Within narrow geographic areas, housing markets assign buyers with different characteristics to indivisible houses that differ by quality. This paper studies housing assignment when a subset of eligible buyers have exclusive access to a subset of houses that form a restricted area. We show that houses in the restricted area can trade at a discount if the matchup of quality and buyer pools is sufficiently different inside versus outside the restricted area. Moreover, the restriction can distort allocations by making eligible buyers choose either higher or lower qualities than ineligible buyers with the same characteristics.

In our leading example, buyers affiliated with Stanford University have exclusive access to houses on campus. We begin by presenting evidence on house prices on and right around Stanford’s campus over the last decade. Using both a simple comparables approach and nearest neighbor matching, we show that houses on campus trade at a substantial discount to similar properties off campus. The discount is relatively smaller for higher quality houses.

We then study the effect of an access restriction in an assignment model with a continuum of houses in which buyer types differ not only by eligibility but also by the marginal utility of house quality. Without the access restriction, our model has an efficient equilibrium in which higher types buy higher quality houses. House prices reflect the relative dispersion of house quality and buyer types. The cost of an additional unit of quality depends on the marginal buyer type; it rises at a faster rate if more distinct buyers must be assigned to similar houses.

When there are more eligible buyers than houses in the restricted area, the efficient equilibrium may survive even with the restriction. Arbitrage by eligible buyers across areas equates prices quality-by-quality as long as the dispersion of quality in the restricted area relative to the dispersion of type (that is, marginal utility) among eligible buyers is everywhere sufficiently similar to the relative dispersion in the economy at large.

Once pairs of distributions are sufficiently different, however, arbitrage across areas becomes impossible and houses in the restricted area trade...
at a discount. We study an example economy in which house quality in the restricted area is relatively low, so eligible buyers who do not buy in the restricted area instead buy higher quality houses outside. The example generates price patterns consistent with those found around Stanford. It also illustrates that a restriction can distort allocations differently at the high and low end of the quality spectrum.

On the one hand, eligible buyers of the best restricted houses buy lower quality houses than non-eligible buyers with the same preferences (and lower quality houses than they would buy if the restriction were lifted). For those high buyer types, the price discount thus provides compensation for compromising on quality inside the restricted area. On the other hand, eligible buyers of the worst restricted houses buy higher quality houses than their peers outside. The price discount helps these low buyer types to buy a better house than they would buy at outside market prices or in the absence of the restriction.

I. House Prices In and Around Stanford

We obtain house prices at the property level from deeds data for 2002–2012. We match deeds to assessor data that contain house characteristics such as lot size, building size, and the number of bathrooms and bedrooms. We restrict attention to a narrow area on the San Francisco Peninsula containing Stanford’s campus (zip code 94305) as well as neighboring areas of Palo Alto and Menlo Park.

Do similar houses trade at different prices on campus? An answer to this question requires estimating the hypothetical price of an on-campus house if it were located off-campus. Figure 1 compares prices of campus homes to predicted prices derived from off-campus comparables. Each market in the figure represents a campus transaction. The horizontal axis measures the transaction price for the campus house, stated in 2012 prices using a simple area index based on median annual prices. The vertical axis measures the median price of comparable off campus houses. We select comparables from transactions that occurred within 180 days based on similarity by building area, lot size as well as the number of bathrooms and bedrooms.

The majority of dots are located above the 45 degree line that would indicate equal pricing on and off campus. Off-campus comparables are thus typically more expensive than the house on campus. This premium is particularly large for condos, marked in light gray and located mostly at the low end of the price distribution. The OLS regression line has an intercept of $668K and a slope coefficient of 0.91 (which is highly significant, but insignificantly different from one). As the transaction price increases, the dollar discount does not vary much, while the percentage discount declines steeply. An interesting exception to this general pattern is found in houses with lots larger than 0.6 acres, marked as stars, which exhibit a particularly large discount even though they are relatively expensive.

In Landvoigt, Piazzesi, and Schneider (2013), we report results from an alternative approach to estimating the hypothetical off-campus value of campus properties. Rather than use median comparable prices, we derive predicted values by nearest neighbor matching as in Caplin et al. (2008). We include not only the above property characteristics, but also latitude, longitude, and neighborhood characteristics at the census
blockgroup level, taken from the American Community Survey. To capture neighborhood effects for a blockgroup, we use the average number of units in a structure, the share of rental units, and the share of households in the highest (topcoded) income bracket. Those variables help predict prices in neighborhoods with diverse individual properties.

The quantitative findings based on this alternative approach confirm the visual impressions from Figure 1. The percentage premium for houses outside campus is highest at the low end of the house quality spectrum. A faculty member who wants a house outside of campus comparable to a house in the bottom quartile of the quality distribution on campus pays 163 percent of what he would pay on campus. This premium declines and reaches zero for high-end houses (in the top quartile of the campus quality distribution, where houses cost more than $2 million).

We estimate the absolute dollar premium for a house outside campus to be roughly constant: $400K across the board.

II. An Assignment Model with Restricted Access

A continuum of houses of measure one has been put up for sale. Houses differ by quality, measured by a one dimensional index \( h \). A share \( \rho \) of houses are located in a restricted area that only a subset of buyers have access to. The distribution of quality inside and outside the restricted area is described by densities \( g_r \) and \( g \). For much of the exposition, we refer to a specific example, based loosely on the Stanford area, that is depicted in Figure 2.\(^4\) In particular, the second panel of the figure shows the house quality densities. The restricted area offers a subset of qualities, with both the highest and lowest qualities missing.

There is a continuum of buyers of measure one. Everyone buys at most one house. A share \( \eta \geq \rho \) of eligible buyers can buy anywhere. The remaining buyers must buy outside the restricted area. Utility from housing does not depend on location: anyone who buys a house of quality \( h \) at price \( p \) receives surplus \( \theta h - p \). Buyers differ by their marginal utility of house quality \( \theta \). The distribution of types \( \theta \) for eligible and other buyers is described by densities \( f_e(\theta) \) and \( f(\theta) \), respectively, plotted in the first panel of Figure 2. The type distribution for eligible buyers is truncated at a point \( \theta_e > 0 \).

An equilibrium consists of buyers’ house choices \( h \) as well as prices for restricted and unrestricted houses \( p_r(h) \) and \( p_u(h) \) so that all

\(^4\) The model presented here is stylized and abstracts from features that are potentially relevant in the Stanford area, such as preference heterogeneity and details of contracting on campus. Its purpose is to show how an assignment model can generate discounts because of differences in buyer and quality populations.

This assumption serves to zero in on the role of distributions on prices. Allowing eligible agents to obtain higher utility from restricted houses introduces an additional force that works to increase house prices in the restricted area. For the application we consider, this force must be weak enough and is omitted.
buyers optimize given prices and markets clear. We consider equilibria such that house quality is strictly increasing in type θ. We further require that all buyers obtain nonnegative surplus from buying a house, so \( p(0) = 0 \). We also have \( p_\gamma(h) \leq p(h) \) in equilibrium since eligible agents do not lose from buying outside the restricted area.

**A. Equilibrium Without an Access Restriction**

The overall distributions of types and houses in the economy, regardless of eligibility, are given by

\[
g_\gamma(\theta) = \eta f_\gamma(\theta) + (1 - \eta) f(\theta),
\]

\[
g_\gamma(h) = \rho g_\gamma(h) + (1 - \rho) g(h).
\]

Throughout we denote cdfs by uppercase letters. Without an access restriction, buyers are assigned to houses according to the strictly increasing QQ plot of \( F_\gamma \) against \( G_\gamma \), that is, \( \theta_\gamma(h) = F_\gamma^{-1}(G_\gamma(h)) \). The optimal choice for a buyer of type θ satisfies the first order condition \( p'(h) = \theta \). The marginal buyer at quality \( h \) prefers a slightly higher (lower) quality house if the price schedule increases by less (more) than \( \theta \) at quality \( h \). Prices follow by integration given the initial condition \( p(0) = 0 \).

The unrestricted assignment is plotted in dark gray in the third panel of Figure 2; it coincides with the light gray line near the boundaries. It is steep when the distribution of types is more dispersed than the distribution of house qualities. Indeed, the slope \( \theta'\gamma(h) \) is given by the density ratio \( \theta'\gamma(h) = g(h)/f(\theta_\gamma(h)) \). When it is high, there are relatively more similar houses close to \( h \) than there are buyers of similar type close to \( \theta_\gamma(h) \). Similar houses must thus be assigned to buyer types with rather different marginal utilities. Prices must then increase at a faster rate \( p'(h) = \rho \theta'\gamma(h) \) near \( h \) to induce those different buyers not to prefer \( h \) itself, as shown in the fourth panel of Figure 2.

**B. Market Clearing with an Access Restriction**

If quality is increasing in type, the assignment must be the same for all buyers of type \( \theta \) who buy outside the restricted area, regardless of whether they are eligible or not. We thus define house quality assignments \( \theta_\gamma(h) \) and \( \theta(h) \) inside and outside the restricted area, respectively. Let \( \tilde{f}_\gamma(\theta) \) denote the (endogenous) density of eligible agents who buy in the restricted area.

Markets must clear at every quality level both inside and outside the restricted area:

\[
\rho g_\gamma(h) = \rho \tilde{f}_\gamma(\theta_\gamma(h)) \theta_\gamma(h),
\]

\[
(1 - \rho) g(h) = (f_\gamma(\theta(h)) - \rho \tilde{f}_\gamma(\theta(h))) \theta(h).
\]

Houses for sale in the restricted area at quality \( h \) must be bought by eligible agents who are assigned those houses in the restricted area. Moreover, houses for sale outside the restricted area must be bought by buyers who are not assigned houses in the restricted area.

In addition, the number of eligible agents who locate outside the restricted area must be nonnegative, that is, for all \( \theta \in [\theta_\gamma, \theta] \)

\[
0 \leq \rho \tilde{f}_\gamma(\theta) \leq \eta f(\theta).
\]

If \( p_\gamma(h) < p(h) \), then the right-hand condition holds with equality at \( \theta = \theta(h) \). All eligible buyers buy in the restricted area when quality is strictly cheaper there. In contrast, if prices are the same across areas at some quality, then eligible buyers are indifferent between areas.

**C. Equilibrium with Equal Prices**

We first ask whether the restriction is binding, that is, whether it makes the unrestricted equilibrium infeasible. Suppose that prices are the same across areas for all quality levels. The equilibrium assignment \( \theta_\gamma \) implies a unique density \( f_\gamma \) that clears the market. The question is whether there are always enough eligible agents to buy the restricted houses at every quality level.

Condition (1) now restricts the slope of the assignment so \( \theta'\gamma(h) \geq \rho g_\gamma(h)/\eta f_\gamma(\theta_\gamma(h)) \).

Since \( \rho \leq \eta \), the condition is always satisfied if the distributions of houses and buyers are identical. If \( \rho = \eta \), it says that the density ratios \( g_\gamma(h)/f_\gamma(\theta) \) and \( g(h)/f(\theta) \) must be equal across areas. This is the knife edge condition that implies equal prices if the two areas were completely segmented markets.

With \( \rho < \eta \), the predictions of the model differ from one with segmented markets: an equal price equilibrium may also exist when the density ratios are different. Indeed, arbitrage by
eligible agents can work to equate prices. For example, suppose the house quality densities are the same. Consider a quality range around \( h \) with many more eligible than ineligible agents. With segmented markets, prices rise less with \( h \) in the restricted area since the relative demand for more expensive houses is lower there. In the present model, some eligible agents can move out of the restricted area and thus equate the relative demands.

D. Price Discounts in the Restricted Area

We now investigate why houses in the restricted area can be strictly cheaper for all quality levels. In this case, if a quality level is available in the restricted area, no eligible buyer will buy it outside. The \( \eta - \rho \) eligible buyers who nevertheless buy outside the restricted area thus choose qualities that are not available inside. The example in Figure 2 has been set up so all that is there is a positive mass of eligible buyers who move outside the restricted area, all of whom buy higher quality houses than those available inside.

The assignment of restricted houses to eligible buyers follows

\[
\theta_r(h) = F_e^{-1}(pG_r(h)/\eta).
\]

In particular, there is a highest type \( \theta^* = \theta_r(\bar{h}_r) = F_e^{-1}(\rho/\eta) \) who is indifferent between buying the highest restricted house \( \bar{h}_r \), at price \( p_r(\bar{h}_r) \) and buying a higher quality \( h^* > \bar{h}_r \) outside the restricted area.

For all types higher than \( \theta^* \), the restriction does not bite and the assignment is given by \( \theta_u(h) \). Below the house quality \( h^* = \theta_u^{-1}(\theta^*) \), outside houses are assigned to ineligible buyers according to

\[
\theta(h) = F^{-1}((1 - \rho)G(h)/(1 - \eta)).
\]

Since \( h^* > \bar{h}_r \), an equilibrium with equal prices cannot exist. Indeed, since assignments are monotonic we must have \( \theta_r(\bar{h}_r) > \theta_u(\bar{h}_r) \) which is incompatible with (1). With the distributions assumed here, the unrestricted assignment asks relatively low types to move into the restricted area. However, not enough of those types are eligible to support an equilibrium with equal prices.

Equilibrium assignments are shown in the third panel of Figure 2. Eligible buyers at the upper end of the restricted area buy lower quality houses than ineligible buyers with the same preferences; for the same threshold marginal utility \( \theta^*_u \), for example, eligible buyers buy \( \bar{h}_r \), while ineligible buyers buy \( h^* > \bar{h}_r \). In contrast, eligible buyers at the lower end of the restricted area buy higher quality houses than ineligible buyers with the same type. Comparison with the unrestricted assignment shows that the highest (lowest) eligible buyers would buy higher (lower) quality houses if the restriction were lifted.

The assignment is brought about by price discounts, as shown in the fourth panel of Figure 2. First order conditions equating the price change to the marginal buyer type hold both inside and outside the restricted area. At quality levels available in the restricted area, prices are found by integration using the indifference of type \( \theta^* \) between \( \bar{h}_r \) and \( h^* \):

\[
p(h) = \int_{0}^{\bar{h}} \theta(\bar{h})d\bar{h},
\]

\[
p_r(h) = p(h) - \int_{h}^{h^*} (\theta_r(\bar{h}) - \theta(\bar{h})) d\bar{h}.
\]

A price discount exists at \( h \) in the restricted area as long as the average assignment between \( h \) and \( h^* \) is higher there. At high qualities, low prices entice relatively high eligible types to buy the relatively low quality houses inside the restricted area. At low qualities, low prices help low eligible types buy relatively high quality houses that are better than those bought by their ineligible counterparts and that they would not buy if the restriction were lifted. Comparison with the unrestricted price shows that the restriction not only lowers price in the restricted area, but also raises them outside.

\[\text{To establish that the resulting prices support an equilibrium, we also need to show that eligible types optimally choose their area. Sufficient conditions for the existence of an equilibrium are provided in Landvoigt, Piazzesi, and Schneider (2013).}\]
including at qualities available in the restricted area itself.

REFERENCES


