The Leverage Cycle

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Abstract

Equilibrium determines leverage, not just interest rates. Variations in leverage cause wild fluctuations in asset prices. This leverage cycle can be damaging to the economy, and should be regulated.

1 Introduction to the Leverage Cycle

At least since the time of Irving Fisher, economists, as well as the general public, have regarded the interest rate as the most important variable in the economy. But in times of crisis, collateral rates (equivalently margins or leverage) are far more important. Despite the cries of newspapers to lower the interest rates, the Fed would sometimes do much better to attend to the economy-wide leverage and leave the interest rate alone. The Fed ought to rethink its priorities.

When a homeowner (or hedge fund or a big investment bank) takes out a loan using say a house as collateral, he must negotiate not just the interest rate, but how much he can borrow. If the house costs \$100 and he borrows \$80 and pays \$20 in cash, we say that the margin or haircut is 20%, and the loan to value is \$80/\$100 = 80%. The leverage is the reciprocal of the margin, namely the ratio of the asset value to the cash needed to purchase it, or \$100/\$20 = 5.

In standard economic theory, the equilibrium of supply and demand determines the interest rate on loans. It would seem impossible that one equation could determine two variables, the interest rate and the margin. But in my theory, supply and demand do determine both the equilibrium leverage (or margin) and the interest rate.

It is apparent from everyday life that the laws of supply and demand can determine both the interest rate and leverage of a loan: the more impatient borrowers are, the higher the interest rate; the more nervous the lenders become, the higher the collateral they demand. But standard economic theory fails to properly capture these effects, struggling to see how a single supply-equals-demand equation for a loan could determine two variables: the interest rate and the leverage. The theory typically ignores the possibility of default (and thus the need for collateral), or else fixes the leverage as a constant, allowing the equation to predict the interest rate.

Yet variation in leverage has a huge impact on the price of assets, contributing to economic bubbles and busts. This is because for many assets there is a class of buyer for whom the asset is more valuable than it is for the rest of the public (standard economic theory, in contrast, assumes that asset prices reflect some fundamental value). These buyers are willing to pay more, perhaps because they are more sophisticated and know better how to hedge their exposure to the assets, or they are more risktolerant, or they simply like the assets more. If they can get their hands on more money through more highly leveraged borrowing (that is, getting a loan with less collateral), they will spend it on the assets and drive those prices up. If they lose wealth, or lose the ability to borrow, they will buy less, so the asset will fall into more pessimistic hands and be valued less.

In the absence of intervention, leverage becomes too high in boom times, and too low in bad times. As a result, in boom times asset prices are too high, and in crisis times they are too low. This is the leverage cycle.

Leverage dramatically increased in the United States from 1999 to 2006. A bank that in 2006 wanted to buy a AAA-rated mortgage security could borrow 98.4% of the purchase price, using the security as collateral, and pay only 1.6% in cash. The leverage was thus 100 to 1.6, or about 60 to 1. The average leverage in 2006 across all of the US$2.5 trillion of so-called 'toxic' mortgage securities was about 16 to 1, meaning that the buyers paid down only $150 billion and borrowed the other $2.35 trillion. Home buyers could get a mortgage leveraged 20 to 1, a 5% down payment. Security and house prices soared.

Today leverage has been drastically curtailed by nervous lenders wanting more collateral for every dollar loaned. Those toxic mortgage securities are now leveraged on average only about 1.5 to 1. Home buyers can now only leverage themselves 5 to 1 if they can get a government loan, and less if they need a private loan. De-leveraging is the main reason the prices of both securities and homes are still falling.

The leverage cycle is a recurring phenomenon. The financial derivatives crisis in 1994 that bankrupted Orange County in California was the tail end of a leverage cycle. So was the emerging markets mortgage crisis of 1998, which brought the Connecticut-based hedge fund Long-Term Capital Management to its knees, prompting an emergency rescue by other financial institutions.

In ebullient times competition drives leverage higher and higher. An investor comes to a hedge fund and says the fund down the block is getting higher returns. The fund manager says the other guy is just leveraging more. The investor responds, well whatever he's doing, he's getting higher returns. Pretty soon both funds are leveraging more. During a crisis, leverage can fall by 50% overnight, and by more over a few days or months. A homeowner who bought his house last year by taking out a subprime mortgage with only 5% down cannot take out a similar loan today without putting down 25% (unless he qualifies for one of the government rescue programs). The odds are great that he wouldn't have the cash to do it, and reducing the interest rate by 1 or 2% won't change his ability to act.

The policy implication of my theory of equilibrium leverage is that the fed should manage system wide leverage, curtailing leverage in normal or ebullient times, and propping up leverage in anxious times.

I fully agree that if agents extrapolate blindly, assuming from past rising prices that they can safely set very small margin requirements, or that falling prices means that it is necessary to demand absurd collateral levels, then the cycle will get much worse. But a crucial part of my leverage cycle story is that every agent is acting perfectly rationally from his own individual point of view. People are not deceived into following illusory trends. They do not ignore danger signs. They do not panic. They look forward, not backward. But under certain circumstances the cycle spirals into a crash anyway. The lesson is that even if people remember this leverage cycle, there will be more leverage cycles in the future, unless the Fed acts to stop them.

The crash always involves the same three elements. First is scary bad news that increases uncertainty. This leads to tighter margins as lenders get more nervous. This in turn leads to falling prices and huge losses by the most optimistic, leveraged buyers. All three elements feed back on each other; the redistribution of wealth from optimists to pessimists further erodes prices, causing more losses for optimists, and steeper price declines, which rational lenders anticipate, leading then to demand more collateral, and so on.

The best way to stop a crash is to prevent it from happening in the first place, by restricting leverage in ebullient times.

To reverse the crash once it has happened requires reversing the three causes. In today's environment, that means stopping foreclosures and the free fall of housing prices. As we shall see, the only reliable way to do that is to write down principal. Second, leverage must be restored to sane, intermediate levels. The fed must step around the banks and lend directly to investors, at more generous collateral levels than the private markets are willling to provide. And third, the Treasury must inject optimistic capital to make up for the lost buying power of the bankrupt leveraged optimists. This might also entail bailing out various crucial players.

My theory is of course not completely original. Over 400 years ago Shakespeare explained that to take out a loan one had to negotiate both the interest rate and the collateral level. It is clear which of the two Shakespeare thought was the more important. Who can remember the interest rate Shylock charged Antonio? But everybody remembers the pound of flesh that Shylock and Antonio agreed on as collateral. The upshot of the play, moreover, is that the regulatory authority (the court) decides that the collateral level Shylock and Antonio agreed upon was socially suboptimal, and the court decrees a different collateral level. The Fed too should sometimes decree different collateral levels.

In more recent times there has been pioneering work on collateral by Bernanke, Gertler, Gilchrist, Kiyotaki and Moore. This work emphasized the feedback from the fall in collateral prices to a fall in borrowing capacity, assuming a constant loan to value ratio. By contrast, my work defining collateral equilibrium, which was first published in 1997, contemporaneously with Kiyotaki and Moore, focused on what determines the *ratios* (LTV, margin, or leverage) and why they change. In practice, I believe the change in ratios has been far bigger and more important than the change in levels. In 2003 I published my leverage cycle theory on the anatomy of crashes and margins (it was an invited address at the 2002 World Econometric Society meetings), arguing that in normal times leverage gets too high, and in bad times leverage is too low. In 2008 I published a paper in the AER with Ana Fostel on leverage cycles and the anxious economy. There we noted that margins do not move in lock step across asset classes, ad that a leverage cycle in one asset class might spread to other unrelated asset classes. In a paper with Bill Zame we describe the general properties of collateral equilibrium. In joint work with Felix Kubler, we show that managing collateral levels can lead to pareto improvements.

1.1 Survey of Literature

In addition to previously mentioned articles, must mention Minsky moments, Kindleberger, Tobin on collateral, and more recently, Brunnermeier and Pedersen, Shin, Allen and Gale, Banerjee, Shleifer and Vishny.

1.2 Why was this leverage cycle worse than previous cycles?

There are a number of elements that played into the leverage cycle crisis of 2007-9 that had not appeared before, which explain why it has been so bad. I will gradually incorporate them into the model. The first I have already mentioned, namely that leverage got higher than ever before.

The second is the invention of the credit default swap. The buyer of "CDS insurance" gets a dollar for every dollar of defaulted principal on some bond. But he is not limited to buying as much insurance as he owns bonds. In fact, he very likely is buying the CDS nowadays because he thinks the bonds are bad and does not want to own them at all. CDS are, despite their names, not insurance at all, but a vehicle for pessimists to leverage their views. Conventional leverage allows optimists to push the price of assets unduly high; CDS allows pessimists to push asset prices unduly low. The standardization of CDS for mortgages in late 2005 led to their trades in large quantities in 2006 and 2007. This I believe was one of the precipitators of the downturn.

Third, this leverage cycle was really a combination of two leverage cyles, in mortage securities and in housing. The two reinforce each other. The tightening margins in securities led to lower security prices, which made it harder to issue new mortgages, which made it harder for homeowners to refinance, which made them more likely to default, which made securities riskier, which made their margins get tighter and so on.

Fourth, and perhaps most important, when promises exceed collateral values, as when housing is "under water" or upside down", there are typically large losses in turning over the collateral. Today subprime bondholders expect only 25% of the loan amount back when they foreclose on a home. A huge number of homes are expected to be foreclosed (some say 8 million). The point will be that even if borrowers and lenders foresee that the loan amount is so large then there will be states in which the collateral is under water, and therefore will cause deadweight losses, they will not be able to prevent themselves from agreeing on such levels.

Fifth, the leverage cycle potentially has a major impact on productive activities. High asset prices means strong incentives for production, and a boon to real construction. The fall in asset prices has a blighting effect on new real activity. This is the essence of Tobin's Q. And it is the real reason why the crisis stage of the leverage cycle is so alarming.

1.3 Outline

I present the basic model of the leverage cycle drawing on my 2003 paper, in which a continuum of investors vary in their optimism. I explain how equilibrium can determine a unique leverage ratio, and I show how dynamic variations in equilibrium leverage arise from changing conditions, and in turn produce dramatic effects on asset prices. This is a context in which the leverage cycle is very easy to see. The welfare implications, on the other hand, do not seem so dire, because with complete markets one would also see large fluctuations in asset prices. I therefore move to a second model, drawn from my 1997 paper, in which probabilities are objectively given, and heterogeneity arises from differences in the utility for housing. Once again leverage is endogenously determined, but now the asset prices are much more volatile than they would be with complete markets, and the welfare situation is very different (though not necessarily Pareto worse). Along the way I add the particular elements mentioned above that explain why this cycle was so bad. The last section combines the two previous approaches to explain the double leverage cycle, in housing and in securities, which is an essential element of our current crisis.

1.4 Crises

Crises do not really occur just from coordination failures. There are no self confirming runs on the bank without some bad news. But not all bad news lead to crises, even when the news is very bad.

Bad news in my view must be of a special kind to cause an adverse move in the leverage cycle. The special bad news must not only lower expectations (as by definition all bad news does), but it must create more uncertainty, and more disagreement. On average news reduces uncertainty, so I have in mind a special, but by no means unusual, kind of news. The idea is that at the beginning, everyone thinks the chances of ultimate failure require too many things to go wrong to be of any substantial probability. There is little uncertainty, and therefore little room for disagreement. Once enough things go wrong to raise the spectre of real trouble, the uncertainty goes way up in everyone's mind, and so does the possibility of disagreement.

An example occurs when output is 1 unless two things go wrong, in which case output becomes .2. If an optimist thinks the chance of each thing going wrong is independent and equal to .1, then it is easy to see that he thinks the chance of ultimate breakdown is .01=(.1)(.1). Expected output for him is .992. In his view ex ante, the variance of final output is $.99(.01)(1-.2) = .0079$. After the first piece of bad new, his expected output drops to .92. But the variance jumps to $.9(.1)(1-.2) = .072$, a tenfold increase.

A pessimist who believes the probability of each piece of bad news is independent and equal to .5 originally thinks the probability of ultimate breakdown is .25=(.5)(.5). Expected output for him is .85. In his view ex ante, the variance of final output is $.75(.25)(1-.2) = .15$. After the first piece of bad new, his expected output drops to .6. But the variance jumps to $.5(.5)(1-.2) = .20$. Note that the expectations differed originally by $.992 - .85 = .142$, but after the bad news the disagreement more than doubles to $.92 - .6 = .32$.

I call the kind of bad news that increases uncertainty and disagreement *scary* news.

The news in the last 18 months has indeed been of this kind. When agency mortgage default losses were less than 1/4%, there was not much uncertainty and not much disagreement. Even if they tripled, they would still be small enough not to matter. Similarly, when subprime mortgage losses (that is losses incurred after homeowners failed to pay, were thrown out of their homes, and the house was sold for less than the loan amount) 3%, they were so far under the bond cushion of 8% that there was not much uncertainty or disagreement about whether the bonds would suffer losses, especially the higher rated bonds (with cushions of 15% or more).

1.5 Anatomy of a Crash

I use my theory of the equilibrium leverage to outline the anatomy of market crashes after the kind of scary news I just described.

i) Assets go down in value on scary bad news.

ii) This causes a big drop in the wealth of the natural buyers (optimists) who were leveraged.

iii) This leads to further loss in asset value, and in wealth for the natural buyers.

iv) Then margin requirements are tightened because of increased uncertainty and disagreement.

v) This causes huge losses in asset values via forced sales.

vi) Many optimists will lose all their wealth and go out of business

vii) There may be spillovers if sales of other assets are forced.

viii) Investors who survive have a great opportunity.

1.6 Natural Buyers

A crucial part of my story is heterogeneity between investors. The natural buyers want the asset more than the general public. This could be for many reasons. The natural buyers could be less risk averse. Or they could have access to hedging techniques the general public does not that make the assets less dangerous for them. Or they could get more utility out of holding the assets. Or they could have access to a production technology that uses the assets more efficiently than the general public. Or they could have special information based on local knowledge. Or they could simply be more optimistic. I have tried nearly all these possibilities at various times in my models. In the real world, the natural buyers are probably made up of a mixture of these categories. But for modeling purposes, the simplest is the last, namely that the natural buyers are more optimistic by nature. They have different priors from the pessimists. I note simply that this perspective is not really so different from the others. Differences in risk aversion in the end just mean different risk adjusted probabilities.

A crucial part of the story then is the heterogeneity between optimists and pessimists. A loss for the optimists is much more important to prices than a loss for the pessimists, because it is the optimists who will be holding the assets and bidding their prices up. The loss of access to capital by the optimists (and the subsequent moving of assets from optimists to pessimists) creates the crash.

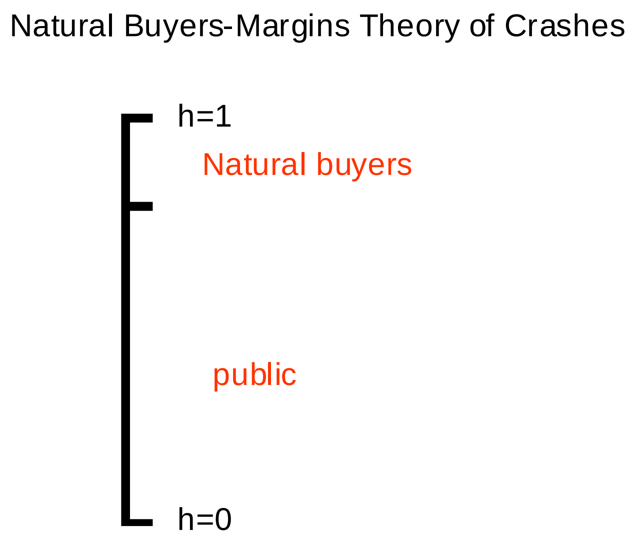
Current events have certainly borne out this heterogeneity hypothesis. When the big banks (who are the classic natural buyers) lost lots of capital through their blunders in the CDO market, that had a profound effect on new investments. Some of that capital was restored by international investments from Singapore and so on, but it was not enough, and it quickly dried up when the initial investments lost money.

Macroeconomists have often ignored the natural buyers hypothesis. For example, some macroeconomists compute the marginal propensity to consume out of wealth, and find it very low. The loss of $250 billion dollars of wealth could not possibly matter much they said, because the stock market has fallen many times by much more and economic activity hardly changed. But that ignores who lost the money.

The natural buyers hypothesis is not original with me. The innovation is in combining it with equilibrium leverage.

I do not presume a cut and dried distinction between natural buyers and the public. I imagine a continuum of agents uniformly arrayed between 0 and 1. Agent h on that continuum thinks the probability of good news is $g(h) = h$, and the probability of bad news is $b(h) = 1 - h$. The higher the h, the more optimistic the agent.

The more optimistic an agent, the more natural a buyer he is. By having a continuum I avoid a rigid categorization of agents. The agents will choose whether to be borrowers and buyers of risky assets, or lenders and sellers of risky assets. There will be some break point h* such that those more optimistic than $h*$, $h > h*$, are on one side of the market and and those with $h < h*$ are on the others side. But this break will be endogenous, depending on the exact circumstances.



2 Borrowing and Asset Pricing

Consider a simple example with one consumption good C, and one asset Y, that can pay either 1 or .2 per unit. Imagine the asset as a mortgage that either pays in full or defaults with recovery .2. But it could also be an oil well that might be a gusher or small. Or a house with good or bad resale value next period. Let every agent own one unit of the asset at time 0 and also one unit of the consumption good at time 0. For simplicity we think of the consumption good as something that can be used immediately in a quantity c that is chosen, or costlessly warehoused (stored) in a quantity denoted by w, like oil or cigarettes or canned food or simply gold (that can be used as fillings) or money. The agents only care about the total expected consumption they get, no matter when they get it. They are not impatient. The difference between the agents is only in the probability each attaches to a good outcome vs default. (All mortgages will either default together or pay off together). I imagine the agents arranged uniformly on a continuum, with agent $h \in [0, 1]$ assigning probability h to the good outcome

See diagram 1.

More formally, denoting the amount of consumption C in state s of by c_s, and the holding in state s of Y by y_s, and the storage of the consumption good at time 0 by

Endogenous Collateral with Heterogeneous Beliefs: A Simple Example

Let each agent $h \in H \subset [0,1]$ assign probability h to s = U and probability $1 - h$ to s = D. Agents with h near 1 are optimists, agents with h near 0 are pessimists.

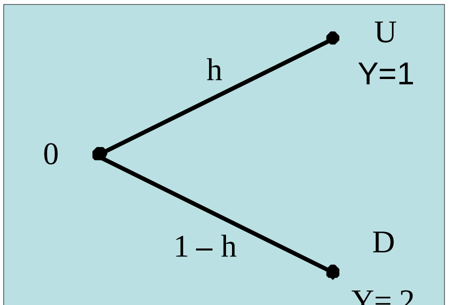


Figure 2

Suppose that 1 unit of Y gives \$1 unit in state U and .2 units in D.

w_0 we have

$$u^h(c_0, y_0, w_0, c_U, c_D) = c_0 + hc_U + (1-h)c_D$$
$$e^h = (e^h_{C_o}, e^h_{Y_o}, e^h_{C_U}, e^h_{C_D}) = (1, 1, 0, 0)$$

Storing goods and holding assets provide no direct utility, they just increase income in the future.

Suppose the price of the asset per unit at time 0 is p, somewhere between 0 and 1. The agents h who believe that

$$h1 + (1-h).2 > p$$

will want to buy the asset, since by paying p now they get something with expected payoff next period greater than p and they are not impatient. Those who think

$$h1 + (1-h).2 < p$$

will want to sell their share of the asset. I suppose there is no short selling, but I allow for borrowing. In the real world it is impossible to short sell many assets other than stocks. Even when it is possible, only a few agents know how, and those typically are the optimistic agents who are most likely to want to buy. So the assumption of no short selling is quite realistic.

If borrowing were not allowed, then the asset would have to be held by a large part of the population. The price of the asset would be .68. Agent $h = .60$ values the asset at $.68 = .60(1) + .40(.2)$. So all those h below .60 will sell all they have, or $.60(1) = .60$ in aggregate. Every agent above .60 will buy as much as he can afford. Each of these agents has just enough wealth to buy $1/.68 \approx 1.5$ more units, hence $.40(1.5) = .60$ units in aggregate. Since the market for assets clears at time 0, this is the equilibrium with no borrowing.

More formally, taking the price of the consumption good in each period to be 1 and the price of Y to be p, we can write the budget set without borrowing for each agent as

$$\begin{aligned} B_N^h(p) = \{&(c_0, y_0, w_0, c_1, c_2) \in \mathbb{R}_+^5 : c_0 + w_0 + p(y_0 - 1) = 1 \\ &c_U = w_0 + y_0 \\ &c_D = w_0 + (.2)y_0\}. \end{aligned}$$

Given the price p, each agent chooses the consumption plan $(c_0^h, y_0^h, w_0^h, c_1^h, c_2^h)$ in $B_N^h(p)$ that maximizes his utility u^h defined above. In equilibrium all markets must clear

$$\begin{aligned} \int_0^1 (c_0^h + w_0^h) dh &= 1 \\ \int_0^1 y_0^h dh &= 1 \\ \int_0^1 c_U^h dh &= 1 + \int_0^1 w_0^h dh \\ \int_0^1 c_D^h dh &= .2 + \int_0^1 w_0^h dh \end{aligned}$$

In this equilibrium agents are indifferent to storing or consuming right away, so we can describe equilibrium as if everyone warehoused and postponed consumption by taking

$$\begin{aligned} p &= .68 \\ (c_0^h, y_0^h, w_0^h, c_1^h, c_2^h) &= (0, 2.5, 0, 2.5, .5) \text{ for } h \geq .60 \\ (c_0^h, y_0^h, w_0^h, c_U^h, c_D^h) &= (0, 0, 1.68, 1.68, 1.68) \text{ for } h < .60. \end{aligned}$$

When loan markets are created, a smaller group of less than 40% of the agents will be able to buy and hold the entire stock of the asset. If borrowing were unlimited, at an interest rate of 0, the single agent at the top would borrow so much that he would buy up all the assets by himself. And then the price of the asset would be 1, since at any price p lower than 1 the agents h just below 1 would snatch the asset away from $h = 1$. But this agent would default, and so the interest rate would not be zero, and the equilibrium allocation needs to be more delicately calculated.

2.1 Incomplete Markets

We shall restrict attention to loans that are non-contingent, that is that involve promises of the same amount φ in both states. It is evident that the equilibrium allocation under this restriction will in general not be Pareto efficient. For example, in the no borrowing equilibrium, everyone would gain from the transfer of $\varepsilon > 0$ units of consumption in state U from each $h < .60$ to each agent with $h > .60$, and the transfer of $3\varepsilon/2$ units of consumption in state D from each $h > .60$ to each agent with $h < .60$. The reason this has not been done in the equilibrium is that there is no asset that can be traded that moves money from U to D or vice versa. We say that the asset markets are incomplete. We shall assume this incompleteness for a long time, until we consider Credit Default Swaps.

2.2 Collateral

We have not yet determined how much people can borrow or lend. In conventional economics they can do as much of either as they like, at the going interest rate. But in real life lenders worry about default. Suppose we imagine that the only way to enforce deliveries is through collateral. A borrower can use the asset itself as collateral, so that if he defaults the collateral can be seized. Of course a lender realizes that if the promise is φ in both states, then he will only receive

$$\begin{aligned}&\min(\varphi, 1) \text{ if good news}\\&\min(\varphi, .2) \text{ if bad news}\end{aligned}$$

The introduction of collateralized loan markets introduces two more parameters: how much can be promised φ, and at what interest rate r?

Suppose that borrowing were arbitrarily limited to $\varphi \leq .2$ per unit of Y, that is suppose agents were allowed to promise at most .2 units of consumption per unit of the collateral Y they put up. That is a natural limit, since it is the biggest promise that is sure to be covered by the collateral. It also greatly simplifies our notation, because then there would be no need to worry about default.

Borrowing gives the most optimistic agents a chance to spend more. And this will push up the price of the asset. But since they can borrow strictly less than the value of the collateral, optimistic spending will still be limited. Each time an agent buys a house, he has to put some of his own money down in addition to the loan amount he can obtain from the collateral just purchased. He will eventually run out of capital.

We can describe the budget set formally with our extra variables.

$$\begin{aligned}B^h_{.2}(p, r) = \{(c_0, y_0, \varphi_0, w_0, c_1, c_2) \in \mathbb{R}^6_+ :\\ c_0 + w_0 + p(y_0 - 1) = 1 + \frac{1}{1+r}\varphi_0 : \varphi_0 \leq .2\\ c_U = w_0 + y_0 - \varphi_0\\ c_D = w_0 + (.2)y_0 - \varphi_0\}.\end{aligned}$$

Note that $\varphi_0 > 0$ means that the agent is making promises in order to borrow money to spend more at time 0. Similarly, $\varphi_0 < 0$ means the agent is buying promises which will reduce his expenditures on consumption and assets in period 0, but enable him to consume more in the future states U and D. Equilibrium is defined by the price and interest rate (p, r) and agent choices $(c_0^h, y_0^h, \varphi_0^h, w_0^h, c_U^h, c_D^h)$ in $B_{.2}^h(p, r)$ that maximizes his utility u^h defined above. In equilibrium all markets must clear

$$\int_0^1 (c_0^h + w_0^h)dh = 1$$
$$\int_0^1 y_0^h dh = 1$$
$$\int_0^1 \varphi_0^h dh = 0$$
$$\int_0^1 c_U^h dh = 1 + \int_0^1 w_0^h dh$$
$$\int_0^1 c_D^h dh = .2 + \int_0^1 w_0^h dh$$

Clearly the no borrowing equilibrium is a special case of the collateral equilibrium, once the limit .2 on promises is replaced by 0.

One can calculate that the equilibrium price of the asset is now .75. At that price agent $h = .69$ is just indifferent to buying. Those $h < .69$ will sell all they have, and those $h > .69$ will buy all they can with their cash and with the money they can borrow. One can check that the top 31% of agents will indeed demand exactly what the bottom 69% are selling.

Who would be doing the borrowing and lending? The top 31% is borrowing to the max, in order to get their hands on what they believe are cheap assetss. The bottom 69% do not need the money for buying the asset, so they are willing to lend it. And what interest rate would they get? 0% interest, because they are not lending all they have in cash. (Their total lending is in fact $.2/.69 = .29 < 1$ per person). Since they are not impatient and they have plenty of cash left, they are indifferent to lending at 0%. Competition among these lenders will drive the interest rate to 0%.

More formally, we can define the equilibrium equations as

$$p = h^*1 + (1 - h^*)(.2)$$
$$p = \frac{(1 - h^*)(1) + .2}{h^*}$$

The first equation says that the marginal buyer h^* is indifferent to buying the asset. The second equation says that the price of Y is equal to the amount of money the agents above h^* spend buying it, divided by the amount of the asset sold. The numerator is then all the top group's consumption endowment, $(1 - h^*)(1)$, plus all they can borrow after they get their hands on all of Y, namely $.2/(1+r) = .2$. The denominator is comprised of all the sales of one unit of Y each by the agents below h^*.

We must also take into account buying on margin. An agent who buys the asset while simultaneously selling as many promises as he can will only have to pay down $p - .2$. His return will be nothing in the down state, because then he will have to turn over all the collateral to pay back his loan. But in the up state he will make a profit of $1 - .2$. So we need a third equation that says that agent h* is indifferent to not buying the asset or buying the asset on margin.

$$p - .2 = h^*(1 - .2)$$

But this equation is automatically satisfied as long as p is set to satisfy the first equation above; simply subtract .2 from both sides. When an agent has constant marginal utility for consumption, he will be indifferent to buying outright and buying on margin. In a later section we shall see that when agents have diminishing marginal utility for consumption, they will strictly prefer to buy on margin.

As we said, the large supply of durable consumption good, no impatience, and no default implies that the equilibrium interest rate must be 0. Solving the two equations above and plugging these into the agent optimization gives equilibrium

$$\begin{aligned} h^* &= .69 \\ (p, r) &= (.75, 0), \\ (c_0^h, y_0^h, \varphi_0^h, w_0^h, c_1^h, c_2^h) &= (0, 3.2, .64, 0, 2.6, 0) \text{ for } h \geq .69 \\ (c_0^h, y_0^h, \varphi_0^h, w_0^h, c_1^h, c_2^h) &= (0, 0, -.3, 1.45, 1.75, 1.75) \text{ for } h < .69. \end{aligned}$$

Notice that the price rises modestly, because there is a modest amount of borrowing. Notice also that even at the higher price, fewer agents hold all the assets (because they can afford to buy on borrowed money).

The lesson here is that the looser the collateral requirement, the higher will be the prices of assets. This has not been properly understood by economists. The conventional view is that the lower is the interest rate, then the higher will asset prices be, because their cash flows will be discounted less. But in the example I just described, where agents are patient, the interest rate will be zero regardless of the lending (up to some huge amount). The fundamentals do not change, but because of a change in lending standards, asset prices rise. Clearly there is something wrong with conventional asset pricing formulas. The problem is that to compute fundamental value, one has to use probablities. But whose probabilities?

The recent run up in asset prices has been attributed to irrational exuberance because conventional pricing formulas based on fundamental values failed to explain it. But the explanation I propose is that it got easier and easier to borrow. We shall return to this momentarily, after we figure out how much people can borrow.

Before turning to the next section, observe that equilibrium leverage is

$$\frac{.75}{(.75-.2)} = 1.4.$$

The loan to value is $.2/.75 = 27\%$, the margin or haircut was $.55/.75 = 73\%$. In the no borrowing equilibrium, leverage was obviously 1.

But leverage cannot yet be said to be endogenous, since we have exogenously fixed the collateral level or maximal promise at .2. Why wouldn't the most optimistic buyers be willing to borrow more, defaulting in the bad state of course, but compensating the lenders by paying a higher interest rate?

2.3 Equilibrium Leverage

Any lender must balance what he foresees getting repaid against the loan amount he gives up at time 0.

So how much can an agent borrow at time 0 using one asset worth p as collateral? In the example where promises were restricted to \$2, agents (at the prevailing interest rate of 0) borrowed \$2.

But why should haircuts be so high? Or equivalently, why should leverage be so low?

Before 1997 there had been virtually no work on equilibrium margins. Collateral was discussed almost exclusively in models without uncertainty. Even now the few writers who try to make collateral endogenous do so by taking an ad hoc measure of risk, like volatility or value at risk, and assume that the margin is some arbitrary function of the riskiness of the repayment.

It is not surprising that economists have had trouble modeling equilibrium haircuts or leverage. We have been taught that the only equilibrating variables are prices. It seems impossible that the demand equals supply equation for loans could determine two variables.

The key is to think of many loans, not one loan. Irving Fisher and then Ken Arrow taught us to index commodities by their location, or their time period, or by the state of nature, so that the same quality apple in different places or different periods might have different prices. So we must index each promise by its collateral. A promise of .2 backed by a house is different from a promise of .2 backed by 2/3 of a house. The former will deliver .2 in both states, but the latter will deliver .2 in the good state and only 1.33 in the bad state. The collateral matters.

Conceptually we must replace the notion of contracts as promises with the notion of contracts as ordered pairs of promises and collateral. Each ordered pair-contract will trade in a separate market, with its own price.

$$Contract_j = (\Pr omise_j, Collateral_j) = (A_j, C_j)$$

The ordered pairs are homogeneous of degree one. A promise of .2 backed by 2/3 of a house is simply 2/3 of a promise of .3 backed by a full house. So without loss of generality, we can always normalize the collateral. In our example we shall focus on contracts in which the collateral C_j is simply one unit of Y.

So let us denote by j the promise of j in both states in the future, backed by the collateral of one unit of Y. We take an arbitrarily large set J of such assets, but include j=.2.

The $j = .2$ promise will deliver .2 in both states, the $j = .3$ promise will deliver .3 after good news, but only .2 after bad news, because it will default there. The promises would sell for different prices, and different prices per unit promised.

Our definition of equilibrium must now incorporate these new promises $j \in J$ and prices π_j. When the collateral is so big that there is no default, $\pi_j = j/(1+r)$, where r is the riskless rate of interest. But when there is default, the price cannot be derived from the riskless interest rate alone. Given the price π_j, and given that the promises are all non-contingent, we can always compute the implied nominal interest rate as $r_j = 1/\pi_j$.

We must distinguish between sales $\varphi_j > 0$ of these promises (that is borrowing) from purchases of these promises $\varphi_j < 0$. The two differ more than in their sign. A sale of a promise obliges the seller to put up the collateral, whereas the buyer of the promise does not bear that burden. The marginal utility of buying a promise will often be much less than the marginal disutillity of selling the same promise, at least if the agent does not otherwise want to hold the collateral.

We can describe the budget set formally with our extra variables.

$$B^h(p, \pi) = \{(c_0, y_0, (\varphi_j)_{j\in J}, w_0, c_1, c_2) \in \mathbb{R}_+^{5+J}:$$

$$c_0 + w_0 + p(y_0 - 1) = 1 + \sum_{j=1}^{J} \varphi_j \pi_j$$

$$\sum_{j=1}^{J} \max(\varphi_j, 0) \leq y_0$$

$$c_U = w_0 + y_0 - \min(1, j) \sum_{j=1}^{J} \varphi_j$$

$$c_D = w_0 + (.2)y_0 - \min(.2, j) \sum_{j=1}^{J} \varphi_j\}.$$

Equilibrium is defined exactly as before, except that now we must have market clearing for all the contracts $j \in J$.

$$\int_0^1 (c_0^h + w_0^h) dh = 1$$
$$\int_0^1 y_0^h dh = 1$$
$$\int_0^1 \varphi_j^h dh = 0, \ \forall j \in J$$
$$\int_0^1 c_U^h dh = 1 + \int_0^1 w_0^h dh$$
$$\int_0^1 c_D^h dh = .2 + \int_0^1 w_0^h dh$$

It turns out that the equilibrium is exactly as before. The only asset that is traded is j=.2, ((.2,.2),1). All the other contracts are priced, but in equilibrium neither bought nor sold. Their prices can be computed by what the value the marginal buyer $h^* = .69$ attributes to them. So the price $\pi_{.3}$of the .3 promise is .27, much more than the price of the .2 promise but less per dollar promised. Similarly the price of a promise of .4 is given below

$$\begin{aligned}
\pi_{.2} &= .69(.2) + .31(.2) = .2 \\
1 + r_{.2} &= .2/.2 = 1.00 \\
\pi_{.3} &= .69(.3) + .31(.2) = .269 \\
1 + r_{.3} &= .3/.269 = 1.12 \\
\pi_{.4} &= .69(.4) + .31(.2) = .337 \\
1 + r_{.3} &= .4/.337 = 1.19
\end{aligned}$$

Thus an agent who wants to borrow .2 using one house as collateral can do so at 0% interest. An agent who wants to borrow .269 with the same collateral can do so by promising a 12% interest. An agent who wants to borrow .337 can do so by promising 19% interest. The puzzle of one equation determining both a collateral level and an interest rate is resolved; each collateral level corresponds to a different interest rate. It is quite sensible that less secure loans with higher defaults will require higher rates of interest.

What then do we make of my claim about "the" equilibrium margin? The surprise is that in this kind of example, with only one dimension of risk and one dimension of disagreement, only one margin will be traded! Everybody will voluntarily trade only the .2 loan, even though they could all borrow or lend different amounts at any other rate.

How can this be? Well the key is that the lenders are people with $h < .69$ who do not want to buy the asset. They are lending instead of buying the asset because they think there is a substantial chance of bad news. It should be no surprise that they do not want to make risky loans, even if they can get a 19% rate instead of a 0% rate, because the risk of default is too high for them. Indeed the risk is perfectly correlated with the asset which they have already shown they do not want. What about the borrowers? Mr h=1 thinks for every .75 he pays on the asset, he can get 1 for sure. Wouldn't he love to be able to borrow more, even at a slightly higher interest rate? The answer is no! In order to borrow more, he has to substitute a .4 loan for a .2 loan. He pays the same amount in the bad state, but pays more in the good state, in exchange for getting more at the beginning. But that is crazy for him. He is the one convinced the good state will occur, so he definitely does not want to pay more just where he values money the most.

Thus the only loans that get traded in equilibrium involve margins just tight enough to rule out default. That depends of course on the special assumption of only two outcomes. But often the two outcomes lenders have in mind are just two. And typically they do set haircuts in a way that makes defaults very unlikely. Recall that in the 1994 and 1998 leverage crises, not a single lender lost money on repo trades. Of course in more general models, one would imagine more than one margin and more than one interest rate emerging in equilibrium.

To summarize, in the usual theory a supply equals demand equation determines the interest rate on loans. In my theory equiilibrium often determines the equilibrium leverage (or margin) as well. It seems surprising that one equation could determine two variables, and to the best of my knowledge I was the first to make th observation (in 1997 and again in 2003) that leverage could be uniquely determined in equilibrium. I showed that the right way to think about the problem of endogenous collateral is to consider a different market for each loan depending on the amount of collateral put up, and thus a different interest rate for each level of collateral. A loan with a lot of collateral will clear in equilibrium at a low interest rate, and a loan with little collateral will clear at a high interest rate. A loan market is thus determined by a pair (promise,collateral), and each pair has its own market clearing price. The question of a unique collateral level for a loan reduces to the less paradoxical sounding, but still surprising, assertion that in equilibrium everybody will choose to trade in the same collateral level for each kind of promise. I proved that this must be the case when there are only two successor states to each state in the tree of uncertainty.

2.3.1 Upshot of equilibrium leverage

We have shown that in this context, equilibrium leverage transforms the purchase of the collateral into the buying of the up Arrow security. Since the buyer of the collateral will simultaneously sell the promise of the down payoff, on net he is just buying the up Arrow security.

2.4 Complete Markets

Suppose there were complete markets. Then the equilibrium would simply be $((p_U, p_D), (x_0^h, w_0^h, x_U^h, x_D^h))$ such that

$$\int_0^1 (c_0^h + w_0^h) dh = 1$$

$$\int_0^1 c_U^h dh = 1 + \int_0^1 w_0^h dh$$

$$\int_0^1 c_D^h dh = .2 + \int_0^1 w_0^h dh$$

$$(x_0^h, w_0^h, x_U^h, x_D^h) \in B^h(p) = \{(x_0, w_0, x_U, x_D) :$$
$$x_0 + w_0 + p_U x_U + p_D x_D \leq 1 + p_U 1 + p_D(.2)\}$$
$$(x_0, w_0, x_U, x_D) \in B^h(p) \Rightarrow u^h(x_0, x_U, x_D) \leq u^h(x_0, x_U, x_D)$$

It is easy to calculate that complete markets equilibrium occurs where $(p_U, p_D) = (.44, .56)$ and agents $h > .44$ spend all of their wealth of 1.55 buying 3.5 units of consumption each in state U and nothing else, giving total demand of $(1 - .44)3.5 = 2.0$ and the bottom .44 agents spend all their wealth buying 2.78 units of x_D each, giving total demand of $.44(2.78) = 1.2$ in total.

The price of Y with complete markets is therefore $p_U 1 + p_D(.2) = .55$, much lower than the incomplete markets, leverage price.

2.5 CDS and the Repo Market

The contract markets we have studied so far are similar to the Repo markets that have played such an important role on Wall Street. In these markets borrowers take their collateral to a dealer and use that to borrow money via non-contingent promises due one day later. The CDS is a much more recent contract.

The invention of the CDS or credit default swap moved the markets closer to complete. In our two state example with plenty of collateral, their introduction actually does lead to the complete markets solution, despite the need of collateral. In general, with more perishable goods, and goods in the future that are not tradable now, the introduction of CDS does not complete the markets.

A CDS is a promise to pay the principal default on a bond. Thus thinking of the asset as paying 1 or .2 if it defaults, the credit default swap would pay .8 in the down state, and nothing anywhere else. In other words, the CDS is tantamount to trading the down Arrow security.

The credit default swap needs to be collateralised. There are only two possible collaterals for it, the security, or the gold. A collateralisable contract promising an Arrow security is particularly simple, because it is obvious we need only consider versions in which the collateral exactly covers the promise. So choosing the normalizations in the most convenient way, there are essentially two CDS contracts to consider, a CDS promising .2 in state D and nothing else, collateralized by the security, or a CDS promising 1 in state D, and nothing else, collateralized by a piece of gold. So we must add these two contracts to the repo contracts we considered earlier.

It is a simple matter to show that the complete markets equilibrium can be implemented via the two CDS contracts. The agents $h > .44$ buy all the security Y and all the gold, and sell the maximal amount of CDS against all that collateral. Since all the goods are durable, this just works out.

2.5.1 The upshot of CDS

In this simple model, the CDS is the mirror image of the repo. It is tantamount therefore, to letting the pessimists leverage. That is why the price of the asset goes down once the CDS is introduced.

Another interesting consequence is that the CDS kills the repo market. Buyers of the asset switch from selling repo contracts against the asset to selling CDS. It is true that since the introduction of CDS in late 2005 into the mortgage market, the repo contracts have steadily declined.

In the next section we ignore CDS and reexamine the repo contracts in a dynamic setting. Then we return to CDS.

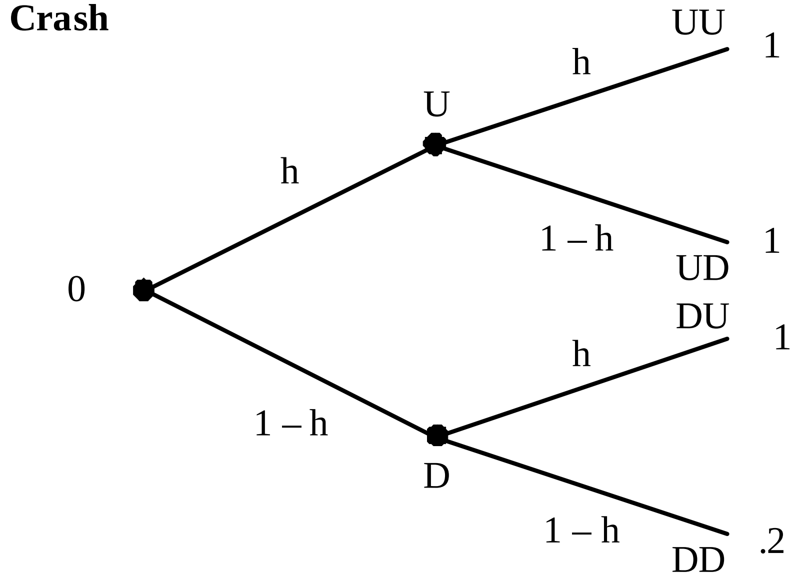
3 The Leverage Cycle

If in the example bad news occurs and the value plummets to .2, there will be a crash. This is a crash in the fundamentals. There is nothing the government can do to avoid it. But the economy is far from the crash in that example. It hasn't happened yet. The marginal buyer thinks the chances of a fundamentals crash are only 31%. The average buyer thinks the fundamentals crash will occur with just 15% probability.

The point of the leverage cycle is that excess leverage followed by excessive deleveraging will cause a crash even before there has been a crash in the fundamentals, and even if there is no subsequent crash in the fundamentals. The crash comes for two reasons. First because the asset price is excessively high in the initial or over-leveraged normal economy. And second, because after deleveraging, the price is even lower than it would have been at those tough margin levels had there never been the over-leveraging in the first place.

So consider the same example but with three periods instead of two. Suppose, as before, that each agent begins in state s=0 with one unit of money and one unit of the asset, and that both are perfectly durable. But now suppose the asset Y pays off after two periods instead of one period. After good news in either period, the asset pays 1 at the end. Only with two pieces of bad news does the asset pay .2. The state space is now S={0,U,D,UU,UD,DU,DD}. We use the notation s^* to denote the immediate predecessor of s.

Diagram 2 here



Suppose agents again have no impatience, but care only about their expected consumption of dollars. Formally, letting c_s be consumption in state s, and letting e_s^h be the initial endowment of the consumption good in state s, and letting y_{0*}^h be the initial endowment of the asset Y before time begins, we have for all $h \in [0,1]$

$$u^h(c_0, c_U, c_D, c_{UU}, c_{UD}, c_{DU}, c_{DD})$$
$$= c_0 + hc_U + (1-h)c_D + h^2 c_{UU} + h(1-h)c_{UD} + (1-h)hc_{DU} + (1-h)^2 c_{DD}$$
$$(e_0^h, y_{0*}^h, e_U^h, e_D^h, e_{UU}^h, e_{UD}^h, e_{DU}^h, e_{DD}^h)$$
$$= (1, 1, 0, 0, 0, 0, 0, 0)$$

We define the dividend of the asset by $d_{UU} = d_{UD} = d_{DU} = 1$, and $d_{DD} = .2$, and $d_s = 0$ for all other s.

The agents are now more optimistic than before, since agent h assigns only a probability of (1-h)^2 to the only state, DD, where the asset pays off .2. The marginal buyer from before, h* = .69, for example, thinks the chances of DD are only $(.31)^2 = .09$. Agent h=.87 thinks the chances of DD are only $(.13)^2 = 1.69\%$. But more importantly, if buyers can borrow short term, their loan at 0 will come due before the catastrophe can happen. It is thus much safer than a loan at D.

Assume that repo loans are one-period loans, so that loan sj promises j in states sU and sD. The budget set can now be written iteratively, for each state s.

$$B^h(p,\pi) = \{(c_s, y_s, (\varphi_{sj})_{j\in J}, w_s)_{s\in S} \in \mathbb{R}_+^{(3+J)(1+S)} : \forall s$$

$$(c_s - e_s^h) + w_s + p_s(y_s - y_{s^*}) = y_{s^*}d_s + \sum_{j=1}^{J} \varphi_{sj}\pi_{sj} - \min(p_s + d_s, j)\sum_{j=1}^{J}\varphi_{s^*j}$$

$$\sum_{j=1}^{J} \max(\varphi_{sj}, 0) \leq y_s\}$$

The crucial question again is how much collateral will the market require at each state s? By the logic we described in the previous section, it can be shown that in every state s, the only promise that will be actively traded is the one that makes the maximal promise on which there will be no default. Since there will be no default on this contract, it trades at the riskless rate of interest r_s per dollar promised. Using this insight we can drastically simplify our notation (as in Fostel-Geanakoplos) by redefining φ_s as the amount of the consumption good promised at state s for delivery in the next period, in states sU and sD. The budget set then becomes

$$B^h(p,r) = \{(c_s, y_s, \varphi_s, w_s)_{s\in S} \in \mathbb{R}_+^{4(1+S)} : \forall s$$

$$(c_s - e_s^h) + w_s + p_s(y_s - y_{s^*}) = y_{s^*}d_s + \sum_{j=1}^{J} \varphi_s \frac{1}{1+r_s} - \varphi_{s^*}$$

$$\varphi_s \leq y_s \min(p_{sU} + d_{sU}, p_{sD} + d_{sD})\}$$

Equilibrium occurs at prices (p, r) such that when everyone optimizes in his budget set by choosing $(c_s^h, y_s^h, \varphi_s^h, w_s^h)_{s\in S}$ the markets clear in each state s

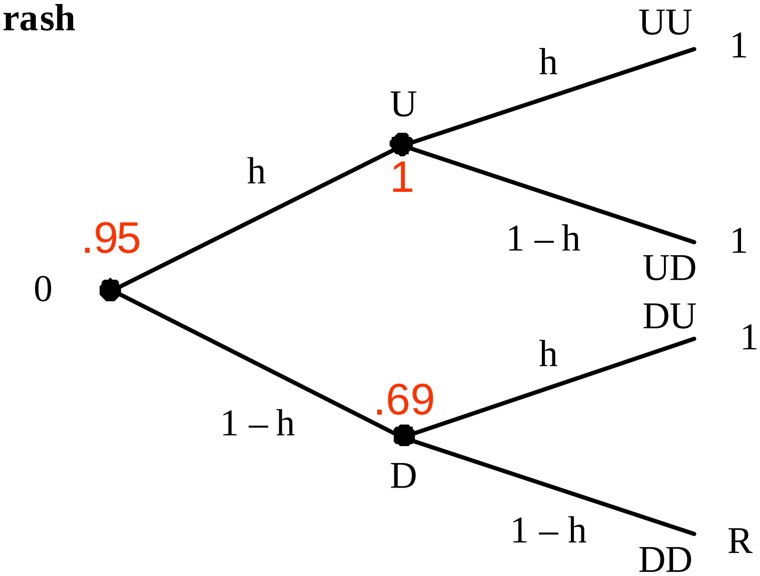
$$\int_0^1 (c_s^h + w_s^h)dh = \int_0^1 e_s^h dh + d_s \int_0^1 y_{s^*}^h dh$$

$$\int_0^1 y_s^h dh = \int_0^1 y_{s^*}^h dh$$

$$\int_0^1 \varphi_s^h dh = 0$$

It will turn out in equilibrium that the interest rate is zero in every state. Thus at time 0, agents can borrow the minimum of the price of Y at U and at D, for every unit of Y they hold at 0. At U agents can borrow 1 unit of the consumption good, for every unit of Y they hold at U. At D they can borrow only .2 units of the consumption good, for every unit of Y they hold at D. In normal times, at 0, there is not very much bad that can happen in the short run. Lenders are therefore willing to lend much more on the same collateral, and leverage can be quite high. Solving the example gives the following prices.

Crash



The price of Y at time 0 of .95 occurs because the marginal buyer is h=.87. Assuming the price of Y is .69 at D and 1 at U, the most that can be promised at 0 using Y as collateral is .69. With an interest rate $r_0 = 0$, that means .69 can be borrowed at 0 using Y as collateral. Hence the top 13% of buyers at time 0 can borrow collectively borrow .69 (since they will own all the assets), and by adding their own .13 of money they can spend .82 on buying the .87 units that are sold by the bottom 87%. The price is .95=.82/.87.

Why is there a crash from 0 to D? Well first there is bad news. But the bad news is not nearly as bad as the fall in prices. The marginal buyer of the asset at time 0, h=.87, thinks there is only a $(.13)^2 = 1.69\%$ chance of ultimate default, and when he gets to D after the first piece of bad news he thinks there is a 13% chance for ultimate default. The news is bad, accounting for a drop in price of about 11 points, but it does not explain a fall in price from .95 to .69 of 26 points. In fact, no agent h thinks the loss in value is nearly as much as 26 points. The biggest optimist h=1 thinks the value is 1 at 0 and still 1 at D. The biggest pessimist h=0 thinks the value is .2 at 0 and still .2 at D. The biggest loss attributable to the bad news of arriving at D is felt by h=.5, who thought the value was .8 at 0 and thinks it is .6 at D. But that drop of .2 is still less than the drop of 26 points in equilibrium.

The second factor is that the leveraged buyers at time 0 all go bankrupt at D. They spent all their cash plus all they could borrow at time 0, and at time D their collateral is confiscated and used to pay off their debts. Without the most optimistic buyers, the price is naturally lower.

Finally, and most importantly, the margins jump from $(.95 - .69)/.95 = 27\%$ to $(.69 - .2)/.69 = 71\%$. In other words, leverage plummets from 3.6 to 1.4.

All three of these factors working together explain the fall in price.

More concretely, let b be the marginal buyer in state D and let a be the marginal buyer in state 0. Then we must have

$$
\begin{aligned}
p_D &= b1 + (1-b)(.2) \\
p_D &= \frac{(1/a)(a-b) + .2}{(1/a)b} = \frac{1.2a - b}{b} \\
a &= \frac{b(1+p_D)}{1.2} \\
p_0 &= \frac{(1-a) + p_D}{a} \\
\frac{a(1-p_D)}{p_0 - p_D} &= a1 + (1-a)\frac{a}{b} \\
\frac{a(1-p_D)}{p_0 - p_D} &= \frac{a1 + (1-a)p_D\frac{a}{b}}{p_0}
\end{aligned}
$$

The first equation says that the price at D is equal to the valuation of the marginal buyer. Because his utility is linear in consumption, he will then also be indifferent to buying on the margin, as we saw in the collateral section. The second equation says that the price at D is equal to the ratio of all the money spent on Y at D, divided by the units sold at D. The top a investors are all out of business at D, so they cannot buy anything. At D the remaining investors in the interval $[0, a)$ each own $1/a$ units of Y and have inventoried or collected $1/a$ dollars. The buyers in the interval $[b, a)$ spend all they have, which is $(1/a)(a - b)$ dollars plus the .2(1) they can borrow on the entire stock of Y. The amount of Y sold at D is $(1/a)b$. This explains the second equation. The third equation just rearranges the terms in the second equation.

The fourth equation is similar to the second. It explains the price of Y at 0 by the amount spent divided by the amount sold. Notice that at 0 it is possible to borrow p_D using each unit of Y as collateral. So the top $(1 - a)$ agents have $(1 - a) + p_D$ to spend on the a units of Y for sale at 0.

The fifth equation equates the marginal utility to a of one dollar, on the right, with the marginal utility of putting one dollar of cash down on a leveraged purchase of Y, on the left. The optimal thing for a to do with a dollar at time 0 is to inventory it. With probablity a, U will be reached and this dollar will be worth a dollar. With probability $1 - a$, D will be reached and a will want to leverage the dollar into as big a purchase of Y as possible. This will result in a gain at D of

$$
\frac{a(1-.2)}{p_D - .2} = \frac{a(1-.2)}{b1 + (1-b)(.2) - .2} = \frac{a}{b}
$$

Hence the marginal utility of a dollar at time 0 is $a1 + (1-a)\frac{a}{b}$, explaining the right hand side of the equation. The marginal utility of leveraging a dollar by buying Y on margin at time 0 can also easily be seen. With $p_0 - p_D$ dollars cas downpayment, one gets a payoff of $(1 - p_D)$ dollars in state U, to which a assigns probability a, explaining the left hand side of the equation.

The last equation says that a is indifferent to buying Y on margin at 0 or buying it for cash. The right hand side shows that by spending p_0 dollars to buy Y at 0, agent a can get a payoff of 1 with probability a, and with probability $(1-a)$ a payoff of P_D dollars at D, which is worth $p_D\frac{a}{b}$ to a. The last equation is a tautological consequence of the previous equation. To see this, note that by rewriting the second to last equation and using the identity $\frac{\alpha}{\beta} = \frac{\gamma}{\delta}$ implies $\frac{\alpha}{\beta} = \frac{\alpha+\gamma}{\beta+\delta}$ we get

$$\begin{aligned}\frac{a(1-p_D)}{p_0 - p_D} &= \frac{p_D[a1 + (1-a)\frac{a}{b}]}{p_D} \\ &= \frac{a(1-p_D) + p_D[a1 + (1-a)\frac{a}{b}]}{p_0 - p_D + p_D} \\ &= \frac{a1 + (1-a)p_D\frac{a}{b}}{p_0}\end{aligned}$$

which is the last equation.

By guessing a value of b, and then iterating through all the equations, one ends up with all the variable specified, and a new value of b. By searching for a fixed point in b, one quickly comes to the solution just described, with the crash from .95 to .69.

In recent times there has been bad news, but according to most modelers the prices of assets today is much lower than would be warranted by the news. There have been numerous bankruptcies of mortgage companies, and even of great investment banks. And the drop in leverage has been enormous.

These kind of events had occured before in 1994 and 1998. The cycle is more severe this time because the leverage was higher.

3.1 Upshot of the Leverage Cycle so far

The wild gyrations in asset prices as equilibrium leverage ebbs and flows is alarming in and of itself. And a second thought must be borne in mind. The natural buyers with high h manifest themselves here as simple optimists. But in reality one should think of them as more sophisticated and knowledgeable buyers who are more risk tolerant because they are better at quantifying the risks. Their eviceration at D leaves the economy in the hands of a much more pessimistic lot, who in the next sections will be much less inclined to invest and produce and be active.

From a strict individualistic welfare point of view, where we do not take a stand on whose probabilities are most accurate, it must be acknowledged that things are not quite as bad as they might seem. With complete markets there would be high volatility as well. The optimists would bet on U, selling their wealth at D, so we would expect the price at D to reflect the opinions of more pessimistic people than average at U or 0, and thus we should expect a big drop in prices at D even with complete markets.

Indeed it is easy to compute the complete markets equilibrium. Nobody would consume until the final period, when all the information had been revealed. So we need only find four prices of consumption at UU,UD,DU, and DD. The supplies of goods are respectively 2,2,2,,1.2, and the most optimistic people will exclusively consume good UU, the next most optimistic will exclusively consume UD and so on. The prices turn out to be $p_{UU} = .29, p_{UD} = .16, p_{DU} = .16, p_{DD} = .39$. This gives a drop of Y from $p_{oY} = .68$ to $p_{DY} = .43$.

The complete markets prices are systematically lower, because effectively complete markets amounts to adding the CDS, which means the pessimists can leverage. As we suggested earlier, though complete markets is always the benchmark when priors are equal, with unequal priors we may have reason to weight some opinions more than others. But in any case, the drop in prices is just as severe with complete markets.

An interesting twist is to assume that the CDS market did not get introduced until the middle period, computing equilibrium with repo markets at time 0 and complete markets from time 1 onwards. Then we get $p_{oY} = .85$ to $p_{DY} = .51$. The sudden introduction of CDS has probably played a bigger role than people realize.

In the next section we drop CDS and return to repo markets, but we analyze the more conventional case of common priors and diminishing marginal utility.

4 Declining Marginal Utility

So far we have assumed that agents had constant marginal utility for consumption. In that case the marginal buyer of a collateral good is indifferent to buying it straight or buying it leveraged. The analysis takes a very interesting turn when we allow for diminishing marginal utility. In the latter case all buyers strictly prefer leveraged buying to cash buying. This has several notable implications for asset pricing and efficiency.

4.1 Example: Borrowing Across Time

We consider an example with two kinds of agents $H = \{A, B\}$, two time periods, and two goods F (food) and H (housing) in each period. For now we shall suppose that there is only one state of nature in the last period.

We suppose that food is completely perishable, while housing is perfectly durable.

We suppose that agent B likes living in a house much more than agent A,

$$u^A(x_{0F},\, x_{0H},\, x_{1F},\, x_{1H}) = x_{0F} + x_{0H} + x_{1F} + x_{1H}\ ,$$

$$u^B(x_{0F},\, x_{0H},\, x_{1F},\, x_{1H}) = 9x_{0F} - 2x_{0F}^2 + 15x_{0H} + x_{1F} + 15x_{1H}\ .$$

Furthermore, we suppose that the endowments are such that agent B is very poor in the early period, but wealthy later, while agent A owns the housing stock

$$e^A = (e^A_{0F},\, e^A_{0H},\, e^A_{1F},\, e^A_{1H}) = (20,\, 1,\, 20,\, 0)\ ,$$

$$e^B = (e^B_{0F},\, e^B_{0H},\, e^B_{1F},\, e^B_{1H}) = (4,\, 0,\, 50,\, 0)\ .$$

We suppose that there are contracts (A_j, C_j) with $A_j = \binom{j}{0}$, promising j units of food in period 1, and no housing, each collateralized by one house $C_j = (0, 1)$ as before. We introduce a new piece of notation D_{1j} to denote the value of actual deliveries of asset j at time 1. Given our norecourse collateral, we know $D_{1j} = \min(jp_{1F}, p_{1H})$.

4.1.1 Arrow–Debreu Equilibrium

If in addition we had a complete set of Arrow securities with infinite default penalties and no collateral requirements, then it is easy to see that there would be a unique equilibrium (in prices and utility payoffs):

$$\begin{aligned} p &= (p_{0F},\, p_{0H},\, p_{1F},\, p_{1H}) = (1,\, 30,\, 1,\, 15)\ , \\ x^A &= (x^A_{0F},\, x^A_{0H},\, x^A_{1F},\, x^A_{1H}) = (22,\, 0,\, 48,\, 0)\ , \\ x^B &= (x^B_{0F},\, x^B_{0H},\, x^B_{1F},\, x^B_{1H}) = (2,\, 1,\, 22,\, 1)\ , \\ u^A &= 70\ ;\quad u^B = 62\ . \end{aligned}$$

Assuming that A consumes food in both periods, the price of food would need to be the same in both periods, since A's marginal utility for food is the same in both periods. We might as well take those prices to be 1. Assuming that B consumes food in the last period, the price of every good that B consumes must then be equal to B's marginal utility for that good. With complete markets, the B agents would be able to borrow as much as they wanted, and they would then have the resources to bid the price of housing up to 30 in period 0 and 15 in period 1.

4.1.2 No Collateral– No contracts Equilibrium

Without the sophisticated financial arrangements involved with collateral or default penalties, there would be nothing to induce agents to keep their promises. Recognizing this, the market would set a price $\pi_j = 0$ for the assets. Agents would therefore not be able to borrow any money. Thus agents of type B, despite their great desire to live in housing, and great wealth in period 1, would not be able to purchase much housing in the initial period. Again it is easy to calculate the unique equilibrium:

$$
\begin{aligned}
&\pi_j = 0 \\
&p = (p_{0F},\, p_{0H},\, p_{1F},\, p_{1H}) = (1,\, 16,\, 1,\, 15)\ , \\
&x^A = (x^A_{0F},\, x^A_{0H},\, x^A_{1F},\, x^A_{1H}) = \left(20 + \frac{71}{32},\, 1 - \frac{71}{32 \cdot 16},\, 35 - \frac{71 \cdot 15}{32 \cdot 16},\, 0\right)\ , \\
&x^B = (x^B_{0F},\, x^B_{0H},\, x^B_{1F},\, x^B_{1H}) = \left(\frac{57}{32},\, \frac{71}{32 \cdot 16},\, 35 + \frac{71 \cdot 15}{32 \cdot 16},\, 1\right)\ , \\
&u^A = 56\ ;\quad u^B \approx 64\ .
\end{aligned}
$$

Agent A, realizing that he can sell the house for 15 in period 1, is effectively paying only $16 - 15 = 1$ to have a house in period 0, and is therefore indifferent to how much housing he consumes in period 0. Agents of type B, on the other hand, spend their available wealth at time 0 on housing until their marginal utility of consumption of x_{0F} rises to $\frac{30}{16}$, which is the marginal utility of owning an extra dollar's worth of housing stock at time 0. That occurs when $9 - 4x^B_{0F} = \frac{30}{16}$, that is, when $x^B_{0F} = \frac{57}{32}$.

4.1.3 Collateral Equilibrium

We now introduce the possibility of collateral, i.e. we suppose the state apparatus is such that the house is confiscated if payments are not made. The unique equilibrium is then:

$$
\begin{aligned}
&D_j = \min(j, 15), \pi_j = \min(j, 15)\ , \\
&p = (p_{0F},\, p_{0H},\, p_{1F},\, p_{1H}) = (1,\, 18,\, 1,\, 15)\ , \\
&x^A = (x^A_{0F},\, x^A_{0H},\, x^A_{1F},\, x^A_{1H}) = (23,\, 0,\, 35,\, 0)\ , \\
&\theta^A_{15} = 15\ ;\ \theta^A_j = 0 \text{ for } j \neq 15;\ \varphi^A_j = 0, \\
&x^B = (x^B_{0F},\, x^B_{0H},\, x^B_{1F},\, x^B_{1H}) = (1,\, 0,\, 35,\, 1)\ , \\
&\theta^B_j = 0\ ;\ \ \varphi^B_{15} = 15\ ;\ \varphi^B_j = 0 \text{ for } j \neq 15; \\
&u^A = 58\ ;\quad u^B = 72\ .
\end{aligned}
$$

The only contract traded is the one $j = 15$ that maximizes the promise that will not be broken. Its price $\pi_{15} = 15$ is given by its marginal utility to its buyer A. Agent B sells the contract, thereby borrowing 15 units of x_{0F}, and uses the 15 units of x_{0F} plus 3 he owns himself to buy 1 unit of the house x_{0H}, at a price of $p_{0H} = 18$. He uses the house as collateral on the loan, paying off in full the 15 units of x_{1F} in period 1. Since, as borrower, agent B gets to consume the housing services while the house is being used as collateral, he gets final utility of 72. Agent A sells all his housing stock, since the best he can do after buying it is to live in it for one year, and then sell it at a price of 15 the next year, giving him marginal utility of 16, less than the price of 18 (expressed in terms of good F).

The most interesting aspect of the collateral equilibrium is the first order condition for the buyer of collateral. The purpose of collateral is to enable people like B, who desperately want housing but cannot afford much (for example in the contract less economy), to buy the housing and live in it by borrowing against the future, using the house as collateral. To the extent that collateral is not a perfect device for borrowing, one might expect that B does not quite get all the housing he needs, and that the marginal utility of housing might end up greater to B than the marginal utility of food. In fact, the opposite is true.

In collateral equilibrium, the marginal utility of a dollar of housing is substantially less than the marginal utility of a dollar of food

$$\frac{MU^B_{x_{0H}}}{p_{0H}} = \frac{30}{18} < \frac{5}{1} = \frac{MU^B_{x_{0F}}}{p_{0F}}$$

So why does B buy housing at all? Because he can buy on margin, i.e. with leverage. He needs to pay only $3 = 18 - 15$ of cash down for the house, getting 15 utiles in period 0, and then he can give the house up in period 1 to repay his loan. This leveraged purchase brings 5 utiles per dollar. This is exactly equal to the marginal utility of food per dollar.

This is a completely general phenomenon. The leveraged purchase brings more marginal utility than the straight cash purchase to any buyer with diminishing marginal utility. We now discuss why.

4.1.4 Liqudity Wedge and Collateral Value

To the extent that collateral is not perfect in solving the borrowing problem, borrowers will be constained from borrowing as much as they would like. The upshot is that the marginal utility today of the price of the contracts the borrowers are selling is much higher than the marginal utility to them of the deliveries they have to make: that is what it means for them to be constrained in their selling of loans, i.e., constrained in their borrowing. In Fostel-Geanakoplos 2008 we called this the liquidity wedge.

In the above example, contract $j = 15$ sells for a price of 15, which gives B marginal utility at time 0 of $(9 - 4x_{0F})15 = 5(15) = 75$. The marginal utility of the deliveries of 15 that B must make at time 1 is $(1)(15) = 15$. This surplus B gains by borrowing explains why he will choose to sell only the contract $j = 15$ that maximizes the amount of money he raises. Selling a contract with j<15 is silly. It deprives B of the oppurtunity to earn more liquidity surplus. Selling contract 16 would not bring any more cash, because contract 16 sells for the same price as contract 15 even though it promises more.

The collateral has a price of 18 relative to food, which is much too high to be explained by its utility relative to food. But as explained in Fostel-Geanakoplos, the price is equal to the payoff value plus the collateral value. Housing does double duty. It enables B agents to get utility by living there, but it also enables B agents to borrow more and to gain more liquidity surplus.

$$\begin{aligned}
p_{0H} &= \text{payoff value} + \text{collateral value} \\
\text{payoff value } &= (MU^B_{x_{0H}} + MU^B_{x_{1H}})(1/\frac{MU^B_{x_{0F}}}{p_{0F}}) = (15+15)/5 = 6 \\
\text{collateral value } &= (MU^B_{x_{0F}}\pi_{15} - MU^B_{x_{1F}}D_{15})(1/\frac{MU^B_{x_{0F}}}{p_{0F}}) = (5 \cdot 15 - 1 \cdot 15)/5 = 12 \\
p_{0H} &= 6 + 12 = 18
\end{aligned}$$

4.1.5 The Failure of "Efficient Markets"

The efficient markets hypothesis essentially says that prices are priced fairly by the market, and that even an uniformed agent should not be afraid to trade, because the prices already incorporate the information acquired by more sophisticated agents. That is true in collateral equilibrium for the contracts, but it is not true of the assets that can be used as collateral. An unsophisticated buyer who did not know how to use leverage would find that he grossly overpaid for housing.

4.1.6 Optimal Collateral Levels?

What would happen if the government simply refused to let borrowers leverage so much, say by prohibiting the trade in contracts for $j > 14$? Although every type B agent wants to leverage up, using $j = 15$, when all the other type B agents are doing the same, he is actually much better off if leverage is limited by government fiat. Then everybody will borrow using asset $j = 14$, and with less buying power, the price of housing will fall. In fact p_{0H} will fall to 17. The downpayment of 3 needed to buy the house is still the same, the consumption of the B types is the same in period 0, and so the marginal utility condition continues to hold. The only difference is that agent B will only have to deliver 14 in period 1 instead of 15. In short the limit on leverage works out as a transfer from A to B.

4.1.7 Why did Housing Prices Rise so much from 1996-2006?

We can put our last observation more directly. Limits on leverage will reduce collateral goods prices, and an expansion of leverage will increase their prices. The remarkable run-up in housing prices in the middle 1990s to the middle 2000s in in my mind less a matter of irrational exuberance than of leverage.

We now consider a more complicated variation of our basic example in which there is uncertainty and default. Now a higher collateral requirement would mean strictly less default, but also lower housing prices. So it is interesting to see which collateral requirement best suits the sellers/lenders.

4.2 Example: Borrowing Across States of Nature, with Default

We consider almost the same economy as before, with two agents A and B, and two goods F (food) and H (housing) in each period. But now we suppose that there are two states of nature $s = 1$ and 2 in period 1, occurring with objective probabilities $(1-\varepsilon)$ and ε, respectively.

As before, we suppose that food is completely perishable and housing is perfectly durable.

We assume

$$u^A(x_{0F},\, x_{0H},\, x_{1F},\, x_{1H},\, x_{2F},\, x_{2H}) = x_{0F} + x_{0H} + (1-\varepsilon)(x_{1F} + x_{1H}) + \varepsilon(x_{2F} + x_{2H})\ ,$$
$$u^B(x_{0F},\, x_{0H},\, x_{1F},\, x_{1H},\, x_{2F},\, x_{2H}) = 9x_{0F}^2 - 2x_{0F}^2 + 15x_{0H} + (1-\varepsilon)(x_{1F} + 15x_{1H}) + \varepsilon(x_{2F} + 15x_{2H})\ .$$

Furthermore, we suppose that

$$e^A = (e^A_{0F},\, e^A_{0H},\, (e^A_{1F},\, e^A_{1H}),\, (e^A_{2F},\, e^A_{2H})) = (20,\, 1,\, (20,0),\, (20,0))\ ,$$
$$e^B = (e^B_{0F},\, e^B_{0H},\, (e^B_{1F},\, e^B_{1H}),\, (e^B_{2F},\, e^B_{2H})) = (4,\, 0,\, (50,0),\, (9,0))\ .$$

To complete the model, we suppose as before that there are assets A_j with $A_{sj} = \binom{j}{0}$, $\forall s \in S$ promising j units of good F in every state $s = 1$ and 2, and no housing. We suppose that the collateral requirement for each contract is one house $C_j = \binom{0}{1}$, as before.

The only difference between this model and the certainty case we had before is that B is poorer in state 2, and so the housing price must drop in state 2. The question is now how leveraged will the market allow B to become? Will it allow B to default?

It turns out that it is very easy to calculate the Arrow–Debreu equilibrium and the collateral equilibrium for arbitrary ε, such as $\varepsilon = 1/4$. But the no collateral equilibrium is given by a very messy formula, so we content ourselves for that case with an approximation when $\varepsilon \approx 0$.

4.2.1 Arrow–Debreu Equilibrium

The unique (in utility payoffs) Arrow–Debreu equilibrium is:

$$p = ((p_{0F},\, p_{0H}),\, (p_{1F},\, p_{1H}),\, (p_{2F},\, p_{2H})) = ((1,\, 30),\, ((1-\varepsilon)(1,15),\, \varepsilon(1,15))\ ,$$
$$x^A = ((x^A_{0F},\, x^A_{0H}),\, (x^A_{1F},\, x^A_{1H}),\, (x^A_{2F},\, x^A_{2H})) = \left((22,0),\, \left(20 + \tfrac{28}{(1-\varepsilon)},\, 0\right),\, (20,0)\right)\ ,$$
$$x^B = ((x^B_{0F},\, x^B_{0H}),\, (x^B_{1F},\, x^B_{1H}),\, (x^B_{2F},\, x^B_{2H})) = \left((2,1),\, \left(50 - \tfrac{28}{1-\varepsilon},\, 1\right),\, (9,1)\right)\ ,$$
$$u^A = 70;\quad u^B = 62 - 41\varepsilon\ .$$

Since agent B is so rich in state 1, he sells off enough wealth from there in exchange for period 0 wealth to bid the price up to his marginal utility of 30. Notice that agent B transfers wealth from period 1 back to period 0 (i.e. he borrows), and by holding the house he also transfers wealth from state 0 to state 2.

4.2.2 No-Collateral Equilibrium

When $\varepsilon > 0$ is very small, we can easily give an approximation to the unique equilibrium with no collateral by starting from the equilibrium in which $\varepsilon = 0$.

$$\pi_j = 0 \ ,$$
$$p = ((p_{0F},\, p_{0H}),\, (p_{1F},\, p_{1H}),\, (p_{2F},\, p_{2H})) = \left((1,16),\, (1,15),\, \left(1,\, \frac{9}{1-\frac{71}{32\cdot 16}}\right)\right)$$
$$\approx ((1,16),\, (1,15),\, (1,10.4)) \ ,$$
$$x^A = ((x^A_{0F},\, x^A_{0H}),\, (x^A_{1F},\, x^A_{1H}),\, (x^A_{2F},\, x^A_{2H})) \approx \left(\left(20+\tfrac{71}{32},\, 1-\tfrac{71}{32\cdot 16}\right),\, \left(35-\tfrac{15\cdot 71}{32\cdot 16},\, 0\right),\, (29,0)\right) \ ,$$
$$x^B = ((x^B_{0F},\, x^B_{0H}),\, (x^B_{1F},\, x^B_{1H}),\, (x^B_{2F},\, x^B_{2H})) \approx \left(\left(\tfrac{57}{32},\, \tfrac{71}{32\cdot 16}\right),\, \left(35+\tfrac{15\cdot 71}{32\cdot 16},\, 1\right),\, (0,1)\right) \ ,$$
$$u^A \approx 56 \ ; \quad u^B \approx 64 \ .$$

4.2.3 Collateral Equilibrium

We can exactly calculate the unique collateral equilibrium by noting that if B promises more in state 2 than the house is worth, then he will default and the house will be confiscated. But after all the agents of type B default in state 2, they will spend all of their endowment e^B_{2F} on good $2H$, giving a price $p_{2H} = 10$. Perhaps surprisingly the equilibrium described below confirms that the B agents do choose to promise more than they can pay in state 2, and the A agents knowingly buy those promises. Indeed the same contract $j = 15$ is traded as when there was certainty and no default. Its price is $\pi_{15} = (1-\varepsilon)15 + \varepsilon 9$ because the rational A agents pay less, anticipating the default in state 2.

$$D_{1j} = \min(j,15); \quad D_{2j} = \min(j,9); \quad \pi_j = (1-\varepsilon)D_{1j} + \varepsilon D_{2j} \ ,$$
$$((p_{0F},\, p_{0H}),\, (p_{1F},\, p_{1H}),\, (p_{2F},\, p_{2H})) = ((1,\, 3+\pi_{15}),\, (1,15),\, (1,9)) \ ,$$
$$x^A = ((x^A_{0F},\, x^A_{0H}),\, (x^A_{1F},\, x^A_{1H}),\, (x^A_{2F},\, x^A_{2H})) = ((23,0),\, (35,0),\, (29,0)) \ ,$$
$$\theta^A_{15} = 15 \ ; \ \theta^A_j = 0 \text{ for } j \neq 15; \ \varphi^A_j = 0 \ ;$$
$$\theta^B_j = 0 \ ; \quad \varphi^B_{15} = 15 \ ; \ \varphi^B_j = 0 \text{ for } j \neq 15 \ ,$$
$$x^B = (x^B_{0F},\, x^B_{0H},\, (x^B_{1F},\, x^B_{1H}),\, (x^B_{2F},\, x^B_{2H})) = ((1,0),\, (35,1),\, (0,1))$$

At the equilibrium prices, each agent of type A is just indifferent to buying or not buying any contract. At these prices any agent of type B reasons exactly as before. Since money is so much more valuable to him at time 0 than it is in the future, he will borrow as much as he can, even if it leads to default in state 2. He will only trade contract $j = 15$.

Thus we see that the free market will not choose levels of collateral which eliminate default. We are left to wonder whether the collateral levels are in any sense optimal for the economy: does the free market arrange for the optimal amount of default?

More generally, we might wonder whether government intervention could improve the functioning of financial markets. After all, the unavailability of collateral might prevent agents from making the promises that would lead to a Pareto improving sharing of future risks. If the government transferred wealth to those agents unable to afford collateral, or subsidized some market to make it easier to get collateral, could the general welfare be improved? The answer, surprisingly, is no, at least under some important restrictions. The following is due to Geanakoplos-Zame (2002)

Constrained Efficiency Theorem: *Each collateral equilibrium is Pareto efficient among the allocations which* (1) *are feasible and* (2) *given whatever period 0 decisions are assigned, respect each agent's budget set at every state s at time 1 at the* ***old*** *equilibrium prices, and* (3) *assume agents will deliver no more on their asset promises than they have to, namely the minimum of the promise and the value of the collateral put up at time 0.*

In particular, if the government redistributes income in period 0, and taxes and subsidizes various markets at time 0, and then allows markets to clear at time 1, then we can be sure that if the time 1 market clearing relative prices are the same as they were at the old equilibrium, then the new allocation cannot Pareto dominate the old equilibrium allocation. In particular, prohibiting contracts $j > 14$ would not be Pareto improving, because in the new equilibrium only contract $j = 14$ would be traded and the price of housing in states 1 and 2 would remain at 15 and 10, respectively.

In the regulated equilibrium we would have $\pi_{14} = (1-\varepsilon)14 + \varepsilon 9$, and the price of housing would fall to $p_{0H} = 3 + \pi_{14}$. Compared to the previous equilibrium, A would lose $(1-\varepsilon)$ utiles because that is the drop in the price p_{0H}. Since he gets no surplus out of trading contracts, the decreased volume of loans does not affect his utility. B puts the same money down in period 0, but delivers one unit of food less in state 1, meaning he gains $(1-\varepsilon)$ utiles.

Our theorem suggests that the free market does not expose this economy to unnecessary risks from default. However, it relies on the hypothesis that future relative prices are not affected by current collateral requirements. In general this will prove to be false, as we shall see shortly.

4.2.4 Excess Volatility

Since the 1929 stock market crash it has been widely argued that low margin requirements can increase the volatility of stock prices. The argument is usually of the following kind: when there is bad news about the stocks, margins are called and the agents who borrowed against the stocks are forced to put them on the market, which lowers their prices still further.

The trouble with this argument is that it does not quite go far enough. In general equilibrium theory, every asset and commodity is for sale at every moment. Hence the crucial step in which the borrowers are forced to put the collateral up for sale has by itself no bite. On the other hand the argument is exactly on the right track.

We argued that using houses or stocks, or mortgage derivatives as collateral for loans (i.e., allowing them to be bought on margin) makes their prices more volatile. The reason is that those agents with the most optimistic view of the assets' future values, or simply the highest marginal utility for their services, will be enabled by buying on margin to hold a larger fraction of them than they could have afforded otherwise. But with bad news for the asset, there is a redistribution of wealth away from the optimists and toward the pessimists who did not buy on margin. The marginal buyer of the stock is therefore likely to be someone less optimistic or less rich than would have been the case had the stock not been purchased on margin, and the income redistribution not been so severe. Thus the fall in price is likely to be more severe than if the stock could not have been purchased on margin.

Our story is borne out vividly in the example when differences stem not from optimism but from heterogeneous tastes for housing. When the housing stock can be purchased on margin (i.e., used as collateral), agents of type B are enabled to purchase the entire housing stock, raising its price from 16 (where it would have been without collateral) to 18. In the bad state these agents default and all their holdings of the housing stock are seized. Although they end up buying back the entire housing stock, their wealth is so depleted that they can only bid up the housing prices to 9.

When there is no collateral the agents of type B can afford to purchase only a fraction $\alpha = 71/(32)(16)$ of the housing stock at time 0. But they own that share free and clear of any debts. Thus when bad news comes, they do not lose anything. They can apply their wealth to purchasing the remaining $1-\alpha$ of the housing stock, which forces the price up to approximately 10.4. Thus when there is no collateral, the housing prices are never as high nor never as low as when the housing stock can be used as collateral.

4.2.5 Under Water Collateral and Foreclosure Costs

Let us change our model in a simple way to account for the fact that foreclosure is a very expensive operation. Suppose that for each dollar the loan exceeds the market price of the collateral, the confiscator of the property must pay a dollar to repair the house and restore it to a condition at which it can be sold at the market price. This means that a house for which the LTV is 160% (the loan is 60% above the market value of the house) would require 60% of its market value be squandererd in repairs from the damages caused by foreclosure. The lender would thus recoup only 40%/160% = 25% of his loan when seizing the house. These numbers are completely consistent with recovery rates on foreclosures today in the subprime housing market.

The question is: if borrowers and lenders are aware of these terrible foreclosure losses, will they nevertheless trade loans which they foresee will create substantial deadweight losses?

In the new equilibrium we can compute that indeed the leverage will be just as big! Again the only traded contract will be $j = 15$. In equilibrium we find that

$$\pi_{15} = (1-\varepsilon)15 + \varepsilon(9 - (15-9))$$
$$p_{0H} = 3 + \pi_{15}$$

The rest of the equilibrium can be guessed as before.

Now we can ask our question again: what would happen if the government regulated leverage in period 0 by prohibiting any contracts with $j > 15 - \eta$?

It is easy to check that only the contract $j = 15 - \eta$ would be traded, and that we would have

$$\pi_{15-\eta} = (1-\varepsilon)(15-\eta) + \varepsilon(9 - (15-\eta-9))$$
$$p_{0H} = 3 + \pi_{15-\eta}$$

The regulated curtailment of leverage would have the effect of reducing housing prices by $(1-\varepsilon)\eta - \varepsilon\eta = (1-2\varepsilon)\eta$, lowering the utility of A by the same amount. The utility of B would rise by $(1-\varepsilon)\eta$, this time giving an increase in the sum of utilities.

There is a limit on how big η can be, however, because if $3 + \pi_{15-\eta}$ falls below $1 + (1-\varepsilon)15 + \varepsilon 9$, then the type A agents will buy the house at time 0.

In the next section, however, we see that there is lots of room to curtail leverage.

5 The Double Leverage Cycle

One of the main causes of the severity of the current leverage cycle is that there are two of them built on each other, in the housing market and in the mortgage securities market. Houses back mortgage securities, hence a crash in housing prices has ramifications for the securities market. But a crash in the price of mortgage securities affects the loans homeowners can get, which in turn affects the housing market.

So consider now a population made up of the type B agents from the second model, who get utility from living in houses, and the agents $h \in [0,1]$ from the first model who only get utility from consumption. The homeowners will issue mortgages in order to borrow the money to buy the houses. These mortgages will be packaged and sold to the optimists who figure the state is very likely to be the good states in which the mortgages pay off in full. The optimists will borrow the money to buy the mortgages in the repo market, using the mortgages as collateral for their loans. The houses thus serve twice as collateral, first backing the homeowner mortgage loans, and then backing the mortgage securities which back the optimists' borrowing.

More concretely, let us consider an economy with 3 time periods 0,1,2 as in our first model, and states of the world 0,U,D,UU,UD,DU,DD. Let there be a continuum of type B agents who begin with 1 unit of canned food and 3.15 units of labor, and no houses at time 0, and 50 units of canned food in UU,UD,DU, and 9 units of canned food in DD, and no other endowments. Suppose that canned food and houses are perfectly durable. Canned food can be eaten at any time, but is durable. We still use the letter F to describe it. We use the letter L to denote leisure. We shall suppose the marginal utility of leisure (that is, the marginal disutility of labor) is denoted by c.

Suppose the type B agents each own a production technology that can take labor at time 0 and transform 18.15 units into a house a time 0 (that will then be perfectly durable). Let type B agents assign probability $(1-\varepsilon)$ to nature moving up at any state, and let their utility be

$$
\begin{aligned}
&u^B(x_{0F}, x_{0H}, x_{0L}, x_{UUF}, x_{UUH}, x_{UDF}, x_{UDH}, x_{DUF}, x_{DUH}, x_{DDF}, x_{DDH},) \\
&= 9x_{0F}^2 - 2x_{0F}^2 + 15x_{0H} + cx_{0L} + (1-\varepsilon)^2(x_{UUF} + 15x_{UUH}) + \\
&(1-\varepsilon)\varepsilon(x_{UDF} + 15x_{UDH}) + \varepsilon(1-\varepsilon)(x_{DUF} + 15x_{DUH}) + \varepsilon^2(x_{DDF} + 15x_{DDH})
\end{aligned}
$$

$$
\begin{aligned}
&((e_{0F}^B, e_{0H}^B, e_{0L}^B), (e_{UUF}^B, e_{UDF}^B, e_{DUF}^B, e_{DDF}^B)) \\
&= ((1, 0, 3.15), (50, 50, 50, 9))
\end{aligned}
$$

Note that if the marginal utility of leisure $c = 0$, then the B agents will always work 3.15 hours. But if c is large, then they may choose not to work at all if the wage is low enough.

Suppose also there is a continuum of agents $h \in [0, 1]$ who are exactly like the agents in our first model, except that instead of owning one unit of X and Y at time 0, they each own 15 units of food and labor at time 0. Agent $h \in [0, 1]$ has utility and endowments

$$
\begin{aligned}
&u^h(x_{0F}, x_{0L}, x_{0L}, x_{UF}, x_{DF}, x_{UUF}, x_{UDF}, x_{DUF}, x_{DDF}) \\
&= x_{0F} + \bar{c}x_{0L} + hx_{UF} + (1-h)x_{DF} \\
&+ h^2 x_{UUF} + h(1-h)x_{UDF} + (1-h)hx_{DUF} + (1-h)^2 x_{DDF}
\end{aligned}
$$

$$
(e_{0F}^h, e_{0H}^h, e_{0L}^h) = (15, 0, 15)
$$

The contracts in the economy are of two types, depending on the collateral. In one kind of contract j, called a mortgage loan of type j, agents can promise the fixed amount j of good F in every one of the last states UU,UD,DU,DD, using one house as collateral. In the other type of contract k,j called a Repo, agents can promise k units of good F in every immediate successor state, using contract j as collateral.

In our economy, the type B agents will borrow money at time 0 by issuing a mortgage, using the house they will be constructing at the same time as collateral. The most optimistic agents in [0,1] will buy those mortgages, thereby lending the B agents the money. Since the mortgages will default in state DD (but not until then), they will be risky. Hence the pessimists will not want to buy those mortgages. But the optmists don't have enough money to buy them all. So they will borrow money from the pessimists by selling repo loans against the mortgages they hold. These safer repo loans will be held by the pessimists. The Repos are one-period loans, unlike the mortgage, which is a two-period loan.

In a moment we shall define the equilibrium formally, but in view of our earlier analysis, it is easy to describe now. We normalize by taking the price of canned food to be 1 in every state. In equilibrium the price of labor in period 0 will be .95. The income of the B agents at time 0 is then $1 + (3.15)(.95) = 4$. By constant returns to scale, the price of the house will then be $(18.15)(.95) = (3.15+15)(.95) = 3+14.25 = 17.25$. The B agents will each buy a house by putting 3 dollars down and borrowing the remaining 14.25 by issuing a mortgage j=15 promising 15 in every state in the last period.

Let us make the hypothesis that if the house is underwater by y dollars when the loan comes due, then y dollars must be wasted in order to restore the house to mint condition to sell on a par with the other houses on the market. In all the terminal states except DD, the prices will not be underwater, and the house will sell for 15. But in DD the house will only sell for 9, which means it will be underwater, and the mortgage will only deliver 9-(15-9)=3 to the mortgage holder after he confiscates the house and sells it, net of the restoration costs.

We see that the terminal payoffs of the mortgage security is thus (15,15,15,3), which is 15 times the terminal price of the security in our first example. The agents of type h each own 15 units of the canned food, exactly 15 times what they owned of the durable consumption good before, and their labor income is exactly enough to buy one mortgage security, again 15 times as valuable as the security in our first example. Hence this equilibrium we are computing is just the one in our first example scaled up by a factor of 15.

In state D the mortgage will be worth 15(.69) dollars. In state 0, the top 13% of agents h will buy the mortgages by issuing repos promising k=15(.69). In state D these optimists will be wiped out, and the mortgages will fall into the hands of more pessimistic investors. Agents $h \in [.61, .87)$ will buy the mortgage securities at D, issuing contract (j,k)=(15,3) in the repo market to obtained borrowed money to help them buy it. Part of the reason the price of mortgages is so low at D is that the payoffs are so bad at DD, which limits not just the value but the amount of leverage that can be obtained at D.

We see that there is a double leverage cycle. At 0 homeowners leveraged too much in the mortgage market, and investors leveraged their mortgage purchases too much in the repo markets. At D there is massive deleveraging in the securities market, and no opportunity for leverage in the housing market, since the homeowners are all under water and cannot sell their houses.

5.1 What's so bad about the leverage cycle?

In the leverage cycle asset prices shoot up and shoot down as leverage changes. This drastic change is unsettling in any real economy, and I would argue is a danger in and of itself. But why is this so bad in welfare terms? In our first example, the same kind of asset gyrations would occur with complete markets, though on a different scale. In the second example, however, asset prices would have remained stable between U and D with complete markets, while with collateral and incomplete markets they drop by 50% between U and D. If complete markets is our benchmark, then we see evidence of a problem already.

But the real danger I believe is that when asset prices collapse, new activity plunges as well. Why would anyone want to buy a new mortgage, to enable a new builder, when the old mortgages are so cheap? Let us re-examine our double leverage cycle.

Imagine now that in addition to the agents already described, there is a new generation of B' agents born at U and at D, with the same utilities over youthful consumption and over terminal consumption as before. Imagine also that we give the h agents 15 more units of labor endowment at U and at D, but no more income of food. Let us allow the new generation to write mortgages just like the old ones did, payable in the terminal states. Will the new generation be able to build houses like the old?

The answer at U is yes, but at D is emphatically no. The most optimistic agents will have already gone broke by D, and the leverage on mortgages is very low, so the price of mortgages will be much lower at D than at 0. Thus the B' agents will not be able to borrow nearly as much to buy the house, and thus will not be able to hire the labor necessary to build it.

Homeowners do recognize that by leveraging so much, they are tumbling further under water at DD, reducing the payoffs to the lender there and thereby reducing the increase in the amount they can borrow by promising more. A key externality they do not recognize, however, is that by doing so they limit how much the optimists can leverage at D, and therefore also at 0, and therefore they fail to recognize that they are reducing the value of the mortgages they want to sell. Similarly, the optimists who leverage so much at 0 overlook the fact that if they had tempered their leverage, they would be ina better position to buy mortgages at D, which would have raised the price of houses at D, which might have led to their further employment at D.

Need to finish calculating this.

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