Exchange rate intervention with options

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Abstract

We consider the problem of a Central Bank that has exchange rate goals. In a partial equilibrium setting, we compare “direct” intervention through sale/purchase of reserves in the currency market with an alternative strategy of intervention with options. The investment bank the Central Bank deals with will have to hedge its position and therefore buy or sell the underlying security (the foreign currency) and bonds. Intervention through options seems to perform better, but the value of the cross-elasticity exchange rate–interest rate is crucial. We provide some grounds for the construction of a full equilibrium model.

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1. Introduction

Most exchange rate regimes require some type of active policy from the Central Bank. This is obvious when the exchange rate is fixed or moves in a target zone, but even in free floating regimes, Central Banks often decide to intervene in order to smooth temporary volatility or for other long-term currency goals (considerations of an over or undervalued currency). Traditional foreign exchange intervention involves buying or selling reserves or the use of monetary policy (see Edison, 1993 for a review of methods and reasons for intervention). Intervention in the forward
foreign exchange market has been argued to be more effective (Eaton and Turnovsky, 1984). This paper is a first attempt to study, in a partial equilibrium setting, the use of currency options as an alternative strategy to achieve exchange rate goals (see Taylor, 1995; Wiseman, 1996 and Breuer, 1999).

When the Central Bank buys or writes options to the investment bank, the investment bank takes a short or long position in options that will have to hedge by constructing a synthetic long or short position in options. Otherwise, the investment bank would be engaging in speculative trading. That is a legitimate strategy that the investment bank might want to adopt, but in this paper we will focus on the additional effect (to speculative positions or warehousing of options) that trading by the Central Bank would have on the investment bank. The importance of hedging is independent of the type of position (short or long) the investment bank holds. In fact, although the potential risk of a long position is limited by the value of the asset, this can mean the total position of the investment bank and, therefore, the lack of a hedge can lead to bankruptcy. A synthetic option is constructed by buying or shortselling the underlying security (the currency) and shortselling or buying a domestic bond (for calls and puts, respectively). If the Central Bank buys a put option on the currency, for example, the investment bank that sells the option will have to replicate a long put in order to hedge its position. Replication of a long put involves sale of the underlying security (the currency). The Central Bank can induce the same effect in the currency markets by selling reserves or by buying puts.

Garber and Spencer (1994) describe the effects of hedging on currency stabilization when the Central Bank uses the interest rate to sustain a fixed exchange rate regime. Malz (1995) studies the effect of hedging positions in a particular type of currency option. Taylor (1995) suggests the use of out-of-the-money puts on the domestic currency as an “insurance policy” to be used in currency crises. Wiseman (1996) also suggests the use of options (in bilateral agreements) as a tool to stabilize the currency. Breuer (1999) considers the more general question of how (if ever) to use options to reduce exchange rate volatility and concludes that “selling” rather than “buying” would be the strategy to follow. The downside to using options is the fact that the Central Bank undertakes financial responsibilities that might produce a larger loss of reserves when the options expire than if direct intervention with reserves had been used. Options are very risky securities because they are equivalent to a highly leveraged position in the underlying asset. If the Central Bank takes a short position in options, the strategy commits the Central Bank until maturity of the options (in theory, it is possible to take the opposite position with the investment bank and, therefore, stop possible future losses, however, this can be costly).

In this paper we consider a foreign exchange and a bonds market and study the use of options as the tool to implement currency stabilization objectives through the hedges of investment banks. The objective of the Central Bank is to smooth exchange rate volatility. Although intervention with options might be useful for other objectives (like defending a currency peg) the possible anticipation of Central Bank intervention in the options market adds a dimension that we do not address in this paper. Additionally, the hedging strategy of the investment bank also involves trades in domestic bonds; therefore, this strategy also affects interest rates (and through them
exchange rates as a second order effect). In fact, interventions that try to sustain the exchange rate (interventions at the weak end) will have to induce sales of the underlying security (foreign currency); additionally, the hedging strategy requires purchases of the domestic bond that will produce a drop of the domestic interest rate (which allows for a tightening of the monetary policy as a complementary action).

In order to compare traditional intervention by buying/selling reserves with the use of options, we develop an appropriate framework and simulate paths of the economy for given parameters. We compare paths of both exchange rates and interest rates under both types of interventions and we perform some comparative static analysis for different parameter values. Overall, intervention with options seems to be more efficient from the point of view of the use of reserves, whose level stays higher, on average. Besides, it gives the Central Bank further maneuvering space by lowering interest rates. The lower is the cross-elasticity of the exchange rate with respect to the interest rate, the larger is the advantage of intervening with options. This is a first attempt to understand the possible effects of using options in foreign exchange intervention. We do not construct a full general equilibrium model but our setting can shed some intuition on how options would work in a general equilibrium model. An extension of our setting to a general equilibrium model (see Edison, 1993 for references) would in principle suggest that, as risky securities, options covary with bonds and would have similar effects to the risk–return combinations available to investors as long-term bonds. In such a setting, the introduction of options would shift the mean-variance frontier towards the left of the investors. The element that would have to be added to that analysis is the fact that options are by nature equivalent to highly leveraged investments. In order to capture the essence of options we would need an equilibrium model where individuals face borrowing constraints. 1

We emphasize that the type of interventions we have in mind in our model are those that correspond to dirty floating or intra-band interventions. In the case of target zones (or fixed rates) the fact that market participants can anticipate Central Bank interventions will have a non-trivial effect on the valuation and hedging of the options. The “signaling channel” (see Edison, 1993) will become more complicated if options are included. In this paper we will focus on the direct effect of Central Bank intervention in the spot rate. Besides, we will implicitly assume a somehow stable environment. Currency crises would affect the prices of the options and hedging strategies. Ideally we would cast the problem in a setting with fully endogenous exchange rate dynamics, where agents’ expectations about Central Bank intervention were modeled. However, currency crises are beyond the scope of this paper.

The paper is organized as follows: in Section 2 we describe the bond and currency markets; in Section 3 we describe the basic objectives of the traditional sterilized intervention (that we call “direct” intervention); in Section 4 we describe the option markets; in Section 5 we explain the mechanics of Central Bank intervention with options and two different versions of this strategy; Section 6 is devoted to the numerical exercise and analysis of the results; we close the paper with some conclusions.

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1 We are grateful to a referee for suggesting this analysis and the main ideas of this discussion.
2. The setting

The equilibrium interest rate is determined in the bond market. Bond demand will be directly related to the interest rate level. Currency demand depends on both the exchange rate (demand from imports/exports) and the interest rate level (foreign demand of domestic bonds). We do not provide microfoundations for the demand and supply that are exogenous. The setting we describe in this section is the setting that would prevail in case there were no Central Bank interventions.

2.1. Bond market

We assume that there is a market of domestic bonds denominated in the local currency. In this setting, bonds are (locally) riskless securities whose price $B$ satisfies:

$$\frac{dB_t}{B_t} = r_t dt,$$

where $r_t$ is the interest rate, to be determined in equilibrium. These securities are free from default risk. We assume that demand ($D^B_t$) and supply ($S^B_t$) are given by:

$$D^B_t = \log r_t + z_{1t},$$

$$S^B_t = B,$$

where $B$ is the fixed supply of bonds, and $z_1$ is a stochastic variable that satisfies,

$$dz_{1t} = \mu_1 dt + \sigma_1 dW_{1t},$$

where $\mu_1$ and $\sigma_1$ are constant and $W_t$ is a standard Brownian motion process. The stochastic shocks to the demand represented by $z_1$ are, for example, political events or international economic changes (an increase in the level of foreign interest rates) that affect the demand of domestic bonds but which we do not model explicitly.

The fact that we assume a constant supply of domestic bonds $B$ has implications for the type of Central Bank intervention we consider. We will be explicit about it in Section 3. In equilibrium $D^B_t = S^B_t$. Solving, the equilibrium interest rate is,

$$r_t = e^{B_t - z_{1t}}.$$

We then obtain a one-factor geometric Brownian process for the interest rate

$$\frac{dr_t}{r_t} = \left(-\mu_1 + \frac{1}{2}(\sigma_1)^2\right)dt + \sigma_1 dW_{1t}.$$

Intervention with options will affect these dynamics of the interest rate.

2.2. Currency markets

We assume that supply and demand of the foreign currency depend on both the interest rate $r$ (determined in the bond market) and the exchange rate $x$. The rationale
is that part of the demand of bonds will depend on the differential between domestic and foreign interest rate (that we implicitly assume to be constant). Investors will shift money in or out of the country, depending on the domestic interest level. Furthermore, imports and exports will depend on the exchange rate. The exchange rate is defined as the number of units of domestic currency that can be bought with one unit of the foreign currency (i.e. an increase in $x$ means depreciation of the domestic currency). Demand ($D^C$) and supply ($S^C$), both expressed in units of the foreign currency, are:

$$S^C_t = \bar{M} + \frac{1}{2}\log x_t + \frac{1}{2}a_1 \log r_t + \frac{1}{2}z_2,$$

$$D^C_t = -\frac{1}{2}\log x_t - \frac{1}{2}a_1 \log r_t - \frac{1}{2}z_2,$$

where $\bar{M}$ is constant, $a_1 > 0$ is a constant coefficient (that, in order to simplify the model, we assume affects demand and supply in the same way) and $z_2$ satisfies,

$$dz_2 = \mu_2 dt + \sigma_2 dW_2,$$

with $\mu_2$ and $\sigma_2$ constant and, $W_2$ a standard Brownian motion process, independent of $W_1$. $\bar{M}$ can be interpreted as an inelastic part of the supply; it is obvious that the results do not change if we interpret $\bar{M}$ as an inelastic component of the demand, that is, we change its sign. The stochastic process $z_2$ represents shocks to the supply/demand; the way it is incorporated into both equations implies that both supply and demand are equally affected by these shocks, but does not imply anything related to its sign; it can be the result of, for example, political events that might affect supply (demand) in a negative (positive) way or vice versa. Similarly, an increase in interest rates will have a positive impact on the demand of domestic bonds and, therefore, a positive impact on the supply of the currency and the opposite effect on the demand of the currency. We also assume symmetry between the effect on supply and on demand. In equilibrium, $S^C = D^C$ which implies

$$x_t = (r_t)^{-a_1 e^{z_2} e^{-\bar{M}}} = e^{-\bar{M} - a_1 \bar{M} + \frac{1}{2}(a_1 \sigma_1)^2 + (\sigma_2)^2} e^{a_1 \sigma W_1 + \sigma_2 W_2}.$$  

The exchange rate is then a two-factor geometric Brownian motion process,

$$\frac{dx_t}{x_t} = \left( a_1 \mu_1 + \mu_2 + \frac{1}{2}(a_1 \sigma_1)^2 + (\sigma_2)^2 \right) dt + a_1 \sigma_1 dW_1 + \sigma_2 dW_2.$$  

One of the factors is idiosyncratic to the foreign exchange market and the other is common to foreign exchange and bond markets. With this model, we guarantee that the exchange rate and the interest rate are not perfectly correlated. Empirical evidence seems to suggest that there is no systematic relationship between interest rates and exchange rates (see, for example, Edison and Pauls, 1993).

The model presented here deviates from Uncovered Interest Parity (UIP): UIP in continuous time requires that (we drop time indicators):

$$\frac{r^F - r}{r^F} = \frac{1}{dt} E^x \frac{dx}{x} = a_1 \mu_1 + \mu_2 + \frac{1}{2}(a_1 \sigma_1)^2 + (\sigma_2)^2,$$
where \( r^F \) represents the interest rate of the foreign country. It is clear that there will be systematic deviations since the left hand side will typically be stochastic (we do not model \( r^F \) but as long as it does not covary perfectly with \( r \), that will be the case) while the right hand side is constant. As in Edison (1993), we define the risk premium as the difference between the left hand side and the right hand side of Eq. (8). Obviously, it can take positive or negative values. The risk premium will be stochastic, but its average size over time will depend on the size of \( a_1 \). For a positive \( \mu_1 \) (which seems the most likely case: small or negative \( \mu_1 \) would mean a large expected increase in interest rates) the risk premium will be inversely related to \( a_1 \): the higher the elasticity of exchange rates to interest rates, the lower the risk premium. A large and positive risk premium would be consistent with a low \( a_1 \), while a small (maybe negative) risk premium would suggest a high \( a_1 \). Additionally, the size of \( a_1 \) will be an indication of the type of ex post deviations from UIP. Tanner (1998) shows that ex post deviations seem to be the result of unanticipated real exchange growth for developed countries and from interest rate differentials for developing countries. The unanticipated real exchange growth in our model is given by \( a_1 \sigma_1 dW_1 + \sigma_2 dW_2 \). Therefore, a large \( a_1 \) would magnify deviations due to unanticipated growth of the exchange rate while a relatively smaller \( a_1 \) would explain interest rate differentials.

3. Direct Central Bank intervention

We describe the benchmark type of intervention that we call “Direct Intervention,” and is based on sale/purchase of reserves. When the exchange rate reaches a given high level \( x \) (“weak” currency) the Central Bank sells reserves, so that the new exchange rate is set at a level \( x - \bar{e} \), with \( \bar{e} \) given. Similarly, when the exchange rate reaches a given low level \( \bar{x} \) (“strong” currency), the Central Bank withdraws an amount of the foreign currency such that the new exchange rate is set at a level \( \bar{x} + \bar{e} \) with \( \bar{e} \) given. For simplicity, we will assume that \( \bar{e} = \bar{e} = \bar{e} \).

However, we do not consider explicitly the case of target zones. Target zone models typically assume that intervention only takes place at the boundaries. We assume that the objective of the Central Bank is to smooth exchange rate volatility by intervening when the exchange rate deviates from the reference exchange rate that the Central Bank estimates will hold in the long term. In our model, \( x \) can be considered as the reservation exchange rate that triggers Central Bank intervention. This exchange rate value is not public and can change from intervention to intervention. There are two reasons for not considering explicitly a target zone model: on one hand its existence would have an effect on the dynamics of the exchange rate. On the other hand, it would also have an effect on the valuation of options (we will be more specific in the next Section) that would become intractable.

We now discuss the monetary effects of the intervention. Central Banks can choose between sterilized and non-sterilized intervention. Sterilized intervention implies that the Central Bank will keep interest rates constant by providing the necessary liquidity or withdrawing the excess liquidity. In non-sterilized intervention the Central Bank changes interest rates in order to achieve the desired exchange rate level. Here we
will assume that the Central Bank buys or sells reserves, but they are never exchanged by bonds. That may or may not involve sterilization, depending on the reason for the pressure on the exchange rate. Suppose, for example, that the exchange rate is “weakening” and the Central Bank decides to intervene by selling reserves. There are two possible reasons for the pressure on the exchange rate: people might be exchanging their holdings of domestic currency for reserves (say an increase in the demand of imported goods); alternatively investors might be exchanging domestic bonds for foreign bonds (as a result of political instability, for example). In the former case, the Central Bank directly provides the reserves requested by the public; this is sterilized intervention. In the latter case the Central Bank will allow interest rates to go up as sellers of bonds have to attract new buyers. The sellers will exchange the proceeds of their sale for reserves that the Central Bank will provide; this is non-sterilized intervention. The effects of open market operations would be additional to those of the direct intervention with reserves or the intervention with options. However, as we will see, our results will not depend on the monetary policy chosen.

Suppose that at moment $t$ the exchange rate reaches the level $\bar{x}$, for $z_2$ and $\bar{r}$ (or, equivalently, $\bar{z}_2$ and $z_1$). That is,

$$\bar{x} = x_t = (\bar{r}_t)^{-a_1} e^{\bar{z}_2 - \bar{M}}. \tag{13}$$

The Central Bank intervenes by injecting an amount $I$ (expressed in units of the foreign currency) that is added to the supply $S_C$ or subtracted from the demand $D_C$. In order to achieve its target, the amount $I$ has to be such that (from (7) and (8))

$$\bar{x} - \mathcal{E} = x_t - \mathcal{E} = (\bar{r}_t)^{-a_1} e^{\bar{z}_2 - \bar{M} - I_t}. \tag{14}$$

From (13) and (14) we get that

$$I_t = -\bar{M} - \log(\bar{x} - \mathcal{E}) - a_1 \log \bar{r}_t + \bar{z}_2. \tag{15}$$

Similarly, if the exchange rate reaches $\bar{x}$, the Central Bank withdraws an amount $J$.

Suppose, then, that the Central Bank has to intervene at points $i = (1, \ldots, n)$ (injecting) and at points $j = (1, \ldots, m)$ (withdrawing). The new equilibrium is,

$$x_t = r_t^{-a_1} e^{z_2 - \bar{M} - \sum_{i=1}^{n} I_i + \sum_{j=1}^{m} J_j}. \tag{16}$$

We will assume that the Central Bank has a total balance of reserves denoted by $\bar{R}$ that sets the limit on the total amount of interventions,

$$\bar{R} + \sum_{j=1}^{m} J_j \geq \sum_{i=1}^{n} I_i. \tag{17}$$

When (17) becomes a strict equality the Central Bank runs out of reserves and cannot intervene anymore. We move to a full floating rate system.
4. Option markets

We assume that there is also an options market where calls and puts on the foreign currency are traded. These are European type options with a fixed maturity of \( T \) years. The price of the options is expressed in units of the domestic currency. Options are instruments with a high level of leverage embedded in them, this is essential to understand the depth of this market and should be a basic component of a general equilibrium model that considered the problem presented in this paper. These options are traded by investment banks that immediately cover their positions by replicating the opposite position in the option with the underlying security (the currency) and the bond. Of course, there might be trades in the options market besides those considered in this paper, but we focus on the “additional” effect induced by the participation of the Central Bank and that might be an increase, a decrease or even a change of sign of the hedge of the investment bank (depending on the specific strategy). The relevant aspect is the net effect on the demand–supply of the currency.

In order to replicate the price of the option and to compute the hedging ratios of investment banks (“delta”) we need a pricing formula. We assume that investment banks use an approximation provided by the adjusted Black and Scholes (1973) formula (see Biger and Hull, 1983, Garman and Kohlhagen, 1983 and Grabbe, 1983). This is in fact an exact formula for an exchange rate that follows a geometric Brownian motion process and constant domestic and foreign interest rates. However, although we implicitly assume that the foreign interest is constant, it is obvious from the model described in Section 2 that neither the exchange rate nor the domestic interest rate satisfy the assumptions required. Nevertheless, as an approximation, we will assume that the formula is employed using the current interest rate (to be determined in equilibrium in the bond market) and the current volatility of the exchange rate (the parameter that explains the dynamics of the exchange rate in equilibrium and that is given by Eq. (11)) as if they were constant. The justification for this simplification is twofold. First, in practice these formulas are used, as no analytic expressions are available. Second, since the model we are using is indeed a simplification of the true dynamics of the economy, we do not think that this assumption has a big impact in terms of our conclusions, because pricing errors will be of second order with respect to the feedback effect that a full equilibrium model would provide. In fact, suppose that at the time of intervention the implied volatility of the options went up because agents anticipated the intervention: options would be more expensive but the hedge ratio (or “delta”, the meaning of this concept is explained below) would also be higher, therefore a smaller number of options would be necessary in order to implement the strategy. Of course, this relationship is not linear, but provides an additional argument in favor of our simplification. For an analysis of the functioning of hedging strategies and the role of investment banks in foreign exchange derivatives markets, see Garber and Spencer (1994), Malz (1995) and Breuer (1999).

From (11), the volatility of the exchange rate \( \sigma_e \) is, \( (\cdot)^\top \) represents “transpose”),

\[
\sigma_e^\top = (a_1 \sigma_1, \sigma_2).
\]  

(18)

In fact, since we have a two-dimensional factor model we would need three securi-
ties to construct a perfect replicating portfolio of the options, but for the same reasons given above we will restrict ourselves to the currency and the domestic bond.

We denote the prices of the call and put $C$ and $P$. For the specific pricing formula, see Garman and Kohlhagen (1983). This formula also provides the basis for the computation of the “delta” of the options, that we denote by $\Delta_C$ (for the call) and $\Delta_P$ (for the put). $\Delta_C$ is the number of units of the underlying security (foreign currency) that have to be bought in order to replicate the call. The difference between the price of that amount of foreign currency and the price of the call will have to be borrowed at the domestic interest rate (equivalent to shortselling domestic bonds). Similarly, $\Delta_P$ represents the number of units of the underlying security that will have to be sold short in order to replicate the put. The cash raised plus the price of the put will have to be invested in the domestic bond. Therefore, after the Central Bank estimates the amount of reserves the intervention would require, the “delta” gives the corresponding number of options. We will assume that investment banks adjust their hedging positions continuously, as $x$, $r$, $\sigma_e$ and time to maturity, change.

A final problem of our model is that when there is intervention, the true volatility of the exchange rate will not be given by the value $\sigma_e$ of (18), but it will depend on agents beliefs about the probability of intervention. However, as we stated in Section 3, we will assume that the value $\overline{x}$ at which intervention takes place is not public and agents do not know that the Central Bank is intervening. A justification would be that the continuous time structure we present is an approximation, and in the truly discrete dynamics of exchange rates, the individuals that participate in the currency markets cannot tell whether the change from $\overline{x}$ to $\overline{x} - \varepsilon$ is the result of Central Bank intervention or the result of a shock to the markets. In the numerical exercise we perform later we will overcome this problem by using historic volatility instead of $\sigma_e$.

Another aspect of the functioning of options that we consider is margin requirements. An investor with a short position in options faces the potential of a financial loss if the option finishes “in-the-money”, that is, has a positive payoff at maturity. The investor will have to deposit collateral in an account as guarantee. The margin changes day to day, with changes in price of the underlying security (“marked to market”). Margins set a limit to the size of the short position.

We will assume that the collateral $m$ necessary to satisfy the margin at $t$ is exactly the amount the option is in-the-money,

\[ m_t = \max(x_t - K, 0), \]

\[ m_t = \max(0, K - x_t), \]

depending on whether the investor has a short position in calls (19) or in puts (20).

Besides the short position in a given option, the Central Bank might have a long position in a different type of option. If this option is in the money, the net risk of the investment bank is lower (since the investment bank is the potential debtor). Long positions in options do not generate “positive” margins. However, securities are commonly used as collateral in margin requirements. In this case, the value of the long position is de facto a riskfree security for the investment bank, since it is the debtor of such security. We will assume that the amount by which the long
options are in the money is subtracted from the margin. By computing the margin as we do in this paper, the collateral is always enough to guarantee the potential net debt of the Central Bank with respect to the investment bank. In practice, the amount by which the option is out of the money is discounted from the required margin. Here we discount the amount by which the long position is in the money.

With respect to the treatment of margins, we will try to keep a balance in the exercise we perform in this paper. It could be argued that Central Banks might be exempt from margin requirements or, at least, get a special treatment. On the other hand, Central Banks of countries with a history of exchange rate crises are likely to have to provide some type of guarantee to investment banks. In any case, since this paper only attempts to be a first approximation to the possible use of options by Central Banks, assuming that they have to satisfy margin requirements is the most conservative setting. As we will see later, the results of our analysis seem to suggest that options can be a useful instrument for Central Banks. The fact that the result holds in a setting where margin requirements are satisfied reinforces the conclusion.

In order to satisfy margin requirements, we assume that the Central Bank uses its reserves on the foreign currency. Therefore, posting reserves as collateral does not have any immediate impact on the exchange rate. As we will explain later, in our exercise we assume that the Central Bank is subject to margin calls. If the Central Bank runs out of reserves, it defaults and the exchange rate moves to a free floating regime: in the case of direct intervention it is obvious that the Central Bank might exhaust its reserves. In the case of intervention with options, the same result would occur if the margin requirements are such that all reserves have to be posted.

5. Central Bank intervention with options

We describe one strategy with options that can have the same immediate effects as direct intervention, but also some second order effects. We assume that reserves are the only asset owned by the Central Bank, in order to abstract from monetary effects.

The Central Bank will buy/write calls or puts from/to an investment bank. In order to hedge its position, the latter will take an equivalent (but with opposite sign) position by constructing a synthetic option with the underlying security and bonds. Both long and short positions are risky and should be hedged. The risk of short positions is that they might end in the money and the investment bank will have to pay the corresponding payoff. The risk of long positions is that they might end up out of the money. In both cases, the goal of the hedge is to avoid possible financial losses.

Typically, end users are long and, therefore, the investment bank has to hedge a short position. If the Central Bank writes a call to the investment bank, the result will be a drop of the hedge (that will induce a sale of the underlying security).

We emphasize that we consider a partial equilibrium setting in this paper. As such, we only consider the effect of the trading in options by the Central Bank additional to all other trades in currency options that would also have an effect in the currency, but that we take as exogenous and implicit in Eqs (7) and (8). With this strategy,
the Central Bank will have a direct effect in the currency markets by inducing an increase in the supply of the underlying security. However, there are also limitations to this strategy: the Central Bank will be subject to margin requirements. We will assume that the Central Bank has to satisfy the restriction expressed in (19, 20). In order to do that, the Central Bank has to post reserves. However, the Central Bank might also have a long position in options that have a positive value. Therefore, denoting by $m^B$ the total amount of collateral the Central Bank has to deposit in order to satisfy margin requirements, and by $L$ the value of the long position in options the Central Bank might have at a given moment, it has to be the case that

$$\frac{L_t}{x_t} - \frac{m^B_t}{x_t} \geq 0.$$

(21)

We divide $L$ and $m$ by the exchange rate because prices of options and, therefore, the value of the long position and the margin in (19) and (20) are expressed in units of the domestic currency. Whenever (21) becomes an equality, the Central Bank cannot intervene any more and the regime moves to a full floating exchange system.

Besides the direct effect on the currency markets, trading in options will have an indirect effect on the currency through the bond market. Replication of the option implies a long or short position in the underlying security and the opposite position in the bond. Whether the Central Bank writes calls or buys puts, the effect on the currency markets will be the sale of the currency that always has to be accompanied by a purchase of domestic bonds. More explicitly, if $C$ is the price of a call option and $\Delta_C$ the delta of the option, $H$, the nominal value of bonds the investment bank will have to buy in order to replicate the short position in call options (it has a long position in calls that it hedges by replicating a short position) will be

$$H_t = \Delta_C x_t - C_t.$$

(22)

This has an effect on the demand for bonds $D^B$ that now becomes

$$D^B_t = \log r_t + z_{1t} + H_t,$$

(23)

and the new equilibrium interest rate will be

$$r_t = e^{B - z_{1t} - H_t}.$$

(24)

Since the investment bank will demand more bonds, the equilibrium interest rate will go down, as there will be a lower outstanding amount of bonds left.

Therefore, by trading options the Central Bank affects the exchange rate in a double way, directly through the currency market and indirectly through the bond market. Consider a trade in options that results in a sale of the currency by the investment bank of size $V$ (units of the foreign currency); given the effect expressed by Eq. (24), the resulting exchange rate after the intervention will be

$$x_t = e^{a_{1t}(-B + z_{1t} + H_t) - M + z_{2t} - V_t}.$$

(25)

Then, in order to get the same impact as an intervention of $I$ units of the foreign currency through direct sale of the currency by the Central Bank, we need
\[ V - a_i H = I. \] (26)

It is clear that as the price of the underlying security changes, the value of \( \Delta_c \) and \( H \) will change, therefore intervention through options is not a one time effect and it will have consequences all throughout the life of the option. It is possible that as a result of the hedge, trading in options will induce additional volatility in the exchange rate. That is the case if the investment banks are net short, as explained by Breuer (1999). One important effect pointed out above is that interventions in the upper bound (weak currency) will yield a lower interest rate level (i.e., the Central Bank sells a call on the currency, the investment bank then undertakes a long position in calls that hedges by replicating a short position in calls). Interventions in the lower bound (strong currency) will produce higher interest rate levels. We emphasize that this type of intervention with options abstracts from all types of monetary policy considerations. It is clear, though, that the Central Bank will have additional tools to complement the intervention with options. For example, since this intervention will lower interest rates, the Central Bank could complement intervention in the upper bound with a tightening of the monetary policy. An additional consequence of this is the fact that the equilibrium interest rate, which in the case of direct intervention satisfied a one-factor model as explained by Eq. (6), will now become a two-factor stochastic variable: The exchange rate changes as a result of changes in \( z_1 \) (through the interest rate) and \( z_2 \) (directly). The investment bank will adjust its hedge and change its position in bonds and, therefore, change the equilibrium interest rate which now will depend (through the hedging strategy) on changes in \( z_2 \).

There are several strategies that could be used. We will focus on a “zero cost” strategy. The Central Bank constructs a combination consisting of writing calls and buying puts on the underlying security (the foreign currency) in such a way that the market price of the portfolio is zero. The potential destabilizing effect of a pure portfolio of put options is studied by Breuer (1999). In order to compute the strategy, first we choose a strike price \( K_\bar{\bar{\alpha}} \). In fact, we have a degree of freedom in choosing the strike price: the amount of foreign currency sold by the investment bank will be the result of the number of options traded by the Central Bank and the delta of each option. The delta depends on the strike price \( K \). The Central Bank can induce the same amount of sale of the foreign currency through an infinite number of possible pairs of numbers of options and strike prices. We now explain the criterion we use in the numerical exercise in order to select the strike price: after the Central Bank trades in options, the exchange rate is altered. This induces an adjustment of the hedge by the investment bank. We select the strike price that requires the lowest readjustment of the hedge by the investment bank after the initial construction of the hedging portfolio and its effect on the exchange rate. This criterion is not based on economic efficiency, but allows tractability of the effects of the intervention. There might be better criteria from an economic point of view. For this strike price we compute the price of the call and put, \( C(K) \) and \( P(K) \), from the formula in Garman and Kohlhagen (1983). Then we compute the number of short calls and long puts that would yield a portfolio of zero cost; obviously, any portfolio with the same ratio of short calls and long puts will have a zero cost. Say that for each single call
we need $\kappa(\bar{K})$ puts, that is, $\kappa(\bar{K}) = [C(\bar{K})/P(\bar{K})]$ (clearly, $\kappa > 0$). Constructing this basic zero cost portfolio formed by one call and $\kappa(\bar{K})$ puts will induce a sale of $\Delta_c(\bar{K}) - \kappa(\bar{K})\Delta_p(\bar{K})$ units of the foreign currency for the hedge of the investment bank, where the deltas are also from Garman and Kohlhagen (1983). We note that $\Delta_p$ is always negative.

Simultaneously, the investment bank will have to buy bonds in order to cover its long position in calls and short position in puts. The nominal amount of bonds to be purchased is given by Eq. (22) for calls and its equivalent for puts. However, we are considering zero cost positions, which means that the cost of the puts will be equal to the cost of the calls and, therefore, the amount of bonds purchased will correspond to the amount of the underlying security sold; that is, suppose that $\eta(\bar{K})$ represents the number of calls that have to be written by the Central Bank and we denote by $V$ the number of units of the underlying (foreign currency) sold for hedging,

$$V = \eta(\bar{K})(\Delta_c(\bar{K}) - \kappa(\bar{K})\Delta_p(\bar{K})), \quad (27)$$

and by $H$ the value of bonds purchased as given by Eq. (22)

$$V, \chi_i = H_t. \quad (28)$$

From Eq. (26), the total impact in the currency value is

$$I_t = V_t - a_1 H_t = (1 - a_1 \chi_t)V_t. \quad (29)$$

In order to determine the exact number $\eta(\bar{K})$ of calls that have to be sold we compute

$$\eta(\bar{K}) = \frac{V}{\Delta_c - \kappa(\bar{K})\Delta_p} = \frac{V}{\Delta_c - \frac{C(\bar{K})}{P(\bar{K})}\Delta_p} = \frac{I}{(1 - a_1 \chi_t)\left(\Delta_c - \frac{C(\bar{K})}{P(\bar{K})}\Delta_p\right)}, \quad (30)$$

where $I$ is determined from Eq. (15).

Besides, it has to be taken into account that the hedging strategy of the investment bank is dynamic. As the price of the underlying security (the currency), its volatility, the interest rate, and time left to maturity change, the price of the calls and puts and their deltas will change; that will force the investment bank to update its strategy and buy or sell the currency and bonds. The outcome of this series of interactions is not obvious. It also has to be taken into account that when the option matures the investment bank will close its position buying the currency and selling bonds.

The limit to intervention is given by the margin requirements of (21). The Central Bank has a long position in calls, therefore $L$ is strictly positive.

6. Numerical exercise

We compare the effects of each type of intervention through a numerical exercise. In Section 2 we cast the model in a continuous time setting. We discretize the continuous time stochastic differential equations of the type of (6) in the following way:
\[
\Delta r_t = r_{t+\Delta t} - r_t = r_t \left( -\mu_1 + \frac{1}{2}(\sigma_1)^2 \right) \Delta t + \sigma_1 \omega_t, \tag{31}
\]

where \( \omega \sim N(0, \sqrt{\Delta t}) \) and \( \Delta t \) is an appropriate time interval. In this exercise we will simulate the dynamics of the exchange rate under each type of intervention. The dynamics of shocks will be common to both types of interventions. However, due to the hedging of the investment bank, the dynamics of the exchange rate and interest rate will be different for each type of intervention.

We first need to simulate two factors (there are two Brownian motion processes explaining the dynamics of the exchange rate), therefore we need \( \omega_1 \) and \( \omega_2 \), using the notation in (31). We will use standard techniques to generate two series of independent pseudo-random numbers and derive from them, the exchange rate, interest rate and reserve level series. We set upper and lower intervention rates \( x^\bar{\text{e}} \) and \( x^\underline{\text{e}} \). For convenience, we will keep those rates constant throughout the whole period considered. We will also assume that \( \varepsilon \) (defined in Section 3) is constant in order to keep our setting as simple as possible. With regard to the volatility of the exchange rate, Eq. (18) gives us the volatility value without intervention. However, when the Central Bank intervenes, it will affect the volatility of the exchange rate. We will compute actual path volatility and assume that the investment bank will use the historic volatility in order to price the option (this will be a better estimate of implied volatility than the expression of Eq. (18)). Since the path we consider represents one year of exchange rate history, we need to generate a previous path in order to compute the starting value (for the time period analyzed) of the volatility of the exchange rate. When the exchange rate surpasses one of the boundaries intervention takes place.

The implementation of the exercise for the case of direct Central Bank intervention (Section 3) is straightforward. With respect to the implementation of Central Bank intervention with options, we follow the procedure described in Section 5. We first choose a strike price and from it derive the number of calls and puts that will yield the desired effect as explained in Section 5. However, a problem in this case is the fact that if we pick a random strike price, the number of calls may be considerably larger or smaller than the number of puts. Since the intervention changes the price of the options, the investment bank has to readjust its hedging portfolio as a result of the price changes; if the number of calls and puts is very unbalanced the effect of the adjustment on the exchange rate can also be important. In order to avoid that problem, we select the strike price in such way that the effect on the exchange rate and, therefore, the prices of the options, will not require an adjustment in the hedging portfolio of the investment bank. In order to find that strike price we set a grid of strike prices and compute the second order effect. We stop checking when we find a strike price that gives rise to a second order effect smaller than a given very small preset level. Then we move to the following time step and compute the new equilibrium exchange rate and interest rate taking into consideration the new shocks (realizations of \( \omega_1 \) and \( \omega_2 \)) and the effects of the hedge of the investment bank.

This exercise provides a framework to study the effects of the two types of intervention considered on the level of reserves, level of interest rates, volatility of the
interest rate and volatility of the exchange rate. In order to study the effects on the reserves level we study the cost in reserves of each type of intervention. In the case of direct Central Bank intervention this problem is straightforward. In the case of intervention with options, it is not as clear since the margins will have an important effect on the level of reserves. As we explained in Section 4, we will assume that the Central Bank has to post reserves in order to guarantee the margin requirements of (19) and (20). We are assuming that, initially, foreign reserves are the only asset in the balance of the Central Bank. In this way we abstract from other monetary policy considerations. However, since the Central Bank also can take long positions in options we will assume that the amount by which the long options are in the money is subtracted from the margin. When the reserves are not enough to satisfy the necessary collateral, we assume that the Central Bank will have to abandon the regime and move to a free floating exchange regime.

We repeat this exercise over one hundred paths and compute the average over those paths. We denote by $x_d$ the exchange rate under direct intervention and by $x_o$ the exchange rate under intervention with options. Similarly, we denote by $r_d$ and $r_o$ the respective interest rate levels and by $c_d$ and $c_o$ the respective levels of reserves. The parameters that we will keep constant throughout the exercise are the following, $\bar{x}/H_{11005} = 0.30$, $\bar{x}/H_{11005} = 0.20$, $\varepsilon = 0.01$, $\mu_1 = 0$, $\mu_2 = 0.2$, $B = 1$, $M = 1$.

We summarize our results on exchange rates, interest rates and reserves in three tables. We consider four statistics. The first two are the mean over the whole path and the standard deviation over that whole path. The values we report for the mean and standard deviation are the averages of the previous statistics over the hundred paths generated, except for the cost in reserves. Since we compute the final cost of reserves, the statistics for the cost of reserves are computed over the hundred paths, i.e., mean of the cost of reserves is the average of final cost of reserves over the one hundred paths generated and similarly for the standard deviation. The maximum and minimum are the maximum and minimum over the hundred paths. We certainly do not expect to converge to any values with a small number of paths. Our objective is to make sure that the qualitative results we report are robust.

Some results are consistent for all values of the parameters considered. Since we assume a positive drift of the exchange rate (tendency towards devaluation) most of the interventions take place at $\bar{x}$, the “weak” boundary. The exchange rate seems to stay, on average, at a higher level when the Central Bank intervenes with options. However, the volatility of the exchange rate is lower under intervention with options. With respect to the interest rate, it stays at a lower level when the Central Bank uses options; besides, it is (in general) less volatile. A result of this is that the Central Bank could also tighten the monetary policy as a complementary action to the intervention through options. Regarding the cost of reserves, it is always lower when the Central Bank uses options. In fact, it is (on average) negative (accumulation of reserves) for all exercises performed. Of course, for some paths (each exercise involves one hundred paths) the Central Bank runs out of reserves. These two results combined (lower interest rates and lower cost of reserves) make our choice of monetary policy irrelevant. We could achieve even lower cost of reserves with higher interest rates. Overall, then, our exercise would seem to suggest that Central Bank
intervention with options is better than the traditional intervention in currency markets.

In Table 1 we study the sensitivity of the statistics to $a_1$, the cross exchange rate–interest rate elasticity. The interest rate level is lower when the Central Bank intervenes with options, but the spread shrinks as the cross-elasticity increases (it is constant for the case of direct intervention, since in this case it does not depend on the elasticity). The reason is that as the exchange rate decreases the value of the calls decreases (the argument works in the opposite direction for the puts); as the value of the calls falls, the hedge of the investment bank requires a smaller long position in domestic bonds and that pushes up the interest rate. With respect to the cost in reserves, intervention with options is always more efficient, but the spread in efficiency decreases as the cross-elasticity parameter increases.

Table 2 analyzes changes in the statistics as the “intrinsic” volatility of the interest rate (that we denote $\sigma_1$) changes. Intervention with options seems to make interest rate volatility less sensitive to changes in $\sigma_1$ as a result of the “feedback” effect resulting from the hedging strategy of the investment bank. The spread of efficiency with respect to the cost in reserves increases as the volatility increases.

Table 3 does the same comparative statics but for $\sigma_2$, the volatility parameter specific to exchange rates. The delta of the portfolio is larger and, therefore, the position in bonds of the investment bank is also larger (yielding smaller interest rates). Volatility of interest rates in the case of intervention with options increases as the volatility increases. The spread in the cost in reserves increases very fast as $\sigma_2$ increases (the cost of reserves increases for the case of direct intervention while the accumulation of reserves increases in the case of intervention with options).

<table>
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<tr>
<th>$a_1$</th>
<th>$x_o$</th>
<th>$x_d$</th>
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<th>$x_d$</th>
<th>$x_o$</th>
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<td>0.27920</td>
<td>0.27556</td>
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<td>0.26206</td>
<td>0.26159</td>
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<td>0.01511</td>
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<td>0.22873</td>
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<td>3.41823</td>
<td>3.42240</td>
<td>3.41823</td>
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<tr>
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<td>2.18359</td>
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<th>$c_d$</th>
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<th>$c_d$</th>
<th>$c_o$</th>
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<tr>
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7. Conclusions

In this paper we have attempted to compare the effects of “direct” Central Bank intervention (using reserves) with intervention using options. The Central Bank we consider is interested in smoothing volatility and intervenes when the exchange rate reaches a reservation level (that is not public: possible anticipation of the intervention would complicate substantially the pricing of the options). We develop a framework
that allows us numerical comparisons between both types of interventions. Our numerical exercise seems to suggest that Central Bank intervention with options is a good alternative to direct Central Bank intervention, but its efficiency depends heavily on the parameters of the economy, in particular the cross-elasticity currency market–bond market. Although we only consider a partial equilibrium setting, it seems that the results are robust enough to grant an opening of the debate.

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