Web Appendix to
Adoption of Standards under Uncertainty

Michael Ostrovsky Michael Schwarz

January 30, 2004

In this appendix we explore some parallels between our results and some recent results on global games.

Frankel, Morris, and Pauzner (2003) show that in global games, different equilibria may be pinned down by vanishingly small noise. They also show that a sufficient condition for an equilibrium in a global game to be robust to the structure of noise is to be a *weighted potential maximizer*, provided that the payoffs are *own-action quasiconcave*. These concepts are defined in Section 6 of FMP as follows.

Definition *A complete information game g is own-action quasiconcave if for all i and opposing action profiles $a_{-i} \in A_{-i}$ and for all constants c, the set $\{a_i : g_i(a_i, a_{-i}) \geq c\}$ is convex.*

Definition *Action profile a^* is a weighted potential maximizer (WP-maximizer) of g if there exists a vector $\xi \in \mathbb{R}^I_+$ and a weighted potential function $v : A \rightarrow \mathbb{R}$ with $v(a^*) > v(a)$ for all $a \neq a^*$, such that for all i, a_i, $a'_i \in A_i$ and $a_{-i} \in A_{-i}$,*

$$v(a_i, a_{-i}) - v(a'_i, a_{-i}) = \xi_i[g_i(a_i, a_{-i}) - g_i(a'_i, a_{-i})].$$

The results of Frankel, Morris, and Pauzner (2002) have parallels in our setting. Namely, the games presented herein indeed have weighted potential maximizers, and are own-action quasiconcave. Changing our setting in such a way that the game no longer has a potential leads to the dependence of equilibrium on the structure of noise.

We focus our attention on the case with no discounting; the results do not change if we consider $\beta < 1$, but presentation gets more complicated.

1 Potentiality of "Noiseless" $\Gamma(1)$

Consider a "noiseless" version of game $\Gamma(1)$, where $t_i = \mu_i$.

$$\begin{aligned}\Pi_i(t_i) &= -c_i(t_* - t_i) - d_i t_* \\ \Pi_i'(t_i) &= c_i - (c_i + d_i)\chi(\text{player } i \text{ is last}),\end{aligned}$$

and therefore payoffs Π_i are own-action quasiconcave.

To show that this is also a weighted potential game, let $v(t) = \sum s_i t_i - t_*$, where s_i is the support ratio of player i. Let $\xi_i = \frac{1}{c_i+d_i}$. Then

$$\begin{aligned}v(t_i, t_{-i}) - v(t_i', t_{-i}) &= [s_i t_i - t_*] - [s_i t_i' - t_*'] \\ &= \frac{[c_i t_i - (c_i + d_i)t_*] - [c_i t_i' - (c_i + d_i)t_*']}{c_i + d_i} \\ &= \xi_i \left(\Pi_i(t_i, t_{-i}) - \Pi_i(t_i', t_{-i})\right).\end{aligned}$$

Thus, $v(t) = [\sum s_i t_i - t_*]$ is a weighted potential function of "noiseless" $\Gamma(1)$. If $\sum s_i < 1$, this function is maximized at a certain value of t (since we assume that target arrival times are bounded from below), and "noisy" $\Gamma(1)$ has a unique equilibrium with adoption. When $\sum s_i > 1$, v is unbounded (adding the same constant τ to all t_i increases v by $(\sum s_i - 1)\,\tau$), and $\Gamma(1)$ has no equilibrium with adoption. When $\sum s_i = 1$, there is a continuum of values of t maximizing function v (since adding the same constant to all target arrival times t_i leaves function v unchanged), and $\Gamma(1)$ has a continuum of equilibria with adoption.

2 Potentiality of the Adoption Game with Gradual Network Externalities and Identical Players

We now show that the game with identical players and gradual network externalities also has a potential function (the proof of its own-action quasiconcavity is very similar to the proof of $\Gamma(1)$'s own-action quasiconcavity, and is thus omitted). Namely, let $v(t) = -[d(1)t^1 + d(2)t^2 + \cdots + d(N)t^N]$, where t^1 is the actual compliance time of the earliest adopter, t^2 is the actual compliance time of the next adopter, and so on. Then

$$\begin{aligned}\frac{\partial v}{\partial t_i} = & -d(1)\chi(\text{player } i \text{ is first}) \\ & -d(2)\chi(\text{player } i \text{ is second}) \\ & -\cdots \\ & -d(N)\chi(\text{player } i \text{ is last}).\end{aligned}$$

The expected net benefit of player i from delaying his compliance time by an infinitesimal amount of time, $\frac{\partial Pi_i}{\partial t_i}$, is also equal to $-d(1)\chi(\text{player } i \text{ is first})-d(2)\chi(\text{player } i \text{ is second}) - \cdots - d(N)\chi(\text{player } i \text{ is last})$, and so $v(t_i, t_{-i}) - v(t_i', t_{-i}) = \Pi_i(t_i, t_{-i}) - \Pi_i(t_i', t_{-i})$. Therefore, v is a potential function of the game with gradual network externalities and identical players. Just like in the game $\Gamma(1)$, one can verify that this potential function has a (unique) maximizer if and only if the corresponding game has a (unique) equilibrium where players adopt the standard.

3 The Game with Gradual Network Externalities and Different Players

A general game with gradual network externalities and different players no longer has a weighted potential function (this can be checked by comparing cross-derivatives $\frac{\partial \Pi_i}{\partial \mu_j}$), and so we do not necessarily expect equilibria to be robust to the form of noise. Indeed, the following counterexample presents a game with gradual network externalities and different payoffs, in which the existence of equilibrium depends on the structure of noise.

There are three players. The net payoff of player i when he arrives kth is equal to $d_i(k)$, given in the table below.

$i \setminus k$	1	2	3
1	-1	0	1
2	-1	0	1
3	-1	-1	2

In the first case, suppose that each player's noise is distributed uniformly on $[0, \epsilon]$. Then it is an equilibrium for all players to target the same arrival time—the expected marginal benefit from deviating by an infinitesimal amount (we will call this "the expected first-day payoff") is zero for each player (proportional to $(-1 + 0 + 1)/3 = 0$ for players 1 and 2 and to $(-1 - 1 + 2)/3 = 0$ for player 3).

In the second case, consider the following form of noise. Players 1 and 2 either arrive very early ($[0, \epsilon]$), or very late ($[100\epsilon, 101\epsilon]$), with equal probabilities. Player 3's noise, on the other hand, is still uniform on $[0, \epsilon]$. Then this game has no equilibrium.

Indeed, suppose there is an equilibrium. It has to be in pure strategies, since, holding other players' strategies constant, a player's expected payoff is concave in his target arrival time. If players 1 and 2 target the same compliance time, pick any one of them; otherwise, pick the one who targets the earlier time. Without loss of generality, assume player 1 is picked. Normalize his earliest possible arrival compliance time to 0. Then it cannot be the case that player 3 ever arrives later than 100ϵ (because that would imply that player 3's target compliance time is greater than 99ϵ and he never arrives before ϵ. That, in turn, would imply that player 1 arrives first more often than he arrives last, making his expected first-day payoff negative, which is impossible in equilibrium). Thus, player 3 always arrives before time 100ϵ. Therefore, for him to be the last one it has to be the case that both players 1 and 2 comply in the earlier of the two intervals, which happens with probability .25. But then player 3's first-day payoff is no more than $.25*2-.75*1 < 0$, which is impossible in equilibrium.

References

[1] Frankel, David, Stephen Morris, and Ady Pauzner (2003), Equilibrium Selection in Global Games with Strategic Complementarities, *Journal of Economic Theory,* 108, 1-44.