can consider a model in which the market maker observes a signal that is independent of all other random variables. The equilibrium in that model will be equivalent to one in which $k_M = 0$.

by analyzing several examples in which the market maker possesses private information about the value of the security and study how this information gets incorporated into the price of the security and how it affects equilibrium trading strategies and the sensitivity of equilibrium prices to market demand.

5.1 Trading "Against" Own Signal

We start with an example of information structure under which a trader who receives a signal about the value of the security trades in the opposite direction: if, based on his information, the expected value of the security is positive, then he shorts the security; if it is negative, then he buys it. Note that since our model is single-period, there cannot be any dynamic incentives to manipulate prices, of the form "I will try to mislead others first, and then take advantage of the mispricing."[15]

Example 1 *The value of the security is distributed as $v \sim N(0,1)$. There are two strategic traders. Trader 1 observes a noisy estimate of v: $\theta_1 = v + \rho_1 \xi$, where $\xi \sim N(0,1)$ is a random variable independent of v, and ρ_1 is a parameter that determines how accurate trader 1's signal is (e.g., if $\rho_1 = 0$, then trader 1 observes v exactly, and if ρ_1 is very large, then trader 1's signal is not very accurate). Trader 2 also observes a noisy estimate of v: $\theta_2 = v + \rho_2 \xi$, with the same "driver" of noise, ξ, as in trader 1's signal, but with a potentially different magnitude of noise, ρ_2. Finally, there is demand from liquidity traders, $u \sim N(0,1)$, which is independent of all other random variables. Formally, the resulting correlation matrix is*

$$\Omega = \left(\begin{array}{c|cc|c} 1 & 1 & 1 & 0 \\ \hline 1 & 1+\rho_1^2 & 1+\rho_1\rho_2 & 0 \\ 1 & 1+\rho_1\rho_2 & 1+\rho_2^2 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array} \right).$$

From the analysis and closed-form characterization in the preceding section, we know that in the unique linear equilibrium the pricing rule is characterized by some $\beta_D > 0$, and the strategies of traders 1 and 2 are characterized by:

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \frac{1}{\beta_D} \Lambda^{-1} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \tag{3}$$

where $\Lambda = \begin{pmatrix} 2+2\rho_1^2 & 1+\rho_1\rho_2 \\ 1+\rho_1\rho_2 & 2+2\rho_2^2 \end{pmatrix}$.

Using the matrix inversion formula and setting $\delta = \frac{1}{\beta_D \cdot \det(\Lambda)}$ (which is positive, since Λ is positive definite), we get

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \delta \begin{pmatrix} 2+2\rho_2^2 & -1-\rho_1\rho_2 \\ -1-\rho_1\rho_2 & 2+2\rho_1^2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \delta \begin{pmatrix} 1+2\rho_2^2-\rho_1\rho_2 \\ 1+2\rho_1^2-\rho_1\rho_2 \end{pmatrix}. \tag{4}$$

[15] For examples of settings in which such dynamic incentives do arise, see Brunnermeier (2005) and Sadzik and Woolnough (2015).

Thus, if $\rho_1 = 2\rho_2 + \frac{1}{\rho_2}$, trader 1 *never trades*, despite θ_1 being informative about the value of the security, and for $\rho_1 > 2\rho_2 + \frac{1}{\rho_2} > 0$, trader 1 *always trades in the direction opposite to his signal* θ_1, despite θ_1 being positively correlated with the value of the security, v. Similarly, if ρ_2 is equal to or greater than $2\rho_1 + \frac{1}{\rho_1}$, then trader 2 does not trade or trades in the direction opposite to his signal.

To get the intuition behind this seemingly puzzling behavior, consider a slight variation of Example 1.

Example 2 *The value of the security is $v \sim N(0,1)$. There are two strategic traders. Trader 1 observes a noisy estimate of v: $\theta_1 = v + \xi$, where $\xi \sim N(0,1)$, independent of v. Trader 2 observes ξ: $\theta_2 = \xi$. The demand from liquidity traders, $u \sim N(0,1)$, is independent of all other random variables. The resulting correlation matrix is*

$$\Omega = \left(\begin{array}{c:cc:c} 1 & 1 & 0 & 0 \\ \hdashline 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right).$$

In this case, $\Lambda = \begin{pmatrix} 4 & 1 \\ 1 & 2 \end{pmatrix}$ and

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \frac{1}{\beta_D}\Lambda^{-1}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \delta\begin{pmatrix} 2 & -1 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \delta\begin{pmatrix} 2 \\ -1 \end{pmatrix} \tag{5}$$

for some $\delta > 0$, and thus trader 2 trades in the direction opposite to his signal. Note that in this example, trader 2 is not informed about the value of the security: his signal ξ is independent of v. However, he is informed about the bias in trader 1's signal, and thus knows in which direction trader 1 is likely to "err" when submitting his demand. Thus, trader 2, by partly "undoing" this error (i.e., trading against it), can in expectation make a positive profit, despite not having any direct information about the value of the security. In a sense, while trader 1 trades on "fundamental" information, trader 2 trades on "technical" information: trader 1's ability to make money is due to his information about the value of the security, while trader 2's ability to make a profit is due to his information about the "mistakes" of other agents in the economy.[16]

In Example 1, the intuition is similar. If ρ_2 is large relative to $2\rho_1 + \frac{1}{\rho_1}$, then the main "chunk" of trader 2's information is about the mistake that trader 1 makes, and not about the fundamental value of the security. This causes trader 2 to want to "undo" that mistake and trade "against" his signal, while trader 1 continues to trade in a natural direction. When $\rho_2 = 2\rho_1 + \frac{1}{\rho_1}$, the incentives of trader 2 to trade on "fundamental" information (the positive correlation of his signal with the

[16]Formally, we say that trader i has "fundamental" information if $Cov(\theta_i, v|\theta_M) \neq 0$, and say that trader i has "technical" information if $Cov(\theta_i, u|\theta_M) \neq 0$ or $Cov(\theta_i, \theta_j|\theta_M) \neq 0$ for some $j \neq i$. If a trader has neither fundamental nor technical information, then in equilibrium he does not trade, and does not make any profit. In Section 3 of the Online Appendix, we formally state and prove this result (Proposition OA.2), and also explore in more detail the dependence and non-dependence of equilibrium trading strategies on various types of information.

value of the security) and on the "technical" information (the positive correlation of his signal with the mistake of trader 1) cancel out, and trader 2 ends up not trading.

Examples 1 and 2 illustrate that a strategic trader's behavior in equilibrium is driven not only by the correlation of his information and the value of the asset, but also by the informational content of his signals relative to the information already contained in the signals and the resulting behavior of other agents—potentially even to the point of reversing the direction of his trade. It is this flexibility that allows strategic traders' information to get fully aggregated and incorporated in prices as market size grows, even for very rich information structures. In contrast, the behavior of liquidity traders is exogenous, and is not endogenously affected by what information it contains. As a result, the information contained in liquidity demand is fully incorporated in market prices only under appropriate correlation structures (see Section 6 for details).

5.2 Information about Liquidity Demand

In this section, we present an example showing what happens when one of the strategic traders does not know anything about the value of the security, but is informed about the amount of liquidity trading. We then compare the equilibrium to that of the standard model without such a trader.

Example 3 *The value of the security is distributed as $v \sim N(0, \sigma_{vv})$, and the demand from liquidity traders is distributed as $u \sim N(0, \sigma_{uu})$, independently of v. There are two strategic traders. Trader 1's signal is equal to v: $\theta_1 = v$. He is fully informed about the value of the security, just like in the standard Kyle model. Trader 2 is uninformed about the value of the security, but has insider information about the demand from liquidity traders: $\theta_2 = u$. Formally, the correlation matrix is*

$$\Omega = \left(\begin{array}{c:c:c:c} \sigma_{vv} & \sigma_{vv} & 0 & 0 \\ \hdashline \sigma_{vv} & \sigma_{vv} & 0 & 0 \\ 0 & 0 & \sigma_{uu} & \sigma_{uu} \\ \hdashline 0 & 0 & \sigma_{uu} & \sigma_{uu} \end{array}\right).$$

The auxiliary matrices in this example are:

$$\Lambda = \begin{pmatrix} 2\sigma_{vv} & 0 \\ 0 & 2\sigma_{uu} \end{pmatrix}, \; A_u = \begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix}, \text{ and } A_v = \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix}.$$

Coefficient b in the quadratic equation is equal to zero, and therefore

$$\begin{aligned} \gamma &= \sqrt{-\frac{c}{a}} = \sqrt{\frac{\sigma_{uu}}{\sigma_{vv}}}, \\ \alpha_1 &= \frac{1}{2}\gamma = \frac{1}{2}\sqrt{\frac{\sigma_{uu}}{\sigma_{vv}}}, \\ \alpha_2 &= -\frac{1}{2}. \end{aligned}$$

For comparison, if the second strategic trader was not present, the model would reduce to the standard model of Kyle (1985), and the equilibrium would be characterized by

$$\gamma = 2\sqrt{\frac{\sigma_{uu}}{\sigma_{vv}}},$$
$$\alpha_1 = \sqrt{\frac{\sigma_{uu}}{\sigma_{vv}}}.$$

In other words, when the second strategic trader (who is informed about the demand from liquidity traders) is present in the market, that trader "takes away" one half of that "liquidity" demand. As a result, the first strategic trader, who knows the value of the security, trades half as much as he would in the absence of that second trader, and the market maker's pricing rule is twice as sensitive. Therefore, for any realization of v and u, the price in the market with the second strategic trader will be exactly the same as that in the market without that trader—and thus the informativeness of prices is not affected in either direction by whether there is a trader in that market who observes the trading flow from liquidity traders. Likewise, the expected loss of liquidity traders is also unaffected by the presence of a trader who observes their demand. Since, by construction, the market maker in expectation breaks even, it has to be the case that the profit of the second strategic trader comes out of the first trader's pocket. In fact, the second trader takes away exactly one half of the first trader's profit.[17] Also, as in Example 2, the second trader is trading on "technical" information, and is only able to make a profit because of the "mistakes" of other agents in the economy.

Example 3 shows that when liquidity demand is fully observed by some strategic traders, they may have an incentive to trade in the opposite direction, effectively removing part of that demand from the market. If the number of such strategic traders grows large, they may end up removing all liquidity demand from the market, potentially hindering information aggregation (and possibly the existence of limit equilibrium) in large markets (see, e.g., Footnote 20). Thus, in Sections 6 and 7 we assume not only that the variance of liquidity demand is positive, but also that it remains positive when we condition it on the signals of large groups of strategic traders and the market maker.

5.3 Informed Market Maker

In the preceding examples, the market maker does not receive any information other than the aggregate demand coming from strategic and liquidity traders. In this subsection, we turn to examples in which the market maker does possess some additional information. We show how this information affects the strategies of other traders and illustrate the interplay between the weight the market maker places on this additional information and the weight she places on market demand.

Our first two examples illustrate that the equilibrium obtained when the market maker has private information is generally not the same as when that information is publicly available (i.e., known both to the market maker and to all strategic traders).[18] This difference will turn out to be

[17] To see this, note that the prices in the two markets are always the same, realization by realization, while the demand of the first strategic trader, in the presence of the second one, is exactly one half of what it would be in the absence of that trader.

[18] Jain and Mirman (1999) and Luo (2001) study extensions of the Kyle (1985) model with a partially informed

important later in the paper, in Section 7, when we study the informativeness of prices as the sizes of some (but not all) groups of strategic traders become large. In that setting, as the sizes of some of the groups become large, the market behaves as if the signals of those groups were observed directly by the market maker—and not as if the signals of those groups were observed publicly.

Example 4 *The value of the security is $v \sim N(0,1)$. There is one strategic trader, who observes signal $\theta = v + \epsilon_1$. The market maker observes signal $\theta_M = v + \epsilon_2$. Variables ϵ_1 and ϵ_2 are distributed normally with mean 0 and variance 1, independently of each other and of all other variables. The demand from liquidity traders is also independently distributed as $u \sim N(0,1)$. Formally, the covariance matrix that describes this information structure is*

$$\Omega = \left(\begin{array}{c:c:c:c} 1 & 1 & 1 & 0 \\ \hdashline 1 & 2 & 1 & 0 \\ \hdashline 1 & 1 & 2 & 0 \\ \hdashline 0 & 0 & 0 & 1 \end{array} \right).$$

Applying the formulas derived in Section 4, we get $\Sigma_{diag} = \Sigma_{\theta\theta} = \Sigma_{MM} = 2$ and $\Sigma_{\theta M} = \Sigma_{\theta v} = \Sigma_{Mv} = 1$. Thus, $\Lambda = 2 + 2 - 1/2 = 7/2$, $A_u = 0$, and $A_v = 2(1 - 1/2)/7 = 1/7$. The coefficients in the quadratic equation for γ are $a = -2/49$, $b = 0$, and $c = 1$, and thus

$$\beta_D = \frac{1}{\gamma} = \frac{\sqrt{2}}{7}.$$

Hence, the strategic trader's behavior is given by

$$\alpha = \frac{1}{\beta_D} A_v = \frac{1}{2}\sqrt{2},$$

and the market maker's sensitivity to her own signal is

$$\beta_M = \Sigma_{MM}^{-1} \left(\Sigma_{Mv} - \Sigma_{\theta M}^T A_v \right) = \frac{3}{7}.$$

Consider now a variation of Example 4, in which the market maker's signal is public information (i.e., known to both the market maker and the strategic trader).

Example 5 *The value of the security is $v \sim N(0,1)$. The market maker observes signal $\theta_M = v + \epsilon_2$. The strategic trader now observes* two *signals, $\theta^1 = v + \epsilon_1$ and $\theta^2 = v + \epsilon_2$. Both ϵ_1 and ϵ_2 are normally distributed with mean 0 and variance 1, independently of each other and of all other variables. The demand from liquidity traders is independently distributed as $u \sim N(0,1)$. The*

market maker and with partially informative public information, respectively. The difference between the two cases can be seen by comparing their results (setting $\sigma_i^2 = 0$ in Luo (2001)). Our Examples 4 and 5 are similar, though not identical, to the models of Jain and Mirman (1999) and Luo (2001). We present the examples to emphasize the distinction between the two cases within the same setup, as this distinction is important for our hybrid-market information aggregation results.

covariance matrix that describes this information structure is now

$$\Omega = \left(\begin{array}{c:cc:c:c} 1 & 1 & 1 & 1 & 0 \\ \hdashline 1 & 2 & 1 & 1 & 0 \\ 1 & 1 & 2 & 2 & 0 \\ \hdashline 1 & 1 & 2 & 2 & 0 \\ \hdashline 0 & 0 & 0 & 0 & 1 \end{array}\right).$$

We now have $\Sigma_{diag} = \Sigma_{\theta\theta} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, $\Sigma_{MM} = 2$, $\Sigma_{\theta M} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\Sigma_{\theta v} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, and $\Sigma_{Mv} = 1$. Thus, $\Lambda = \begin{pmatrix} 7/2 & 1 \\ 1 & 2 \end{pmatrix}$, $\Lambda^{-1} = 1/6 \begin{pmatrix} 2 & -1 \\ -1 & 7/2 \end{pmatrix}$, $A_v = \Lambda^{-1} \left(\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot 1/2 \right) = 1/6 \begin{pmatrix} 1 \\ -1/2 \end{pmatrix}$, and $A_u = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

The coefficients of the quadratic equation on γ are now:

$$a = -1/36 \begin{pmatrix} 1 \\ -1/2 \end{pmatrix}^T \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ -1/2 \end{pmatrix} = -1/24, \; b = 0, \; \text{and } c = 1,$$

and thus

$$\beta_D = \frac{1}{\gamma} = \frac{\sqrt{6}}{12},$$

the strategic trader's behavior is given by

$$\alpha = \frac{1}{\beta_D} A_v = \begin{pmatrix} \frac{\sqrt{6}}{3} \\ -\frac{\sqrt{6}}{6} \end{pmatrix},$$

and the market maker's sensitivity to her own signal is now given by

$$\beta_M = \Sigma_{MM}^{-1} \left(\Sigma_{Mv} - \Sigma_{\theta M}^T A_v \right) = \frac{1}{2}.$$

The equilibria in these two examples are substantively different: the sensitivities of the market maker to the aggregate demand and to her own signal are different, and the sensitivity of the strategic trader's demand to signal θ^1 is different as well. We can also compute the expected profits that the strategic trader makes in these two markets (and thus the losses of liquidity traders): in the first example, the expected profit is $\sqrt{2}/7$, while in the second one it is greater: $\sqrt{6}/12$. These differences illustrate the point that having the market maker observe a signal is substantively different from having that signal observed publicly.

Our next example considers the case in which a strategic trader's information is strictly worse than the information available to the market maker.

Example 6 *Let ν_1, ν_2, ϵ_1, ϵ_2, and u be independent random variables, each distributed normally with mean 0 and variance 1. The value of the security is $v = \nu_1 + \nu_2$. The demand from liquidity traders is u. There are two partially informed strategic traders and a partially informed market maker. Trader 1's signal is $\theta_1 = \nu_1 + \epsilon_1$. Trader 2's signal is $\theta_2 = \nu_2 + \epsilon_2$. Market maker's signal* *is $\theta_M = \nu_2$. Note that while trader 1 possesses some "exclusive" information about the value of the security, trader 2 does not (because ν_2 is observed by the market maker, and ϵ_2 is pure noise). Formally, the correlation matrix is*

$$\Omega = \left(\begin{array}{c:cc:c:c} 2 & 1 & 1 & 1 & 0 \\ \hdashline 1 & 2 & 0 & 0 & 0 \\ 1 & 0 & 2 & 1 & 0 \\ \hdashline 1 & 0 & 1 & 1 & 0 \\ \hdashline 0 & 0 & 0 & 0 & 1 \end{array}\right).$$

The auxiliary matrices in this example are:

$$\Lambda = \begin{pmatrix} 4 & 0 \\ 0 & 3 \end{pmatrix}, \; A_u = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \text{ and } A_v = \begin{pmatrix} \frac{1}{4} \\ 0 \end{pmatrix}.$$

Therefore, in this case, we have

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \frac{1}{\beta_D} \begin{pmatrix} \frac{1}{4} \\ 0 \end{pmatrix},$$

and so $\alpha_2 = 0$. Thus, trader 2 does not trade in equilibrium. This illustrates a more general phenomenon: in equilibrium, a strategic trader cannot make a positive profit (and does not trade) if his information is the same as or worse than (in the information-theoretic sense) that of the market maker.[19]

Our final example considers a sequence of markets, indexed by the number of strategic traders, m. All traders receive the same information, which is imperfectly correlated with both the value of the asset and the market maker's information.

Example 7 *The value of the security, v, the demand from liquidity traders, u, and two information shocks, ϵ_1 and ϵ_2, are all distributed normally with mean 0 and variance 1, independently of each other. There are m identically informed strategic traders and a partially informed market maker. Each strategic trader observes a signal $\theta_1 = v+\epsilon_1$. The market maker observes a signal $\theta_M = v+\epsilon_2$. Formally (indexing all matrices by the number of strategic traders in the market, m), the correlation matrix is*

$$\Omega^m = \left(\begin{array}{c:cccc:c:c} 1 & 1 & 1 & \cdots & 1 & 1 & 0 \\ \hdashline 1 & 2 & 2 & \cdots & 2 & 1 & 0 \\ 1 & 2 & 2 & \cdots & 2 & 1 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 1 & 2 & 2 & \cdots & 2 & 1 & 0 \\ \hdashline 1 & 1 & 1 & \cdots & 1 & 2 & 0 \\ \hdashline 0 & 0 & 0 & \cdots & 0 & 0 & 1 \end{array}\right).$$

[19] See Proposition OA.2 in Section 3 of the Online Appendix for a formal statement and proof of this result.

The auxiliary matrices are:

$$\Lambda^m = \begin{pmatrix} 3\frac{1}{2} & 1\frac{1}{2} & \cdots & 1\frac{1}{2} & 1\frac{1}{2} \\ 1\frac{1}{2} & 3\frac{1}{2} & \cdots & 1\frac{1}{2} & 1\frac{1}{2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1\frac{1}{2} & 1\frac{1}{2} & \cdots & 3\frac{1}{2} & 1\frac{1}{2} \\ 1\frac{1}{2} & 1\frac{1}{2} & \cdots & 1\frac{1}{2} & 3\frac{1}{2} \end{pmatrix}, \text{ so that } (\Lambda^m)^{-1} = \begin{pmatrix} \frac{3m+1}{6m+8} & \frac{-3}{6m+8} & \cdots & \frac{-3}{6m+8} & \frac{-3}{6m+8} \\ \frac{-3}{6m+8} & \frac{3m+1}{6m+8} & \cdots & \frac{-3}{6m+8} & \frac{-3}{6m+8} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{-3}{6m+8} & \frac{-3}{6m+8} & \cdots & \frac{3m+1}{6m+8} & \frac{-3}{6m+8} \\ \frac{-3}{6m+8} & \frac{-3}{6m+8} & \cdots & \frac{-3}{6m+8} & \frac{3m+1}{6m+8} \end{pmatrix},$$

$$A_u^m = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}, \text{ and } A_v^m = \begin{pmatrix} \frac{1}{3m+4} \\ \vdots \\ \frac{1}{3m+4} \end{pmatrix}.$$

Coefficient b in the quadratic equation is equal to zero, and so

$$\begin{aligned} \gamma^m &= \sqrt{-\frac{c}{a}} = \frac{3m+4}{\sqrt{2m}}, \\ \alpha_i^m &= \gamma^m A_{vi}^m = \frac{1}{\sqrt{2m}}, \\ \beta_M^m &= \frac{1}{2}\big(1 - \frac{m}{3m+4}\big) = \frac{2m+4}{6m+8} = \frac{m+2}{3m+4}. \end{aligned}$$

Note that the weight β_M that the market maker places on her own signal is not constant in m. If there were no strategic traders at all, and only noise traders ($m = 0$, although strictly speaking that case is not allowed by our general setup), it would be equal to $\frac{1}{2} = \frac{Cov(v,\theta_M)}{Var(\theta_M)}$. As m grows, this weight is monotonically decreasing (converging to $\frac{1}{3}$ in the limit). Intuitively, as m grows, an increasingly large fraction of the market maker's information about the value of the security is also contained in the strategic demand, and can be extracted from it by the market maker—thus leaving a smaller part for the signal θ_M that the market maker observes directly.

The second observation concerns the informativeness of prices. Take any m, and consider a realization of θ_1, θ_M, and u. In this realization, demand D is equal to $m\alpha_i^m\theta_1 + u = \frac{m}{\sqrt{2m}}\theta_1 + u$, and the market price P set by the market maker is equal to $\beta_D^m D + \beta_M^m\theta_M = \frac{m}{3m+4}\theta_1 + \frac{m+2}{3m+4}\theta_M + \frac{\sqrt{2m}}{3m+4}u$. Now, fix the realization of random variables, and let the number of strategic traders, m, grow to infinity. Then price P converges to $\frac{1}{3}\theta_1 + \frac{1}{3}\theta_M$. But notice that this expression is precisely the expected value of the asset, v, conditional on the information available in the market: u is uninformative, because it is independent of all other random variables, and

$$\begin{aligned} E[v|\theta_1,\theta_M] &= Cov\left(v, \begin{pmatrix} \theta_1 \\ \theta_M \end{pmatrix}\right)^T Var\left(\begin{pmatrix} \theta_1 \\ \theta_M \end{pmatrix}\right)^{-1} \begin{pmatrix} \theta_1 \\ \theta_M \end{pmatrix} \\ &= \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} \theta_1 \\ \theta_M \end{pmatrix} \\ &= \frac{1}{3}\theta_1 + \frac{1}{3}\theta_M. \end{aligned}$$

Hence, as the number of strategic traders becomes large, their information and the information of the market maker get incorporated into the market price with precisely the weights that a Bayesian observer with access to *all* information available in the market would assign. In other words, as the number of strategic traders becomes large, all information available in the market is aggregated and revealed by the market price. In the next section, we show that this is not a coincidence: the information aggregation result holds very generally.

6 Information Aggregation in Large Markets

We now turn to the second main result of our paper: the aggregation of dispersed information when the number of traders becomes large.

Consider a sequence of markets, indexed by $m = 1, 2, \ldots$. Every market is in the general framework of Section 3. In every market, there are n groups of strategic traders, with at least one trader in each group. Index i, $1 \leq i \leq n$, now denotes a group of traders. The size of group i in market m is denoted by $\ell_i^{(m)}$. Every trader j in group i receives a k_i-dimensional signal $\theta_i + \xi_{i,j}$, where θ_i denotes the signal component common to all traders in group i and $\xi_{i,j}$ denotes the idiosyncratic component of trader j. We denote by $\theta = (\theta_1; \ldots; \theta_n)$ the vector of common components of the signals, and denote by Ω the variance-covariance matrix of vector $\mu = (v; \theta; \theta_M; u)$. The idiosyncratic components $\xi_{i,j}$ are distributed identically across the traders in group i, with each $\xi_{i,j}$ distributed according to a k_i-dimensional normal distribution with mean 0 and variance Σ_i^ξ. Every $\xi_{i,j}$ is independent of all other random variables in the model. We place no restrictions on the covariance matrices Σ_i^ξ. In particular, we allow for the degenerate case $\Sigma_i^\xi = 0$, which corresponds to an environment in which all traders in group i receive the same signal θ_i.

We assume that Ω and $\Sigma_1^\xi, \ldots, \Sigma_n^\xi$ are the same for all markets m. The number of traders in each group, however, changes with m: specifically, we assume that for every i, $\lim_{m\to\infty} \ell_i^{(m)} = \infty$, i.e., all groups become large as m becomes large. We do not impose any restrictions on the rates of growth of those groups: e.g., the sizes of some groups may grow much faster than those of other groups.

We slightly strengthen one of the two conditions on matrix Ω made in Section 3, replacing Assumption 2 with the following:[20]

Assumption 2L $Var(u|\theta, \theta_M) > 0$.

It follows from Theorem 1 that for each m, there exists a unique linear equilibrium in the corresponding market. Let $p^{(m)}$ denote the random variable that is equal to the resulting price in the unique linear equilibrium of market m.

We can now state and prove our main result on information aggregation in large markets. If the

[20]Under the original Assumptions 1 and 2, information may not get aggregated as markets become large. To see that, consider a modification of the example introduced in footnote 12. Value $v \sim N(0, 1)$. There are m strategic traders with the same signal $\theta_1 = v$. The demand of liquidity traders is $u = -v$. Then in the unique linear equilibrium, the demand of each strategic trader is equal to θ_1/m, the aggregate demand of all strategic trader is equal to $\theta_1 = v = -u$, the aggregate demand of all traders is equal to zero, and thus the equilibrium price is also always equal to zero, for any m. Thus, there is no information aggregation of any kind in the limit as m becomes large. See Section 1 of the Online Appendix for a formal derivation of these results.

demand from liquidity traders is positively correlated with the true value of the asset (conditional on other signals), then prices in large markets aggregate all available information: $p^{(m)}$ converges to $E[v|\theta, \theta_M, u]$. If liquidity demand is negatively correlated with the true value of the asset, then prices in large markets aggregate all available information *except* that contained in liquidity demand: $p^{(m)}$ converges to $E[v|\theta, \theta_M]$. If liquidity demand is uncorrelated with the true value of the asset, then both statements are true: $p^{(m)}$ converges to $E[v|\theta, \theta_M, u] = E[v|\theta, \theta_M]$.

Theorem 2

- *If* $Cov(u, v|\theta, \theta_M) \geq 0$*, then* $\lim_{m\to\infty} E\left[\left(p^{(m)} - E[v|\theta, \theta_M, u]\right)^2\right] = 0$.
- *If* $Cov(u, v|\theta, \theta_M) \leq 0$*, then* $\lim_{m\to\infty} E\left[\left(p^{(m)} - E[v|\theta, \theta_M]\right)^2\right] = 0$.

In Appendix B, we prove Theorem 2 for the special case in which the variance-covariance matrix of random vector $(\theta; \theta_M; u)$ is full rank. This additional assumption guarantees that certain matrices remain invertible in the limit as m becomes large, which in turn allows us to give a direct proof of the theorem without technical complications. However, this special case rules out some interesting possibilities (e.g., one type of traders knowing strictly more than another type of traders), so in the Online Appendix (Section 5), we provide the full proof of Theorem 2, which does not rely on this simplifying assumption.

The intuition for the information aggregation result is that, when the number of informed traders of each type is large, the information of each strategic trader has to be (almost) fully incorporated into the market price, since otherwise each trader of that type would be able to make a non-negligible profit, which cannot happen in equilibrium. Also, as the size of every group i grows, the idiosyncratic noise in the aggregate demand from that group vanishes, leaving only the "informative" part of the demand that is driven by the common component θ_i.[21] The signal of the market maker gets incorporated into the market price by construction. Finally, with liquidity demand, the situation is more subtle. When liquidity demand is positively correlated with the asset value ($Cov(u, v|\theta, \theta_M) > 0$), equilibrium strategies and market depth adjust precisely in a way that makes liquidity demand get incorporated into the market price "correctly," i.e., with the same weight as it would be incorporated into the market price by a Bayesian observer who was fully informed about all the random variables in the model (except value v). As a result, price $p^{(m)}$ converges to $E[v|\theta, \theta_M, u]$, and so all information available in the market is incorporated into the market price. However, when liquidity demand is negatively correlated with the value of the asset

[21]The fact that the idiosyncratic components in signals have no impact on equilibrium outcomes in large markets is parallel to the results in McLean and Postlewaite (2002) and McLean et al. (2005) in which the agents with such idiosyncratic components in signals have non-redundant information, but become "informationally small" as markets become large: adding the information of an extra agent to the information of others does not significantly impact the Bayesian estimate of the value of the security. Note, however, that "informational smallness" by itself is not sufficient for our results. In an economy without idiosyncratic components in signals, agents become "informationally small" as soon as the size of each group i is at least two. However, information is generally not aggregated in our setting in finite markets, even if the size of each group is two or greater and all the agents in each group i receive the same signal θ_i.

($Cov(u, v|\theta, \theta_M) < 0$), this cannot happen. In equilibrium, aggregate demand always enters the market price with a positive sign (sensitivity β_D is positive). Thus, liquidity demand also enters the market price with a positive sign. However, a fully informed Bayesian observer would put a *negative* weight on liquidity demand—which cannot happen in any linear equilibrium, for any parameter values. So what happens instead as m becomes large is that the variance of the aggregate demand from informed traders grows to infinity (in contrast to the case $Cov(u, v|\theta, \theta_M) > 0$, in which it converges to a finite value). And thus, as m grows, liquidity demand u has less and less impact on the market price, and in the limit it has no impact at all: price $p^{(m)}$ converges to $E[v|\theta, \theta_M]$. The same happens in the case $Cov(u, v|\theta, \theta_M) = 0$, for the same reason, but in that case $E[v|\theta, \theta_M]$ is equal to $E[v|\theta, \theta_M, u]$, and so price $p^{(m)}$ does converge to the expected value of the asset given all the information available in the market.

Another way to get intuition about the result is to notice that as a particular group i becomes large, its aggregate behavior converges to that of a single agent who is trying to minimize the expected square of the difference between the true value of the asset and its market price, $E\left[(v-p)^2\right]$.[22] By construction, the market maker is also trying to minimize $E\left[(v-p)^2\right]$ (subject to the constraint that the sensitivity of price to aggregate demand, β_D, is positive). Thus, as market size grows large, the system in essence behaves as a game with n partially informed traders (each corresponding to a particular group i and receiving the signal θ_i) and a market maker, all of whom have the same objective function: to minimize the expected square of the mispricing. The commonality of objective functions implies that the profile of policies by these $n+1$ agents that minimizes the expected squared mispricing will be an equilibrium of this limit game. When $Cov(u, v|\theta, \theta_M)$ is positive, the profile of policies that minimizes the expected squared mispricing is the one that sets the price $p = E[v|\theta, \theta_M, u]$, incorporating all the information available in the market. When $Cov(u, v|\theta, \theta_M)$ is negative or zero, setting the price at $p = E[v|\theta, \theta_M, u]$ is impossible, since that would require setting $\beta_D \leq 0$, which is not allowed. In fact, since β_D has to be positive, any profile of strategies by the $n+1$ agents has to put positive weight on u in forming the price—which the $n+1$ agents do not want to do. So they will want to set β_D to be infinitesimally small, and then

[22]To see this, fix a market and the corresponding equilibrium, and consider a group i with ℓ traders, all of whom observe the same signal $\theta_i \in \mathbb{R}$ (the cases with multidimensional information or idiosyncratic components in signals are more notationally cumbersome, but the conclusions are the same). Suppose in equilibrium each of these traders, after observing realization $\widetilde{\theta}_i$, submits demand $d^* = \alpha_i \widetilde{\theta}_i$. Let $p_{-i} = p - \beta_D\left(\ell \alpha_i \theta_i\right)$ denote the random variable corresponding to what the price in the market would have been if all traders in group i demanded zero instead of submitting the demands prescribed by the equilibrium. Conditional on observing $\widetilde{\theta}_i$, each agent in group i submits demand d that is maximizing $d \times \left(E\left[v - p_{-i}|\widetilde{\theta}_i\right] - \beta_D(\ell-1)d^* - \beta_D d\right)$, which implies $d = \frac{1}{2\beta_D}\left(E\left[v - p_{-i}|\widetilde{\theta}_i\right] - \beta_D(\ell-1)d^*\right)$. Since in equilibrium $d = d^*$, by rearranging the terms we get $\beta_D(\ell+1)d^* = E\left[v - p_{-i}|\widetilde{\theta}_i\right]$. The aggregate demand of group i is equal to ℓd^*, and so as $\ell \to \infty$, the aggregate demand of group i converges to $\frac{1}{\beta_D} E\left[v - p_{-i}|\widetilde{\theta}_i\right]$. Now suppose we instead have a single trader i who observes a realization $\widetilde{\theta}_i$ of signal θ_i and whose objective is to minimize $E\left[(v-p)^2\right]$. We have $E\left[(v-p)^2|\widetilde{\theta}_i\right] = E\left[(v - p_{-i} - \beta_D d)^2|\widetilde{\theta}_i\right] = E\left[(v-p_{-i})^2|\widetilde{\theta}_i\right] - 2d\beta_D E\left[v - p_{-i}|\widetilde{\theta}_i\right] + \beta_D^2 d^2$. The first term of this sum does not depend on d, and so the expression is minimized at $d = \frac{1}{\beta_D} E\left[v - p_{-i}|\widetilde{\theta}_i\right]$—which is precisely the quantity to which the aggregate demand of group i converges in the original game as ℓ becomes large.

adjust the strategies of the n partially informed traders accordingly, to get price p to be close to $E[v|\theta, \theta_M]$—which provides the infimum of the square of the mispricing given the constraint $\beta_D > 0$. Note that this intuition also illustrates that mathematically, there is no asymmetry between the cases of $Cov(u, v|\theta, \theta_M) > 0$ and $Cov(u, v|\theta, \theta_M) < 0$, and the difference in predictions for those cases arises from the economic incentives of the agents. Namely, if in the original game the goal of the strategic traders was to *lose* as much money as possible, the limit game with the $n + 1$ traders would in fact be the same as in our original case, except that the constraint would be $\beta_D < 0$. And so all information would get aggregated in the case $Cov(u, v|\theta, \theta_M) < 0$ (and market depth would remain bounded), and only information contained in θ and θ_M, but not that contained in u, would get aggregated in the case $Cov(u, v|\theta, \theta_M) > 0$ (and market depth would go to infinity).

The information aggregation result in Theorem 2 raises some natural questions. The first one is to what extent it matters that the variance of liquidity traders' demand $u^{(m)}$ remains constant as markets become large. What would happen if that variance also grew together with the number of strategic traders? Of course, the profits made by the strategic traders and their equilibrium strategies would be affected. It turns out, however, that equilibrium prices would remain unchanged. Specifically, for a given market, if liquidity demand were scaled by some factor ρ, the equilibrium strategies of all strategic traders would also get rescaled by the same factor ρ, the sensitivity of market maker's pricing rule to the aggregate demand, β_D, would get rescaled by $1/\rho$, and the equilibrium prices would thus stay the same. Proposition OA.4 in Section 4 of the Online Appendix formally proves these statements. This result, in turn, immediately implies that the conclusion of Theorem 2 would not be affected if we allowed liquidity demand $u^{(m)}$ to scale as a function of m.

Another question is whether the presence of a Bayesian market maker is critical for information aggregation. Is it important that there is an agent in the economy who is accurately setting prices based on the information available to her? To answer this question, in Section 8 of the paper we consider a model of Cournot competition, which can be viewed as an analogue of our main model with one key difference: the Bayesian market maker is replaced with a mechanical market maker whose sensitivity to aggregate demand, β, is exogenously fixed, instead of being determined endogenously in equilibrium. We find that the presence of a Bayesian market maker is not critical for information aggregation: Proposition 2 in Section 8.3 shows that as the number of firms grows, the outcome (equilibrium price and total quantity produced) of Cournot competition with information dispersed among the firms converges to that of Cournot competition in which all firms have access to all information.

Finally, a natural question is what happens if some groups remain "small," while others grow "large." How does the information of these two types of groups get incorporated into equilibrium prices? The next section addresses this question.

7 Information in "Hybrid" Markets

In many situations, some "scarce" information about the value of a security is known by only a small number of traders, perhaps just one, while some other information, while not publicly available, may be more "abundant," and may be observed by a large number of traders. In this section, we explore how these two types of information get incorporated into market prices in equilibrium.

It is intuitive that due to market impact and the resulting strategic considerations, "scarce" information will not be fully incorporated into market prices, and the traders possessing this information will make positive profits, while "abundant" information will be almost fully incorporated into market prices (and the traders possessing it will make vanishingly small profits). What is less immediate is the interplay between these two types of information, and how they get combined with the information observed directly by the market maker and the information contained in liquidity demand. In particular, a seemingly natural conjecture is that "abundant" information will enter the price essentially as a public signal, observed by everyone in the economy. Our last result shows that this is not the case: instead, "abundant" information, in the limit, enters into market prices in the same way as if it were directly observed by the market maker—but not by the strategic traders observing "scarce" information. As Examples 4 and 5 in Section 5 illustrate, this is substantively different from the case in which "abundant" information is observed by all the agents in the economy.

Formally, using the notation introduced in Section 6, suppose that for some $s \geq 1$, the sizes of the groups $i = 1, \ldots, s < n$ remain constant as m varies, i.e., $\ell_i^{(m)} = \ell_i$ for some ℓ_i, while for $i = s+1, \ldots, n$, the size of group i grows to infinity, i.e., $\ell_i^{(m)} \to \infty$. We will refer to groups $i = 1, \ldots, s$ as "small groups," and to groups $i = s+1, \ldots, n$ as "large groups."

Every trader j of a small group i receives signal θ_i. Every trader j of a large group i receives signal $\theta_i + \xi_{i,j}$, where θ_i is the signal component common to all traders of group i, and $\xi_{i,j}$ is the idiosyncratic component of trader j, independently distributed according to a normal distribution with mean 0 and variance-covariance matrix Σ_i^{ξ}.[23]

Throughout this section, let θ_S be the vector of signals of the small groups, i.e., $\theta_S = (\theta_1; \ldots; \theta_s)$, and let θ_L be the vector of common components of the signals of the large groups, i.e., $\theta_L = (\theta_{s+1}; \ldots; \theta_n)$.

We make two assumptions:

Assumption 1H $\quad Cov(v, \theta_S | \theta_L, \theta_M) \neq 0.$

Assumption 2H $\quad Var(u | \theta_L, \theta_M) > 0.$

The first assumption states that at least one of the small groups has some information about the value of the asset that is neither included in the information observed by the market maker

[23] Note that the assumption that all traders in the same small group i receive the same signal θ_i is without loss of generality: one small group of size ℓ_i in which traders also receive idiosyncratic components with nonzero variance can be represented as ℓ_i small groups of size one.

nor contained in the joint information of the large groups (even as the large groups grow infinitely large and the common components of their signals can be extracted from the joint information without the noise from idiosyncratic components). This assumption is analogous to Assumption 1 of Section 3, ensuring that even in the limit, some information about the value of the security remains "scarce." The second assumption states that the information of the market maker and the joint information of large groups is not sufficient to fully learn the demand from liquidity traders, even in the limit as the size of the large groups grows to infinity. This assumption is analogous to Assumption 2L of Section 6.

Our next result shows that under Assumptions 1H and 2H, equilibrium prices in the above sequence of markets converge to the equilibrium price that would obtain in an alternative market, in which only the small groups of traders are present (with the same information as in the original markets, θ_S), and in which the market maker directly observes both her original signal θ_M and the informative, common components of signals observed by the large groups of traders in the original markets, θ_L. Let $\{p^{(m)}\}$ denote the sequence of random variables that are equal to the prices in the linear equilibria of the original sequences of markets indexed by m. Let $p^{(alt)}$ denote the random variable that corresponds to the equilibrium price obtained in the alternative market.

Theorem 3 $\lim_{m\to\infty} E\left[\left(p^{(m)} - p^{(alt)}\right)^2\right] = 0.$

In Appendix C, we prove Theorem 3 for the special case in which the variance-covariance matrix of random vector $(\theta_S; \theta_L; \theta_M; u)$ is full rank. As in the proof of the special case of Theorem 2 in Appendix B, this additional assumption simplifies the proof by guaranteeing that certain matrices remain invertible in the limit as m becomes large. However, this special case rules out some interesting possibilities (e.g., some small groups know some elements of the common components of signals of some large groups), and so in the Online Appendix (Section 6), we provide the full proof of Theorem 3, which does not rely on this simplifying assumption. The techniques used in the proofs are similar to those used in the proofs of Theorem 2 in the special and general cases, except that the presence of small groups requires a separate treatment, because for the traders in those groups, strategic incentives do not vanish in the limit. Also, note that unlike in Theorem 2, the result in Theorem 3 does not depend on the sign of the covariance of liquidity demand with the other random variables in the model. The reason for that is that in the large-market case, in the case of $Cov(u, v|\theta, \theta_M) \le 0$, as the market was getting larger, market maker's sensitivity β_D was converging to zero, removing the impact of u on the market price. In the hybrid-market case, even as some groups become large, there are still some groups that remain small and whose traders thus possess "scarce" information which would have allowed them to make infinite profits if β_D converged to zero. So in the hybrid-market case, β_D remains bounded away from zero even in the limit, regardless of the sign of $Cov(u, v|\theta, \theta_M)$.

We conclude this section with a final observation. As we saw in Examples 4 and 5 in Section 5, the expected profit of an informed agent can be strictly higher when he observes the signal of the market maker than when he does not, because observing the information of the market maker allows the informed trader to better use the part of his information that is *not* known to the market maker. In the case of "hybrid" markets, Theorem 3 shows that equilibria converge to those that would obtain if the information of "large" groups was observed by the market maker, but *not publicly*, so that the "small" groups do not observe that information. This situation may create incentives for trading information. Indeed, if the traders of the small groups were to obtain information from some of the traders of the large groups, those small-group traders could increase their expected profit by a non-negligible amount. At the same time, in the limit, large-group traders make zero profits anyway, so they would not lose anything by sharing this information with the small-group traders. Thus, if trading information is allowed, the large-group information may end up being purchased by small-group traders, and thus the market, in the limit, may behave as if that information was observed publicly. We leave the formal analysis of this intuition to future research.

8 Information Aggregation under Cournot Competition

The main model we study in the paper has many "moving parts". In particular, it has three types of agents: in addition to fully optimizing strategic traders, it has mechanical liquidity traders and Bayesian market makers. It is thus natural to ask which of these components are the driving forces behind our main result on information aggregation (Theorem 2 in Section 6). Would the result break down without a market maker who explicitly sets prices to be equal to the expected value of security conditional on the information available to him? Is it essential that there are liquidity traders present in the model who in expectation lose money and by doing so "subsidize" trading and information discovery?

To shed light on these questions, in this section we consider a model that contains neither Bayesian market makers nor liquidity traders, but is otherwise closely related to the main model of our paper. (As we explain in footnote 26 below, the model of this section can be equivalently viewed as a model of trading with a mechanical market maker whose sensitivity to demand is exogenously fixed.) The model we consider in this section is asymmetric Cournot competition, in which firms observe imperfect (and generally different) signals about the intercept of the market demand function (an analogue of the value of the security v in the main model of the paper) and the question we address is, again, whether this asymmetric information gets aggregated as the market grows large.[24] We show that information does indeed get aggregated as the number of firms increases—the total quantity and price in the market converge to those that would obtain if

[24] In a recent paper, Bergemann et al. (2015) also compare informational properties of trading under Cournot competition, in which the slope of the price response is exogenously fixed, and of trading in a setting in the spirit of Kyle (1985), in which the slope of the price response is endogenously determined by a Bayesian market maker (as well as in a setting of demand function competition in the spirit of Kyle (1989), which we do not consider). Their focus, however, is different from ours: while we study information aggregation in the limit as the number of strategic traders becomes large, Bergemann et al. (2015) keep the number of players fixed and study the spaces of possible equilibrium outcomes under general information structures, the equivalences of these outcomes under different equilibrium notions (Bayes Correlated Equilibrium and Bayes Nash Equilibrium), and the properties of these spaces (such as the first and second moments of the equilibrium distributions of players' actions).

all the firms had access to all available information.[25]

8.1 Model

There are n firms in the market for a good. Each firm has a constant marginal cost of production c per unit of the good, and no fixed costs. The demand function for the good is not initially known to the firms. Rather, if the firms in aggregate produce Q units of the good, the resulting market price will be

$$p = v - \beta Q,$$

where $\beta > 0$ is the commonly known slope of the inverse demand function, and v is the uncertain intercept of that function.

Prior to making a production decision, each firm i observes a signal $\theta_i \in \mathbb{R}^{k_i}$ (where as before, $k_i \geq 1$ is the dimensionality of the signal). Vector $\theta = (\theta_1; \dots; \theta_n)$ summarizes the signals of all the firms. We assume that the vector $(\theta; v)$ is drawn randomly from the multivariate normal distribution with expected value $(0; \bar{v})$ and variance-covariance matrix Ω. We further assume, without loss of generality, that every variance-covariance matrix for signal θ_i of strategic trader i is full rank.

The timing of the game is as follows. After observing its signal θ_i, each firm i decides to produce quantity $q_i(\theta_i)$ of the good. The total amount produced is thus $Q = \sum_{i=1}^{n} q_i(\theta_i)$. The resulting market price is $p(v, Q) = v - \beta Q$. The realized payoff of firm i is $(p - c)q_i$.

8.2 Linear Equilibrium

As before, we restrict attention to linear equilibria, i.e., those of the form $q_i(\theta_i) = \alpha_i^T \theta_i + \delta_i$ for all i and some profile of vectors $\alpha_i \in \mathbb{R}^{k_i}$ and $\delta_i \in \mathbb{R}$. For notational simplicity, we denote a linear equilibrium by these linear coefficients, and define $\alpha = (\alpha_1; \dots; \alpha_n)$ and $\delta = (\delta_1; \dots; \delta_n)$.

Proposition 1 *In the Cournot competition game, there exists a unique linear equilibrium.*

The proof of Proposition 1 is in Appendix D. The proof is, in essence, a simplified version of the proof of Theorem 1, except that it only involves the analogues of Steps 2 and 3 from that proof. Step 1 (market maker's Bayesian updating) is not needed, because there is no market maker in the current model, and the price impact of each individual unit of supply, β, is exogenously fixed, instead of being endogenously determined by the market maker. Recall that Step 3 of the proof of Theorem 1 allowed us to express all the equilibrium strategies of the traders as a function of a single

[25] This result is closely related to the results of Li (1985) and Palfrey (1985), who also observe that under constant marginal costs of production, Cournot competition efficiently aggregates distributed information as the number of firms becomes large. The key difference between our result and those of Li (1985) and Palfrey (1985) is that we allow for an arbitrary matrix of correlations of the firms' signals, while Li (1985) and Palfrey (1985) require the distribution of signals to be symmetric. Note that the assumption of constant marginal costs is important for the results Li (1985) and Palfrey (1985), and thus also for our information aggregation result. As Vives (1988) shows, if production costs are quadratic (and thus marginal costs are increasing in quantity instead of being constant), the market does not converge to the full-information outcome as the number of firms becomes large, even if the firms are ex ante identical, both in their cost structures and in the informativeness and correlation structures of their signals.

parameter—the inverse of the market maker's sensitivity to aggregate demand, β_D. Steps 4 and 5 then derived a quadratic equation in that parameter and showed that it has a unique positive root. In the Cournot competition setting, the sensitivity β is fixed exogenously, and so the analogue of Step 3 concludes the proof.[26]

The closed-form solutions no longer involve the roots of a quadratic equation, and take the following form. Each firm i's strategy is given by

$$q_i(\theta_i) = \alpha_i^T \theta_i + \delta_i,$$

where for each i,

$$\delta_i = \beta^{-1} \frac{\bar{v} - c}{n+1},$$

and vector α is given by

$$\alpha = \beta^{-1} (\Sigma_{\theta\theta} + \Sigma_{diag})^{-1} \Sigma_{\theta v},$$

where matrices $\Sigma_{\theta\theta}$, Σ_{diag}, and $\Sigma_{\theta v}$ are defined as before. Note that the formula for vector α (as a function of price sensitivity β and the three matrices) is essentially the same as that in our main model (in the case when the market maker does not observe any private signals; see Section 4.2), except for the terms related to liquidity demand u that has no counterpart in our Cournot competition model. Of course, the key difference between the two formulas is that in the main model of our paper, sensitivity β_D is derived endogenously, while in the Cournot competition model, sensitivity β is exogenously fixed.

8.3 Information Aggregation in Large Markets

We now turn to the behavior of markets with a large number of participants. Our modeling approach is analogous to that in Section 6. Specifically, consider a sequence of markets, indexed by $m = 1, 2, \ldots$. The inverse demand function is the same in all markets m: $p(Q) = v - \beta Q$. In every market, there are n groups of firms, with at least one firm in each group. The groups are indexed by $i = 1, \ldots, n$, and each group i in market m consists of $\ell_i^{(m)}$ firms, with $\ell_i^{(m)} \to \infty$ as $m \to \infty$.

Each firm j in group i receives signal $\theta_i + \xi_{i,j} \in \mathbb{R}^{k_i}$, where θ_i is the common signal component of all firms in group i, and $\xi_{i,j}$ is the idiosyncratic component of firm j. As before, we assume that the random vector $(\theta_1; \ldots; \theta_n; v)$ is distributed normally with mean $(0; \ldots; 0; \bar{v})$ and variance-covariance matrix Ω. We also assume that the variance-covariance matrix of random vector $\theta = (\theta_1; \ldots; \theta_n)$ is positive definite. Every $\xi_{i,j}$ is drawn from the normal distribution with mean zero and variance Σ_i^ξ, independently of all the other random variables in the model. We do not impose any restrictions

[26] To see the parallels between the two models more directly, consider the version of the model in Section 3 in which the market maker does not observe any direct signals, and the version of the Cournot competition in which the marginal cost c is zero. In the former, the realized payoff of an individual trader i from submitting demand d_i is $d_i(v - p) = d_i(v - \beta_D(d_i + \sum_{j \neq i} d_j + u))$. In the latter, the realized payoff of an individual firm i from producing q_i units of the good is $q_i(v - \beta Q) = q_i(v - \beta(q_i + \sum_{j \neq i} q_j))$. So the Cournot competition setting can be viewed as a version of the main model of the paper in which liquidity demand is fixed at zero, and the market maker is mechanical, with the sensitivity exogenously fixed at $\beta_D = \beta$, instead of being endogenously determined by the Bayes rule.

on Σ_i^ξ, and in particular we allow for the possibility $\Sigma_i^\xi = 0$, in which case all the firms in group i observe identical signals.

As a benchmark, we also consider a sequence of alternative markets with the number of firms growing to infinity (for concreteness, let the number of firms in market m equal $N^{(m)} = \sum_{i=1}^n \ell_i^{(m)}$), but with a much simpler information structure: all "common components" of all signals are known to all firms. Formally, each firm j observes the same signal $\theta = (\theta_1; \ldots; \theta_n)$. In this sequence of alternative markets, all information is shared by all firms, and as the number of firms increases, the outcomes (i.e., the total quantity produced and the equilibrium price) converge to the perfectly competitive equilibrium. Our next proposition shows that the outcomes in the original sequence of markets also converge to the same perfectly competitive outcome, thus aggregating all the information distributed among the firms. Formally, let $Q^{(m)}$ and $p^{(m)}$ denote the random variables corresponding to the total quantity produced and the price realized in the original market m, and let $Q^{(alt,m)}$ and $p^{(alt,m)}$ denote the random variables corresponding to the total quantity produced and the price realized in the alternative market m where all the firms observe the same joint signal θ.

Proposition 2 $\lim_{m\to\infty} E[(Q^{(m)} - Q^{(alt,m)})^2] = 0$ *and* $\lim_{m\to\infty} E[(p^{(m)} - p^{(alt,m)})^2] = 0$.

The proof of Propostion 2 is in Appendix D. The proof proceeds by showing that in both sequences (original and alternative), for any realization of signals θ, the total quantity produced converges to the quantity produced in the perfectly competitive market with the intercept of the demand function equal to $E[v|\theta]$: $Q^*(\theta) = (E[v|\theta] - c)/\beta$. The result for the convergence of prices is then immediate.

Proposition 2 illustrates that the main driving force behind the information aggregation results in our paper is not the presence of a market maker who sets prices in an "intelligent" way, but rather the fact that the individual actions of informed players get aggregated (via the aggregate demand in the main model of our paper, and via the aggregate production in Cournot competition). The "aggregate action" of each group of players reflects that group's common signal, and these "aggregate actions" of the groups are then further aggregated by the marketplace with the appropriate weights. This "aggregation of actions" feature is important for our information aggregation results. In Section 7 of the Online Appendix, we provide a simple example of a Beauty Contest game in which dispersed information does not get aggregated in the limit, even though that game shares many of the features with the models considered above (normally distributed signals, linear best responses, and the uniqueness of linear equilibrium that can be characterized in closed form).

9 Conclusion

This paper studies trading behavior and the properties of prices in informationally complex markets. Our main framework generalizes the single-period version of the linear-normal model of Kyle (1985), allowing for multiple differentially informed strategic traders and for essentially arbitrary correlations among the random variables involved in the model: the value of the traded asset, the signals of the strategic traders and competitive market makers, and the demand from liquidity traders.

In this framework, we establish two main results.

First, we show that there always exists a unique linear equilibrium. We characterize the equilibrium analytically and express all agents' equilibrium behavior in closed form.

Second, we show that as the number of informed agents becomes large, their information gets fully aggregated and incorporated into market prices (with the full informational efficiency of the market requiring an additional assumption of non-negative covariance of the true value of the security and the demand from liquidity traders, conditional on the signals of strategic traders and the market maker). Crucially, our general framework allows us to avoid imposing the usual symmetry restrictions that are typically made in the literature on the strategic foundations of information aggregation in large markets.

Our paper leaves a number of open questions and directions for future research. One question is to what extent our analysis can be generalized to a dynamic setting, in which trading takes place over multiple periods, and each strategic trader takes into account the impact of his trading on his future arbitrage opportunities. As we discuss in the introduction, Bernhardt and Miao (2004) study a multi-period model of trading with rich information structures, closely related to our setting. They provide a characterization (necessary and sufficient conditions) of linear equilibria in their setting, and show that just as in our static setting, (1) if all strategic traders use linear strategies, then the resulting market maker's pricing rule is also linear, and (2) if the market maker is using a linear pricing rule, and all traders except i use linear trading strategies, then the best response of trader i is also a linear strategy. However, showing that the resulting system of equations has a solution remains an open question. Our results suggest one possible approach to attacking this question. Specifically, consider a dynamic version of our game in which the agents put a discount factor δ^t on the profits they make in period t, where $\delta \in [0, 1]$. Let $\delta = 0$, i.e., consider a myopic version of the "dynamic" game. Then it is immediate from our results that a linear equilibrium in this "myopic dynamic" game exists, is unique, and can be easily characterized in closed form.[27] Note also that this equilibrium is strict, payoffs are continuous in the discount factor, and for any discount factor, best responses and price response functions are all linear. So we conjecture that if the discount factor changes from zero by a small amount, the linear equilibrium will get perturbed continuously, and will not disappear (and new linear equilibria will not appear). This line of attack may make it possible to show that in the fully dynamic case, a linear equilibrium exists and is unique for a sufficiently small discount factor—or for a sufficiently large number of strategic traders having the same information (which, just like a small discount factor, would weaken dynamic incentives to conceal information). This approach may also provide a unified

[27] This observation is a direct corollary of Theorem 1 and the facts that (1) in every period $t \geq 2$, when the traders and the market maker incorporate the additional information that they observed in earlier periods, the information structure is still jointly normal, and (2) in each period the "effective" game that the agents play is static, because of the zero discount factor.

computational technique for finding equilibria in these dynamic markets when the discount factor is large and the number of strategic traders is small: one can start with the case of the zero discount factor, obtain the equilibrium in closed form, and then start gradually increasing the discount factor in small increments, recomputing the equilibrium for each small increment.

Second, the fact that our model admits explicit closed-form solutions for various objects of interest (equilibrium trading strategies, market maker's pricing behavior, expected profits and losses of various agents, trading volume, and so on) for every profile of the model's primitives makes it "embeddable" as part of richer settings and games. For instance, one can study pre-trading investment in costly acquisition of information (about the fundamentals of the traded security, about liquidity demand, or about the information of other strategic traders), mergers among the agents, or information sharing and trading among them. One can also consider the case of endogenous participation by liquidity traders, by considering a model with several different types of liquidity traders (e.g., retail investors, pension funds, insurance companies, etc.) whose demands may be differentially correlated with the value of the asset and/or with the informed traders' or the market maker's signals, and who choose to participate in the market only if their expected losses do not exceed certain thresholds.

Finally, the tractability of our model may also extend, at least to some degree, to other related settings, such as those with risk-averse traders (with CARA utilities, to preserve the linear-quadratic structure of the game), costly trading (with quadratic trading costs), multiple securities or trading venues (with traders having mean-variance preferences over payoffs), or partially informed liquidity traders (who have some exogenous liquidity demand to buy or sell securities, but who also take into account the expected profits and losses from their trading when deciding on the size of their order).

We leave the exploration of these extensions and generalizations to future research.

Appendix A: Proof of Theorem 1

The proof of Theorem 1 is constructive. For convenience, it is broken into several steps. Step 1 expresses the linear relationship implied by condition (i) of the definition of equilibrium, that price must be equal to the expected value of the security conditional on the information available to the market maker. Step 2 derives the best response of a strategic trader to a linear pricing rule and linear strategies of other strategic traders, and shows that this best response is linear. It also establishes that in equilibrium, coefficient β_D has to be positive. Step 3 summarizes the equations in Steps 1 and 2 and reorganizes them in a system of three "almost" linear equations (they are all linear if one scalar variable, $\gamma = 1/\beta_D$, is fixed). Step 4 reduces this system of equations to one quadratic equation in γ. Step 5 shows that this quadratic equation has exactly one positive root, thus completing the proof.

Step 1. Let $\alpha = (\alpha_1; \ldots; \alpha_n)$ be a profile of linear strategies for the strategic traders. Each α_i in this profile is a vector $(\alpha_i^1; \ldots; \alpha_i^{k_i}) \in \mathbb{R}^{k_i}$, corresponding to linear strategy

$$\begin{aligned} d_i(\theta_i) &= \alpha_i^1 \theta_i^1 + \cdots + \alpha_i^{k_i} \theta_i^{k_i} \\ &= \alpha_i^T \theta_i, \end{aligned}$$

where $\theta_i^1, \ldots, \theta_i^{k_i}$ are the elements of vector $\theta_i \in \mathbb{R}^{k_i}$.

Take any linear pricing rule $(\beta_M; \beta_D)$, $\beta_M \in \mathbb{R}^{k_M}$, $\beta_D \in \mathbb{R}$. For convenience, let vector $\beta = (\beta_M; \beta_D)$ summarize the pricing rule and let random vector $\eta = (\theta_M; D = \alpha^T \theta + u)$ denote the information available to the market maker when she sets the price. Then for this pricing rule to be consistent with profile α, condition (i) of the definition of equilibrium requires that

$$\beta^T \eta = E[v|\eta],$$

which is equivalent to the following condition:[28]

$$Cov(v, \eta) = \beta^T Var(\eta).$$

Expressing $Cov(v, \eta)$ and $Var(\eta)$ using the notation introduced in Section 4.1, we thus get the following equivalent characterization of condition (i) of the definition of equilibrium:

$$\left(\beta_M^T, \beta_D\right) \begin{pmatrix} \Sigma_{MM} & \Sigma_{\theta M}^T \alpha + \Sigma_{Mu} \\ \alpha^T \Sigma_{\theta M} + \Sigma_{Mu}^T & \alpha^T \Sigma_{\theta\theta} \alpha + 2\Sigma_{\theta u}^T \alpha + \sigma_{uu} \end{pmatrix} = \left(\Sigma_{vM}, \Sigma_{\theta v}^T \alpha + \sigma_{vu}\right). \tag{6}$$

Step 2. We now consider the optimization problem of a strategic trader i. Suppose he observes signal realization $\widetilde{\theta}_i$ of signal θ_i, and subsequently submits demand d. Assuming that other traders $j \neq i$ follow linear strategies α_j, and that the market maker follows a linear pricing rule $(\beta_M; \beta_D)$, the expected profit of trader i from submitting demand d when observing realization $\widetilde{\theta}_i$ is equal to

$$E\left[d \left(v - \beta_M^T \theta_M - \beta_D \left(d + \sum_{j \neq i} \alpha_j^T \theta_j + u \right) \right) \middle| \theta_i = \widetilde{\theta}_i \right]. \tag{7}$$

Using the fact that d is a choice variable, and thus d and d^2 are constants from the point of view of taking expectations, we can rewrite equation (7) as

$$d \cdot E\left[v - \beta_M^T \theta_M - \beta_D \left(\sum_{j \neq i} \alpha_j^T \theta_j + u \right) \middle| \theta_i = \widetilde{\theta}_i \right] - d^2 \cdot \beta_D. \tag{8}$$

Now, if $\beta_D < 0$, trader i can make an arbitrarily large expected profit, and no single d maximizes it—hence, β_D cannot be negative in equilibrium.

If $\beta_D = 0$, and $E\left[v - \beta_M^T \theta_M \middle| \theta_i = \widetilde{\theta}_i \right] \neq 0$, then again trader i can make an arbitrarily large expected profit, and no single d maximizes it. But it follows from Assumption 1 in the model[29]

[28] To see the equivalence, note first that $\beta^T \eta = E[v|\eta] \implies Cov(v, \eta) = Cov(E[v|\eta], \eta) = Cov(\beta^T \eta, \eta) = \beta^T Var(\eta)$. To go in the opposite direction, note that $Cov(v, \eta) = \beta^T Var(\eta) = Cov(\beta^T \eta, \eta) \implies Cov(v - \beta^T \eta, \eta) = 0$. Since variables $v - \beta^T \eta$ and η are jointly normal, $Cov(v - \beta^T \eta, \eta) = 0$ implies that they are independent, and thus for every realization $\widetilde{\eta}$ of random variable η, $E[v - \beta^T \eta | \eta = \widetilde{\eta}] = E[v - \beta^T \eta] = 0$, which implies that for every realization $\widetilde{\eta}$, $E[v|\eta = \widetilde{\eta}] = E[\beta^T \eta | \eta = \widetilde{\eta}] = \beta^T \widetilde{\eta}$.

[29] Assumption 1 says that at least one strategic trader i has some useful information beyond that contained in the

that for at least one trader i, for at least some (in fact, for almost all) realizations $\widetilde{\theta}_i$, we have $E\left[v - \beta_M^T \theta_M \middle| \theta_i = \widetilde{\theta}_i\right] \neq 0$—hence, β_D cannot be equal to zero in equilibrium.

Finally, if $\beta_D > 0$, then there is a unique d maximizing the expected profit:

$$d^* = \frac{1}{2\beta_D} E\left[v - \beta_M^T \theta_M - \beta_D \left(\sum_{j\neq i} \alpha_j^T \theta_j + u\right) \middle| \theta_i = \widetilde{\theta}_i\right] \tag{9}$$

$$= \frac{1}{2\beta_D}\left(\Sigma_{iv}^T - \beta_M^T \Sigma_{iM}^T - \beta_D \left(\sum_{j\neq i} \alpha_j^T \Sigma_{ij}^T + \Sigma_{iu}^T\right)\right) \Sigma_{ii}^{-1} \widetilde{\theta}_i, \tag{10}$$

where equation (10) is the standard projection/signal extraction formula, which can be used because of the joint normality of the relevant variables. Note that d^* is a linear function of $\widetilde{\theta}_i$, and vector α_i is uniquely determined by pricing rule $(\beta_M; \beta_D)$ and strategies α_j for $j \neq i$.

Step 3. It therefore follows from the arguments in Steps 1 and 2 that profile of strategies α and pricing rule $(\beta_M; \beta_D)$ form a linear equilibrium if and only if $\beta_D > 0$ and the following two conditions hold:

(i) $\left(\beta_M^T, \beta_D\right) \begin{pmatrix} \Sigma_{MM} & \Sigma_{\theta M}^T \alpha + \Sigma_{Mu} \\ \alpha^T \Sigma_{\theta M} + \Sigma_{Mu}^T & \alpha^T \Sigma_{\theta\theta} \alpha + 2\Sigma_{\theta u}^T \alpha + \sigma_{uu} \end{pmatrix} = \left(\Sigma_{vM}, \Sigma_{\theta v}^T \alpha + \sigma_{vu}\right);$

(ii) for all i, $\alpha_i^T = \frac{1}{2\beta_D}\left(\Sigma_{iv}^T - \beta_M^T \Sigma_{iM}^T - \beta_D \left(\sum_{j\neq i} \alpha_j^T \Sigma_{ij}^T + \Sigma_{iu}^T\right)\right) \Sigma_{ii}^{-1}$.

We will now show that there is a unique profile (α, β) satisfying these conditions, thus proving the existence and uniqueness of linear equilibrium.

First, we re-write condition (ii), for all i, as:

$$2\Sigma_{ii}\alpha_i = \frac{1}{\beta_D}(\Sigma_{iv} - \Sigma_{iM}\beta_M) - \sum_{j\neq i} \Sigma_{ij}\alpha_j - \Sigma_{iu}$$

or equivalently

$$\Sigma_{ii}\alpha_i + \sum_{j=1}^{n} \Sigma_{ij}\alpha_j = \frac{1}{\beta_D}(\Sigma_{iv} - \Sigma_{iM}\beta_M) - \Sigma_{iu}. \tag{11}$$

"Stacking" equations (11) for all i one under another, and rewriting the resulting system of equations in matrix form using the notation defined in Section 4.1, we obtain the following condition (equivalent to condition (ii)):

$$(\Sigma_{diag} + \Sigma_{\theta\theta})\alpha = \gamma\Sigma_{\theta v} - \Sigma_{\theta M}\beta_M' - \Sigma_{\theta u}, \tag{12}$$

where for convenience we define $\gamma = 1/\beta_D$, $\beta_M' = \beta_M/\beta_D$.

Next, using this notation, and transposing the matrix equation in condition (i), that condition can be written as a system of two equations:

$$\Sigma_{MM}\beta_M' + \Sigma_{\theta M}^T \alpha + \Sigma_{Mu} = \gamma\Sigma_{Mv}, \tag{13}$$

$$\alpha^T \Sigma_{\theta M}\beta_M' + \Sigma_{uM}\beta_M' + \alpha^T \Sigma_{\theta\theta}\alpha + 2\Sigma_{\theta u}^T \alpha + \sigma_{uu} = \gamma(\Sigma_{\theta v}^T \alpha + \sigma_{vu}). \tag{14}$$

market maker's signal: $Cov(v, \theta|\theta_M) \neq 0$.

Step 4. We will now solve the system of equations (12), (13), and (14). Equation (13) allows us to express β'_M as a function of α and γ:

$$\beta'_M = \Sigma_{MM}^{-1}\left(\gamma\Sigma_{Mv} - \Sigma_{\theta M}^T\alpha - \Sigma_{Mu}\right). \tag{15}$$

We then plug this expression of β'_M into equation (12):

$$(\Sigma_{diag} + \Sigma_{\theta\theta})\alpha = \gamma\Sigma_{\theta v} - \Sigma_{\theta M}\Sigma_{MM}^{-1}\left(\gamma\Sigma_{Mv} - \Sigma_{\theta M}^T\alpha - \Sigma_{Mu}\right) - \Sigma_{\theta u},$$

or, isolating α on the left-hand side and collecting the terms with γ,

$$\left(\Sigma_{diag} + \Sigma_{\theta\theta} - \Sigma_{\theta M}\Sigma_{MM}^{-1}\Sigma_{\theta M}^T\right)\alpha = \left(\Sigma_{\theta v} - \Sigma_{\theta M}\Sigma_{MM}^{-1}\Sigma_{Mv}\right)\gamma - \left(\Sigma_{\theta u} - \Sigma_{\theta M}\Sigma_{MM}^{-1}\Sigma_{Mu}\right).$$

Note that

$$\begin{aligned}
\Sigma_{\theta\theta} - \Sigma_{\theta M}\Sigma_{MM}^{-1}\Sigma_{\theta M}^T &= Var(\theta) - Cov(\theta,\theta_M)Var(\theta_M)^{-1}Cov(\theta_M,\theta) \\
&= Var(\theta|\theta_M),
\end{aligned}$$

where the last equation follows from the standard projection formula for multivariate normal distributions. Thus, matrix $\Sigma_{\theta\theta} - \Sigma_{\theta M}\Sigma_{MM}^{-1}\Sigma_{\theta M}^T$ is positive semidefinite, and matrix $\Sigma_{diag} + \Sigma_{\theta\theta} - \Sigma_{\theta M}\Sigma_{MM}^{-1}\Sigma_{\theta M}^T$ is positive definite (and thus invertible). Letting

$$\begin{aligned}
\Lambda &= \Sigma_{diag} + \Sigma_{\theta\theta} - \Sigma_{\theta M}\Sigma_{MM}^{-1}\Sigma_{\theta M}^T, \\
A_u &= \Lambda^{-1}\left(\Sigma_{\theta u} - \Sigma_{\theta M}\Sigma_{MM}^{-1}\Sigma_{Mu}\right), \\
A_v &= \Lambda^{-1}\left(\Sigma_{\theta v} - \Sigma_{\theta M}\Sigma_{MM}^{-1}\Sigma_{Mv}\right),
\end{aligned}$$

we can express α as a linear function of γ:

$$\alpha = \gamma A_v - A_u.$$

Plugging this expression into equation (15), we can also express β'_M as a linear function of γ:

$$\begin{aligned}
\beta'_M &= \Sigma_{MM}^{-1}\left(\gamma\Sigma_{Mv} - \Sigma_{\theta M}^T(\gamma A_v - A_u) - \Sigma_{Mu}\right) \\
&= \gamma\Sigma_{MM}^{-1}\left(\Sigma_{Mv} - \Sigma_{\theta M}^T A_v\right) - \Sigma_{MM}^{-1}\left(\Sigma_{Mu} - \Sigma_{\theta M}^T A_u\right).
\end{aligned}$$

Using these expressions, we can now rewrite equation (14) as a quadratic equation of just one scalar variable, γ:

$$a\gamma^2 + b\gamma + c = 0, \tag{16}$$

where

$$\begin{aligned}
a &= A_v^T\Sigma_{\theta M}\Sigma_{MM}^{-1}\left(\Sigma_{Mv} - \Sigma_{\theta M}^T A_v\right) + A_v^T\Sigma_{\theta\theta}A_v - \Sigma_{\theta v}^T A_v, \\
b &= -A_v^T\Sigma_{\theta M}\Sigma_{MM}^{-1}\left(\Sigma_{Mu} - \Sigma_{\theta M}^T A_u\right) - A_u^T\Sigma_{\theta M}\Sigma_{MM}^{-1}\left(\Sigma_{Mv} - \Sigma_{\theta M}^T A_v\right) \\
&\quad +\Sigma_{uM}\Sigma_{MM}^{-1}\left(\Sigma_{Mv} - \Sigma_{\theta M}^T A_v\right) - 2A_v^T\Sigma_{\theta\theta}A_u + 2\Sigma_{\theta u}^T A_v + \Sigma_{\theta v}^T A_u - \sigma_{vu}, \\
c &= A_u^T\Sigma_{\theta M}\Sigma_{MM}^{-1}\left(\Sigma_{Mu} - \Sigma_{\theta M}^T A_u\right) - \Sigma_{uM}\Sigma_{MM}^{-1}\left(\Sigma_{Mu} - \Sigma_{\theta M}^T A_u\right) \\
&\quad +A_u^T\Sigma_{\theta\theta}A_u - 2\Sigma_{\theta u}^T A_u + \sigma_{uu}.
\end{aligned}$$

Therefore, finding a linear equilibrium is equivalent to finding a positive root of equation (16).

To prove that this equation has a unique such root, we first simplify the expressions for a, b, and c. (For the proof, it is sufficient to simplify a and c, but getting a simplified expression for b is useful for deriving an explicit analytic characterization of the equilibrium.) Starting with a:

$$\begin{aligned}
a &= A_v^T \Sigma_{\theta M} \Sigma_{MM}^{-1} \left(\Sigma_{Mv} - \Sigma_{\theta M}^T A_v \right) + A_v^T \Sigma_{\theta\theta} A_v - \Sigma_{\theta v}^T A_v, \\
&= A_v^T \left[\left(\Sigma_{\theta M} \Sigma_{MM}^{-1} \Sigma_{Mv} - \Sigma_{\theta v} \right) + \left(\Sigma_{\theta\theta} - \Sigma_{\theta M} \Sigma_{MM}^{-1} \Sigma_{\theta M}^T \right) A_v \right] \\
&= A_v^T \left[\left(-\Lambda A_v \right) + \left(\Lambda - \Sigma_{diag} \right) A_v \right] \\
&= -A_v^T \Sigma_{diag} A_v.
\end{aligned}$$

Next,

$$\begin{aligned}
b = {} & -A_v^T \Sigma_{\theta M} \Sigma_{MM}^{-1} \left(\Sigma_{Mu} - \Sigma_{\theta M}^T A_u \right) - A_u^T \Sigma_{\theta M} \Sigma_{MM}^{-1} \left(\Sigma_{Mv} - \Sigma_{\theta M}^T A_v \right) \\
& + \Sigma_{uM} \Sigma_{MM}^{-1} \left(\Sigma_{Mv} - \Sigma_{\theta M}^T A_v \right) - 2 A_v^T \Sigma_{\theta\theta} A_u + 2 \Sigma_{\theta u}^T A_v + \Sigma_{\theta v}^T A_u - \sigma_{vu}, \\
= {} & 2 A_v^T \left(\Sigma_{\theta u} - \Sigma_{\theta M} \Sigma_{MM}^{-1} \Sigma_{Mu} \right) + A_u^T \left(\Sigma_{\theta v} - \Sigma_{\theta M} \Sigma_{MM}^{-1} \Sigma_{Mv} \right) \\
& + 2 A_v^T \left(\Sigma_{\theta M} \Sigma_{MM}^{-1} \Sigma_{\theta M}^T - \Sigma_{\theta\theta} \right) A_u + \Sigma_{uM} \Sigma_{MM}^{-1} \Sigma_{Mv} - \sigma_{uv} \\
= {} & 2 A_v^T \Lambda A_u + A_u^T \Lambda A_v \\
& + 2 A_v^T \left(\Sigma_{\theta M} \Sigma_{MM}^{-1} \Sigma_{\theta M}^T - \Sigma_{\theta\theta} \right) A_u + \Sigma_{uM} \Sigma_{MM}^{-1} \Sigma_{Mv} - \sigma_{uv} \\
= {} & A_v^T \left(2 \Sigma_{diag} + \Lambda \right) A_u + \Sigma_{uM} \Sigma_{MM}^{-1} \Sigma_{Mv} - \sigma_{uv}.
\end{aligned}$$

Finally,

$$\begin{aligned}
c = {} & A_u^T \Sigma_{\theta M} \Sigma_{MM}^{-1} \left(\Sigma_{Mu} - \Sigma_{\theta M}^T A_u \right) - \Sigma_{uM} \Sigma_{MM}^{-1} \left(\Sigma_{Mu} - \Sigma_{\theta M}^T A_u \right) \\
& + A_u^T \Sigma_{\theta\theta} A_u - 2 \Sigma_{\theta u}^T A_u + \sigma_{uu} \\
= {} & - \left(\Sigma_{uM} - A_u^T \Sigma_{\theta M} \right)^T \Sigma_{MM}^{-1} \left(\Sigma_{Mu} - \Sigma_{\theta M}^T A_u \right) \\
& + A_u^T \Sigma_{\theta\theta} A_u - 2 \Sigma_{\theta u}^T A_u + \sigma_{uu} \\
= {} & \begin{pmatrix} A_u \\ -1 \end{pmatrix}^T C \begin{pmatrix} A_u \\ -1 \end{pmatrix},
\end{aligned}$$

where

$$\begin{aligned}
C &= \begin{pmatrix} \Sigma_{\theta\theta} & \Sigma_{\theta u} \\ \Sigma_{\theta u}^T & \sigma_{uu} \end{pmatrix} - \begin{pmatrix} \Sigma_{\theta M} \\ \Sigma_{uM} \end{pmatrix} \Sigma_{MM}^{-1} \begin{pmatrix} \Sigma_{\theta M} \\ \Sigma_{uM} \end{pmatrix}^T \\
&= Var\left((\theta; u) \right) - Cov\left((\theta; u), \theta_M \right) Var\left(\theta_M \right)^{-1} Cov\left(\theta_M, (\theta; u) \right) \\
&= Var\left((\theta; u) | \theta_M \right).
\end{aligned}$$

Thus,

$$c = Var(A_u^T \theta - u | \theta_M).$$

Step 5. We will now determine the signs of coefficients a and c.

Matrix Σ_{diag} is positive definite, by construction. Vector A_v is not equal to zero: matrix Λ^{-1} is positive definite, and vector $\Sigma_{\theta v} - \Sigma_{\theta M} \Sigma_{MM}^{-1} \Sigma_{Mv} = Cov(\theta, v | \theta_M)$ is not equal to zero (by Assumption 1 of the model). Thus, $a = -A_v^T \Sigma_{diag} A_v < 0$.

To determine the sign of coefficient c, recall that we have shown in Step 4 that $c = Var(A_u^T\theta - u|\theta_M)$. So if we show that $c \neq 0$, it will immediately follow that $c > 0$.

If $A_u = 0$, then $c \neq 0$ follows from Assumption 2 of the model (which says that the market maker does not perfectly observe liquidity demand: $Var(u|\theta_M) > 0$).

Suppose $A_u \neq 0$. It is convenient to introduce an auxiliary random variable, ϕ, drawn randomly from the normal distribution with mean zero and covariance matrix Σ_{diag}, independent of all other random variables in the model. Note that matrix A_u now has a simple interpretation:

$$A_u = Var(\theta + \phi|\theta_M)^{-1}Cov(\theta, u|\theta_M) = Var(\theta + \phi|\theta_M)^{-1}Cov(\theta + \phi, u|\theta_M).$$

Let $\epsilon = u - A_u^T(\theta + \phi)$. Then $c = Var(\epsilon + A_u^T\phi|\theta_M)$. To show that $c \neq 0$, it is thus sufficient to show that $\epsilon + A_u^T\phi$ is not constant, conditional on θ_M. To show that, consider $Cov(\epsilon + A_u^T\phi, A_u^T(\theta + \phi)|\theta_M) = Cov(\epsilon, A_u^T(\theta + \phi)|\theta_M) + Cov(A_u^T\phi, A_u^T(\theta + \phi)|\theta_M)$.

First, $Cov(\epsilon, A_u^T(\theta + \phi)|\theta_M) = Cov(u - A_u^T(\theta + \phi), A_u^T(\theta + \phi)|\theta_M) = Cov(u, \theta + \phi|\theta_M)A_u - A_u^TVar(\theta + \phi|\theta_M)A_u = 0$.

Second, $Cov(A_u^T\phi, A_u^T(\theta + \phi)|\theta_M) = Var(A_u^T\phi|\theta_M) = A_u^T\Sigma_{diag}A_u$, which is not equal to zero, because $A_u \neq 0$ and Σ_{diag} is positive definite. Therefore, $Cov(\epsilon + A_u^T\phi, A_u^T(\theta + \phi)|\theta_M) \neq 0$, and thus $\epsilon + A_u^T\phi$ is not constant conditional on θ_M, and so $c > 0$.

Thus, $a < 0$, $c > 0$, and hence equation (16) has exactly one positive root. Therefore, there exists a unique linear equilibrium.

Appendix B: Proof of Theorem 2 (Special Case)

In this Appendix, we prove Theorem 2 in the special case in which the covariance matrix of random vector $(\theta; \theta_M; u)$ is full rank. In the Online Appendix (Section 5), we provide the full proof of Theorem 2, without making this simplifying assumption.

Step 1. Consider first a specific market m, and, for convenience, drop superscript (m). We know there exists a unique linear equilibrium. It then has to be the case that in this equilibrium, any two strategic traders in the same group have the same linear strategy (otherwise, by swapping the strategies of these two traders, we would be able to obtain a different linear equilibrium). Denote by α_i the aggregate demand multiplier, in equilibrium, of group i; i.e., given signal $\theta_i + \xi_{i,j}$ of trader j in group i, the trader submits demand $\frac{1}{\ell_i}\alpha_i^T(\theta_i + \xi_{i,j})$.

For the remainder of this proof, we define the variables $\xi_i = \frac{1}{\ell_i}\sum_j \xi_{i,j}$, $\xi = (\xi_1; \ldots; \xi_n)$, and the matrices

$$\Sigma^\xi = Var(\xi) = \begin{pmatrix} \frac{1}{\ell_1}\Sigma_1^\xi & 0 & \ldots & \ldots \\ 0 & \frac{1}{\ell_2}\Sigma_2^\xi & 0 & \ldots \\ \vdots & 0 & \ddots & 0 \\ 0 & \ldots & 0 & \frac{1}{\ell_n}\Sigma_n^\xi \end{pmatrix}$$

and

$$\widehat{\Sigma}_{diag} = \begin{pmatrix} \frac{1}{\ell_1}(\Sigma_{11}+\Sigma_1^\xi) & 0 & \dots & \dots \\ 0 & \frac{1}{\ell_2}(\Sigma_{22}+\Sigma_2^\xi) & 0 & \dots \\ \vdots & 0 & \ddots & 0 \\ 0 & \dots & 0 & \frac{1}{\ell_n}(\Sigma_{nn}+\Sigma_n^\xi) \end{pmatrix}.$$

With this notation, the equilibrium condition (i) in Step 1 of of the proof of Theorem 1—the market maker's inference given her information—becomes:

$$\left(\beta_M^T, \beta_D\right) \begin{pmatrix} \Sigma_{MM} & \Sigma_{\theta M}^T\alpha + \Sigma_{Mu} \\ \alpha^T\Sigma_{\theta M} + \Sigma_{Mu}^T & \alpha^T(\Sigma_{\theta\theta}+\Sigma^\xi)\alpha + 2\Sigma_{\theta u}^T\alpha + \sigma_{uu} \end{pmatrix} = \left(\Sigma_{vM}, \Sigma_{\theta v}^T\alpha + \sigma_{vu}\right)$$

where we observe that the only modification is in the variance of the overall demand, which is now written

$$\begin{aligned} Var\left(\sum_{i,j}\frac{\alpha_i^T}{\ell_i}(\theta_i+\xi_{i,j})+u\right) &= \alpha^T\Sigma_{\theta\theta}\alpha + 2\Sigma_{\theta u}^T\alpha + \sigma_{uu} + \sum_i \frac{\alpha_i^T\Sigma_i^\xi\alpha_i}{\ell_i} \\ &= \alpha^T(\Sigma_{\theta\theta}+\Sigma^\xi)\alpha + 2\Sigma_{\theta u}^T\alpha + \sigma_{uu}. \end{aligned}$$

The equations (13) and (14) that capture condition (i) then become slightly different:

$$\Sigma_{MM}\beta_M' + \Sigma_{\theta M}^T\alpha + \Sigma_{Mu} = \gamma\Sigma_{Mv}, \tag{17}$$

$$\alpha^T\Sigma_{\theta M}\beta_M' + \Sigma_{uM}\beta_M' + \alpha^T(\Sigma_{\theta\theta}+\Sigma^\xi)\alpha + 2\Sigma_{\theta u}^T\alpha + \sigma_{uu} = \gamma(\Sigma_{\theta v}^T\alpha + \sigma_{vu}). \tag{18}$$

The equilibrium condition (ii) in Step 3 of the proof of Theorem 1—the best response of a strategic trader—is also slightly different. In this new notation, it becomes: for all i,

$$\frac{1}{\ell_i}\alpha_i^T = \frac{1}{2\beta_D}\left(\Sigma_{iv}^T - \beta_M^T\Sigma_{iM}^T - \beta_D\left(\sum_{j\neq i}\alpha_j^T\Sigma_{ij}^T + \frac{\ell_i - 1}{\ell_i}\alpha_i^T\Sigma_{ii}^T + \Sigma_{iu}^T\right)\right)(\Sigma_{ii}+\Sigma_i^\xi)^{-1},$$

which is equivalent to:

$$\frac{\Sigma_{ii}}{\ell_i}\alpha_i + \frac{2\Sigma_i^\xi}{\ell_i}\alpha_i + \sum_j \Sigma_{ij}\alpha_j = \beta_D^{-1}[\Sigma_{iv} - \Sigma_{iM}\beta_M] - \Sigma_{iu}.$$

Similarly to equation (12) in the proof of Theorem 1, this condition can be rewritten as

$$(\widehat{\Sigma}_{diag} + \Sigma_{\theta\theta} + \Sigma^\xi)\alpha = \gamma\Sigma_{\theta v} - \Sigma_{\theta M}\beta_M' - \Sigma_{\theta u} \tag{19}$$

where γ and β_M' are defined as before.

Next, again by analogy with the proof of Theorem 1, we define

$$\begin{aligned} \widehat{\Lambda} &= \widehat{\Sigma}_{diag} + \Sigma_{\theta\theta} + \Sigma^\xi - \Sigma_{\theta M}\Sigma_{MM}^{-1}\Sigma_{\theta M}^T, \\ \widehat{A}_u &= \widehat{\Lambda}^{-1}\left(\Sigma_{\theta u} - \Sigma_{\theta M}\Sigma_{MM}^{-1}\Sigma_{Mu}\right), \\ \widehat{A}_v &= \widehat{\Lambda}^{-1}\left(\Sigma_{\theta v} - \Sigma_{\theta M}\Sigma_{MM}^{-1}\Sigma_{Mv}\right), \end{aligned}$$

and then finding a linear equilibrium is equivalent to solving the quadratic equation

$$a\gamma^2 + b\gamma + c = 0,$$

where

$$
\begin{aligned}
a &= -\widehat{A}_v^T \widehat{\Sigma}_{diag} \widehat{A}_v, \\
b &= \widehat{A}_v^T \left(2\widehat{\Sigma}_{diag} + \widehat{\Lambda}\right) \widehat{A}_u + \Sigma_{uM}\Sigma_{MM}^{-1}\Sigma_{Mv} - \sigma_{uv}, \\
c &= Var(\widehat{A}_u^T(\theta+\xi) - u|\theta_M) = Var(\widehat{A}_u^T\theta - u|\theta_M) + \widehat{A}_u^T \Sigma^{\xi} \widehat{A}_u.
\end{aligned}
$$

Since by definition $\gamma = 1/\beta_D$, solving the above quadratic equation is equivalent to solving the quadratic equation

$$
c\beta_D^2 + b\beta_D + a = 0,
$$

which turns out to be a more convenient characterization that we will proceed with. Similarly to the proof of Theorem 1, we also have a simple expression for the vector of strategies α:

$$
\alpha = \widehat{A}_v/\beta_D - \widehat{A}_u.
$$

Step 2. Let us now consider the entire sequence of markets, and restore superscript (m) for the variables. From the simplifying assumption that $Var(\theta;\theta_M;u)$ is full rank, it follows that both $Var(\theta|\theta_M)$ and $Var(\theta_M|\theta)$ are full rank, and thus invertible.

As $m \to \infty$, $\widehat{\Sigma}_{diag}^{(m)} \to 0$ and $\Sigma^{\xi,(m)} \to 0$. Thus,

$$
\begin{aligned}
\widehat{\Lambda}^{(m)} &\to \Sigma_{\theta\theta} - \Sigma_{\theta M}\Sigma_{MM}^{-1}\Sigma_{\theta M}^T = Var(\theta|\theta_M), \\
\widehat{A}_u^{(m)} &\to Var(\theta|\theta_M)^{-1}\left(\Sigma_{\theta u} - \Sigma_{\theta M}\Sigma_{MM}^{-1}\Sigma_{Mu}\right) = Var(\theta|\theta_M)^{-1}Cov(\theta,u|\theta_M), \\
\widehat{A}_v^{(m)} &\to Var(\theta|\theta_M)^{-1}\left(\Sigma_{\theta v} - \Sigma_{\theta M}\Sigma_{MM}^{-1}\Sigma_{Mv}\right) = Var(\theta|\theta_M)^{-1}Cov(\theta,v|\theta_M).
\end{aligned}
$$

Therefore, using that $\widehat{A}_u^T \Sigma^{\xi,(m)} \widehat{A}_u \to 0$,

$$
\begin{aligned}
a^{(m)} &\to 0, \\
b^{(m)} &\to Cov(v,\theta|\theta_M)Var(\theta|\theta_M)^{-1}Var(\theta|\theta_M)Var(\theta|\theta_M)^{-1}Cov(\theta,u|\theta_M) + \Sigma_{uM}\Sigma_{MM}^{-1}\Sigma_{Mv} - \sigma_{uv} \\
&= Cov(v,\theta|\theta_M)Var(\theta|\theta_M)^{-1}Cov(\theta,u|\theta_M) - Cov(u,v|\theta_M) \\
&= -Cov(u,v|\theta,\theta_M), \\
c^{(m)} &\to Var\left(Cov(u,\theta|\theta_M)Var(\theta|\theta_M)^{-1}\theta - u|\theta_M\right) + 0 \\
&= Var\left(E[u|\theta,\theta_M] - u|\theta_M\right) \\
&= Var\left(u|\theta,\theta_M\right).
\end{aligned}
$$

Note that these convergence results imply that $\beta_D^{(m)}$ converges to some finite value, since $\lim_{m\to\infty} c^{(m)} = Var(u|\theta,\theta_M) > 0$ (where the last inequality is due to Assumption 2L). If $Cov(u,v|\theta,\theta_M) > 0$, then $\lim_{m\to\infty}\beta_D^{(m)} = Var(u|\theta,\theta_M)^{-1}Cov(u,v|\theta,\theta_M)$. If $Cov(u,v|\theta,\theta_M) \le 0$, then $\lim_{m\to\infty}\beta_D^{(m)} = 0$. We now consider the limiting behavior of price $p^{(m)}$ in these two cases separately.

Step 3, Case $Cov(u, v|\theta, \theta_M) > 0$. Note first that

$$\begin{aligned}
E[v|\theta, \theta_M, u] &= E[v|\theta_M] \\
&\quad + Cov(v, \theta|\theta_M) Var(\theta|\theta_M)^{-1} (\theta - E[\theta|\theta_M]) \\
&\quad + Cov(v, u|\theta, \theta_M) Var(u|\theta, \theta_M)^{-1} (u - E[u|\theta, \theta_M]) \\
&= E[v|\theta_M] \\
&\quad + Cov(v, \theta|\theta_M) Var(\theta|\theta_M)^{-1} (\theta - E[\theta|\theta_M]) \\
&\quad + Cov(v, u|\theta, \theta_M) Var(u|\theta, \theta_M)^{-1} \\
&\qquad \times \left(u - E[u|\theta_M] - Cov(u, \theta|\theta_M) Var(\theta|\theta_M)^{-1} (\theta - E[\theta|\theta_M])\right).
\end{aligned}$$

Thus, $E[v|\theta, \theta_M, u]$ is a linear function of θ, θ_M, and u:

$$E[v|\theta, \theta_M, u] = w_M^T \theta_M + w_\theta^T \theta + w_u u,$$

where weights w are as follows:

$$\begin{aligned}
w_M^T &= Cov(v, \theta_M) Var(\theta_M)^{-1} \\
&\quad - Cov(v, \theta|\theta_M) Var(\theta|\theta_M)^{-1} Cov(\theta, \theta_M) Var(\theta_M)^{-1} \\
&\quad - Cov(v, u|\theta, \theta_M) Var(u|\theta, \theta_M)^{-1} Cov(u, \theta_M) Var(\theta_M)^{-1} \\
&\quad + Cov(v, u|\theta, \theta_M) Var(u|\theta, \theta_M)^{-1} Cov(u, \theta|\theta_M) Var(\theta|\theta_M)^{-1} Cov(\theta, \theta_M) Var(\theta_M)^{-1}; \\
w_\theta^T &= Cov(v, \theta|\theta_M) Var(\theta|\theta_M)^{-1} \\
&\quad - Cov(v, u|\theta, \theta_M) Var(u|\theta, \theta_M)^{-1} Cov(u, \theta|\theta_M) Var(\theta|\theta_M)^{-1}; \\
w_u &= Cov(v, u|\theta, \theta_M) Var(u|\theta, \theta_M)^{-1}.
\end{aligned}$$

Next, price $p^{(m)}(\theta, \xi^{(m)}, \theta_M, u)$ in market m can be expressed as

$$\begin{aligned}
p^{(m)}(\theta, \xi^{(m)}, \theta_M, u) &= \beta_M^{(m)T} \theta_M + \beta_D^{(m)} \left(\alpha^{(m)T} (\theta + \xi^{(m)}) + u\right) \\
&= \beta_M^{(m)T} \theta_M + \beta_D^{(m)} \alpha^{(m)T} (\theta + \xi^{(m)}) + \beta_D^{(m)} u.
\end{aligned}$$

To prove the statement of the theorem for this case, note that

$$\begin{aligned}
E\left[\left(p^{(m)}(\theta, \xi^{(m)}, \theta_M, u) - E[v|\theta, \theta_M, u]\right)^2\right] &= (\beta_D^{(m)})^2 \alpha^{(m)T} \Sigma^{\xi,(m)} \alpha^{(m)} \\
&\quad + \begin{pmatrix} \beta_M^{(m)} - w_M \\ \beta_D^{(m)} \alpha^{(m)} - w_\theta \\ \beta_D^{(m)} - w_u \end{pmatrix}^T Var\left(\begin{pmatrix} \theta_M \\ \theta \\ u \end{pmatrix}\right) \begin{pmatrix} \beta_M^{(m)} - w_M \\ \beta_D^{(m)} \alpha^{(m)} - w_\theta \\ \beta_D^{(m)} - w_u \end{pmatrix}.
\end{aligned}$$

Noting that $\Sigma^{\xi,(m)} \to 0$, it is sufficient to show that as m grows, $\beta_D^{(m)} \to w_u$, $\beta_D^{(m)} \alpha^{(m)} \to w_\theta$, and $\beta_M^{(m)} \to w_M$.

The first convergence result is immediate:

$$\lim_{m \to \infty} \beta_D^{(m)} = Var(u|\theta, \theta_M)^{-1} Cov(u, v|\theta, \theta_M) = w_u.$$

Next:

$$\begin{aligned}
\lim_{m\to\infty} \beta_D^{(m)} \alpha^{(m)} &= \lim_{m\to\infty} \widehat{A}_v^{(m)} - \beta_D^{(m)} \widehat{A}_u^{(m)} \\
&= Var(\theta|\theta_M)^{-1} Cov(\theta, v|\theta_M) \\
&\quad -Var(u|\theta,\theta_M)^{-1} Cov(u,v|\theta,\theta_M) Var(\theta|\theta_M)^{-1} Cov(\theta,u|\theta_M) \\
&= w_\theta.
\end{aligned}$$

Finally:

$$\begin{aligned}
\lim_{m\to\infty} \beta_M^{(m)} &= \lim_{m\to\infty} \Sigma_{MM}^{-1} \left(\Sigma_{Mv} - \Sigma_{\theta M}^T \widehat{A}_v^{(m)}\right) - \beta_D^{(m)} \Sigma_{MM}^{-1} \left(\Sigma_{Mu} - \Sigma_{\theta M}^T \widehat{A}_u^{(m)}\right) \\
&= \Sigma_{MM}^{-1} \left(\Sigma_{Mv} - \Sigma_{\theta M}^T Var(\theta|\theta_M)^{-1} Cov(\theta, v|\theta_M)\right) \\
&\quad - Var(u|\theta,\theta_M)^{-1} Cov(u,v|\theta,\theta_M) \Sigma_{MM}^{-1} \left(\Sigma_{Mu} - \Sigma_{\theta M}^T Var(\theta|\theta_M)^{-1} Cov(\theta,u|\theta_M)\right) \\
&= Var(\theta_M)^{-1} Cov(\theta_M, v) \\
&\quad - Var(\theta_M)^{-1} Cov(\theta,\theta_M)^T Var(\theta|\theta_M)^{-1} Cov(\theta, v|\theta_M) \\
&\quad - Var(u|\theta,\theta_M)^{-1} Cov(u,v|\theta,\theta_M) Var(\theta_M)^{-1} Cov(\theta_M, u) \\
&\quad + Var(u|\theta,\theta_M)^{-1} Cov(u,v|\theta,\theta_M) Var(\theta_M)^{-1} Cov(\theta,\theta_M)^T Var(\theta|\theta_M)^{-1} Cov(\theta,u|\theta_M) \\
&= w_M.
\end{aligned}$$

Step 3, Case $\boldsymbol{Cov(u,v|\theta,\theta_M) \le 0}$. In this case, note that

$$\begin{aligned}
E[v|\theta,\theta_M] &= E[v|\theta_M] \\
&\quad + Cov(v,\theta|\theta_M) Var(\theta|\theta_M)^{-1} \left(\theta - E[\theta|\theta_M]\right).
\end{aligned}$$

Thus, $E[v|\theta,\theta_M]$ is a linear function of θ and θ_M:

$$E[v|\theta,\theta_M] = w_M^T \theta_M + w_\theta^T \theta,$$

where weights w are as follows:

$$\begin{aligned}
w_M^T &= Cov(v,\theta_M) Var(\theta_M)^{-1} \\
&\quad - Cov(v,\theta|\theta_M) Var(\theta|\theta_M)^{-1} Cov(\theta,\theta_M) Var(\theta_M)^{-1}; \\
w_\theta^T &= Cov(v,\theta|\theta_M) Var(\theta|\theta_M)^{-1}.
\end{aligned}$$

As before, price $p^{(m)}(\theta, \xi^{(m)}, \theta_M, u)$ in market m can be expressed as

$$\begin{aligned}
p^{(m)}(\theta, \xi^{(m)}, \theta_M, u) &= \beta_M^{(m)T} \theta_M + \beta_D^{(m)} \left(\alpha^{(m)T}(\theta + \xi^{(m)}) + u\right) \\
&= \beta_M^{(m)T} \theta_M + \beta_D^{(m)} \alpha^{(m)T}(\theta + \xi^{(m)}) + \beta_D^{(m)} u.
\end{aligned}$$

As in Step 2, noting that $Var(\xi^{(m)}) \to 0$, to prove the statement of the theorem for this case, it is thus sufficient to show that as m grows, $\beta_D^{(m)} \to 0$, $\beta_D^{(m)} \alpha^{(m)} \to w_\theta$, and $\beta_M^{(m)} \to w_M$.

The first convergence result, $\beta_D^{(m)} \to 0$, was proven at the end of Step 2 above.

Next,

$$\begin{aligned}
\lim_{m\to\infty} \beta_D^{(m)}\alpha^{(m)} &= \lim_{m\to\infty} \widehat{A}_v^{(m)} - \beta_D^{(m)}\widehat{A}_u^{(m)} \\
&= Var(\theta|\theta_M)^{-1}Cov(\theta, v|\theta_M) \\
&\quad - \left[\lim_{m\to\infty} \beta_D^{(m)}\right] Var(\theta|\theta_M)^{-1}Cov(\theta, u|\theta_M) \\
&= Var(\theta|\theta_M)^{-1}Cov(\theta, v|\theta_M) \\
&= w_\theta.
\end{aligned}$$

Finally,

$$\begin{aligned}
\lim_{m\to\infty} \beta_M^{(m)} &= \lim_{m\to\infty} \Sigma_{MM}^{-1}\left(\Sigma_{Mv} - \Sigma_{\theta M}^T \widehat{A}_v^{(m)}\right) - \beta_D^{(m)}\Sigma_{MM}^{-1}\left(\Sigma_{Mu} - \Sigma_{\theta M}^T\widehat{A}_u^{(m)}\right) \\
&= \lim_{m\to\infty} \Sigma_{MM}^{-1}\left(\Sigma_{Mv} - \Sigma_{\theta M}^T\widehat{A}_v^{(m)}\right) \\
&= Var(\theta_M)^{-1}Cov(\theta_M, v) \\
&\quad - Var(\theta_M)^{-1}Cov(\theta, \theta_M)^T Var(\theta|\theta_M)^{-1}Cov(\theta, v|\theta_M) \\
&= w_M.
\end{aligned}$$

Appendix C: Proof of Theorem 3 (Special Case)

In this Appendix, we prove Theorem 3 in the special case in which the covariance matrix of random vector $(\theta_S; \theta_L; \theta_M; u)$ is full rank. In the Online Appendix (Section 6), we provide the full proof of Theorem 3, without making this simplifying assumption.

Step 1. In addition to the markets indexed $m = 1, 2, \ldots$, we consider the alternative market which includes s groups of traders $i = 1, \ldots, s$. The size of group i is ℓ_i. Each trader j of group i receives signal θ_i. In this alternative market, the market maker receives signal $(\theta_L; \theta_M)$. We use superscript (m) when we refer to the variables of the market m, and we use superscript (alt) when we refer to the variables in the alternative market. By Theorem 1 a unique linear equilibrium exists for each market m and for the alternative market. Recall that $\ell_i^{(m)}$ is constant in m for $i \leq s$.

For $i = s+1, \ldots, n$, we define $\xi_i^{(m)} = (\sum_j \xi_{i,j})/\ell_i^{(m)}$ and $\xi_L^{(m)} = (\xi_{s+1}^{(m)}; \ldots; \xi_n^{(m)})$. For $i \leq s$ we let by convention $\Sigma_i^\xi = 0$, $\xi_i^{(m)} = 0$, and $\xi_S^{(m)} = (\xi_1^{(m)}; \ldots; \xi_s^{(m)})$. Let $\xi^{(m)} = (\xi_S^{(m)}; \xi_L^{(m)})$.

Also, as before, we define

$$\widehat{\Sigma}_{diag}^{(alt)} = \begin{pmatrix} \frac{1}{\ell_1}\Sigma_{11} & 0 & \cdots & 0 \\ 0 & \frac{1}{\ell_2}\Sigma_{22} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & \frac{1}{\ell_s}\Sigma_{ss} \end{pmatrix}$$

and

$$\widehat{\Sigma}_{diag}^{(m)} = \begin{pmatrix} \frac{1}{\ell_1^{(m)}}(\Sigma_{11} + \Sigma_1^{\xi}) & 0 & \cdots & 0 \\ 0 & \frac{1}{\ell_2^{(m)}}(\Sigma_{22} + \Sigma_2^{\xi}) & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & \frac{1}{\ell_n^{(m)}}(\Sigma_{nn} + \Sigma_n^{\xi}) \end{pmatrix}.$$

We also define

$$\Sigma^{\xi,(m)} = \begin{pmatrix} \frac{1}{\ell_1^{(m)}}\Sigma_1^{\xi} & 0 & \cdots & \cdots \\ 0 & \frac{1}{\ell_2^{(m)}}\Sigma_2^{\xi} & 0 & \cdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & \frac{1}{\ell_n^{(m)}}\Sigma_n^{\xi} \end{pmatrix}.$$

Note that $\Sigma^{\xi,(m)}$ is the variance-covariance matrix of $\xi^{(m)}$. Also note that as $m \to \infty$, $\Sigma^{\xi,(m)} \to 0$ and $\widehat{\Sigma}_{diag}^{(m)} \to \widehat{\Sigma}_{diag}^{(\infty)}$, where we define

$$\widehat{\Sigma}_{diag}^{(\infty)} = \begin{pmatrix} \widehat{\Sigma}_{diag}^{(alt)} & 0 \\ 0 & 0 \end{pmatrix}.$$

We could proceed by showing various convergence results directly, by matrix manipulation, as in the proof of the special case of Theorem 2 in Appendix B. However, it turns out that the proof becomes simpler and more intuitive if instead we follow the methodology of the proof of the general case of Theorem 2 in the online appendix, introduce auxiliary random variables, and interpret various matrices in the proof as covariance matrices of various combinations of these auxiliary random variables and the random variables in the model.

Specifically, for each market m, we introduce a random vector $\widehat{\theta}^{(m)}$, which is independent of the other random variables in the model, and is distributed normally with mean 0 and covariance matrix $\widehat{\Sigma}_{diag}^{(m)}$. We also introduce a random vector $\widehat{\theta}_S$, which is independent of the other random variables in the model, and is distributed normally with mean 0 and covariance matrix $\widehat{\Sigma}_{diag}^{(alt)}$. Finally, we introduce a random vector $\widehat{\theta}^{(\infty)}$, which is defined as $\widehat{\theta}^{(\infty)} = (\widehat{\theta}_S; 0)$, and is therefore distributed normally with mean 0 and covariance matrix $\widehat{\Sigma}_{diag}^{(\infty)}$.

First let us focus on the linear equilibrium in market m. We have

$$\begin{aligned} \widehat{\Lambda}^{(m)} &= \widehat{\Sigma}_{diag}^{(m)} + \Sigma^{\xi,(m)} + Var(\theta|\theta_M) = Var(\theta + \xi^{(m)} + \widehat{\theta}^{(m)}|\theta_M), \\ \widehat{A}_u^{(m)} &= (\widehat{\Lambda}^{(m)})^{-1} Cov(\theta, u|\theta_M), \\ \widehat{A}_v^{(m)} &= (\widehat{\Lambda}^{(m)})^{-1} Cov(\theta, v|\theta_M). \end{aligned}$$

Finding the linear equilibrium is equivalent to solving the quadratic equation

$$c^{(m)}(\beta_D^{(m)})^2 + b^{(m)}\beta_D^{(m)} + a^{(m)} = 0,$$

where

$$\begin{aligned}
a^{(m)} &= -(\widehat{A}_v^{(m)})^T \widehat{\Sigma}_{diag}^{(m)} \widehat{A}_v^{(m)}, \\
b^{(m)} &= (\widehat{A}_v^{(m)})^T \left(2\widehat{\Sigma}_{diag}^{(m)} + \widehat{\Lambda}^{(m)}\right) \widehat{A}_u^{(m)} - Cov(u, v|\theta_M), \\
c^{(m)} &= Var((\widehat{A}_u^{(m)})^T(\theta + \xi^{(m)}) - u|\theta_M).
\end{aligned}$$

Similarly, there exists a unique linear equilibrium of the alternative market. Let

$$\begin{aligned}
\widehat{\Lambda}^{(alt)} &= \widehat{\Sigma}_{diag}^{(alt)} + Var(\theta_S|\theta_M, \theta_L) = Var(\theta_S + \widehat{\theta}_S|\theta_M, \theta_L), \\
\widehat{A}_u^{(alt)} &= (\widehat{\Lambda}^{(alt)})^{-1} Cov(\theta_S, u|\theta_M, \theta_L), \\
\widehat{A}_v^{(alt)} &= (\widehat{\Lambda}^{(alt)})^{-1} Cov(\theta_S, v|\theta_M, \theta_L).
\end{aligned}$$

Finding the linear equilibrium is equivalent to solving the quadratic equation

$$c^{(alt)}(\beta_D^{(alt)})^2 + b^{(alt)}\beta_D^{(alt)} + a^{(alt)} = 0,$$

where

$$\begin{aligned}
a^{(alt)} &= -(\widehat{A}_v^{(alt)})^T \widehat{\Sigma}_{diag}^{(alt)} \widehat{A}_v^{(alt)}, \\
b^{(alt)} &= (\widehat{A}_v^{(alt)})^T \left(2\widehat{\Sigma}_{diag}^{(alt)} + \widehat{\Lambda}^{(alt)}\right) \widehat{A}_u^{(alt)} - Cov(u, v|\theta_M, \theta_L), \\
c^{(alt)} &= Var((\widehat{A}_u^{(alt)})^T \theta_S - u|\theta_M, \theta_L).
\end{aligned}$$

The equilibrium price in market m is

$$\begin{aligned}
p^{(m)} &= (\beta_M^{(m)})^T \theta_M + \beta_D^{(m)} \left((\alpha^{(m)})^T \theta^{(m)} + u\right) \\
&= (\beta_M^{(m)})^T \theta_M + \beta_D^{(m)} \left((\alpha_S^{(m)})^T \theta_S + (\alpha_L^{(m)})^T (\theta_L^{(m)} + \xi_L^{(m)}) + u\right),
\end{aligned}$$

where we "decompose" the vector $\alpha^{(m)}$ as $\alpha^{(m)} = \left(\alpha_S^{(m)}; \alpha_L^{(m)}\right)$.

The equilibrium price in the alternative market is

$$\begin{aligned}
p^{(alt)} &= (\beta_M^{(alt)})^T \theta_M^{(alt)} + \beta_D^{(alt)} \left((\alpha^{(alt)})^T \theta_S + u\right) \\
&= (\beta_{M,M}^{(alt)})^T \theta_M + (\beta_{M,L}^{(alt)})^T \theta_L + \beta_D^{(alt)} \left((\alpha^{(alt)})^T \theta_S + u\right),
\end{aligned}$$

where $\theta_M^{(alt)} = (\theta_M; \theta_L)$ and $\beta_M^{(alt)}$ is "decomposed" as $\beta_M^{(alt)} = (\beta_{M,M}^{(alt)}; \beta_{M,L}^{(alt)})$.

We will show in Step 2 that $\beta_D^{(m)} \to \beta_D^{(alt)}$, and then in Step 3 we will show that $\beta_M^{(m)} \to \beta_{M,M}^{(alt)}$, $\beta_D^{(m)}\alpha_L^{(m)} \to \beta_{M,L}^{(alt)}$, and $\beta_D^{(m)}\alpha_S^{(m)} \to \beta_D^{(alt)}\alpha^{(alt)}$. By the same argument as in Step 3 of the proof of the special case of Theorem 2 in Appendix B, showing these four convergence results is sufficient to prove the statement of Theorem 3.

Step 2. First, we show that the coefficients of the quadratic equation that $\beta_D^{(m)}$ satisfies converge to those of the quadratic equation that $\beta_D^{(alt)}$ satisfies. As the coefficient on $(\beta_D^{(alt)})^2$ in the latter equation is positive (as shown in Step 5 on the proof of Theorem 1 in Appendix A), this convergence implies that $\beta_D^{(m)}$ converges to $\beta_D^{(alt)}$.

Step 2(a). We first show that $a^{(m)} \to a^{(alt)}$. We have

$$\widehat{\Sigma}^{(m)}_{diag} \to \widehat{\Sigma}^{(\infty)}_{diag} := Var((\widehat{\theta}_S; 0)),$$

thus

$$\widehat{\Lambda}^{(m)} \to \widehat{\Lambda}^{(\infty)} := Var((\theta_S + \widehat{\theta}_S; \theta_L)|\theta_M),$$

and, as $\widehat{\Lambda}^{(\infty)}$ is positive definite (which follows from Assumption 2H),

$$\begin{aligned}
\widehat{A}^{(m)}_v \to \widehat{A}^{(\infty)}_v &:= (\widehat{\Lambda}^{(\infty)})^{-1} Cov(\theta, v|\theta_M) \\
&= Var((\theta_S + \widehat{\theta}_S; \theta_L)|\theta_M)^{-1} Cov((\theta_S + \widehat{\theta}_S; \theta_L), v|\theta_M).
\end{aligned}$$

This identity implies that for any (fixed) vectors $\widetilde{\theta}_S$ (of the same dimension as random vector θ_S) and $\widetilde{\theta}_L$ (of the same dimension as random vector θ_L), we have

$$(\widehat{A}^{(\infty)}_v)^T (\widetilde{\theta}_S; \widetilde{\theta}_L) = E[v|\theta_M = 0, \theta_S + \widehat{\theta}_S = \widetilde{\theta}_S, \theta_L = \widetilde{\theta}_L]. \tag{20}$$

Now, note that

$$\begin{aligned}
a^{(m)} \to a^{(\infty)} &:= -(\widehat{A}^{(\infty)}_v)^T \widehat{\Sigma}^{(\infty)}_{diag} \widehat{A}^{(\infty)}_v \\
&= -(\widehat{A}^{(\infty)}_v)^T Var((\widehat{\theta}_S; 0)) \widehat{A}^{(\infty)}_v \\
&= -Var\left((\widehat{A}^{(\infty)}_v)^T (\widehat{\theta}_S; 0) \right).
\end{aligned}$$

Likewise, for any (fixed) vector $\widetilde{\theta}_S$ (of the same dimension as θ_S), we have

$$(\widehat{A}^{(alt)}_v)^T \widetilde{\theta}_S = E[v|\theta_M = 0, \theta_S + \widehat{\theta}_S = \widetilde{\theta}_S, \theta_L = 0]. \tag{21}$$

Also,

$$\begin{aligned}
a^{(alt)} &= -(\widehat{A}^{(alt)}_v)^T \widehat{\Sigma}^{(alt)}_{diag} \widehat{A}^{(alt)}_v \\
&= -(\widehat{A}^{(alt)}_v)^T Var(\widehat{\theta}_S) \widehat{A}^{(alt)}_v \\
&= -Var\left((\widehat{A}^{(alt)}_v)^T \widehat{\theta}_S \right).
\end{aligned}$$

Equations (20) and (21) imply that for every realization $\widetilde{\theta}_S$ of random vector $\widehat{\theta}_S$,

$$\begin{aligned}
(\widehat{A}^{(\infty)}_v)^T (\widetilde{\theta}_S; 0) &= E[v|\theta_M = 0, \theta_S + \widehat{\theta}_S = \widetilde{\theta}_S, \theta_L = 0] \\
&= (\widehat{A}^{(alt)}_v)^T \widetilde{\theta}_S,
\end{aligned}$$

and thus

$$Var\left((\widehat{A}^{(\infty)}_v)^T (\widehat{\theta}_S; 0) \right) = Var\left((\widehat{A}^{(alt)}_v)^T \widehat{\theta}_S \right)$$

and so $a^{(m)} \to a^{(\infty)} = a^{(alt)}$.

Step 2(b). Next, we show that $b^{(m)} \to b^{(alt)}$. In the limit,

$$b^{(m)} \to b^{(\infty)} := (\widehat{A}^{(\infty)}_v)^T \left(2\widehat{\Sigma}^{(\infty)}_{diag} + \widehat{\Lambda}^{(\infty)} \right) \widehat{A}^{(\infty)}_u - Cov(u, v|\theta_M),$$

where

$$\widehat{A}^{(\infty)}_u := \lim_{m \to \infty} \widehat{A}^{(m)}_u = (\widehat{\Lambda}^{(\infty)})^{-1} Cov(\theta, u|\theta_M).$$

Similarly to equations (20) and (21) above, for any fixed vectors $\widetilde{\theta}_S$ and $\widetilde{\theta}_L$, we have

$$(\widehat{A}_u^{(\infty)})^T(\widetilde{\theta}_S;\widetilde{\theta}_L) = E[u|\theta_M = 0, \theta_S + \widehat{\theta}_S = \widetilde{\theta}_S, \theta_L = \widetilde{\theta}_L]; \tag{22}$$

$$(\widehat{A}_u^{(alt)})^T\widetilde{\theta}_S = E[u|\theta_M = 0, \theta_S + \widehat{\theta}_S = \widetilde{\theta}_S, \theta_L = 0]. \tag{23}$$

Note that

$$\begin{aligned}(\widehat{A}_v^{(\infty)})^T\widehat{\Sigma}_{diag}^{(\infty)}\widehat{A}_u^{(\infty)} &= (\widehat{A}_v^{(\infty)})^T Var((\widehat{\theta}_S;0))\widehat{A}_u^{(\infty)} \\ &= Cov\left((\widehat{A}_v^{(\infty)})^T(\widehat{\theta}_S;0), (\widehat{A}_u^{(\infty)})^T(\widehat{\theta}_S;0)\right)\end{aligned}$$

and

$$\begin{aligned}(\widehat{A}_v^{(alt)})^T\widehat{\Sigma}_{diag}^{(alt)}\widehat{A}_u^{(alt)} &= (\widehat{A}_v^{(alt)})^T Var(\widehat{\theta}_S)\widehat{A}_u^{(alt)} \\ &= Cov\left((\widehat{A}_v^{(alt)})^T\widehat{\theta}_S, (\widehat{A}_u^{(alt)})^T\widehat{\theta}_S\right).\end{aligned}$$

By equations (20)–(23), for any realization $\widetilde{\theta}_S$ of random vector $\widehat{\theta}_S$,

$$\begin{aligned}(\widehat{A}_v^{(\infty)})^T(\widetilde{\theta}_S;0) &= (\widehat{A}_v^{(alt)})^T\widetilde{\theta}_S \quad \text{and} \\ (\widehat{A}_u^{(\infty)})^T(\widetilde{\theta}_S;0) &= (\widehat{A}_u^{(alt)})^T\widetilde{\theta}_S,\end{aligned}$$

and so

$$(\widehat{A}_v^{(\infty)})^T\widehat{\Sigma}_{diag}^{(\infty)}\widehat{A}_u^{(\infty)} = (\widehat{A}_v^{(alt)})^T\widehat{\Sigma}_{diag}^{(alt)}\widehat{A}_u^{(alt)}.$$

Next, note that

$$(\widehat{A}_v^{(\infty)})^T\widehat{\Lambda}^{(\infty)}\widehat{A}_u^{(\infty)} = Cov(v,(\theta_S+\widehat{\theta}_S;\theta_L)|\theta_M)[Var((\theta_S+\widehat{\theta}_S;\theta_L)|\theta_M)]^{-1}Cov((\theta_S+\widehat{\theta}_S;\theta_L),u|\theta_M)$$

and so

$$(\widehat{A}_v^{(\infty)})^T\widehat{\Lambda}^{(\infty)}\widehat{A}_u^{(\infty)} - Cov(u,v|\theta_M) = -Cov(u,v|\theta_M,\theta_L,\theta_S+\widehat{\theta}_S).$$

Similarly,

$$(\widehat{A}_v^{(alt)})^T\widehat{\Lambda}^{(alt)}\widehat{A}_u^{(alt)} = Cov(v,\theta_S+\widehat{\theta}_S|\theta_M,\theta_L)[Var(\theta_S+\widehat{\theta}_S|\theta_M,\theta_L)]^{-1}Cov(\theta_S+\widehat{\theta}_S,u|\theta_M,\theta_L),$$

and so

$$(\widehat{A}_v^{(alt)})^T\widehat{\Lambda}^{(alt)}\widehat{A}_u^{(alt)} - Cov(u,v|\theta_M,\theta_L) = -Cov(u,v|\theta_M,\theta_L,\theta_S+\widehat{\theta}_S).$$

Therefore, we have

$$\begin{aligned}b^{(m)} \to b^{(\infty)} &= 2(\widehat{A}_v^{(\infty)})^T\widehat{\Sigma}_{diag}^{(\infty)}\widehat{A}_u^{(\infty)} + \left((\widehat{A}_v^{(\infty)})^T\widehat{\Lambda}^{(\infty)}\widehat{A}_u^{(\infty)} - Cov(u,v|\theta_M)\right) \\ &= 2(\widehat{A}_v^{(alt)})^T\widehat{\Sigma}_{diag}^{(alt)}\widehat{A}_u^{(alt)} - Cov(u,v|\theta_M,\theta_L,\theta_S+\widehat{\theta}_S) \\ &= b^{(alt)}.\end{aligned}$$

Step 2(c). Finally, we show that $c^{(m)} \to c^{(alt)}$. We have

$$c^{(m)} \to c^{(\infty)} := Var((\widehat{A}_u^{(\infty)})^T\theta - u|\theta_M)$$

and

$$c^{(alt)} = Var((\widehat{A}_u^{(alt)})^T\theta_S - u|\theta_M, \theta_L).$$

Let random variable χ be the residual from the projection of u on $(\theta_S + \widehat{\theta}_S; \theta_L; \theta_M)$. By construction, χ is orthogonal to θ_L and θ_M and thus, by the properties of the normal distribution, is independent of those two random variables. Recall that $\widehat{\theta}_S$ was also chosen to be independent of θ_L and θ_M.

Next,

$$\begin{aligned} Var((\widehat{A}_u^{(\infty)})^T\theta - u|\theta_M) &= Var\left(u - (\widehat{A}_u^{(\infty)})^T(\theta_S + \widehat{\theta}_S; \theta_L) + (\widehat{A}_u^{(\infty)})^T(\widehat{\theta}_S; 0)|\theta_M\right) \\ &= Var\left(\chi + (\widehat{A}_u^{(\infty)})^T(\widehat{\theta}_S; 0)|\theta_M\right) \end{aligned}$$

and

$$\begin{aligned} Var((\widehat{A}_u^{(alt)})^T\theta_S - u|\theta_M, \theta_L) &= Var\left(u - (\widehat{A}_u^{(alt)})^T(\theta_S + \widehat{\theta}_S) + (\widehat{A}_u^{(alt)})^T\widehat{\theta}_S|\theta_M, \theta_L\right) \\ &= Var\left(\chi + (\widehat{A}_u^{(alt)})^T\widehat{\theta}_S|\theta_M, \theta_L\right). \end{aligned}$$

Since χ and $\widehat{\theta}_S$ are both independent of θ_M and θ_L, we have

$$Var\left(\chi + (\widehat{A}_u^{(\infty)})^T(\widehat{\theta}_S; 0)|\theta_M\right) = Var\left(\chi + (\widehat{A}_u^{(\infty)})^T(\widehat{\theta}_S; 0)\right)$$

and

$$Var\left(\chi + (\widehat{A}_u^{(alt)})^T\widehat{\theta}_S|\theta_M, \theta_L\right) = Var\left(\chi + (\widehat{A}_u^{(alt)})^T\widehat{\theta}_S\right).$$

Take any realizations $\widetilde{\chi}$ and $\widetilde{\theta}_S$ of random variables χ and $\widehat{\theta}_S$. From equations (22) and (23) in Step 2(b), we have

$$\begin{aligned} \widetilde{\chi} + (\widehat{A}_u^{(\infty)})^T(\widetilde{\theta}_S; 0) &= \widetilde{\chi} + E[u|\theta_M = 0, \theta_S + \widehat{\theta}_S = \widetilde{\theta}_S, \theta_L = 0] \\ &= \widetilde{\chi} + (\widehat{A}_u^{(alt)})^T\widetilde{\theta}_S, \end{aligned}$$

and so

$$Var\left(\chi + (\widehat{A}_u^{(\infty)})^T(\widehat{\theta}_S; 0)\right) = Var\left(\chi + (\widehat{A}_u^{(alt)})^T\widehat{\theta}_S\right)$$

and thus

$$c^{(m)} \to c^{(\infty)} = c^{(alt)}.$$

Step 3. We now show that $\beta_M^{(m)} \to \beta_{M,M}^{(alt)}$, $\beta_D^{(m)}\alpha_L^{(m)} \to \beta_{M,L}^{(alt)}$, and $\beta_D^{(m)}\alpha_S^{(m)} \to \beta_D^{(alt)}\alpha^{(alt)}$. The arguments below rely on Assumption 2H, which implies that various conditional expectations that we compute below are guaranteed to be well-defined. They also rely on the result we showed in the previous step, $\beta_D^{(\infty)} = \beta_D^{(alt)}$.

First, note that for any $\widetilde{\theta}_S$, $(\alpha_S^{(m)})^T\widetilde{\theta}_S = (\alpha^{(m)})^T(\widetilde{\theta}_S;0)$, and so

$$\begin{aligned}\lim_{m\to\infty}\beta_D^{(m)}(\alpha^{(m)})^T(\widetilde{\theta}_S;0) &= \beta_D^{(\infty)}\left((\widehat{A}_v^{(\infty)})^T/\beta_D^{(\infty)} - (\widehat{A}_u^{(\infty)})^T\right)(\widetilde{\theta}_S;0)\\ &= E[v-\beta_D^{(\infty)}u|\theta_M=0,\theta_S+\widehat{\theta}_S=\widetilde{\theta}_S,\theta_L=0]\\ &= \beta_D^{(alt)}(\alpha_S^{(alt)})^T\widetilde{\theta}_S.\end{aligned}$$

Thus, $\beta_D^{(m)}\alpha_S^{(m)} \to \beta_D^{(alt)}\alpha^{(alt)}$.

Next, we have

$$\beta_M^{(m)} \to \beta_M^{(\infty)} := \Sigma_{MM}^{-1}\left(\Sigma_{Mv} - \Sigma_{\theta M}^T A_v^{(\infty)}\right) - \beta_D^{(\infty)}\Sigma_{MM}^{-1}\left(\Sigma_{Mu} - \Sigma_{\theta M}^T A_u^{(\infty)}\right),$$

and so for any $\widetilde{\theta}_M$, we have

$$(\beta_M^{(\infty)})^T\widetilde{\theta}_M = E[v-\beta_D^{(\infty)}u|\theta_M=\widetilde{\theta}_M,\theta_S+\widehat{\theta}_S=0,\theta_L=0].$$

Also, similarly to the above expression for $\beta_D^{(\infty)}\alpha_S^{(\infty)}$, for any $\widetilde{\theta}_L$, we have

$$\beta_D^{(\infty)}(\alpha_L^{(\infty)})^T\widetilde{\theta}_L = E[v-\beta_D^{(\infty)}u|\theta_M=0,\theta_S+\widehat{\theta}_S=0,\theta_L=\widetilde{\theta}_L].$$

Thus,

$$(\beta_M^{(\infty)};\beta_D^{(\infty)}\alpha_L^{(\infty)})^T(\widetilde{\theta}_M;\widetilde{\theta}_L) = E[v-\beta_D^{(\infty)}u|\theta_M=\widetilde{\theta}_M,\theta_S+\widehat{\theta}_S=0,\theta_L=\widetilde{\theta}_L].$$

Analogously to the expression for $(\beta_M^{(\infty)})^T\widetilde{\theta}_M$, we also have

$$\begin{aligned}(\beta_M^{(alt)})^T(\widetilde{\theta}_M;\widetilde{\theta}_L) &= E[v-\beta_D^{(alt)}u|\theta_S+\widehat{\theta}_S=0,(\theta_M;\theta_L)=(\widetilde{\theta}_M;\widetilde{\theta}_L)]\\ &= (\beta_M^{(\infty)};\beta_D^{(\infty)}\alpha_L^{(\infty)})^T(\widetilde{\theta}_M;\widetilde{\theta}_L).\end{aligned}$$

Thus, $\beta_M^{(m)} \to \beta_{M,M}^{(alt)}$ and $\beta_D^{(m)}\alpha_L^{(m)} \to \beta_{M,L}^{(alt)}$, and combining all the convergence results above and using the same argument as in Step 3 of the proof of the special case of Theorem 2 in Appendix B, we conclude the proof of Theorem 3.

Appendix D: Proofs of Propositions in Section 8

D.1 Proof of Proposition 1

Fix a firm i, and suppose every firm $j \neq i$ plays according to a linear strategy

$$q_j(\theta_j) = \alpha_j^T\theta_j + \delta_j.$$

Suppose firm i observes realization $\widetilde{\theta}_i$ of signal θ_i. The expected payoff of firm i from producing q units of the good is then equal to

$$E\left[q\left(v-\beta\left(q+\sum_{j\neq i}\left(\alpha_j^T\theta_j+\delta_j\right)\right)-c\right)\middle|\theta_i=\widetilde{\theta}_i\right],$$

which can be rewritten as

$$q\cdot E\left[v-\beta\left(\sum_{j\neq i}\left(\alpha_j^T\theta_j+\delta_j\right)\right)-c\,\middle|\,\theta_i=\widetilde{\theta}_i\right]-q^2\cdot\beta.$$

Since by assumption $\beta > 0$, there is a unique q maximizing the expected profit:

$$q^* = \frac{1}{2\beta} E\left[v - \beta \left(\sum_{j \neq i} \left(\alpha_j^T \theta_j + \delta_j \right) \right) - c \,\middle|\, \theta_i = \widetilde{\theta}_i \right] \tag{24}$$

$$= \frac{1}{2\beta} \left(\bar{v} + \Sigma_{iv}^T \Sigma_{ii}^{-1} \theta_i - \beta \left(\sum_{j \neq i} \left(\alpha_j^T \Sigma_{ij}^T \Sigma_{ii}^{-1} \theta_i + \delta_j \right) \right) - c \right), \tag{25}$$

where in the second line we re-use our earlier notation for various covariance matrices.

Thus, if all firms other than i use strategies linear in their signals, firm i's (unique) best response strategy is also linear in its signal. Moreover, the intercept and the slope of that strategy are uniquely determined. Specifically, for the intercept we have

$$\delta_i = \frac{1}{2\beta} \left(\bar{v} - \beta \sum_{j \neq i} \delta_j - c \right), \tag{26}$$

and for the slope we have

$$\alpha_i^T = \frac{1}{2\beta} \left(\Sigma_{iv}^T \Sigma_{ii}^{-1} - \beta \sum_{j \neq i} \alpha_j^T \Sigma_{ij}^T \Sigma_{ii}^{-1} \right). \tag{27}$$

For the intercepts, multiplying both sides of equation (26) by 2β and moving one of the $\beta\delta_i$ terms under the summation sign, we get

$$\beta\delta_i = \bar{v} - \beta \sum_{j=1}^{n} \delta_j - c,$$

and so all δ_i are equal, and are pinned down by the formula

$$\delta_i = \frac{\bar{v} - c}{\beta(n+1)}.$$

For the slopes α_i, we follow manipulations analogous to those in the proof of Theorem 1: multiply both sides of equation (27) by $2\beta\Sigma_{ii}$ (on the right), move one of the $\beta\alpha_i^T\Sigma_{ii}$ terms under the summation sign, transpose the equation, and "stack" the resulting equations for all i on top of each other. The resulting system of equation can be rewritten as

$$\beta\Sigma_{diag}\alpha = \Sigma_{iv} - \beta\Sigma_{\theta\theta}\alpha,$$

and so the vector of slopes α is pinned down by the formula

$$\alpha = \frac{1}{\beta} \left(\Sigma_{\theta\theta} + \Sigma_{diag} \right)^{-1} \Sigma_{iv},$$

because our assumptions imply that matrix $(\Sigma_{\theta\theta} + \Sigma_{diag})$ is invertible.

D.2 Proof of Proposition 2

Consider first the original sequence of markets and fix a particular market m—though for simplicity, we will drop the superscript (m) until further notice. By Proposition 1, there exists a unique linear equilibrium. To explicitly characterize this equilibrium, we use the arguments and the notation almost identical to those in Step 1 of the proof of Theorem 2 in Appendix B.

Specifically, by symmetry, any two firms in the same group use the same linear strategy in equilibrium. Denote by α_i the *aggregate* supply multiplier of group i, and by δ_i the *aggregate* intercept of group i. Thus, a specific firm j in group i, after observing its signal $\theta_i + \xi_{i,j}$, will produce quantity

$$\frac{1}{\ell_i}\alpha_i^T(\theta_i + \xi_{i,j}) + \frac{1}{\ell_i}\delta_i.$$

As in the proof of Theorem 2, let $\xi_i = \frac{1}{\ell_i}\sum_j \xi_{i,j}$ (the average idiosyncratic term in group i), let $\xi = (\xi_1; \ldots; \xi_n)$, and define matrices Σ^ξ and $\widehat{\Sigma}_{diag}$ as

$$\Sigma^\xi = Var(\xi) = \begin{pmatrix} \frac{1}{\ell_1}\Sigma_1^\xi & 0 & \ldots & \ldots \\ 0 & \frac{1}{\ell_2}\Sigma_2^\xi & 0 & \ldots \\ \vdots & 0 & \ddots & 0 \\ 0 & \ldots & 0 & \frac{1}{\ell_n}\Sigma_n^\xi \end{pmatrix}$$

and

$$\widehat{\Sigma}_{diag} = \begin{pmatrix} \frac{1}{\ell_1}(\Sigma_{11} + \Sigma_1^\xi) & 0 & \ldots & \ldots \\ 0 & \frac{1}{\ell_2}(\Sigma_{22} + \Sigma_2^\xi) & 0 & \ldots \\ \vdots & 0 & \ddots & 0 \\ 0 & \ldots & 0 & \frac{1}{\ell_n}(\Sigma_{nn} + \Sigma_n^\xi) \end{pmatrix}.$$

It is immediate from Proposition 1 that $\delta_i = \frac{\ell_i(\bar{v}-c)}{\beta(N+1)}$. For vector α, writing down the first-order conditions for all firms j of all groups i, and combining them in the same way as in Step 1 of the proof of Theorem 2 in Appendix B, we get the expression

$$\left(\widehat{\Sigma}_{diag} + \Sigma_{\theta\theta} + \Sigma^\xi\right)\alpha = \beta^{-1}\Sigma_{\theta v}. \tag{28}$$

(Note that equation (28) is almost identical to equation (19) in Step 1 of the proof of Theorem 2, except that the latter also contains the terms related to the signal observed by the market maker and the demand from liquidity traders. Since in the current model we do not have either, equation (28) does not include these terms.)

Let us now again write the market indices explicitly, so that

$$\alpha^{(m)} = \beta^{-1}\left(\Sigma_{\theta\theta} + \widehat{\Sigma}_{diag}^{(m)} + (\Sigma^\xi)^{(m)}\right)^{-1}\Sigma_{\theta v},$$

and

$$\delta^{(m)} = \frac{\ell_i\,(\bar{v} - c)}{\beta\left(N^{(m)} + 1\right)}.$$

The total quantity produced in market m, as a function of θ and $\xi^{(m)}$, is then

$$\begin{aligned} Q^{(m)} &= \beta^{-1}\left(\left(\Sigma_{\theta\theta}+\widehat{\Sigma}^{(m)}_{diag}+(\Sigma^{\xi})^{(m)}\right)^{-1}\Sigma_{\theta v}\right)^{T}\theta \\ &+ \beta^{-1}\left(\left(\Sigma_{\theta\theta}+\widehat{\Sigma}^{(m)}_{diag}+(\Sigma^{\xi})^{(m)}\right)^{-1}\Sigma_{\theta v}\right)^{T}\xi^{(m)} \\ &+ \frac{N^{(m)}(\bar{v}-c)}{\beta\left(N^{(m)}+1\right)}. \end{aligned}$$

As m goes to infinity, $\frac{N^{(m)}(\bar{v}-c)}{\beta\left(N^{(m)}+1\right)}$ converges to $\beta^{-1}(\bar{v}-c)$, and matrices $\widehat{\Sigma}^{(m)}_{diag}$ and $(\Sigma^{\xi})^{(m)}$ converge to zero. Moreover, $\xi^{(m)} \xrightarrow{L^2} 0$. Thus, as m goes to infinity,

$$\begin{aligned} Q^{(m)} &\xrightarrow{L^2} \beta^{-1}\left(\left(\Sigma_{\theta\theta}^{-1}\Sigma_{\theta v}\right)^{T}\theta+(\bar{v}-c)\right) \\ &= \beta^{-1}\left(E[v|\theta]-c\right). \end{aligned}$$

For the alternative sequence of markets, note that each alternative market m can be viewed as a special case of the "original" market, with just one group $i=1$, and with no idiosyncratic components of signals within the group (i.e., $\Sigma_i^{\xi}=0$). Thus, the above derivation applies to this special case, and so for the alternative sequence, we also have

$$Q^{(alt,m)} \xrightarrow{L^2} \beta^{-1}\left(E[v|\theta]-c\right),$$

and so

$$Q^{(m)} - Q^{(alt,m)} \xrightarrow{L^2} 0.$$

Moreover, since $p^{(m)} = v - \beta Q^{(m)}$ and $p^{(alt,m)} = v - \beta Q^{(alt,m)}$, we immediately get

$$p^{(m)} - p^{(alt,m)} \xrightarrow{L^2} 0.$$

References

Admati, A. R. and P. Pfleiderer (1988). A theory of intraday patterns: Volume and price variability. *Review of Financial studies 1*(1), 3–40.

Allen, B. E. (1981). Generic existence of completely revealing equilibria for economies with uncertainty when prices convey information. *Econometrica 49*(5), 1173–1199.

Allen, B. E. and J. S. Jordan (1998). The existence of rational expectations equilibrium: A retrospective. In M. Majumdar and R. Radner (Eds.), *Organizations with Incomplete Information*, pp. 42–60. Cambridge: Cambridge University Press.

Anderson, R. M. and H. Sonnenschein (1982). On the existence of rational expectations equilibrium. *Journal of Economic Theory 26*(2), 261–278.

Babus, A. and P. Kondor (2013). Trading and information diffusion in over-the-counter markets. Working paper.

Back, K. (1992). Insider trading in continuous time. *Review of Financial Studies 5*(3), 387–409.

Back, K., C. H. Cao, and G. A. Willard (2000). Imperfect competition among informed traders. *Journal of Finance 55*(5), 2117–2155.

Bagnoli, M., S. Viswanathan, and C. Holden (2001). On the existence of linear equilibria in models of market making. *Mathematical Finance 11*(1), 1–31.

Bergemann, D., T. Heumann, and S. Morris (2015). Information and market power. Working paper.

Bernhardt, D. and J. Miao (2004). Informed trading when information becomes stale. *Journal of Finance 59*(1), 339–390.

Biais, B., D. Martimort, and J.-C. Rochet (2000). Competing mechanisms in a common value environment. *Econometrica 68*(4), 799–837.

Biais, B., D. Martimort, and J.-C. Rochet (2013). Corrigendum to "Competing mechanisms in a common value environment". *Econometrica 81*(1), 393–406.

Brunnermeier, M. K. (2005). Information leakage and market efficiency. *Review of Financial Studies 18*, 417–457.

Caballé, J. and M. Krishnan (1994). Imperfect competition in a multi-security market with risk neutrality. *Econometrica 62*(3), 695–704.

Colla, P. and A. Mele (2010). Information linkages and correlated trading. *Review of Financial Studies 23*(1), 203–246.

Dridi, R. and L. Germain (2009). Noise and competition in strategic oligopoly. *Journal of Financial Intermediation 18*(2), 311 – 327.

Dubey, P., J. Geanakoplos, and M. Shubik (1987). The revelation of information in strategic market games. *Journal of Mathematical Economics 16*(2), 105–137.

Foster, F. D. and S. Viswanathan (1994). Strategic trading with asymmetrically informed traders and long-lived information. *Journal of Financial and Quantitative Analysis 29*, 499–518.

Foster, F. D. and S. Viswanathan (1996). Strategic trading when agents forecast the forecasts of others. *Journal of Finance 51*(4), 1437–1478.

Foucault, T. and L. Lescourret (2003). Information sharing, liquidity and transaction costs. Mimeo.

Glycopantis, D. and N. C. Yannelis (2005). *Differential Information Economies.* Springer.

Golosov, M., G. Lorenzoni, and A. Tsyvinski (2014). Decentralized trading with private information. *Econometrica 82*(3), 1055–1091.

Grossman, S. (1976). On the efficiency of competitive stock markets where traders have diverse information. *Journal of Finance 31*(2), 573–585.

Hayek, F. A. (1945). The use of knowledge in society. *American Economic Review 35*(4), 519–530.

Hellwig, M. F. (1980). On the aggregation of information in competitive markets. *Journal of Economic Theory 22*(3), 477–498.

Hellwig, M. F. (1982). Rational expectations equilibrium with conditioning on past prices: A mean-variance example. *Journal of Economic Theory 26*(2), 279–312.

Holden, C. W. and A. Subrahmanyam (1992). Long-lived private information and imperfect competition. *The Journal of Finance 47*(1), 247–270.

Jain, N. and L. J. Mirman (1999). Insider trading with correlated signals. *Economics Letters 65*(1), 105–113.

Jordan, J. S. and R. Radner (1982). Rational expectations in microeconomic models: An overview. *Journal of Economic Theory 26*(2), 201–223.

Kremer, I. (2002). Information aggregation in common value auctions. *Econometrica 70*(4), 1675–1682.

Kreps, D. M. (1977). A note on fulfilled expectations equilibria. *Journal of Economic Theory 14*(1), 32–43.

Kyle, A. S. (1985). Continuous auctions and insider trading. *Econometrica 53*(6), 1315–1336.

Kyle, A. S. (1989). Informed speculation with imperfect competition. *Review of Economic Studies 56*(3), 317–355.

Li, L. (1985). Cournot oligopoly with information sharing. *RAND Journal of Economics 16*(4), 521–536.

Luo, S. (2001). The impact of public information on insider trading. *Economics Letters 70*(1), 59–68.

Madrigal, V. (1996). Non-fundamental speculation. *Journal of Finance 51*(2), 553–578.

McLean, R., J. Peck, and A. Postlewaite (2005). On price-taking behavior in asymmetric information economies. *Essays in Dynamic General Equilibrium Theory: Festschrift for David Cass, Edited by A. Citanna, J. Donaldson, H. Polemarchkis, P. Siconolfi, and S. Spear, Springer Verlag*, 129–142.

McLean, R. and A. Postlewaite (2002). Informational size and incentive compatibility. *Econometrica 70*(6), 2421–2453.

Milgrom, P. R. (1981). Rational expectations, information acquisition, and competitive bidding. *Econometrica 49*(4), 921–943.

Nöldeke, G. and T. Tröger (2001). Existence of linear equilibria in the Kyle model with multiple informed traders. *Economics Letters 72*(2), 159 – 164.

Nöldeke, G. and T. Tröger (2006). A characterization of the distributions that imply existence of linear equilibria in the Kyle-model. *Annals of Finance 2*(1), 73 – 85.

Ostrovsky, M. (2012). Information aggregation in dynamic markets with strategic traders. *Econometrica 80*(6), 2595–2647.

Palfrey, T. R. (1985). Uncertainty resolution, private information aggregation and the cournot competitive limit. *The Review of Economic Studies 52*(1), 69–83.

Pasquariello, P. (2007). Imperfect competition, information heterogeneity, and financial contagion. *Review of Financial Studies 20*(2), 391–426.

Pesendorfer, W. and J. M. Swinkels (1997). The loser's curse and information aggregation in common value auctions. *Econometrica 65*(6), 1247–1282.

Radner, R. (1979). Rational expectations equilibrium: Generic existence and the information revealed by prices. *Econometrica 47*(3), 655–678.

Reny, P. J. and M. Perry (2006). Toward a strategic foundation for rational expectations equilibrium. *Econometrica 74*(5), 1231–1269.

Rochet, J.-C. and J.-L. Vila (1994). Insider trading without normality. *Review of Economic Studies 61*(1), 131–152.

Röell, A. (1990). Dual-capacity trading and the quality of the market. *Journal of Financial Intermediation 1*(2), 105–124.

Rostek, M. and M. Weretka (2012). Price inference in small markets. *Econometrica 80*(2), 687–711.

Rostek, M. and J. H. Yoon (2014). Inference design. Working paper.

Sadzik, T. and C. Woolnough (2015). Multidimensional uncertainty and hypersensitive asset prices. Working paper.

Sarkar, A. (1995). Dual trading: Winners, losers, and market impact. *Journal of Financial Intermediation 4*(1), 77–93.

Vives, X. (1988). Aggregation of information in large cournot markets. *Econometrica 56*(4), 851–876.

Vives, X. (2008). *Information and Learning in Markets: The Impact of Market Microstructure.* Princeton, NJ: Princeton University Press.

Wilson, R. (1977). A bidding model of perfect competition. *Review of Economic Studies 44*(3), 511–518.

Wolinsky, A. (1990). Information revelation in a market with pairwise meetings. *Econometrica 58*(1), 1–23.