Term Structure of Interest Rates and the Macroeconomy

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- Overview
 - issues
 - old approaches
 - some current approaches
 - future research
- "No Arbitrage Taylor Rules"

joint with Andrew Ang and Sen Dong, Columbia University

Issues

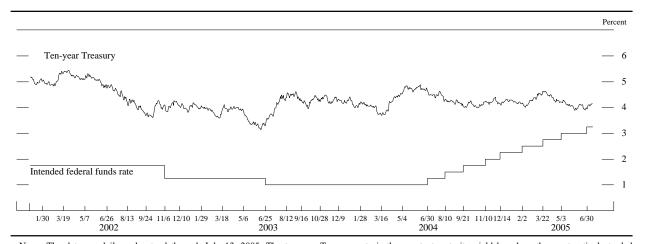
• Information contained in the term structure

for business cycle measurement

spread between short & long Treasuries, corporate bond spreads

- ★ leading indicators
 - Stock and Watson 1989 leading index
 - Monetary policy
 - * 2 famous books in color: Green Book & Blue Book
 - * Greenspan's Monetary Policy Report to Congress on July 20, 2005
 - * Conundrum!?

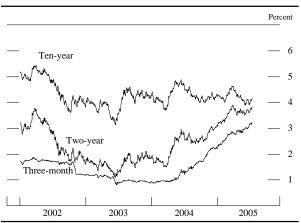
Selected interest rates



Note: The data are daily and extend through July 13, 2005. The ten-year Treasury rate is the constant-maturity yield based on the most actively traded securities. The dates on the horizontal axis are those of FOMC meetings.

Source: Department of the Treasury and the Federal Reserve.

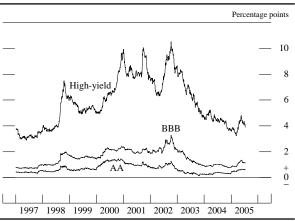
Interest rates on selected Treasury securities



Note: The data are daily and extend through July 13, 2005. Source: Department of the Treasury.

has fallen about 30 basis points over this period. A second possible explanation is investors' willingness to accept smaller risk premiums on long-term securities amid

Spreads of corporate bond yields over comparable off-the-run Treasury yields



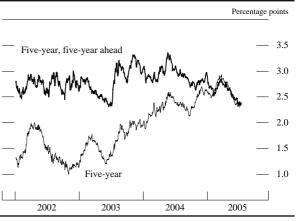
Note: The data are daily and extend through July 13, 2005. The high-yield index is compared with the five-year Treasury yield, and the BBB and AA indexes are compared with the ten-year Treasury yield.

SOURCE: Merrill Lynch AA and BBB indexes and Merrill Lynch Master II high-yield index.

Spreads of yields on investment-grade corporate debt over those on comparable-maturity Treasury securities

- information for policy makers: what is the market expecting?
 - inflationspread between Treasuries and TIPS
 - ★ liquidity, risk premia?
 - next recession
 spread between short and long Treasuries
 why univariate regression?
 - "what we are going to do" fed funds futures
 - ★ risk premia?
 - "how uncertain are they about what we are going to do": implied volatility from interest-rate options
 - ★ Black-Scholes?

TIPS-based inflation compensation



Note: The data are daily and extend through July 13, 2005. Based on a comparison of the yield curve for Treasury inflation-protected securities (TIPS) to the nominal off-the-run Treasury yield curve.

SOURCE: Federal Reserve Board calculations based on data provided by the Federal Reserve Bank of New York and Barclays.

- effects of monetary policy
 - if the Fed increases the target for the short rate by 25 bp, how much will long term rates go up?
 - identification of "monetary policy shocks" $\text{proxy } E_t\left[r_{t+1}\right] \text{ with high-frequency data on fed funds futures}$ $r_{t+1} E_t\left[r_{t+1}\right] = \text{"shock"}$
 - ★ risk premia?
 - impulse responses effects on investment, output, prices, etc.
- how should the Fed conduct monetary policy? e.g. more "transparency"?
- premia on bonds are they large? do they vary over the business cycle?
 compared with equity premia?

Benchmark — expectations hypothesis

$$y_t^{(n)} = \text{time-}t \text{ yield on bond with } n \text{ periods to go}$$

$$= \frac{1}{n} E_t \left[\sum_{i=0}^{n-1} r_{t+i} \right]$$

• Sargent 1969 imposes cross equation restrictions on a VAR with

$$Y_t = \begin{pmatrix} y_t^{(1)} \\ y_t^{(2)} \\ \vdots \end{pmatrix} \text{ where } y_t^{(1)} = r_t$$

- assumes risk neutrality
 - ★ why is the equity premium so high?
- ignores Jensen's inequality terms
 - ★ Campbell 1985 are big, especially in the 1970s and for long bonds

Benchmark — expectations hypothesis ctd.

• standard practice at the Fed: e.g., futures rates = expected rates in the future

nominal rate = real rate + expected inflation
 assume real rate is constant

⇒ Treasuries move because expected inflation moves

e.g. Fama and Schwert 1977 – predict stock returns with "expected inflation"
 "expected inflation" = nominal rate

..... term structure model

no arbitrage implies that we can compute bond prices recursively

$$P_t^{(n)} = E_t \left[M_{t+1} P_{t+1}^{(n-1)} \right]$$

starting at $P_t^{(1)} = \exp(-r_t)$

- ullet implied by no arbitrage there exists an M
- holds in most DSGE models

..... term structure model

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$$P_t^{(n)} = E_t \left[M_{t+1} P_{t+1}^{(n-1)} \right]$$

starting at $P_t^{(1)} = \exp(-r_t)$

Affine

- 1. linear short rate: $r_t = \delta_0 + \delta_1^\top X_t$
- 2. linear dynamics: $X_t = \mu + \phi X_{t-1} + \Sigma \varepsilon_t$, $\varepsilon_t \sim N(0, I)$
- 2. linear risk premia: $M_{t+1} = \exp\left(-r_t \frac{1}{2}\lambda_t^{\top}\lambda_t \lambda_t^{\top}\varepsilon_{t+1}\right)$

$$\lambda_t = l_0 + l_1 X_t$$

Affine term structure model ctd.

Result: $y_t^{(n)} = a_n + b_n X_t$, where a_n , b_n solve ordinary difference equations

which depend on $(\delta_0, \delta_1, \mu, \phi, \Sigma)$ and (l_0, l_1)

VAR unstricted dynamic system

:

state space system fewer dimensions, fewer parameters

•

term structure model consistency of a_n, b_n with expectations

i discrete time (Ang & Piazzesi 2003): many AR lags

:

DSGE more restrictions

standard preferences:

"bond premium puzzle", predictability of bond returns

Term structure model → DSGE model

- small VAR for some macro variables & interest rates
- same variables are factors
- model guides predictions:
 - macro variables help forecasting interest rates (Ang & Piazzesi 2003)
 - nominal short rate does better at forecasting GDP growth than term spreads in particular: low r forecasts high GDP growth (Ang, Wei & Piazzesi 2005)
 - * contradicts OLS regressions where reverse is true
 - * verified in out-of-sample forecasts
 - longest nominal short rate is the best predictors
 - always include lagged GDP growth, at least for short forecasting horizons

Term structure model → DSGE model

- large countercyclical risk premia
 - "tent-shape" function of forward rates
 - matters for using fed funds futures for forecasting and defining monetary policy shocks
- "Great Inflation"
 - Volcker was unlucky Greenspan was lucky about the size of shocks
 heteroskedasticity Pearson and Sun 1994, Buraschi and Jiltsov 2005
 regime switches with constant mean parameters Sims 2004, Sims and Zha
 2004, Ang and Bekaert 2005,
 - Volcker and Greenspan conducted policy in different ways
 regime switches in mean parameters Bansal and Zhou 2002, Bansal, Tauchen and Zhou 2004

- Investors in the 1970s did not see Greenspan coming structural breaks – subsample estimations
 Rudebusch and Wu 2005, Ang, Dong & Piazzesi 2004
- Heterogeneous expectations about inflation
 old households expect low inflation, young households expect high inflation
 Piazzesi and Schneider 2005
- What happens after Greenspan???
 tradesports.com: Bernanke 34%, Feldstein 16%, Hubbard 14%, Taylor 2.5%
 do long-term bond prices correctly price in inflation expectations?

Hybrid models

"IS curve" derived from Euler equation, but pricing kernel is flexible
 Rudebusch and Wu 2004

DSGE model → term structure model

- need to take a stance on inflation
 - money in the (nonseparable) utility function Bakshi and Chen 1996
 - taxes Buraschi and Jiltsov 2005
 - exogenous process fixed by the monetary authority –
 CIR 1985, Bekaert and Grenadier 2001, Wachter 2005
- "fancy preferences" explains predictability and matches up with equity predictability Wachter 2005

"No Arbitrage Taylor Rules"

• Prices & yields of long-term bonds embed expectations about the future

$$y_t^{(n)} = \text{time-}t \text{ yield on bond with } n \text{ periods to go}$$

$$= \frac{1}{n} E_t \left[\sum_{i=0}^{n-1} r_{t+i} \right] + \text{term premium (+ Jensen's inequality terms)}$$

- implied by the absence of arbitrage
- holds in equilibrium of most DSE models
- ullet Nominal short rate r_{t+i} is set using Taylor rule (+ possibly shock)
- Advantages
 - understand term structure movements in terms of policy expectations
 - estimate policy rules with panel data on yields

Fix the affine term structure model.....

$$r_t = \delta_0 + \delta_1^\top X_t$$

where
$$X_t = (g_t, \pi_t, f_t^u)^{\top}$$

$$g_t = \mathsf{GDP}\ \mathsf{growth}$$

$$\pi_t = \text{inflation}$$

$$f_t^u = \text{latent factor}$$

$$X_t = \begin{pmatrix} f_t^o \\ f_t^u \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} + \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix} \begin{pmatrix} f_{t-1}^o \\ f_{t-1}^u \end{pmatrix} + \begin{pmatrix} u_t^1 \\ u_t^2 \end{pmatrix}$$

.....and consider different policy rules

Fix the affine term structure model.....

$$r_t = \delta_0 + \delta_1^{\top} X_t$$
, where $X_t = (g_t, \pi_t, f_t^u) = (f_t^o, f_t^u)$

.....and consider different policy rules

a.) Taylor rule (Taylor 1993)

•
$$r_t = \gamma_0 + \gamma_{1,g} g_t + \gamma_{1,\pi} \pi_t + \varepsilon_t^{MP,T}$$

- ullet recursive identification: g_t and π_t don't react within the quarter Christiano, Eichenbaum & Evans 1996
- find structural parameters γ :

$$- \gamma_0 = \delta_0, \, \gamma_{1,g} = \delta_{1,g}, \, \gamma_{1,\pi} = \delta_{1,\pi}$$

$$-\varepsilon_t^{MP,T} = \delta_{1u} f_t^u$$

Fix the affine term structure model.....

$$r_t = \delta_0 + \delta_1^\top X_t$$
, where $X_t = (g_t, \pi_t, f_t^u)^\top$

$$X_{t} = \begin{pmatrix} f_{t}^{o} \\ f_{t}^{u} \end{pmatrix} = \begin{pmatrix} \mu_{1} \\ \mu_{2} \end{pmatrix} + \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix} \begin{pmatrix} f_{t-1}^{o} \\ f_{t-1}^{u} \end{pmatrix} + \begin{pmatrix} u_{t}^{1} \\ u_{t}^{2} \end{pmatrix}$$

.....and consider different policy rules

- b.) Backward-looking Taylor rule (Clarida, Gali & Gertler 1998 and others)
 - includes current and lagged macro variables and short rates:

$$r_{t} = \gamma_{0} + \gamma_{1,g} g_{t} + \gamma_{1,\pi} \pi_{t} + \gamma_{2,g} g_{t-1} + \gamma_{2,\pi} \pi_{t-1} + \gamma_{2,r} r_{t-1} + \varepsilon_{t}^{MP,B}$$

• find structural parameters γ :

$$-\gamma_0, \gamma_{1,g} = \delta_{1,g}, \gamma_{1,\pi} = \delta_{1,\pi}, \dots \gamma_{2,r} = \phi_{22}$$

$$-\varepsilon_t^{MP,B} = \delta_{1,u} u_t^2$$

- c.) Finite-Horizon Forward-looking Taylor rule (Clarida and Gertler 1997 and others)
 - include future expected inflation and GDP growth

$$r_t = \gamma_0 + \gamma_{1,g} \ E_t \left[g_{t+k,k} \right] + \gamma_{1,\pi} \ E_t \left[\pi_{t+k,k} \right] + \varepsilon_t^{MP,F}$$
 where

$$E_{t} \left[g_{t+k},_{k} \right] = \frac{1}{k} E_{t} \left[\sum_{i=1}^{k} g_{t+i} \right]$$

$$E_{t} \left[\pi_{t+k,k} \right] = \frac{1}{k} E_{t} \left[\sum_{i=1}^{k} \pi_{t+i} \right]$$

ullet find structural parameters γ by noting that

$$E_t [X_{t+1}] = \mu + \phi X_t$$

- d.) Infinite-Horizon Forward-Looking Rule
 - Fed discounts at rate β

$$r_t = \gamma_0 + \gamma_{1,g} E_t \left[\sum_{i=1}^{\infty} \beta^i g_{t+i} \right] + \gamma_{1,\pi} E_t \left[\sum_{i=1}^{\infty} \beta^i \pi_{t+i} \right] + \varepsilon_t^{MP,F}$$

Estimation Method

Baysian MCMC and Gibbs Sampling

ullet handles measurement error $\varepsilon_t^{(n)}$ on all yields

$$\widehat{y}_t^{(n)} = y_t^{(n)} + u_t^{(n)}$$

- handles non-linear parameter restrictions
 - no arbitrage restrictions
 - additionally, forward-looking rules restrictions
- handles more flexible parametrization than maximum likelihood
- impose stationarity with prior
- \bullet quarterly data 1952-2002 on $g_t = \text{GDP}$ growth, $\pi_t = \text{CPI}$ inflation, and CRSP yields

Estimation Results

- Term structure model
 - Model matches (Table 3)
 - * unconditional moments
 - * autocorrelations
 - Latent factor is highly persistent and highly correlated with the longest yield
 - Model matches predictability regressions of excess returns
- Structural
 - Variance decompositions
 - Policy rules + shocks

Predictability results

LHS = return from buying the n-period bond at t and selling at t+1 in excess of the 1-period riskfree rate

	Data				Model			
	g_t	π_t	$y_{t}^{(20)}$	R^2	g_t	π_t	$y_{t}^{(20)}$	R^2
			0.22 (.10)		04 (.05)	04 (.07)	_	0.04
_			1.13 (.45)			96 (.39)		0.06

Risk premia

- are countercyclical: low when GDP and inflation is high, long rates are low
- increase with maturity
- -2/3 of the variance in expected excess returns explained by macro variables

Variance decompositions

Macro variables explain

- roughly 1/3 of the yield variance
- almost all of the variance in yield spreads (especially inflation)

Variance Decompositions (in %, CEE ordering)

	yi	eld leve	els	yield spreads			
maturity	g	π	f^u	g	π	f^u	
1 quarter	12.5	28.7	58.8				
1 year	12.9	25.2	62.0	.5	87.3	12.2	
3 years	13.0	21.2	65.8	.2	92.4	7.4	
5 years	13.0	19.8	67.2	.6	96.0	3.4	

Policy Rules ctd.

Taylor rule:
$$r_t = \gamma_0 + \gamma_{1,g}g_t + \gamma_{1,\pi}\pi_t + \varepsilon_t^{MP,T}$$

	Full Sample		Pre-8	32:Q4	Post-83:Q1		
	OLS	Model	OLS	Model	OLS	Model	
const	.01	.01	.06	.01	.01	.01	
	(.001)	(.001)	(.001)	(.05)	(.002)	(.001)	
g_t	.04	.06	.004	.05	.24	.03	
	(.07)	(.01)	(.08)	(.02)	(.10)	(.04)	
π_t	.64	.28	.68	.27	.61	.24	
	(.08)	(.03)	(.08)	(.03)	(.13)	(.05)	

Policy Rules ctd.

Backward-looking Taylor rule

	const	g_t	π_t	g_{t-1}	π_{t-1}	r_{t-1}	R^2
OLS		.07 (.03)	.18	01	08 (.04)	.88	.89
Model	.01 (.00)				20 (.03)		.96
Taylor	.01 (.00)	.06 (.01)	.28 (.03)				

$$r_t = (1 - .92)(.001 + .72g_t + 3.61\pi_t - .16g_{t-1} - 2.52\pi_{t-1}) + .92r_{t-1} + \varepsilon_t^{\mathsf{MP,B}}$$

Long-run response to inflation: 3.61-2.52=1.09

Policy Rules ctd.

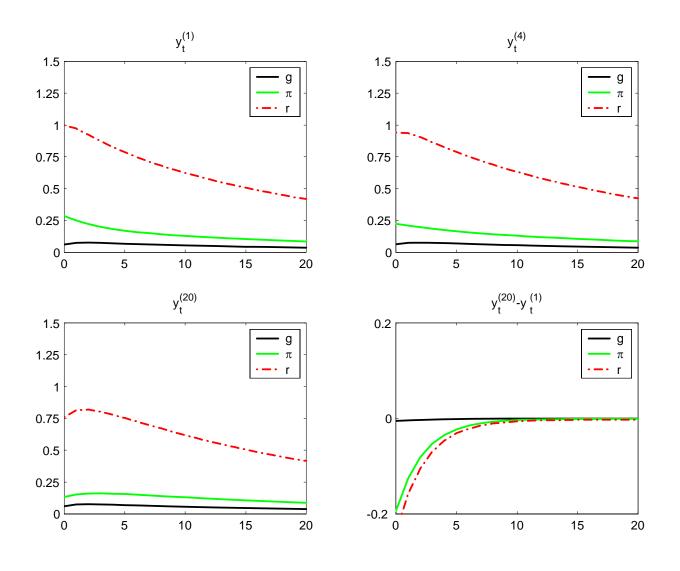
Forward-Looking, Infinite Horizon

$$r_t = \gamma_0 + \gamma_{1,g} E_t \left[\sum_{i=1}^{\infty} \beta^i g_{t+i} \right] + \gamma_{1,\pi} E_t \left[\sum_{i=1}^{\infty} \beta^i \pi_{t+i} \right] + \varepsilon_t^{MP,F}$$

Taylor Rule
$$\gamma_{1,g} \quad \gamma_{1,\pi} \quad \beta$$
 $k = \infty \quad .02 \quad .10 \quad .94$
 $(.01) \quad (.01) \quad (.01)$

 $\beta = .94$ corresponds to an effective horizon of 4.1 years.

Impulse Responses



Conclusions

- Embed various Taylor rules in an arbitrage-free setup:
 original Taylor rules, backward and forward looking rules.
- Panel data approach improves estimates of policy rules
- Baysian estimation methods help us to estimate more flexible dynamics.

Find that macro variables – esp. inflation – explain a large fraction of the variation

- yields
- yield spreads
- expected returns on bonds